

MIAPP

May 24, 2016

Carving out 2D Superconformal Field Theories

Xi Yin

Harvard University

Lin, Shao, Simmons-Duffin, Wang, XY, 1511.04065

Lin, Shao, Wang, XY, work in progress

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How much do we know about two-dimensional conformal field theories?

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quite a bit ...

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quite a bit ...

but not nearly as much as we'd like!

Things we know

1. Exact constructions: WZW models, coset models, rational CFTs, integrable RG flows, orbifolds.
2. Exact solvable irrational CFTs (noncompact): Liouville theory, noncompact coset models (cigar CFT and generalizations)
3. Moduli space of exactly marginal deformations of SCFTs: Calabi-Yau sigma models, D1-D5 CFT
4. Conformal perturbation theory
5. RG flows from UV Lagrangian: GLSM

Things we don't know

1. What is the spectrum of non-BPS operators of a Calabi-Yau model at a generic point in its moduli space?
2. Same question for D1-D5 CFT
3. Are there meromorphic CFTs of $c=24k$ and gap $k+1$ (or just >2) in the weights of Virasoro primaries, for integer $k>1$?
4. Are there CFTs of large central charge c and large gap in the dimension (or twist) of primary operators?

What can conformal bootstrap teach us in two dimensions?



We will apply the conformal bootstrap to unitary superconformal theories. In this talk, I will describe explicit results for

1. $(4,4)$ SCFT, $c=6$
2. $(2,2)$ SCFT, $c=3$ and 9

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1. $(4,4)$ SCFT, $c=6$ (e.g. NLSM on $K3$)
2. $(2,2)$ SCFT, $c=3$ and 9 (e.g. NLSM on $CY3$)

1. (4,4) SCFT, $c=6$
2. (2,2) SCFT, $c=3$ and 9

Our goal is to constrain (and possibly determine) the spectrum of non-BPS operators in such theories.

Importantly, we will investigate the dependence of the spectrum on the moduli of exactly marginal deformations.

1. (4,4) SCFT, $c=6$
2. (2,2) SCFT, $c=3$ and 9

Our starting point is the OPE of BPS operators. We will constrain the spectrum of non-BPS operators that appear in the OPE of a pair of BPS operators, by studying the 4-point function of BPS operators.

The 2d (small) $N=4$ superconformal algebra

1. $c=6k$ Virasoro + 4 supercurrents + $SU(2)$ R-current at level k
2. Representations labeled by (h,j) of primary
3. non-BPS rep: $h>j, j=0, 1/2, \dots, (k-1)/2$.
4. BPS rep: $h=j=0, 1/2, \dots, k/2$.

The 2d (small) $N=4$ superconformal algebra

$c=6, k=1$ case

Only BPS representations are: $h=j=0$ (vacuum) and $h=j=1/2$ (marginal deformations)

Non-BPS representations obey $h > j=0$.

Combining left and right N=4 SCA

c=6, k=1 case

Identity: $(h, j; \tilde{h}, \tilde{j}) = (0, 0; 0, 0)$

1/2 BPS: $(h, j; \tilde{h}, \tilde{j}) = (\frac{1}{2}, \frac{1}{2}; \frac{1}{2}, \frac{1}{2})$

1/4 BPS: $(h, j; \tilde{h}, \tilde{j}) = (s, 0; \frac{1}{2}, \frac{1}{2}), \quad s \geq 1 \quad \text{or L-R exchanged}$

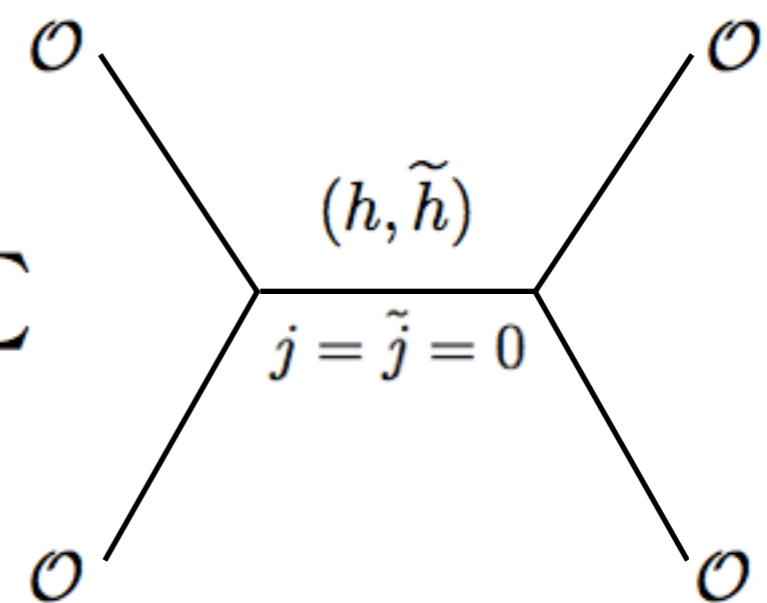
non-BPS: $(h, j; \tilde{h}, \tilde{j}) = (h, 0; \tilde{h}, 0), \quad h - \tilde{h} = s$

We will consider the 4-point function of 1/2-BPS operators.

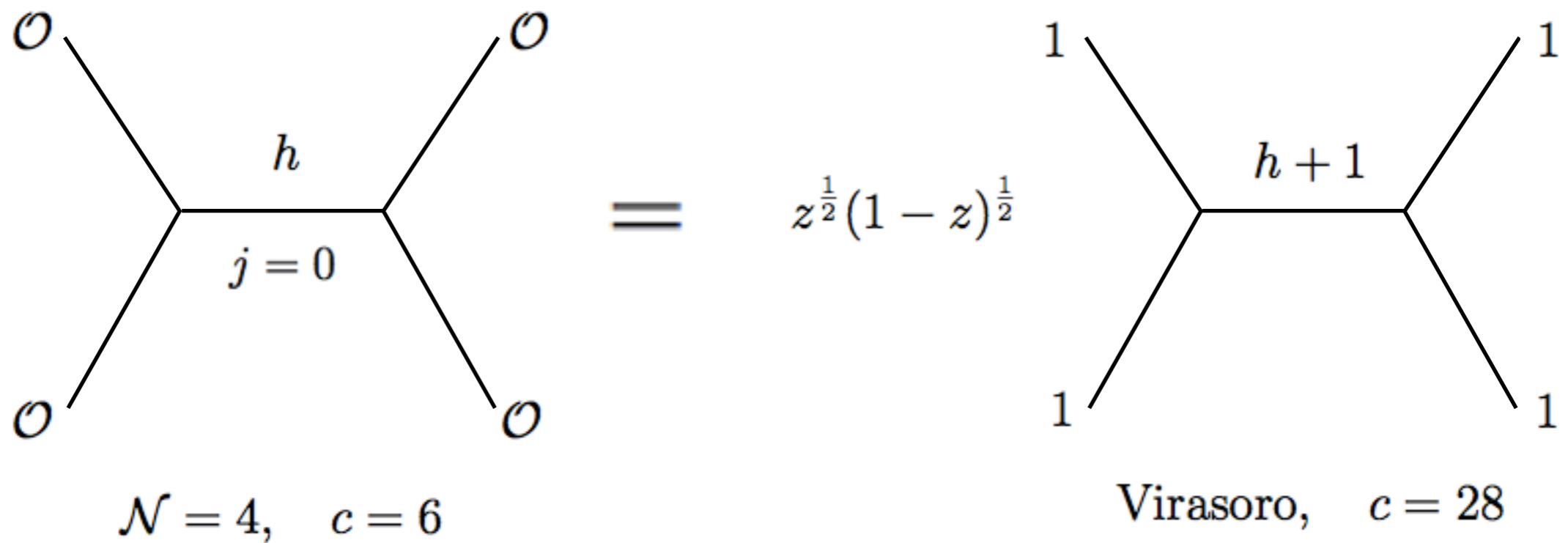
A simplification (special to $k=1$ theories): 1/4-BPS operators absent in the superconformal block decomposition (follows from SC Ward identities).

Convenient to work with 1/2-BPS RR operators, related by spectral flow, with

$$(h, j; \tilde{h}, \tilde{j}) = \left(\frac{1}{4}, 0; \frac{1}{4}, 0\right)$$

$$\langle \mathcal{O}(z, \bar{z}) \mathcal{O}(0) \mathcal{O}(1) \mathcal{O}(\infty) \rangle = \sum$$


Key ingredient 1: N=4 BPS superconformal block
(c=6, k=1 case)



Sketch of argument



A_1 $\mathcal{N} = 4$ cigar CFT

$$\frac{SL(2)_2/U(1)}{\mathbb{Z}_2}$$

Sketch of argument

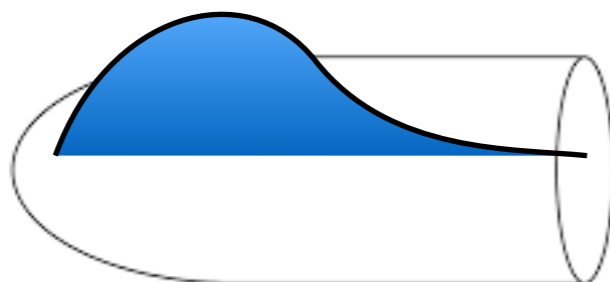


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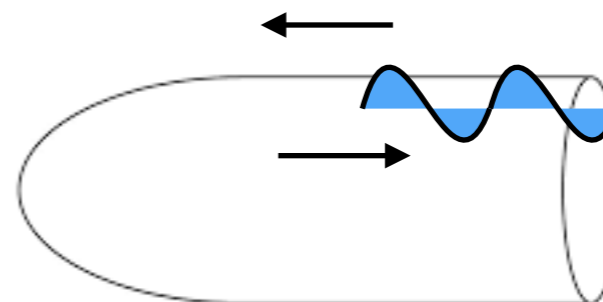
$$\frac{SL(2)_2/U(1)}{\mathbb{Z}_2}$$

Operator spectrum

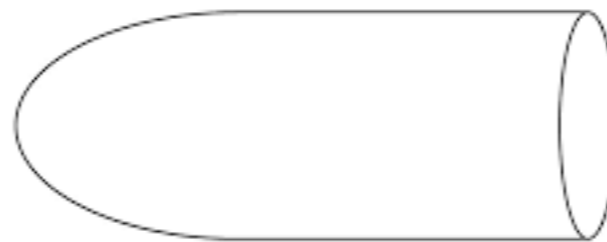
normalizable



non-normalizable



Sketch of argument

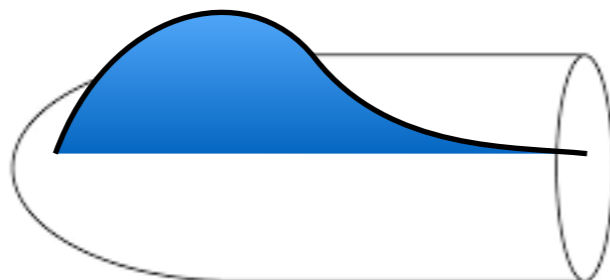


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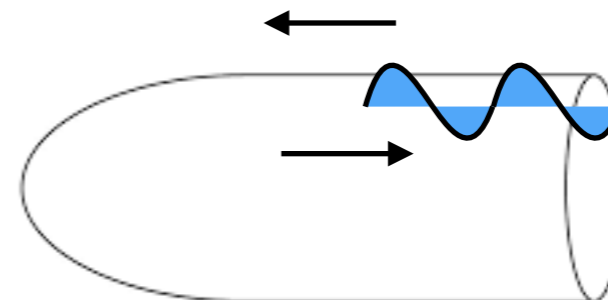
Operator spectrum

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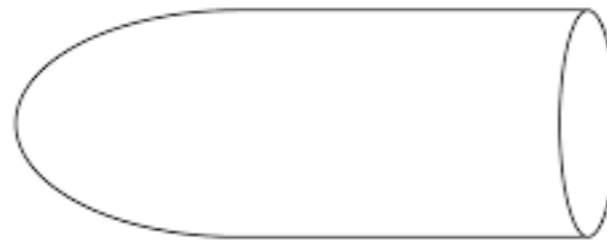
BPS and non-BPS discrete states

non-normalizable



non-BPS continuum

Sketch of argument



$$\frac{SL(2)_2/U(1)}{\mathbb{Z}_2}$$

A_1 $\mathcal{N} = 4$ cigar CFT

4-point function of continuum operators can be computed from $SL(2)$ WZW model, which is then related to a 6-point function in the bosonic Liouville theory of central charge 28 [Ribault-Teschner '05]

Sketch of argument



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The BPS 4-point function is extracted from the residue at a pole of the analytically continued 4-point function of continuum operators, which is reduced to a 4-point function in the bosonic Liouville theory of central charge 28. This identification was used to study the UV completion of 6D super-Yang-Mills amplitudes by little string theory. [Chang-Lin-Shao-Wang-XY, '14]

Sketch of argument



$$\frac{SL(2)_2/U(1)}{\mathbb{Z}_2}$$

A_1 $\mathcal{N} = 4$ cigar CFT

It happens that the $\mathcal{N}=4$ (or $\mathcal{N}=2$) superconformal block decomposition of the cigar CFT is identical to the Virasoro conformal block decomposition of the bosonic Liouville correlator of $c=28$ (a la Ribault-Teschner).

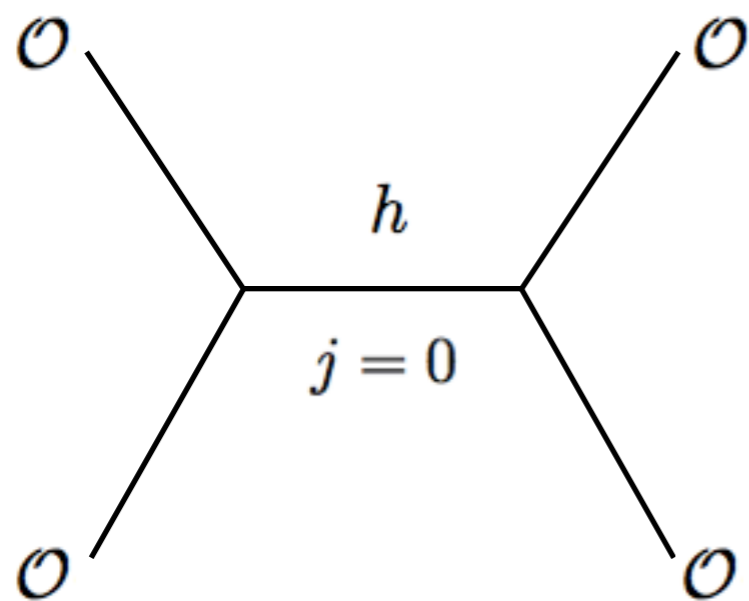
Sketch of argument



A_1 $\mathcal{N} = 4$ cigar CFT

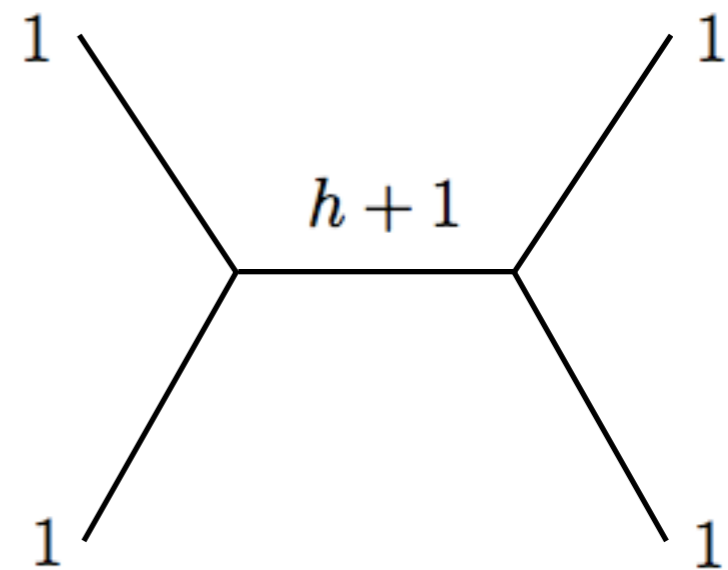
$$\frac{SL(2)_2/U(1)}{\mathbb{Z}_2}$$

Hence, we arrive at (also verified by computer algebra)



$\mathcal{N} = 4, \quad c = 6$

$$= z^{\frac{1}{2}}(1 - z)^{\frac{1}{2}}$$

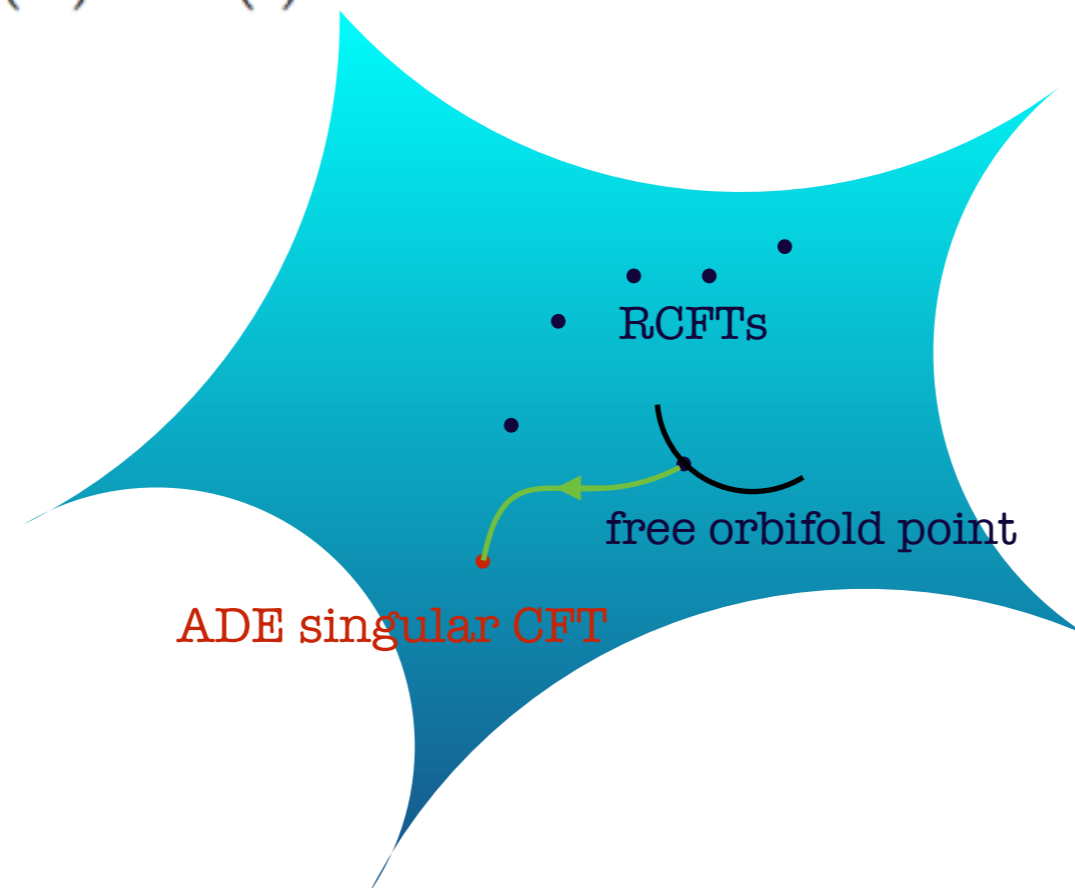


Virasoro, $c = 28$

Key ingredient 2: An exact result on the integrated BPS 4-point function (as function of moduli)

The moduli space of K3 CFT

$$\text{Aut}(\Gamma_{20,4}) \backslash SO(20,4) / SO(20) \times SO(4)$$



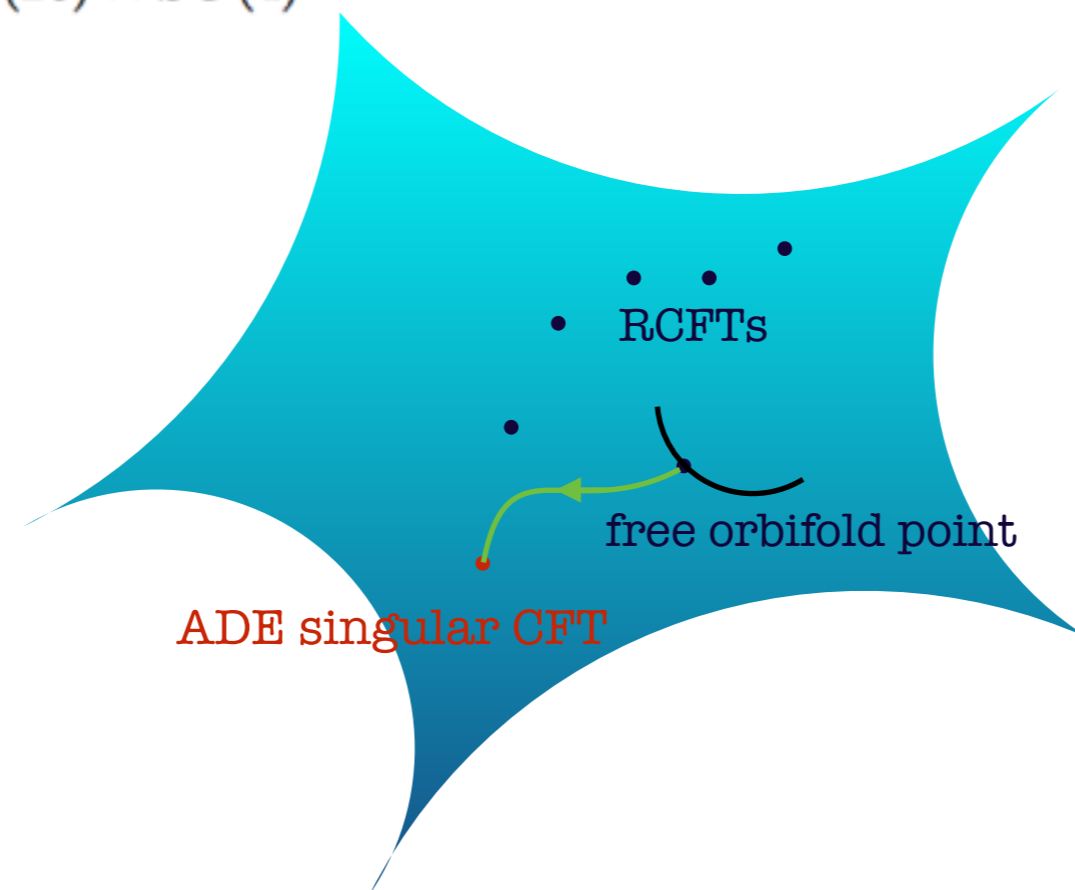
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The moduli space of K3 CFT

$$\text{Aut}(\Gamma_{20,4}) \backslash SO(20,4) / SO(20) \times SO(4)$$

parameterized by lattice embedding

$$\Gamma_{20,4} \subset \mathbb{R}^{20,4}$$



Key ingredient 2: An exact result on the integrated BPS 4-point function (as function of moduli)

$$\int \frac{d^2 z}{|z(1-z)|} \langle \mathcal{O}_i(z, \bar{z}) \mathcal{O}_j(0) \mathcal{O}_k(1) \mathcal{O}_\ell(\infty) \rangle = \frac{\partial^4}{\partial y^i \partial y^j \partial y^k \partial y^\ell} \Big|_{y=0} \int_{\mathcal{F}} d^2 \tau \frac{\Theta_\Lambda(y|\tau, \bar{\tau})}{\eta(\tau)^{24}}$$

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BPS operators $\mathcal{O}_i \longleftrightarrow$ deformations of embedding of lattice Λ

$$\Theta_\Lambda(y|\tau, \bar{\tau}) = e^{\frac{\pi}{2\tau_2} y^2} \sum_{\ell \in \Lambda} e^{\pi i \tau \ell_L^2 - \pi i \bar{\tau} \ell_R^2 + 2\pi i \ell_L \cdot y} \quad y \in \mathbb{R}^{20}$$

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LHS = H^4 effective coupling of type IIB string on K3 at tree level

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RHS = solution to a set of second order differential equations on the moduli space that follow from 6D supersymmetry Ward identities

[Lin-Shao-Wang-XY, '15]

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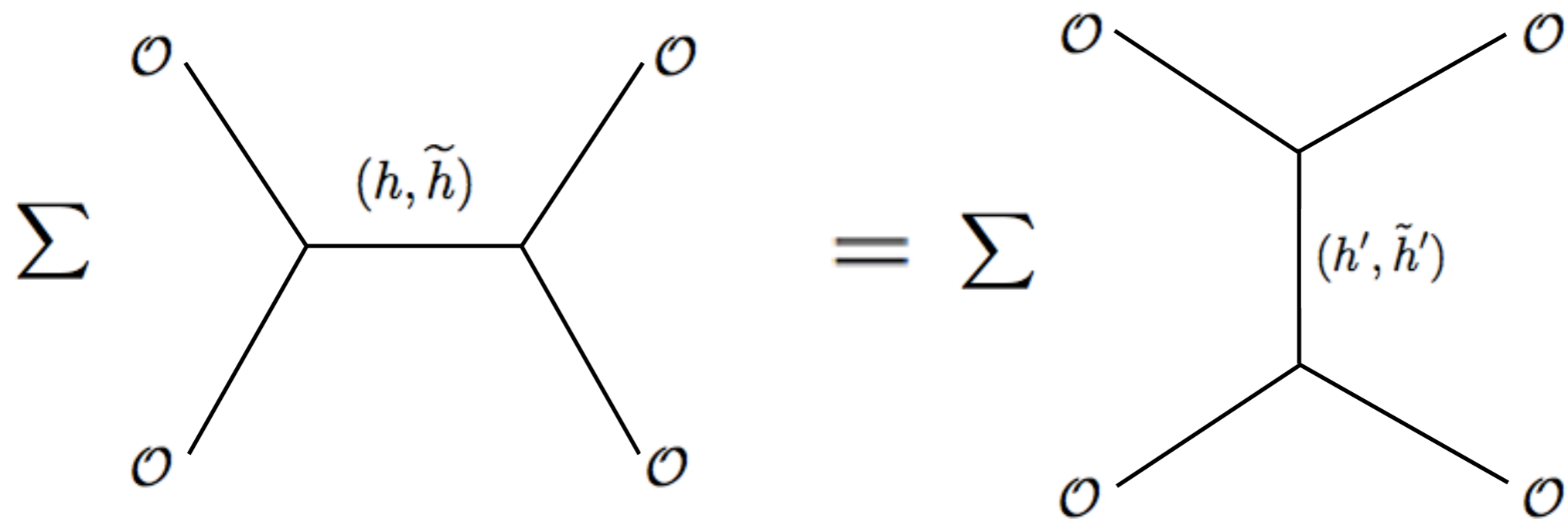
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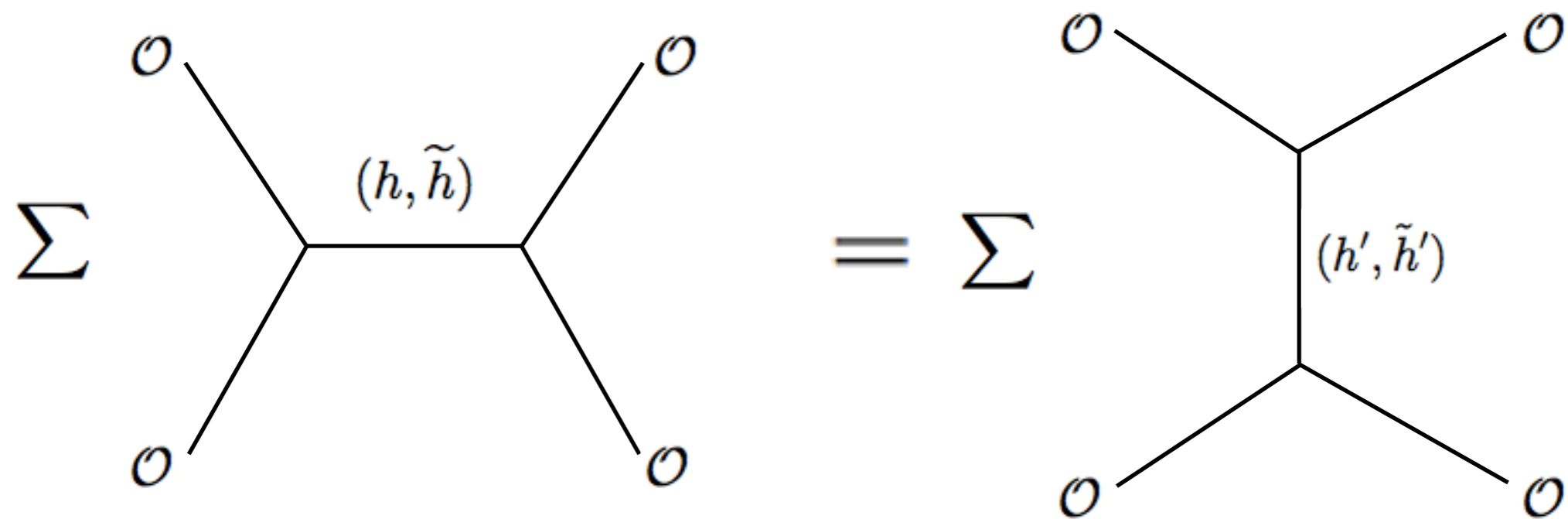
[Lin-Shao-Wang-XY, '15]

computed as an exact function of the moduli, then fed into the bootstrap machine.

Conformal bootstrap: the crossing equation

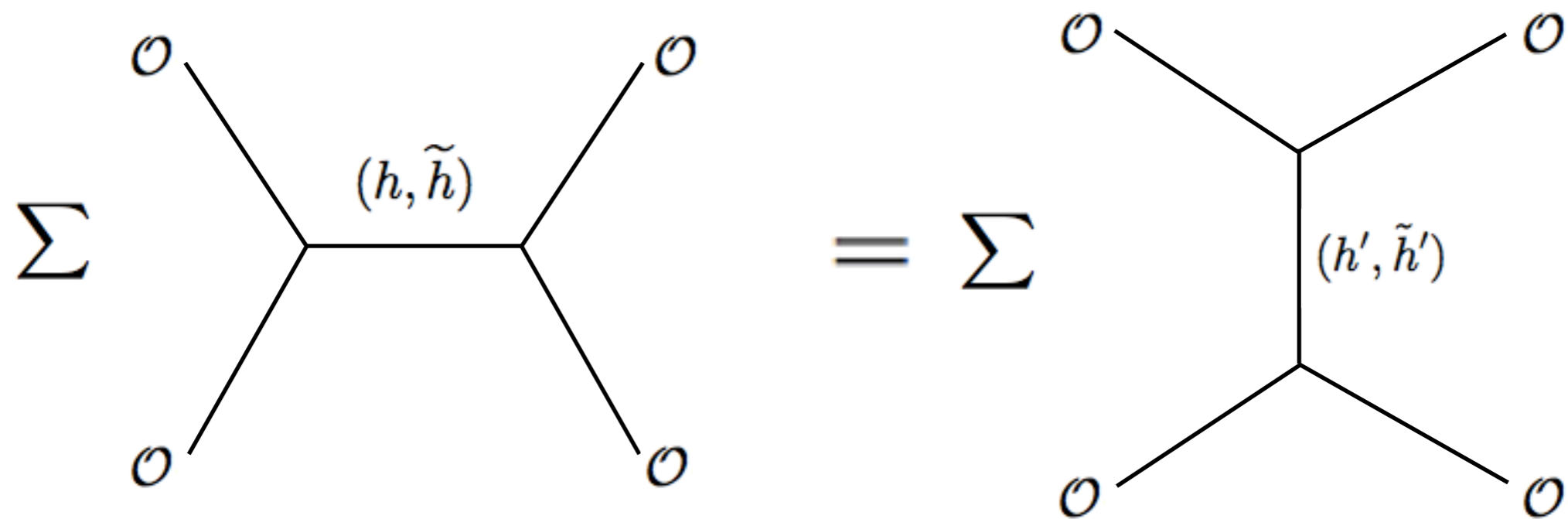


Conformal bootstrap: the crossing equation



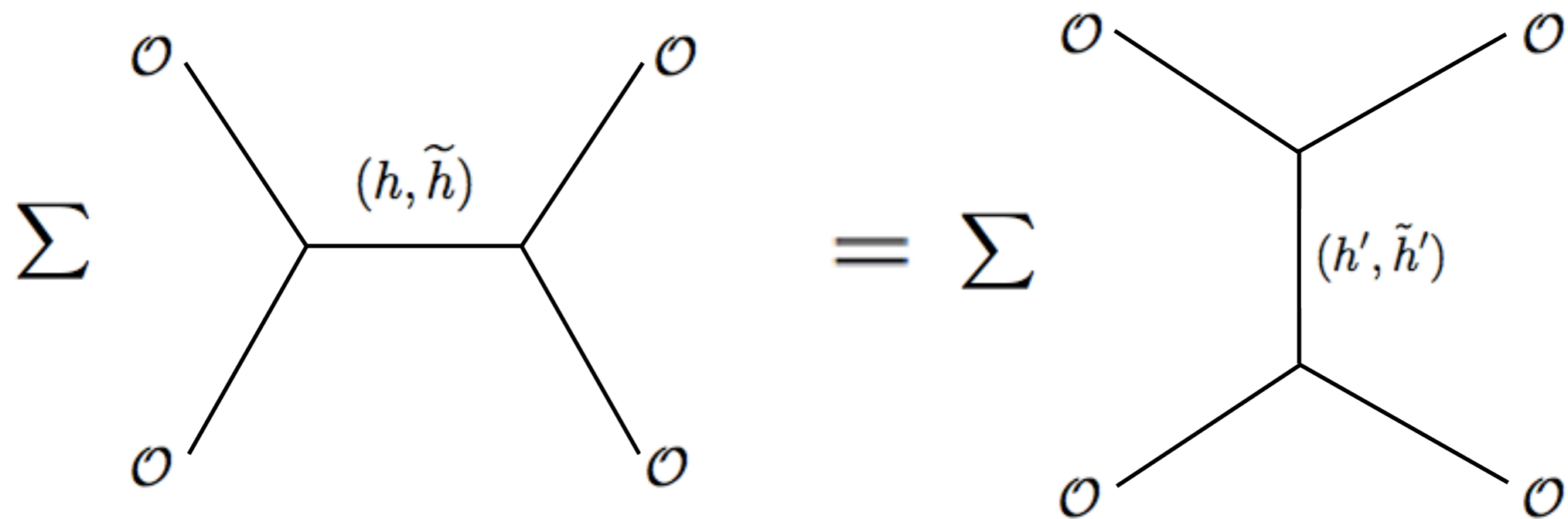
$$0 = \sum_{h_L, h_R} C_{h_L, h_R}^2 \left[\mathcal{F}_{h_L}^R(z) \overline{\mathcal{F}_{h_R}^R(z)} - \mathcal{F}_{h_L}^R(1-z) \overline{\mathcal{F}_{h_R}^R(1-z)} \right]$$

Conformal bootstrap: the crossing equation



$$0 = \sum_{\Delta, s} C_{h_L, h_R}^2 \alpha [\mathcal{F}_{h_L}^R(z) \overline{\mathcal{F}_{h_R}^R(z)} - \mathcal{F}_{h_L}^R(1-z) \overline{\mathcal{F}_{h_R}^R(1-z)}]$$

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$$\alpha_{m,n} = \partial_z^m \bar{\partial}_z^n \Big|_{z=1/2}$$

Conformal bootstrap: the crossing equation

To bound the gap in the spectrum of non-BPS primaries that appear in the OPE of a pair of BPS operators, seek a linear functional (linear combination of $\alpha_{m,n}$'s) such that

$$\alpha \left[\mathcal{F}_{h_L}^R(z) \overline{\mathcal{F}_{h_R}^R(z)} - \mathcal{F}_{h_L}^R(1-z) \overline{\mathcal{F}_{h_R}^R(1-z)} \right] > 0$$

for $\Delta = s = 0$ and all $\Delta > \Delta_{gap}^{trial}$, $s \in 2\mathbb{Z}$

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If such a linear functional α exists, the gap must lie below Δ_{gap}^{trial}

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If such a linear functional α exists, the gap must lie below Δ_{gap}^{trial}

We would then have derived an upper bound on the gap.

A few words on the numerics

1. The bosonic Virasoro blocks are calculated to high orders (~ 40) in the q -expansion using Zamolodchikov's recurrence relation. Here q is the modular parameter of the 2-fold covering torus over the 4-punctured sphere.

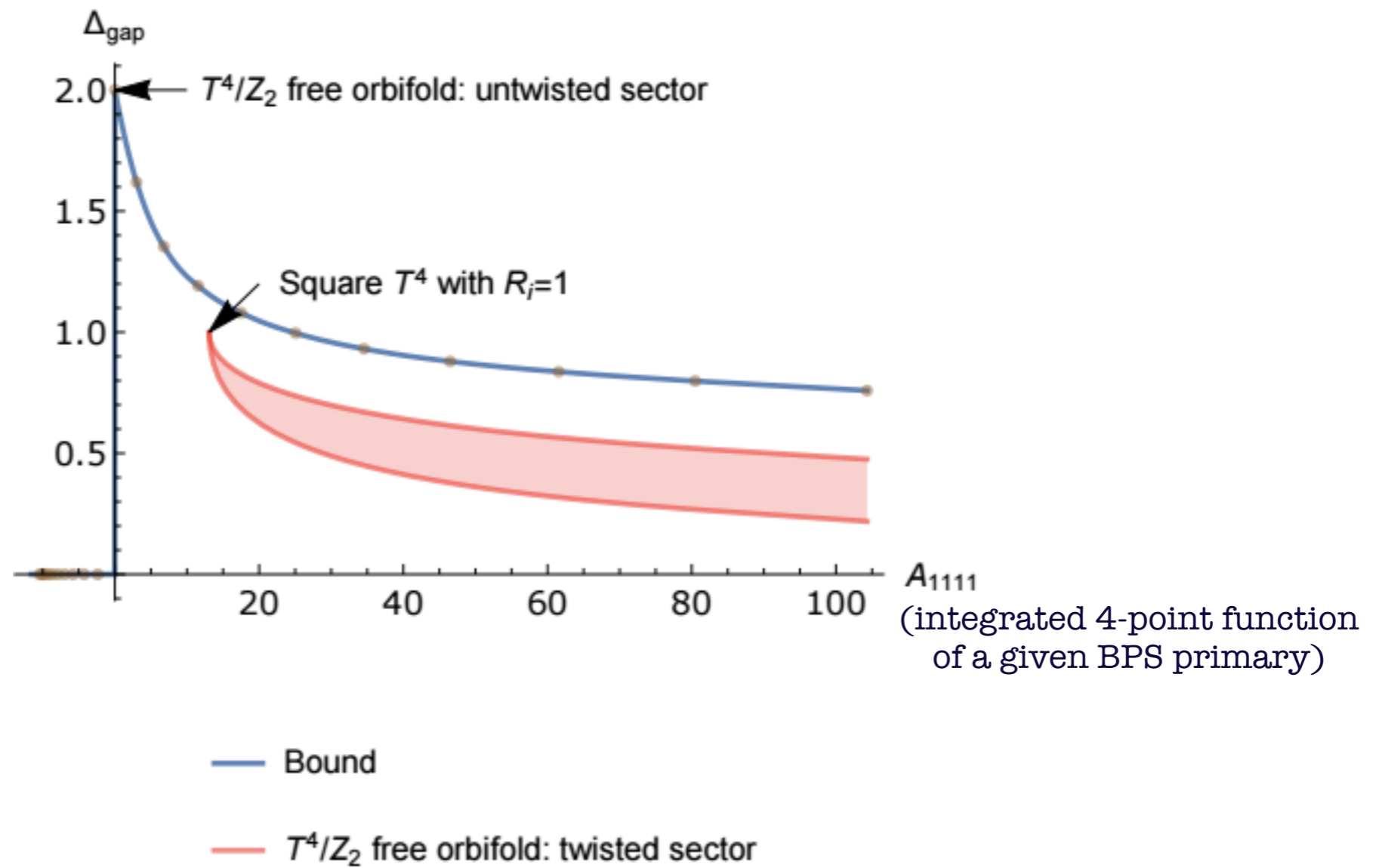
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2. The integrated BPS 4-point function gives a linear equation on the OPE coefficients squared C_{h_L, h_R}^2 . Using crossing symmetry, the relevant integral of the superconformal block is reduced to that over a suitable eye-shaped domain in the q -disc, in order to preserve certain positivity property of the coefficients at large operator dimension and allow for consistent order truncation numerically.

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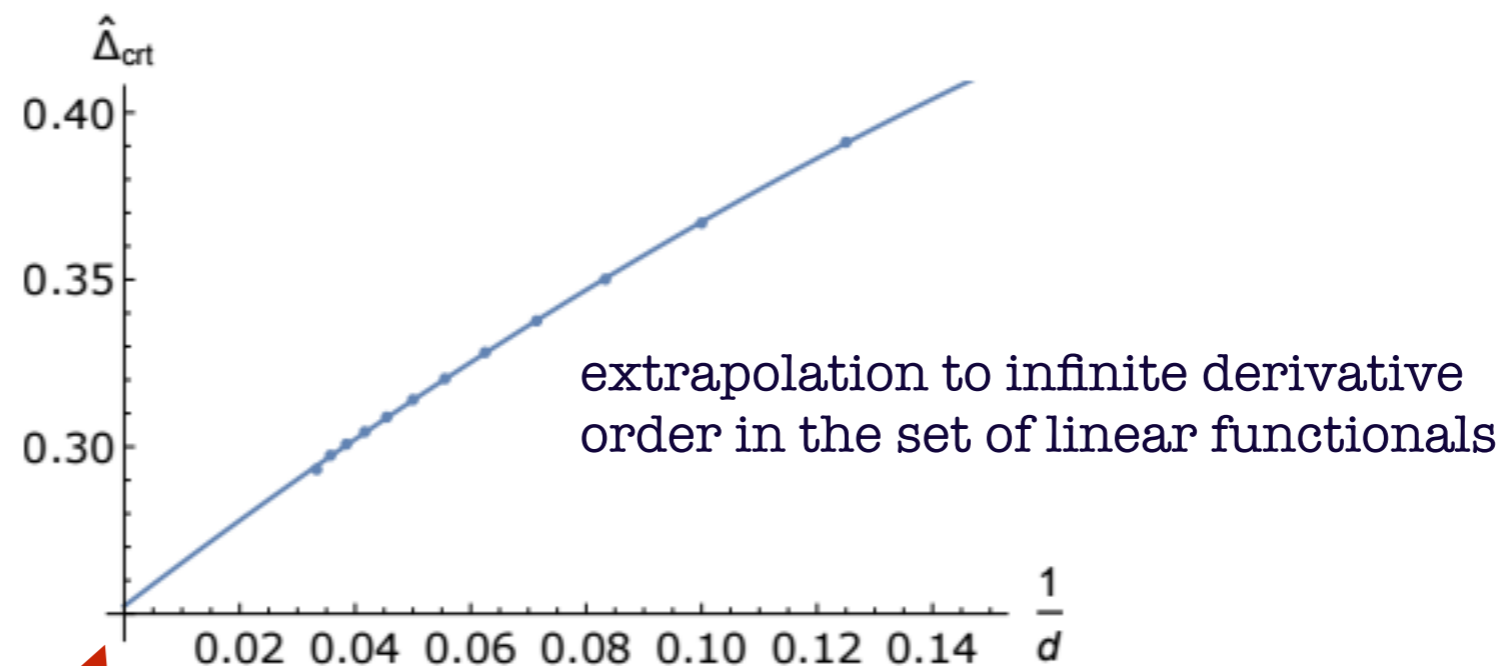
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3. The set of linear functionals on the crossing equation is taken to include derivatives up to ~ 30 th order, and extrapolated numerically to infinite derivative orders.

The gap in the non-BPS operator spectrum of the K3 CFT



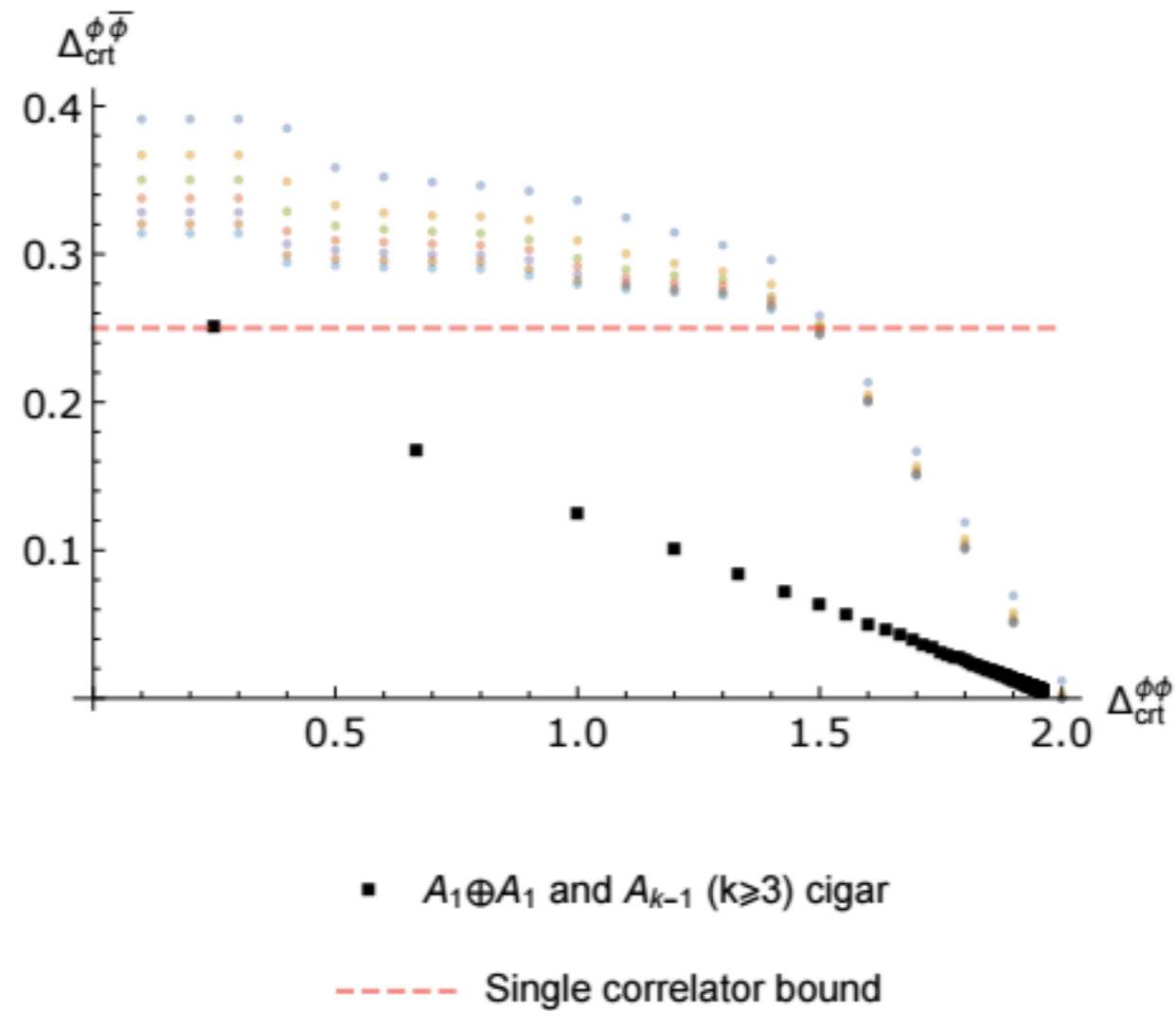
The development of continuum

(when the integrated BPS 4-point function diverges)

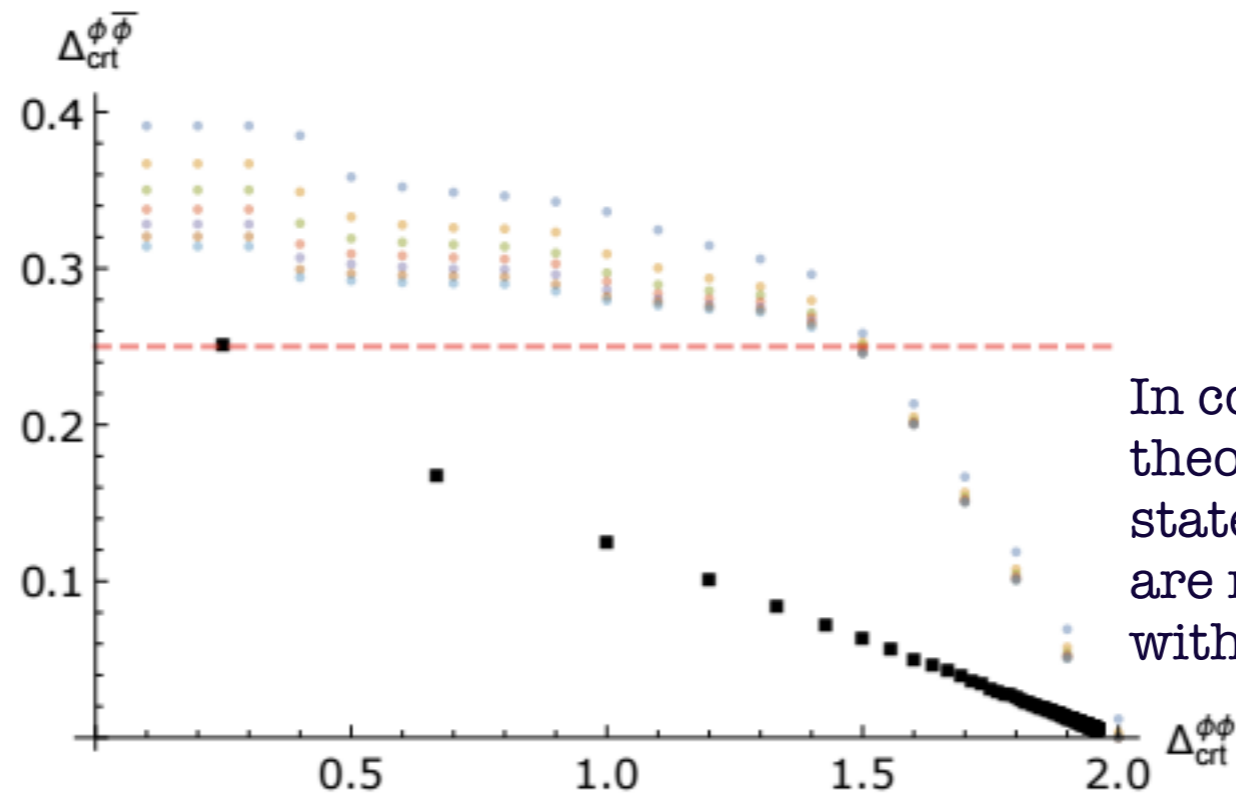


quadratic fit	0.252
A_1 cigar	0.25

Bounding the gap in the non-BPS operators appearing in the OPE of mixed BPS operators



Bounding the gap in the non-BPS operators appearing in the OPE of mixed BPS operators



In comparison with A_k cigar theories, discrete non-BPS states below the continuum are needed for consistency with bootstrap bounds.

- $A_1 \oplus A_1$ and A_{k-1} ($k \geq 3$) cigar
- Single correlator bound

$N=2$ superconformal bootstrap

Representations are labeled by conformal weight h and R-charge q of the primary. Chiral and anti-chiral operators obey $h=q/2$ and $h=-q/2$ respectively. Non-BPS primaries obey $h > |q|/2$.

$N=2$ superconformal bootstrap

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We will explore the OPE of BPS operators (chiral-chiral or chiral-anti-chiral) carrying R-charge q or $-q$.

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Our goal will be to constrain the spectrum of non-BPS operators in the OPE.

Structure of OPE

Chiral-Anti-Chiral (of equal and opposite R-charge):
the OPE contains only non-BPS primaries that are
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Chiral-Chiral: the OPE may contain chiral primaries of R-charge $2q$ (whose coefficient is governed by the chiral ring), descendants of non-BPS primaries of R-charge $2q-1$, and when $q < 1/2$, descendants of anti-chiral primaries of R-charge $2q-1$.

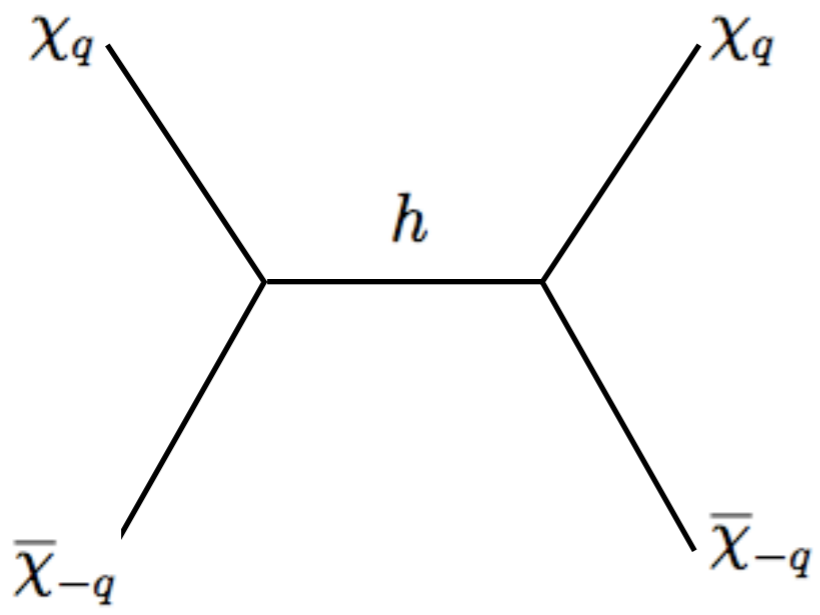
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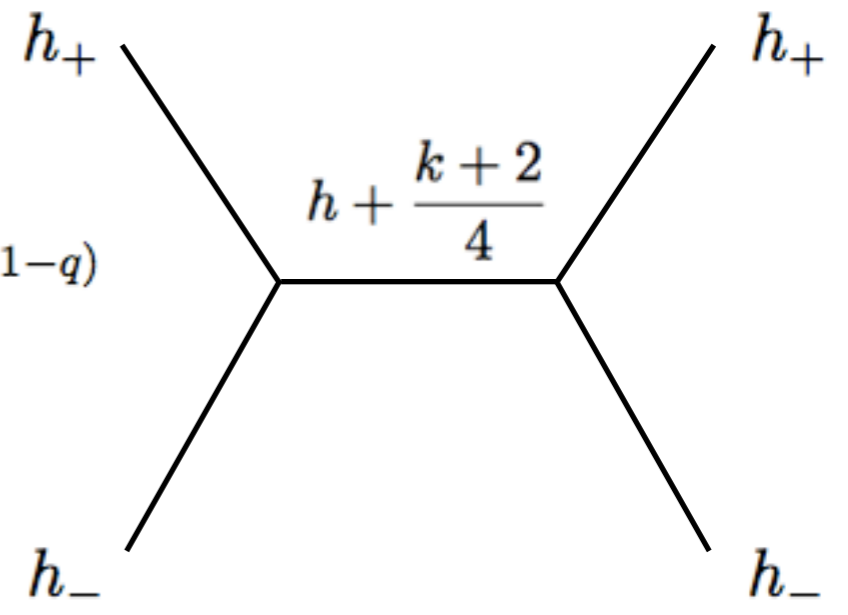
The superconformal blocks of BPS channels are simply given by limits of non-BPS channel blocks.

N=2 BPS superconformal blocks: chiral-anti-chiral channel



$$\mathcal{N} = 2, \quad c = \frac{3(k+2)}{k}$$

$$= (z(1-z))^{\frac{k}{2}q(1-q)}$$



$$\text{Virasoro, } c = 13 + 6k + \frac{6}{k}$$

N=2 BPS superconformal blocks: chiral-anti-chiral channel

$$\begin{array}{c}
 \chi_q \quad \chi_q \\
 \diagdown \quad \diagup \\
 \text{---} h \text{---} \\
 \diagup \quad \diagdown \\
 \bar{\chi}_{-q} \quad \bar{\chi}_{-q}
 \end{array}
 = (z(1-z))^{\frac{k}{2}q(1-q)}
 \begin{array}{c}
 h_+ \quad h_+ \\
 \diagdown \quad \diagup \\
 \text{---} h + \frac{k+2}{4} \text{---} \\
 \diagup \quad \diagdown \\
 h_- \quad h_-
 \end{array}$$

$\mathcal{N} = 2, \quad c = \frac{3(k+2)}{k}$
Virasoro, $c = 13 + 6k + \frac{6}{k}$

The non-BPS intermediate primary has weight h and vanishing R-charge.

N=2 BPS superconformal blocks: chiral-chiral channel

$$\begin{array}{ccc}
 \begin{array}{c} \chi_q \quad \bar{\chi}_{-q} \\ \diagdown \quad / \\ \text{---} h \text{---} \\ / \quad \diagdown \\ \chi_q \quad \bar{\chi}_{-q} \end{array} & = & z^{\frac{k}{2}q^2} (1-z)^{\frac{k}{2}q(1-q)} \begin{array}{c} h_+ \quad h_- \\ \diagdown \quad / \\ \text{---} h + kq(1-q) + \frac{1}{2} \text{---} \\ / \quad \diagdown \\ h_+ \quad h_- \end{array} \\
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 \end{array}$$

N=2 BPS superconformal blocks: chiral-chiral channel

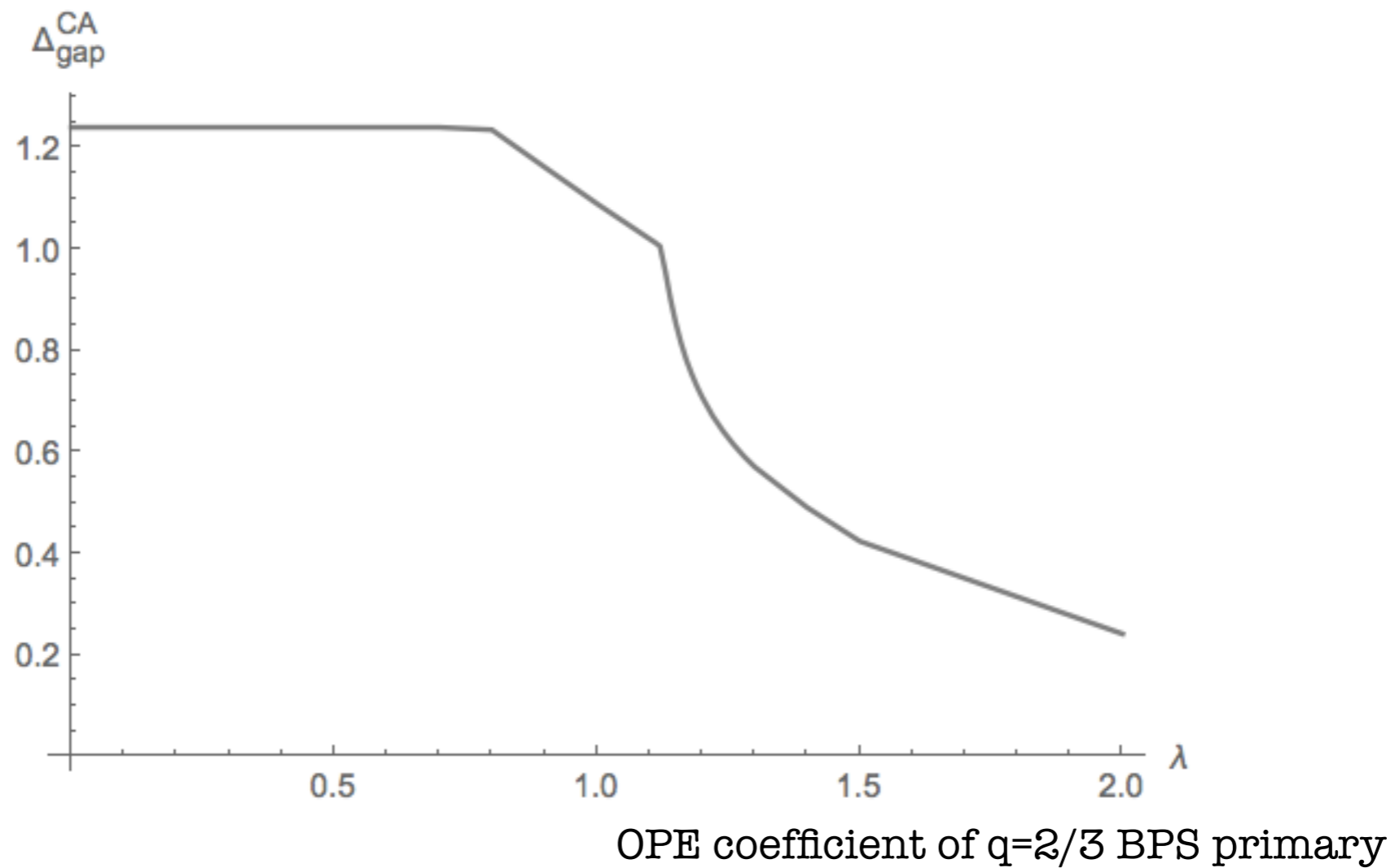
$$\begin{array}{c}
 \chi_q \quad \bar{\chi}_{-q} \\
 \diagdown \quad \diagup \\
 \text{---} h \text{---} \\
 \diagup \quad \diagdown \\
 \chi_q \quad \bar{\chi}_{-q}
 \end{array}
 = z^{\frac{k}{2}q^2} (1-z)^{\frac{k}{2}q(1-q)}
 \begin{array}{c}
 h_+ \quad h_- \\
 \diagdown \quad \diagup \\
 \text{---} h + kq(1-q) + \frac{1}{2} \text{---} \\
 \diagup \quad \diagdown \\
 h_+ \quad h_-
 \end{array}$$

$\mathcal{N} = 2, \quad c = \frac{3(k+2)}{k}$
 $\text{Virasoro}, \quad c = 13 + 6k + \frac{6}{k}$

The non-BPS intermediate primary has weight h and R-charge $2q-1$.

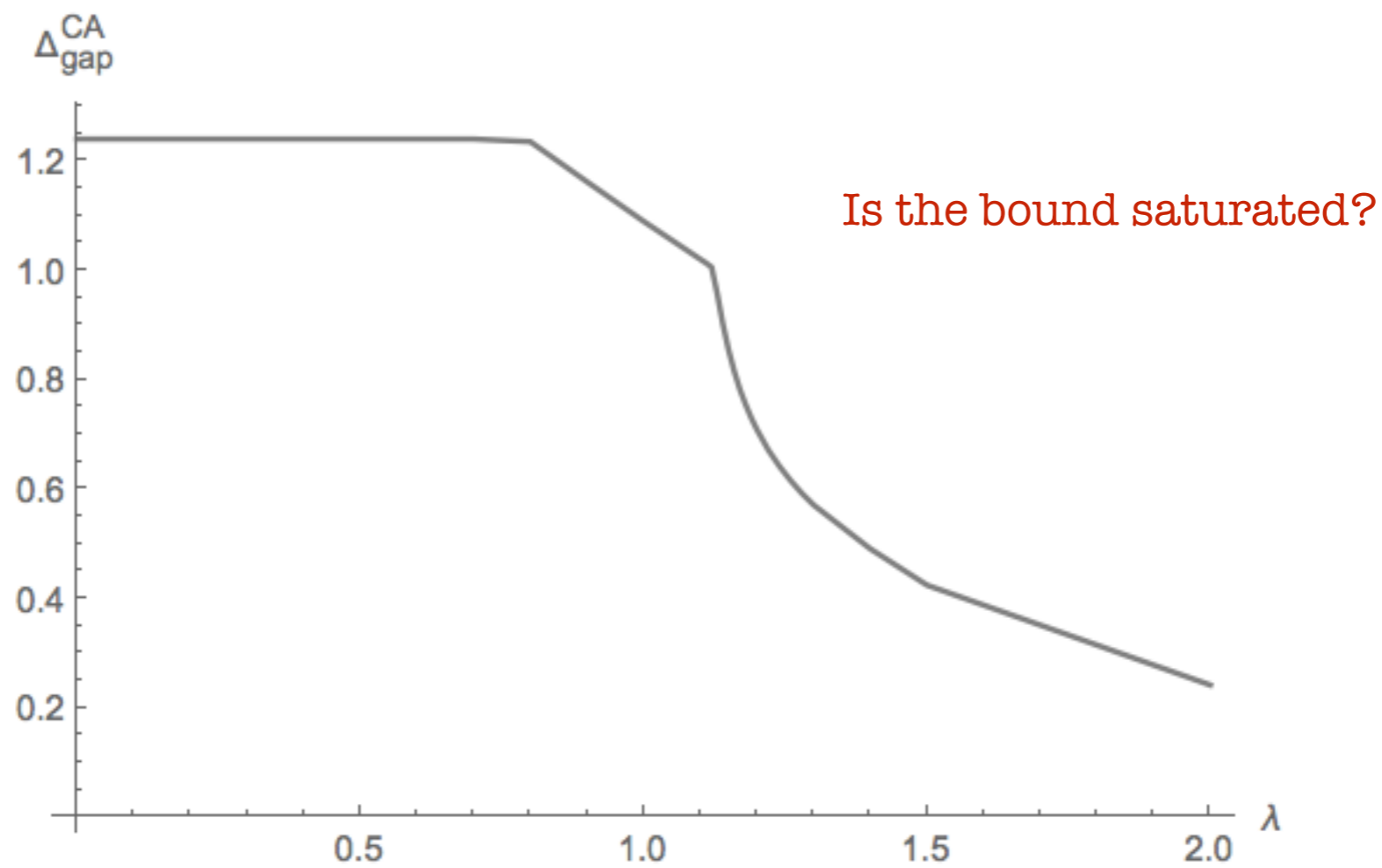
$N=(2,2)$ Bootstrap

$c=3$, chiral-anti-chiral OPE, $q=1/3$



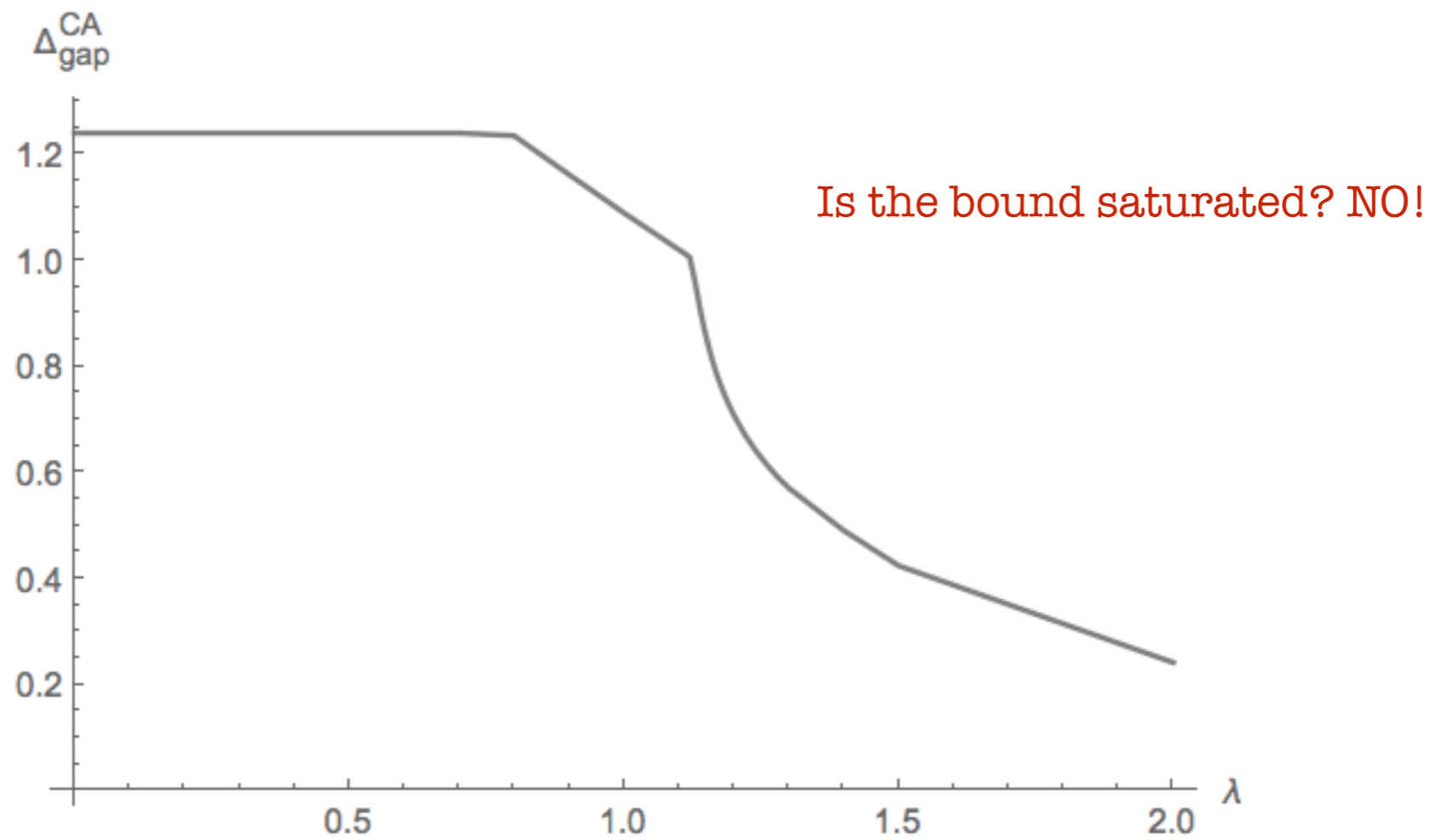
$N=(2,2)$ Bootstrap

$c=3$, chiral-anti-chiral OPE, $q=1/3$



$N=(2,2)$ Bootstrap

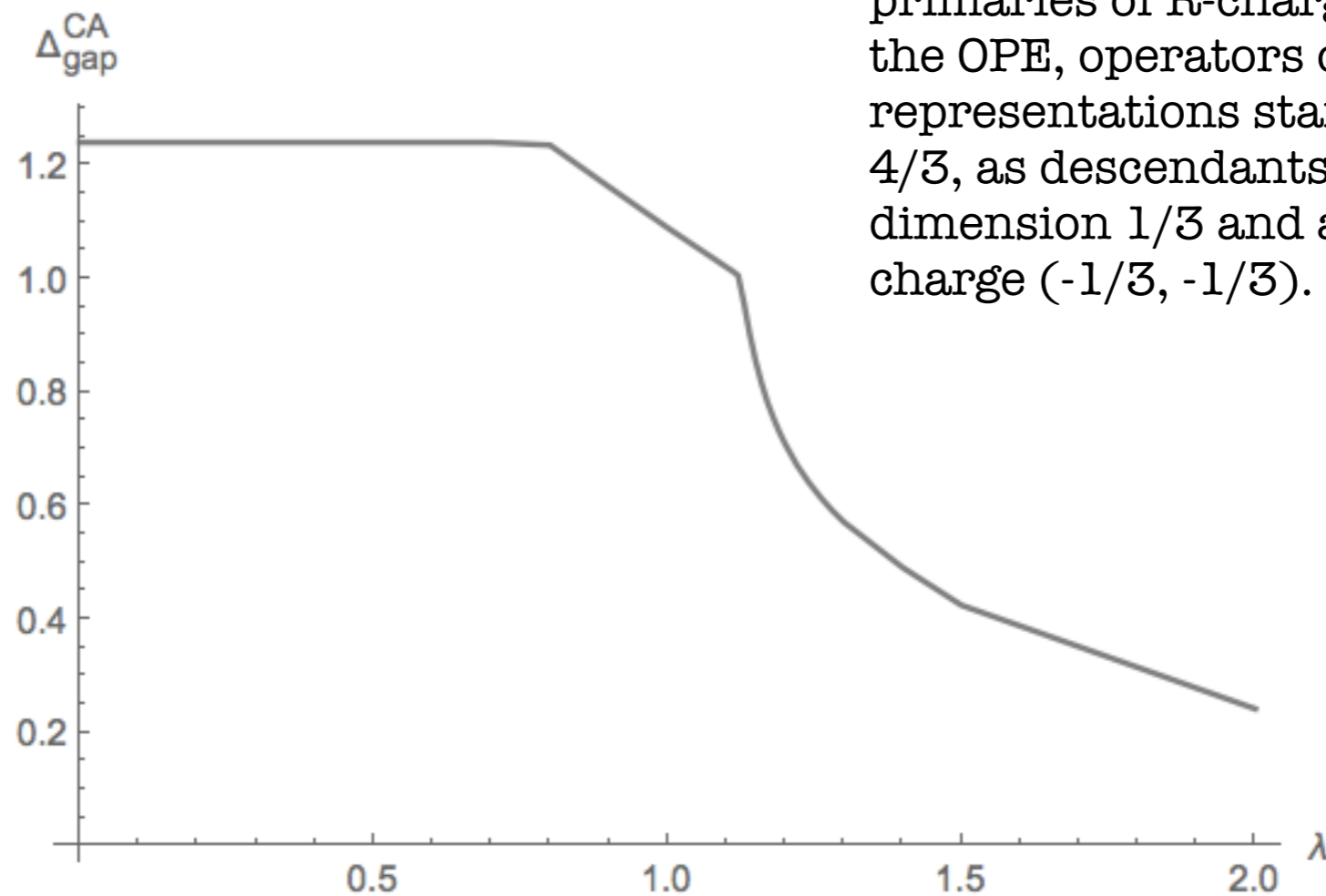
$c=3$, chiral-anti-chiral OPE, $q=1/3$



$N=(2,2)$ Bootstrap

$c=3$, chiral-anti-chiral OPE, $q=1/3$

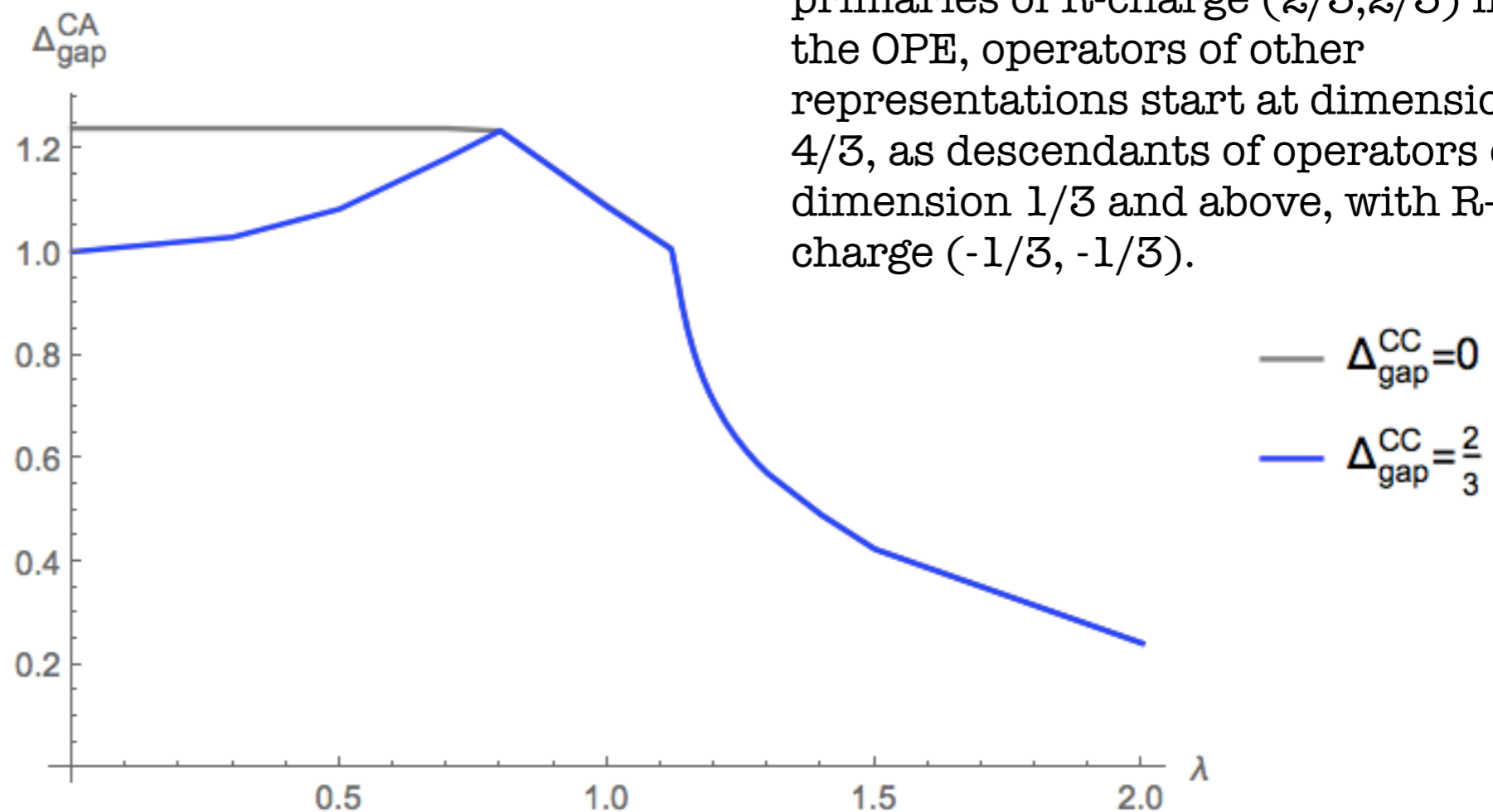
While there are dimension $2/3$ BPS primaries of R-charge $(2/3, 2/3)$ in the OPE, operators of other representations start at dimension $4/3$, as descendants of operators of dimension $1/3$ and above, with R-charge $(-1/3, -1/3)$.



$N=(2,2)$ Bootstrap

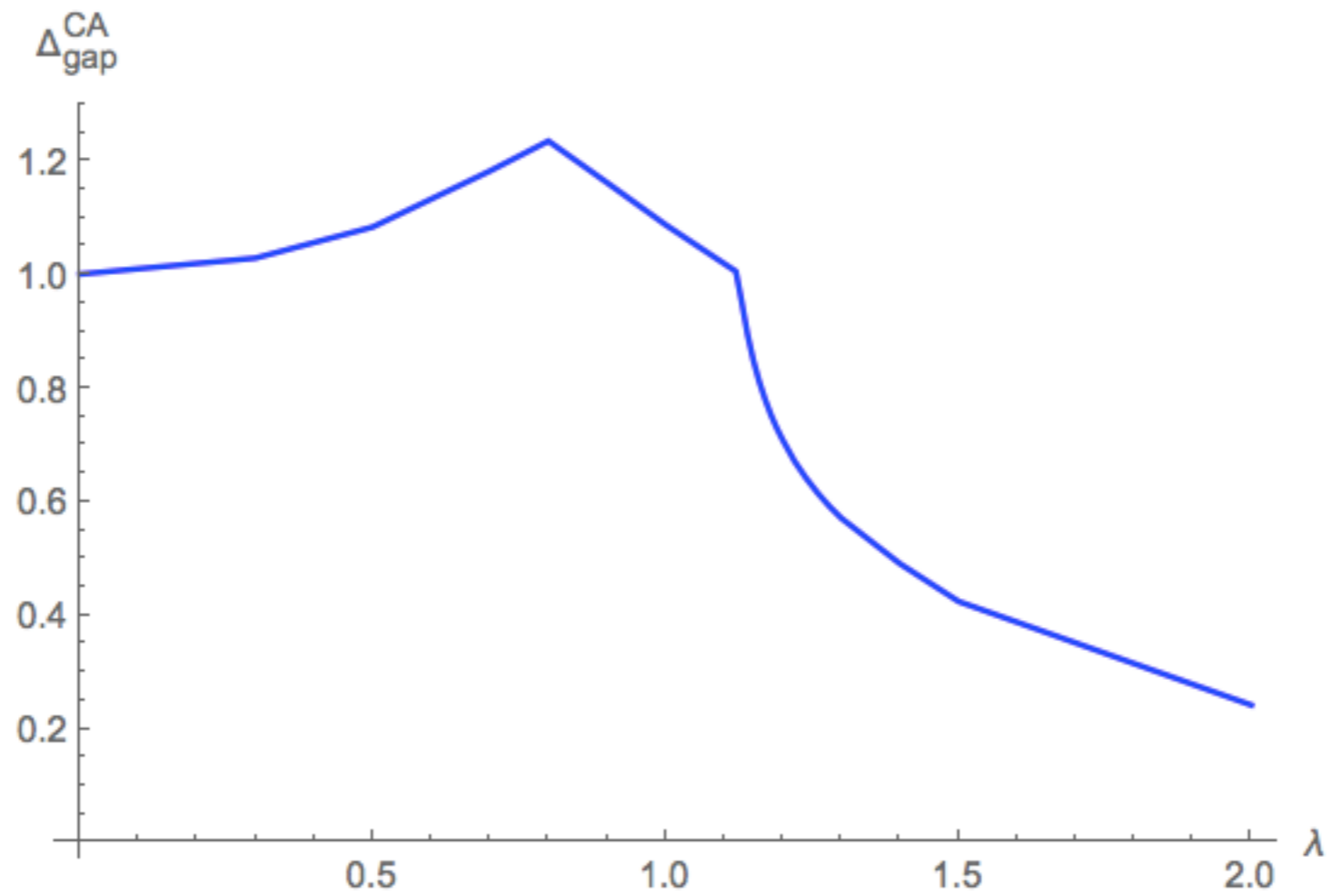
$c=3$, chiral-anti-chiral OPE, $q=1/3$

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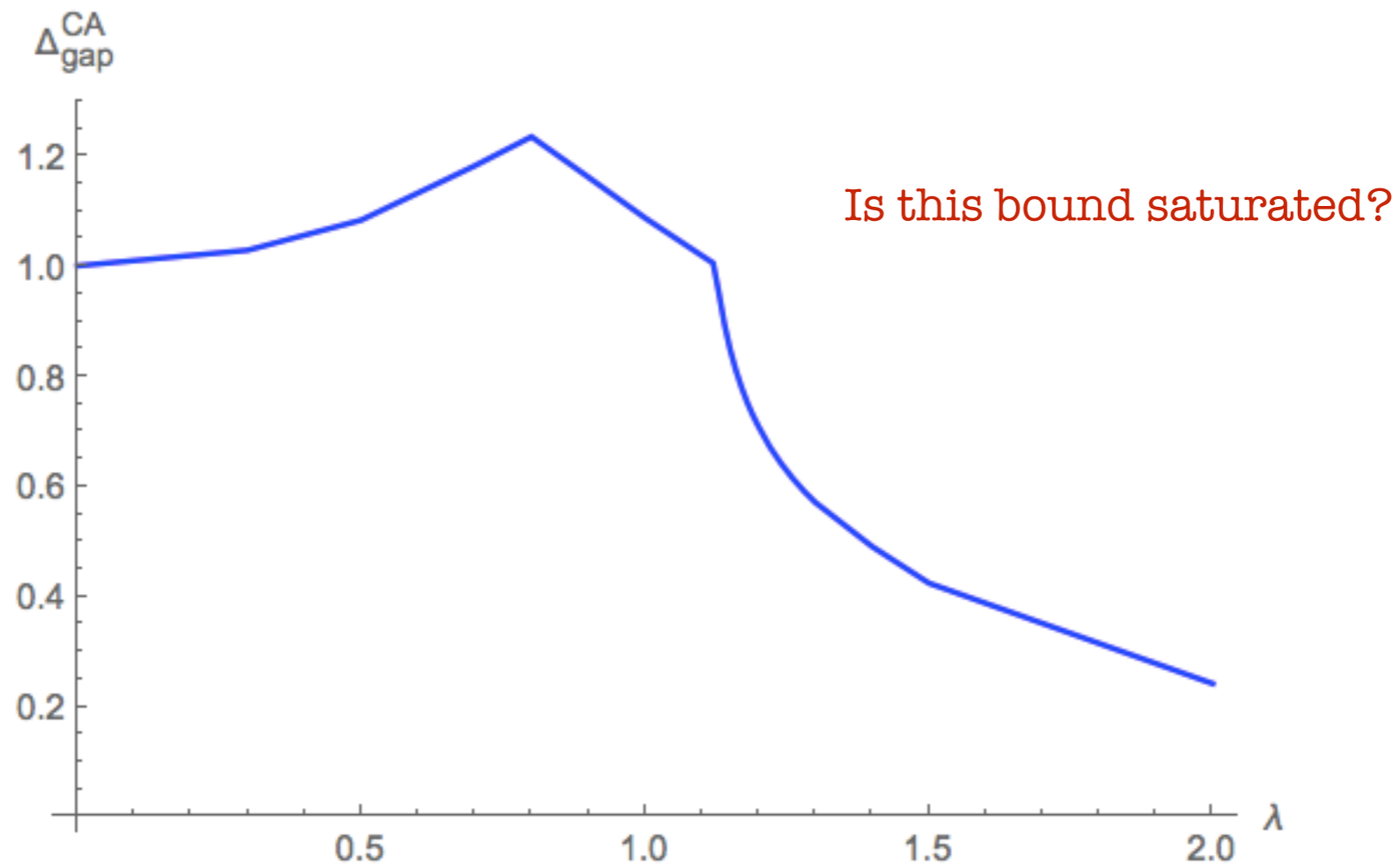
$N=(2,2)$ Bootstrap

$c=3$, chiral-anti-chiral OPE, $q=1/3$



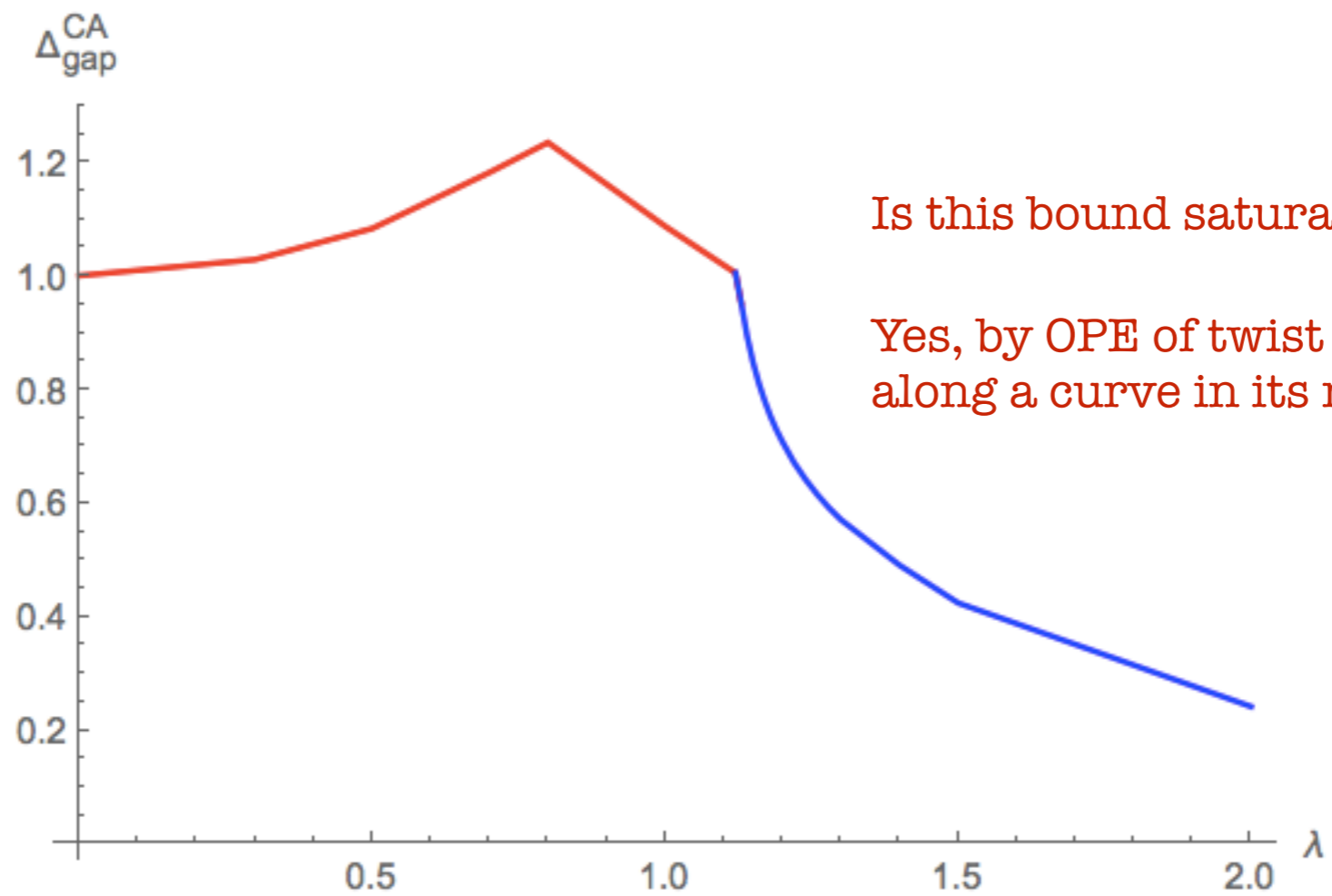
$N=(2,2)$ Bootstrap

$c=3$, chiral-anti-chiral OPE, $q=1/3$



$N=(2,2)$ Bootstrap

$c=3$, chiral-anti-chiral OPE, $q=1/3$

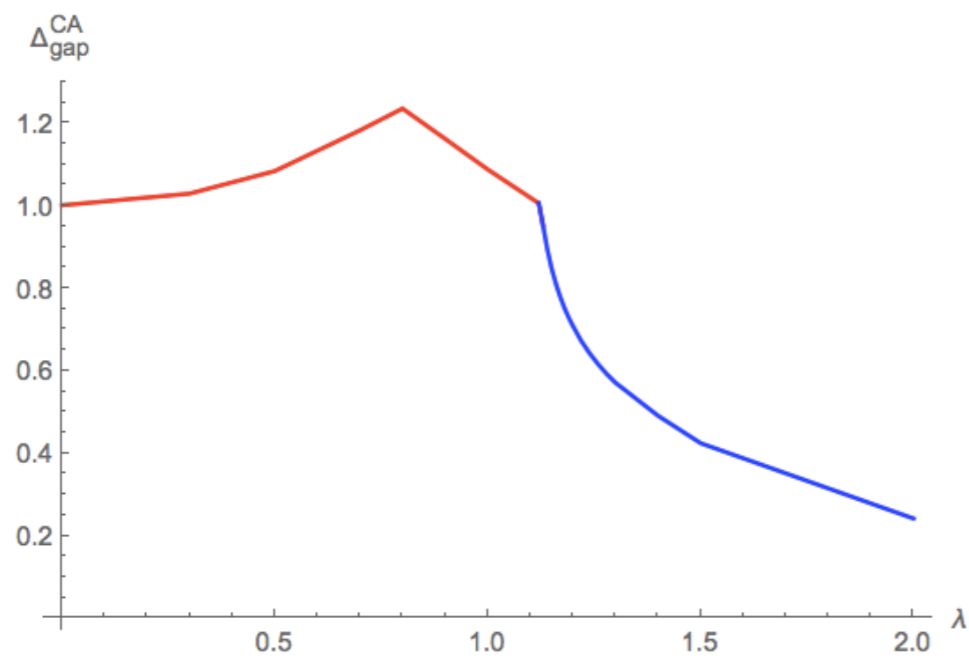


Is this bound saturated?

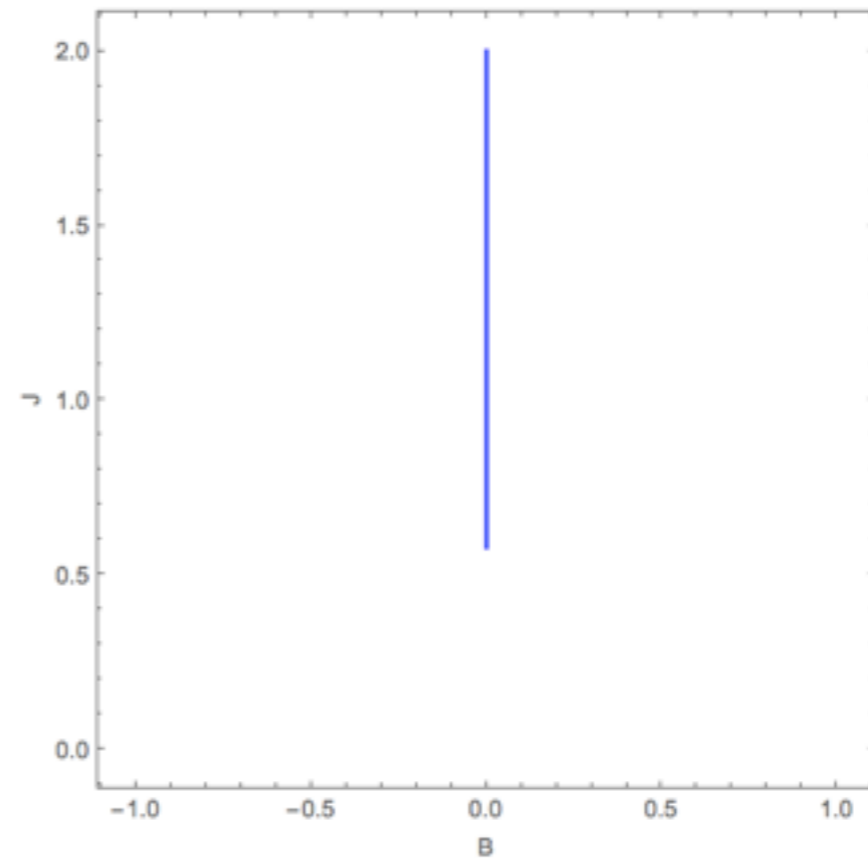
Yes, by OPE of twist fields in T^2/Z_3 along a curve in its moduli space!

$\mathcal{N}=(2,2)$ Bootstrap

$c=3$, chiral-anti-chiral OPE, $q=1/3$



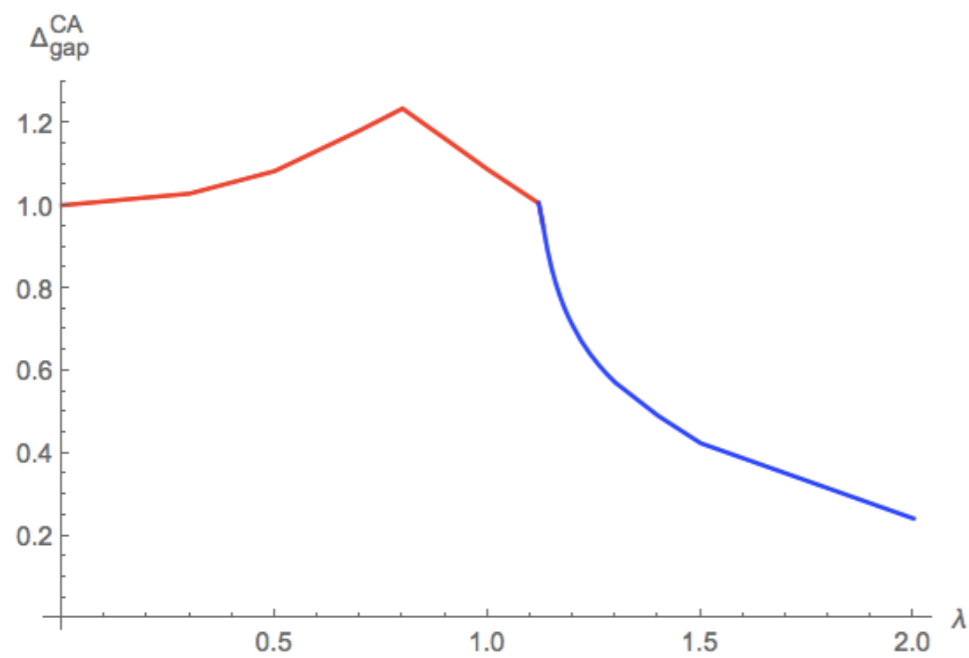
bootstrap bound



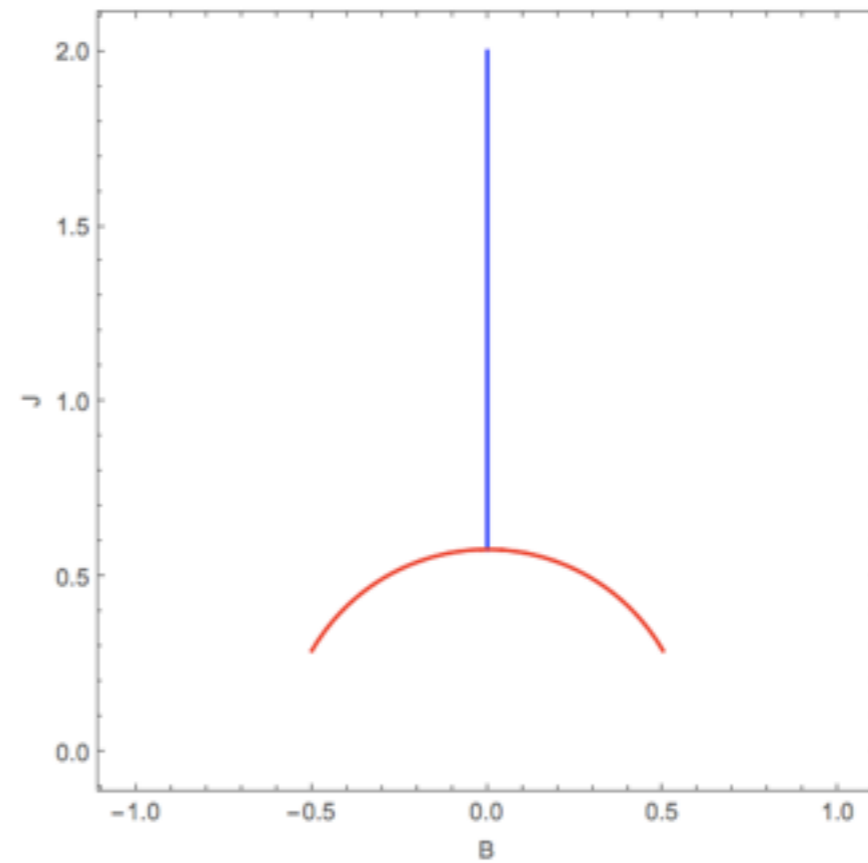
moduli space

$\mathcal{N}=(2,2)$ Bootstrap

$c=3$, chiral-anti-chiral OPE, $q=1/3$



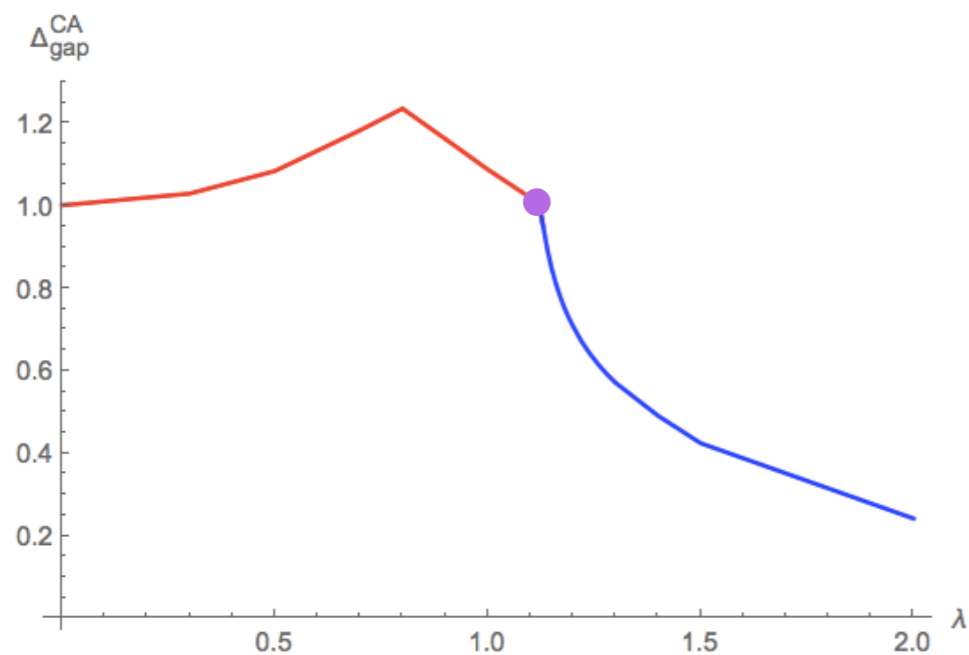
bootstrap bound



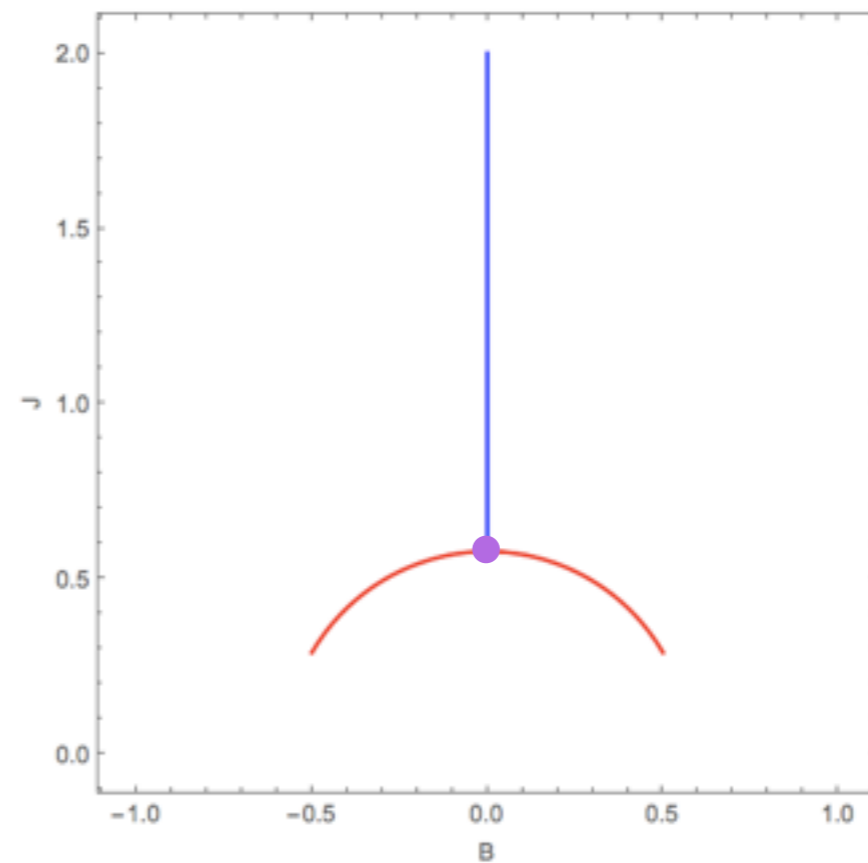
moduli space

$\mathcal{N}=(2,2)$ Bootstrap

$c=3$, chiral-anti-chiral OPE, $q=1/3$



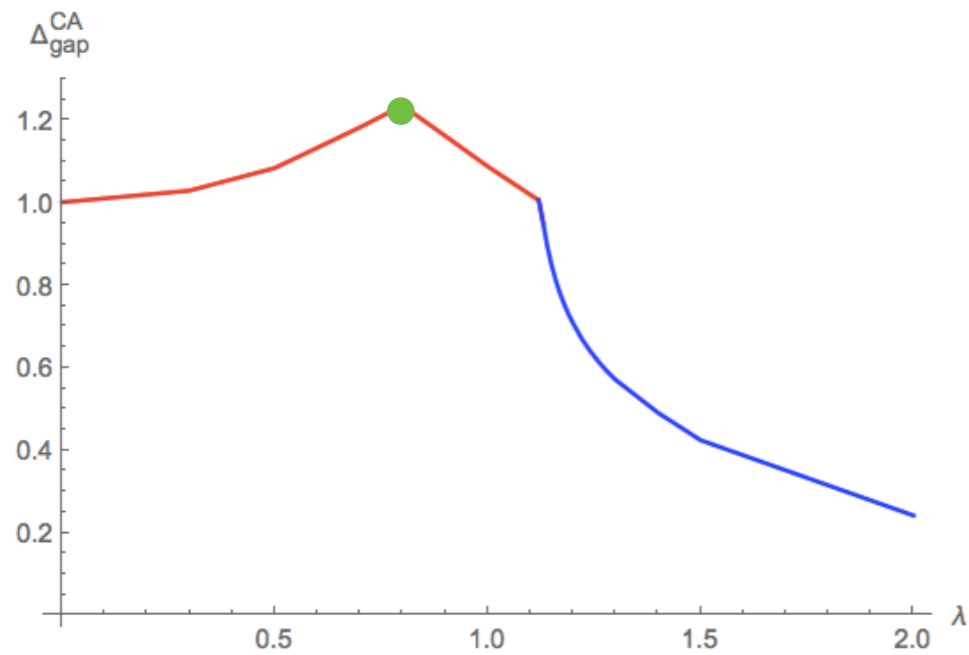
bootstrap bound



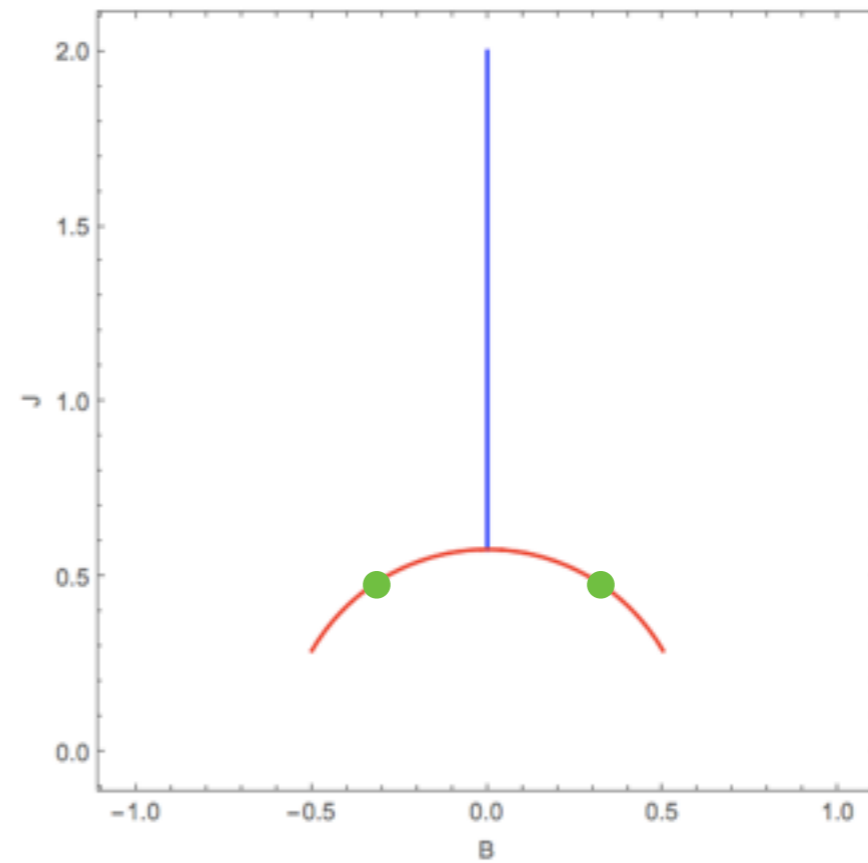
moduli space

$\mathcal{N}=(2,2)$ Bootstrap

$c=3$, chiral-anti-chiral OPE, $q=1/3$



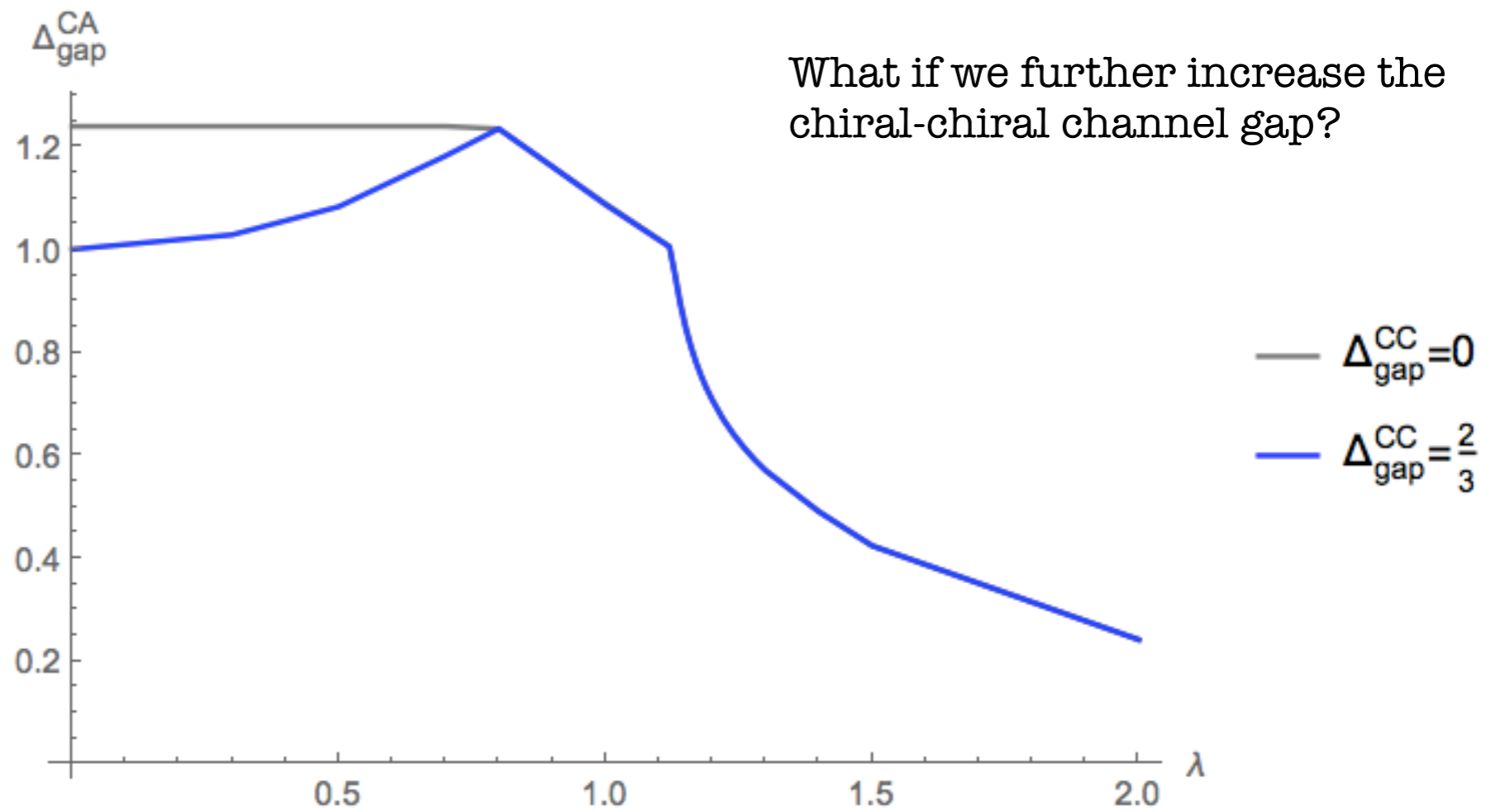
bootstrap bound



moduli space

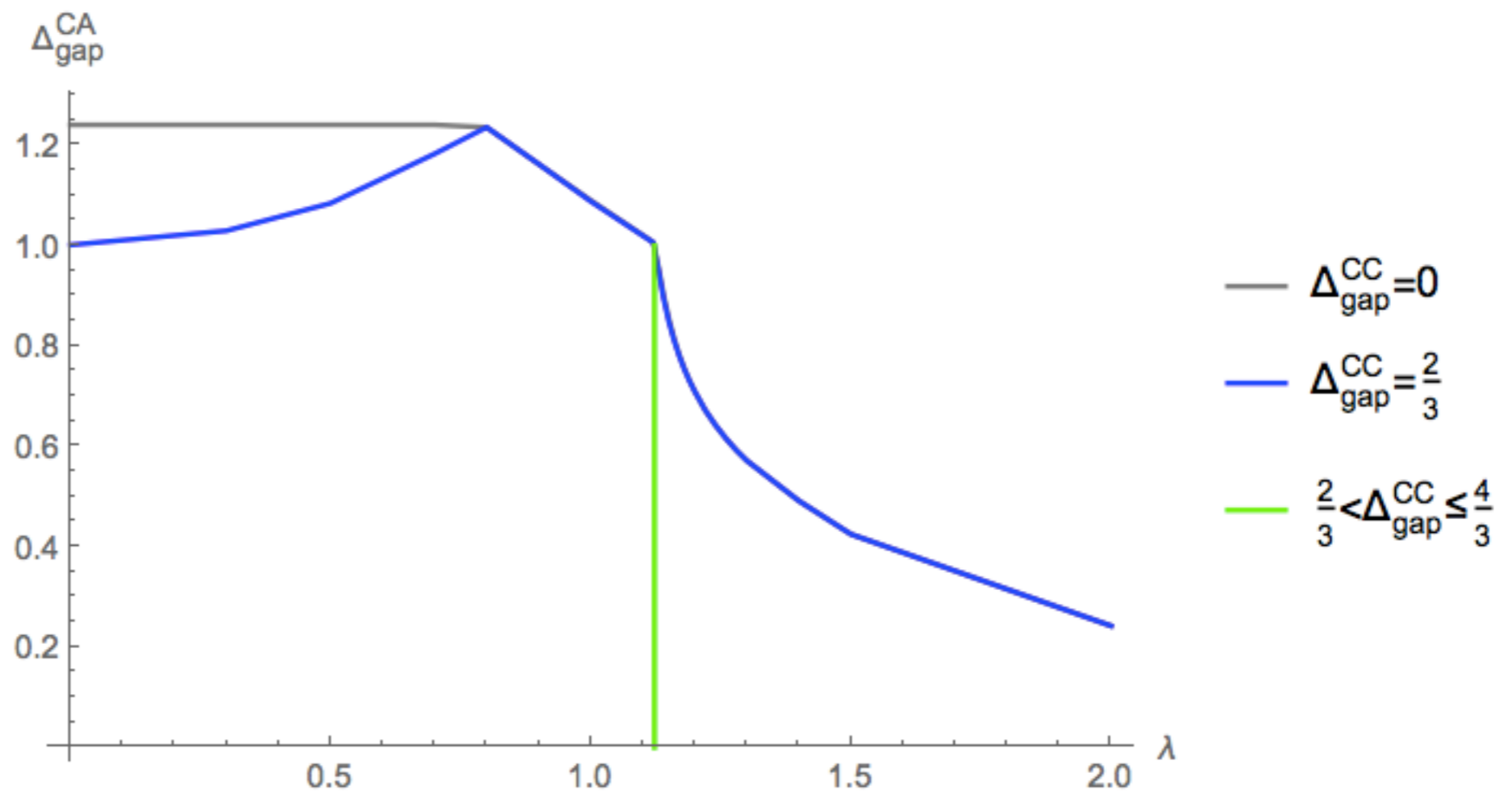
$N=(2,2)$ Bootstrap

$c=3$, chiral-anti-chiral OPE, $q=1/3$



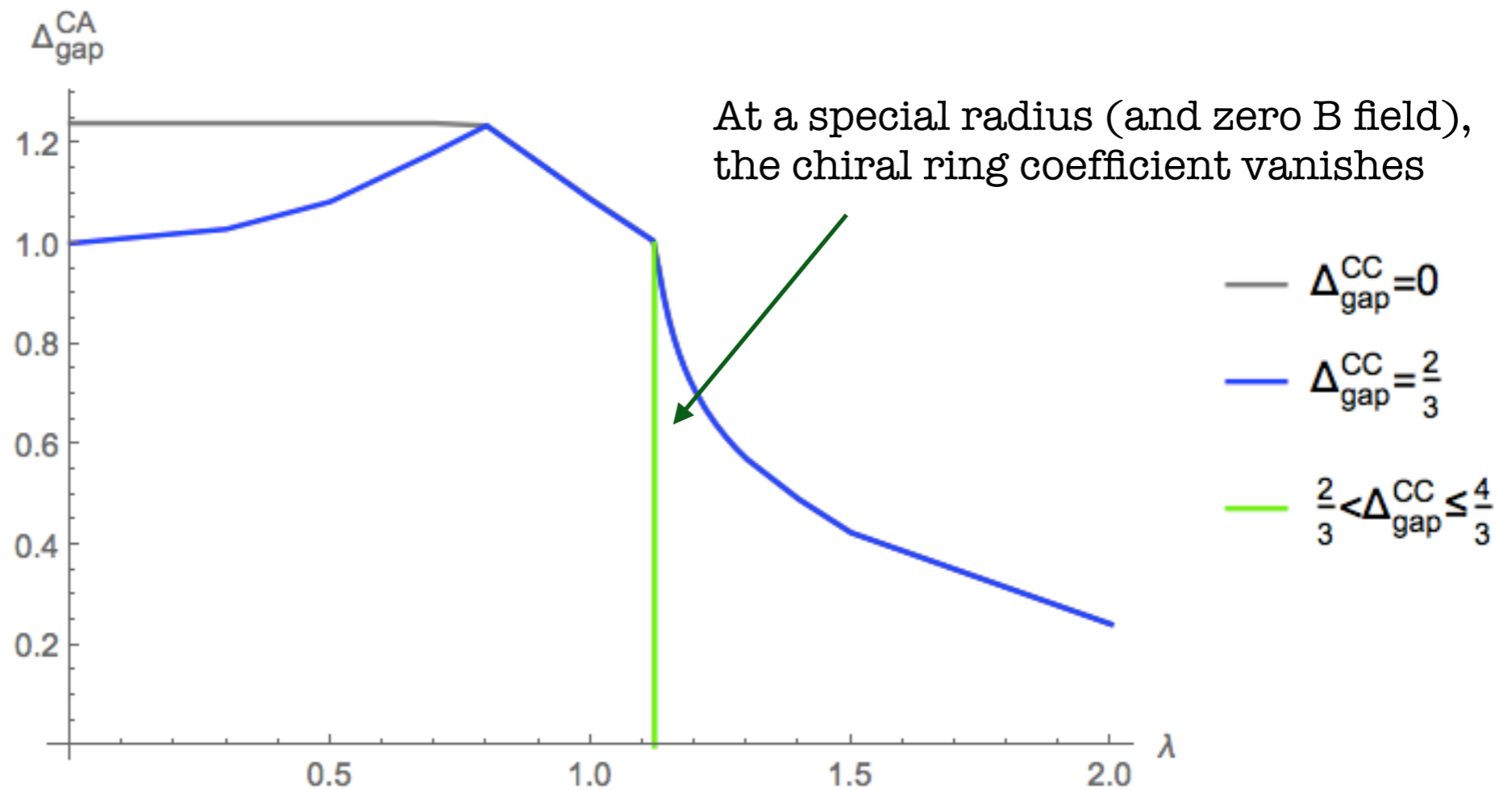
$N=(2,2)$ Bootstrap

$c=3$, chiral-anti-chiral OPE, $q=1/3$



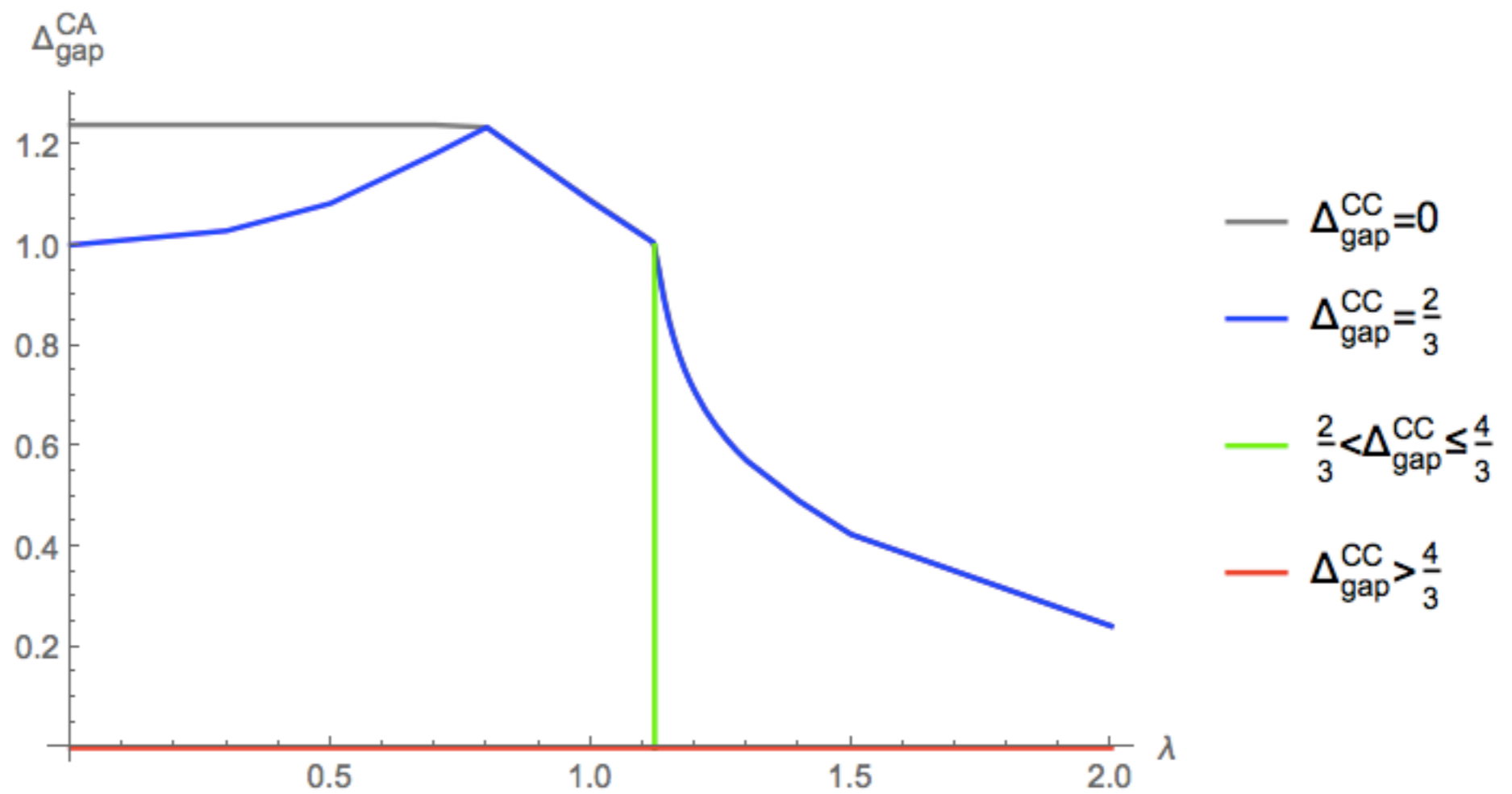
$N=(2,2)$ Bootstrap

$c=3$, chiral-anti-chiral OPE, $q=1/3$



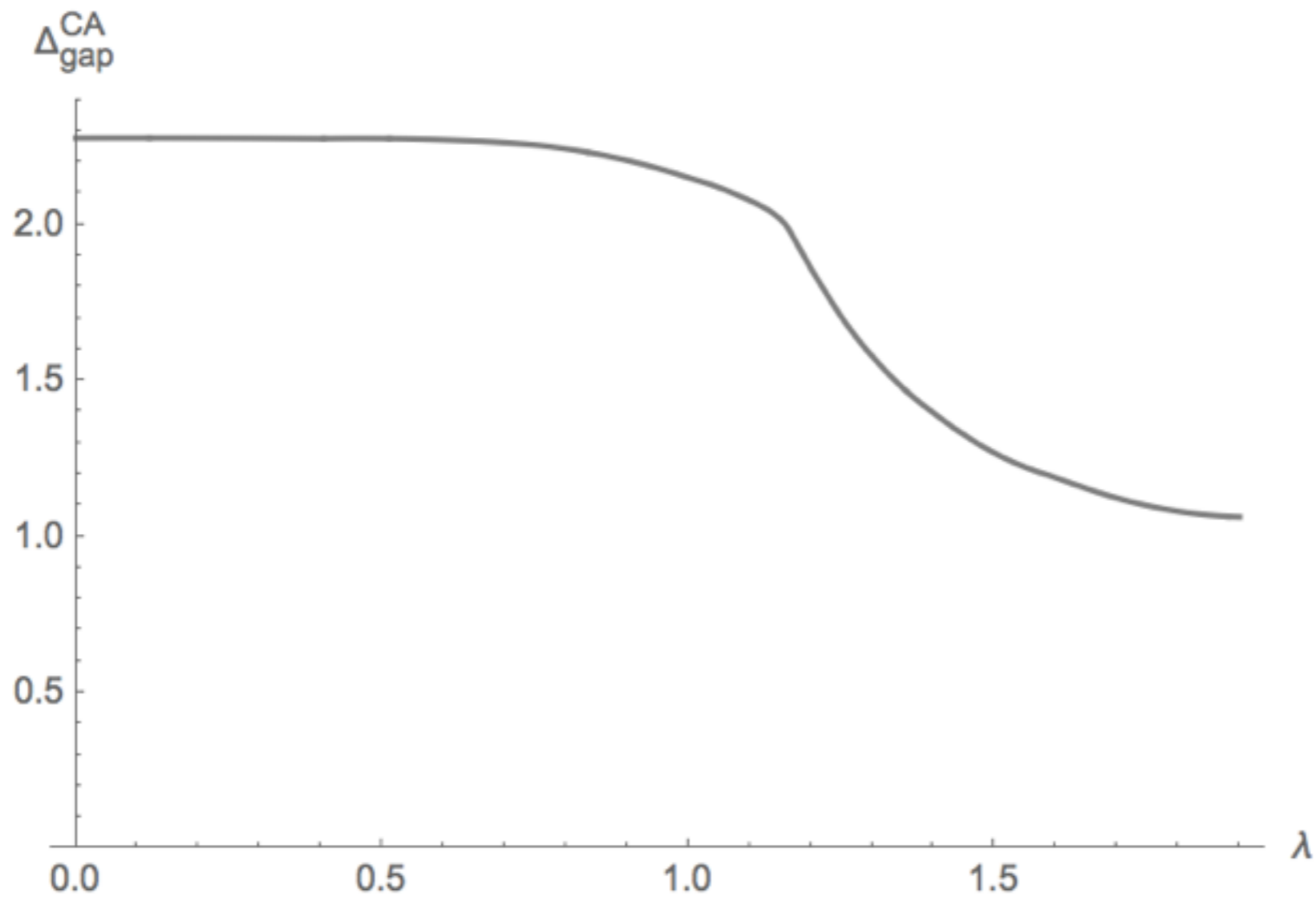
$N=(2,2)$ Bootstrap

$c=3$, chiral-anti-chiral OPE, $q=1/3$



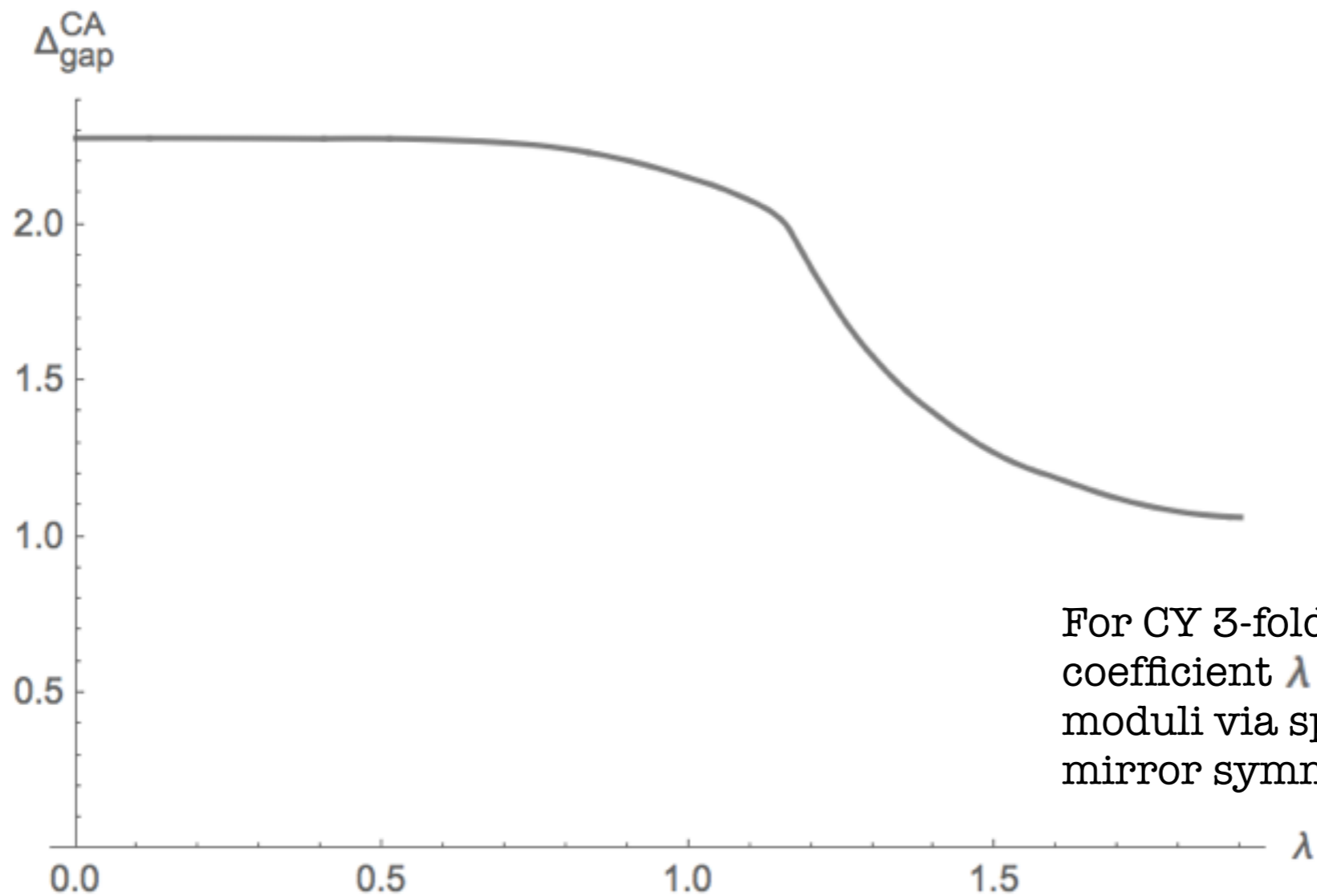
$N=(2,2)$ Bootstrap

$c=9$, chiral-anti-chiral OPE, $q=1$



$N=(2,2)$ Bootstrap

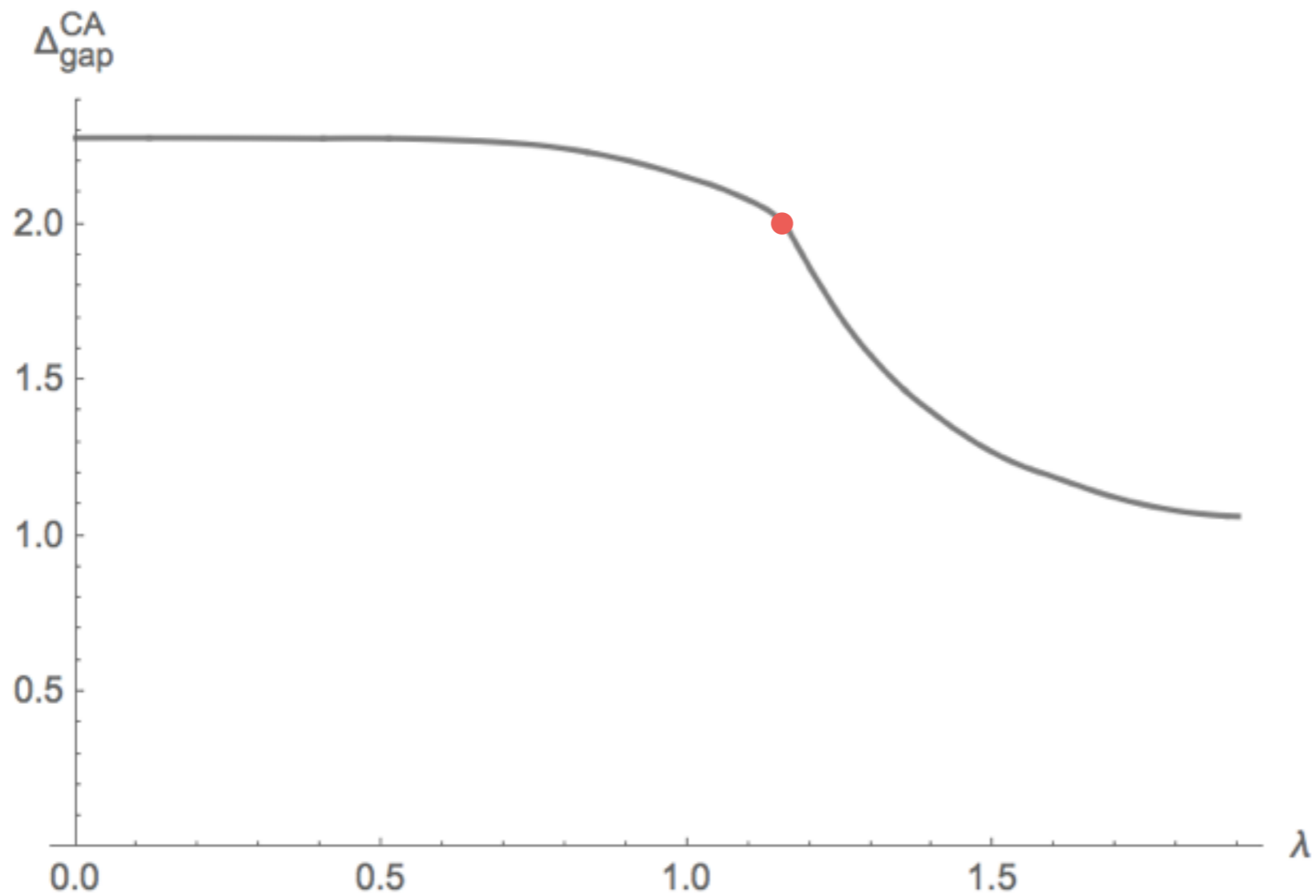
$c=9$, chiral-anti-chiral OPE, $q=1$



For CY 3-fold models, the BPS OPE coefficient λ is a known function of moduli via special geometry and mirror symmetry.

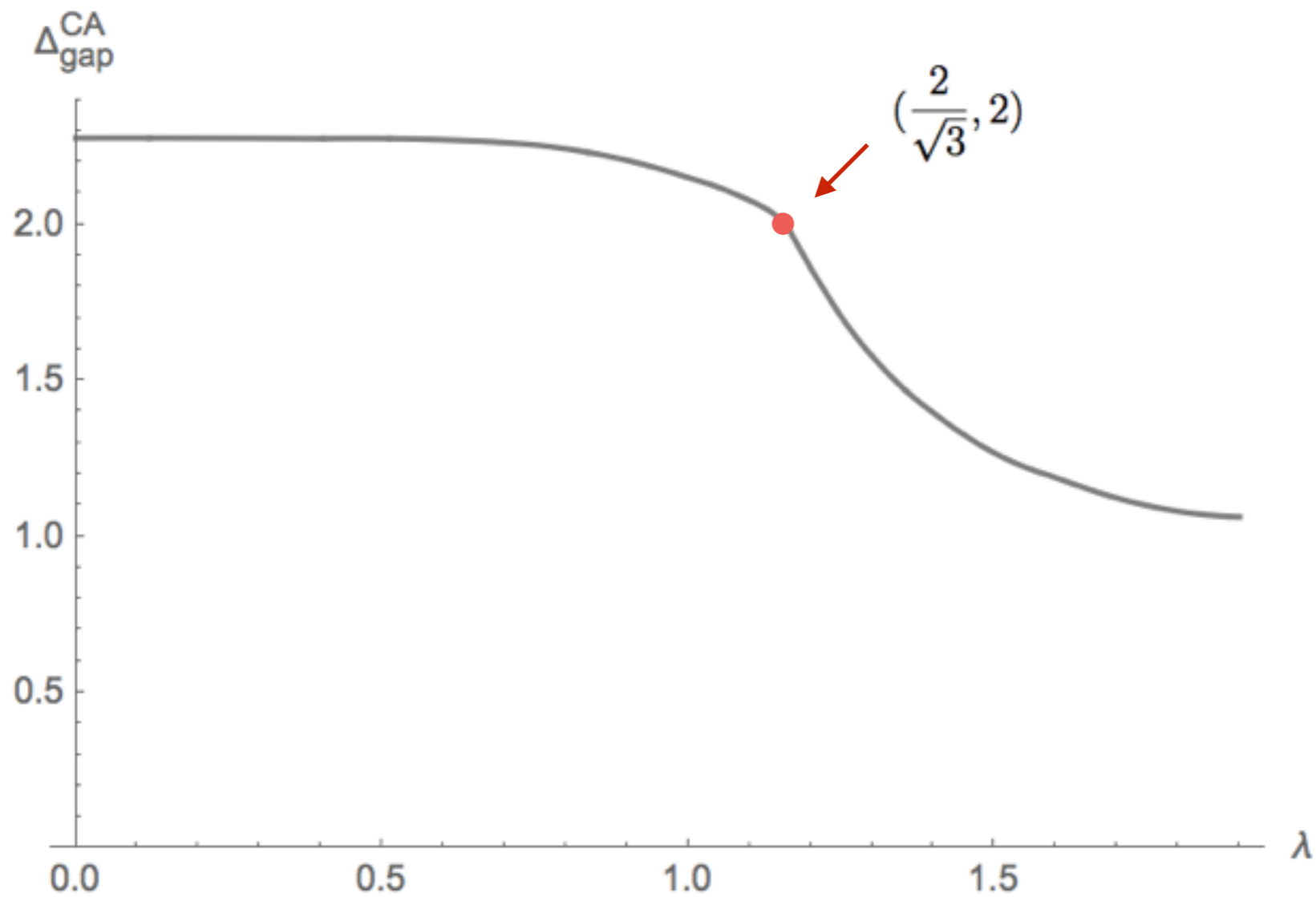
$N=(2,2)$ Bootstrap

$c=9$, chiral-anti-chiral OPE, $q=1$



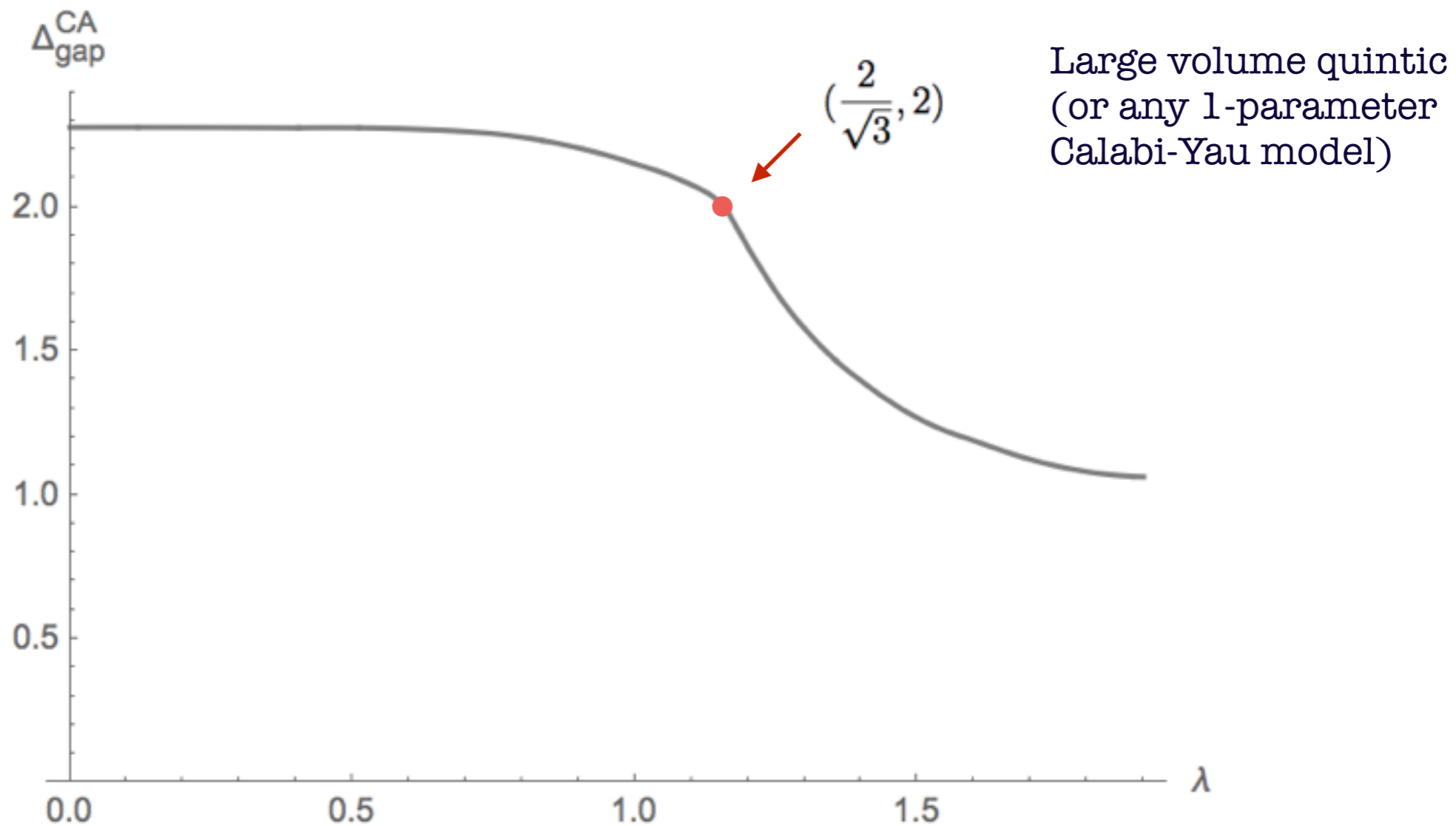
$N=(2,2)$ Bootstrap

$c=9$, chiral-anti-chiral OPE, $q=1$



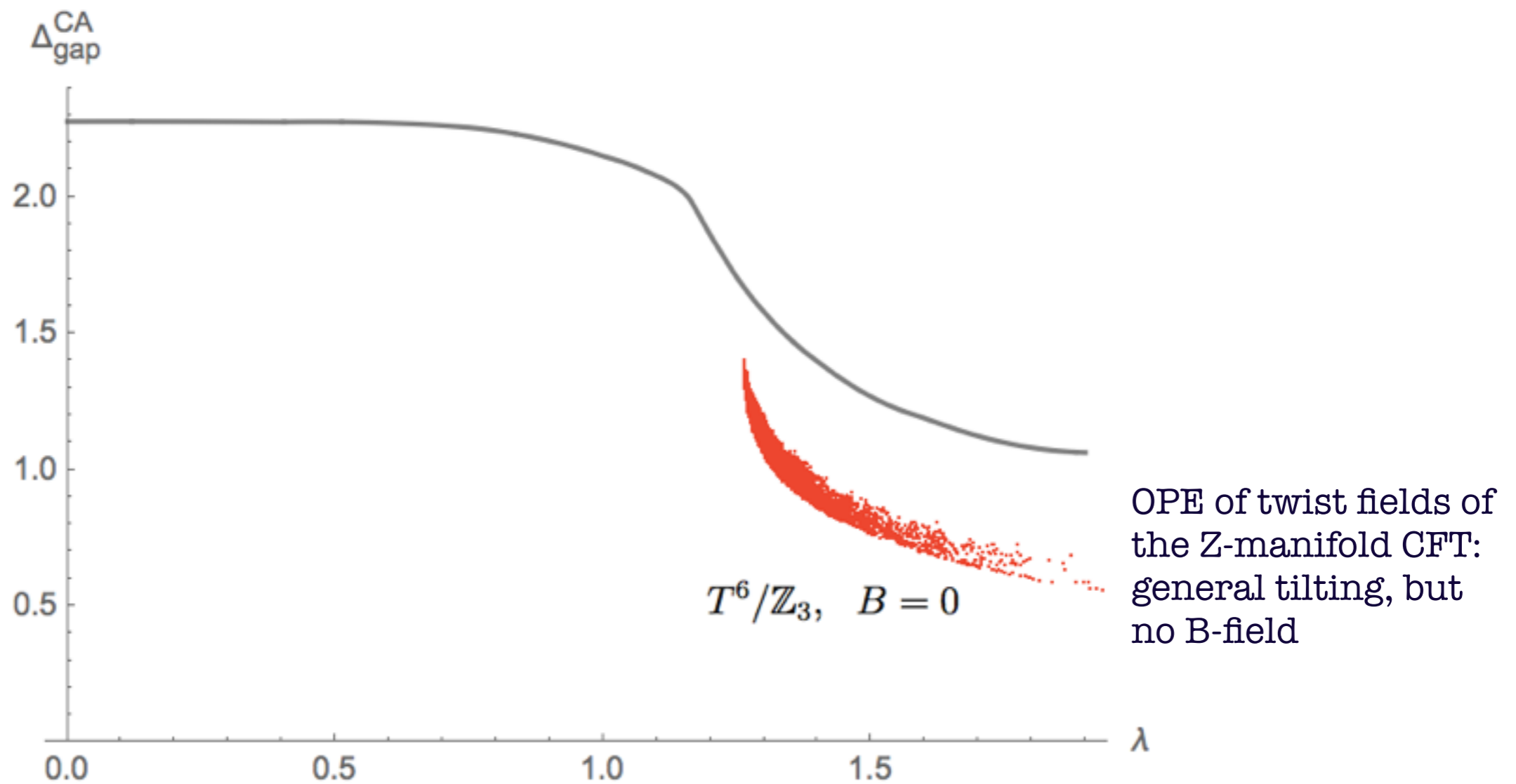
$N=(2,2)$ Bootstrap

$c=9$, chiral-anti-chiral OPE, $q=1$



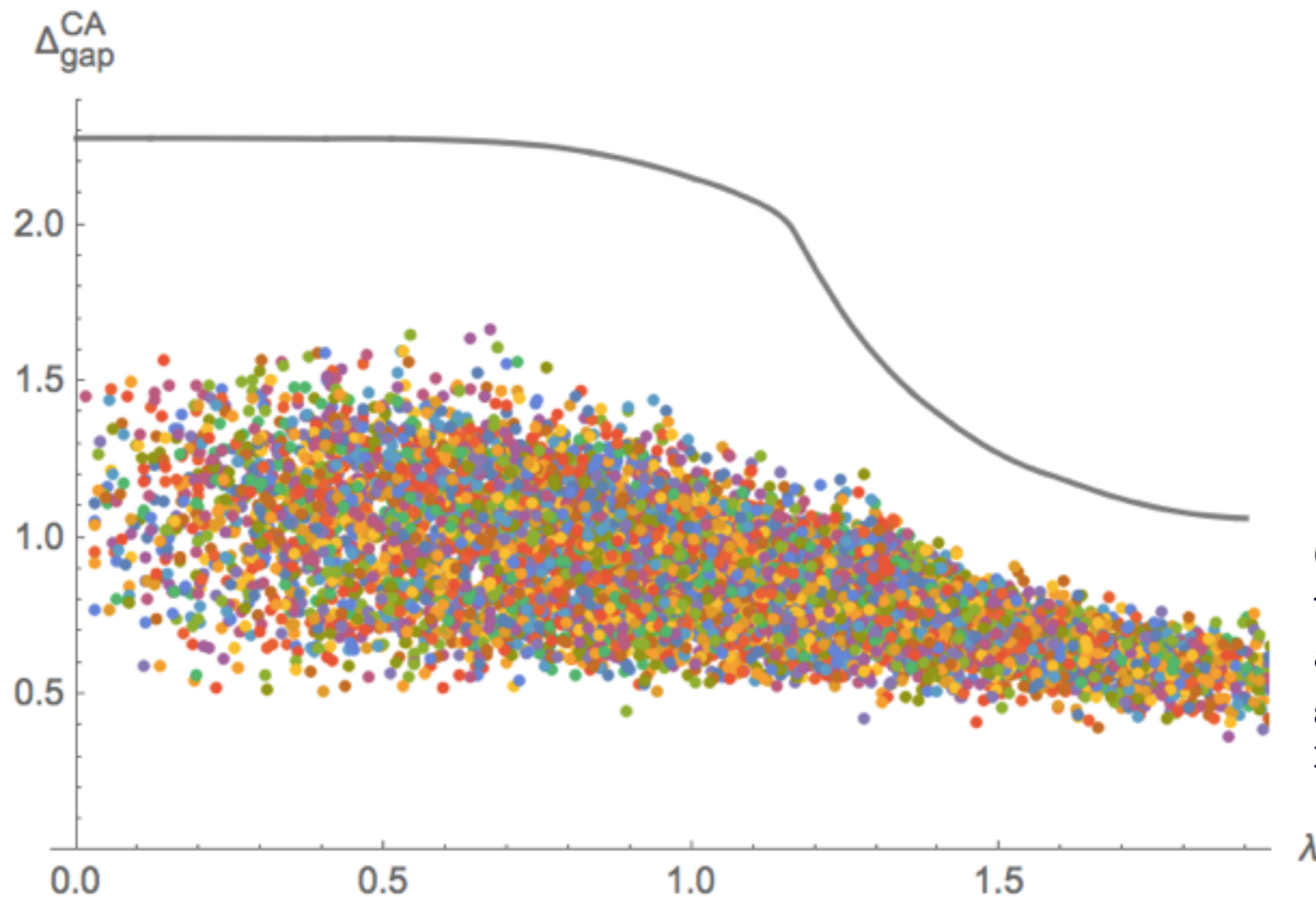
$N=(2,2)$ Bootstrap

$c=9$, chiral-anti-chiral OPE, $q=1$



$N=(2,2)$ Bootstrap

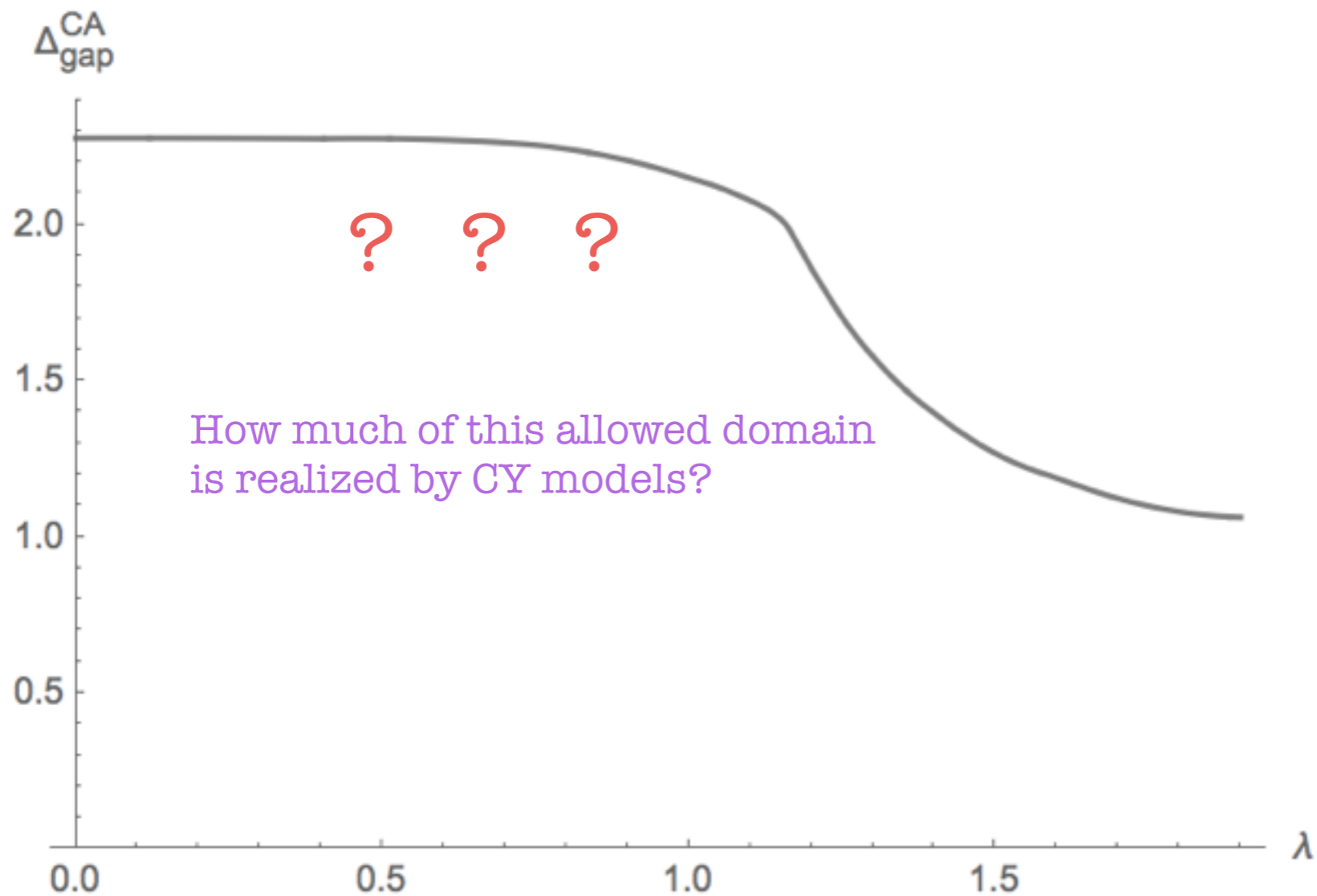
$c=9$, chiral-anti-chiral OPE, $q=1$



OPE of twist fields of the Z-manifold CFT: a sampling over general tilting and flat B-field

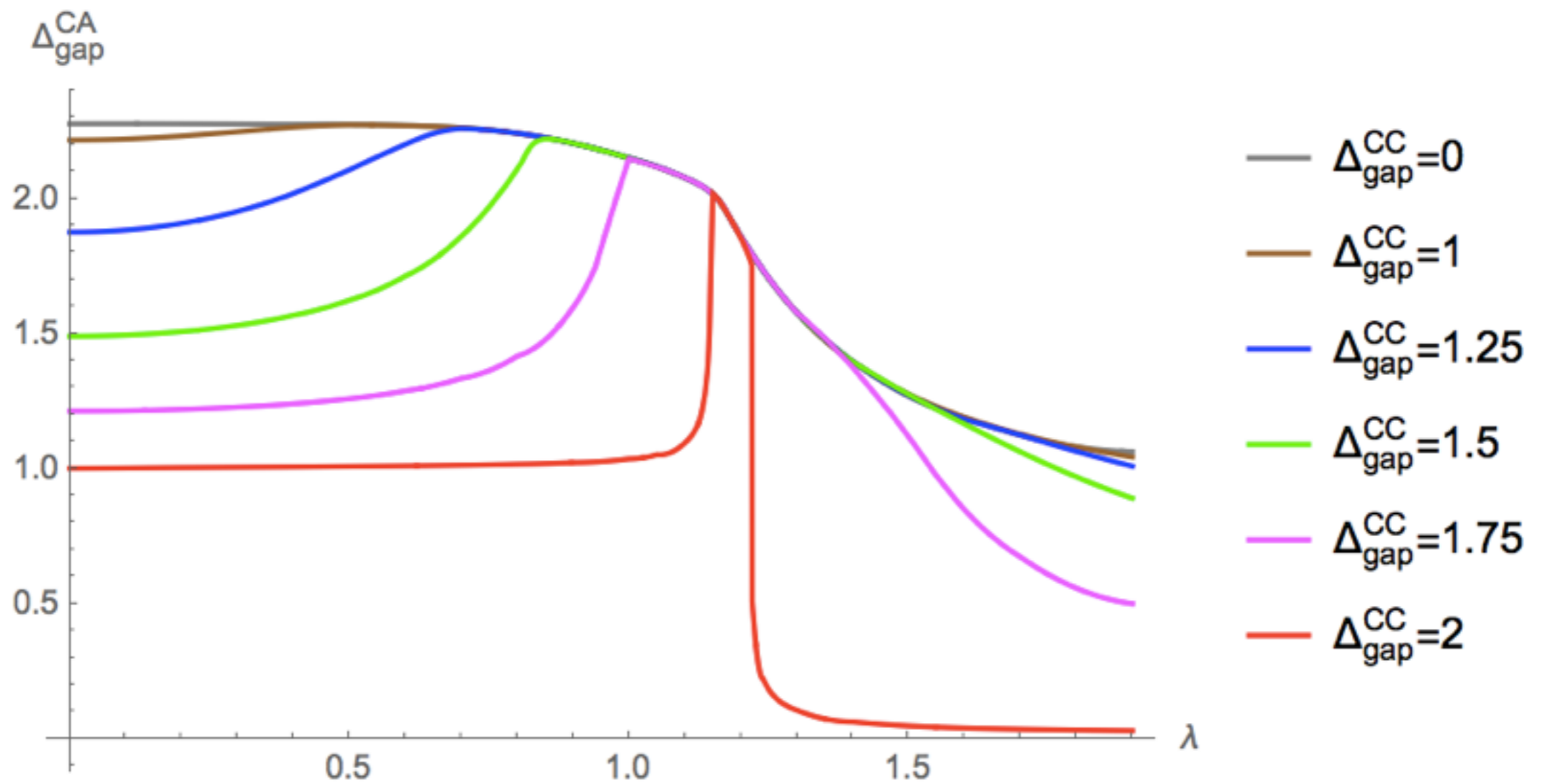
$N=(2,2)$ Bootstrap

$c=9$, chiral-anti-chiral OPE, $q=1$



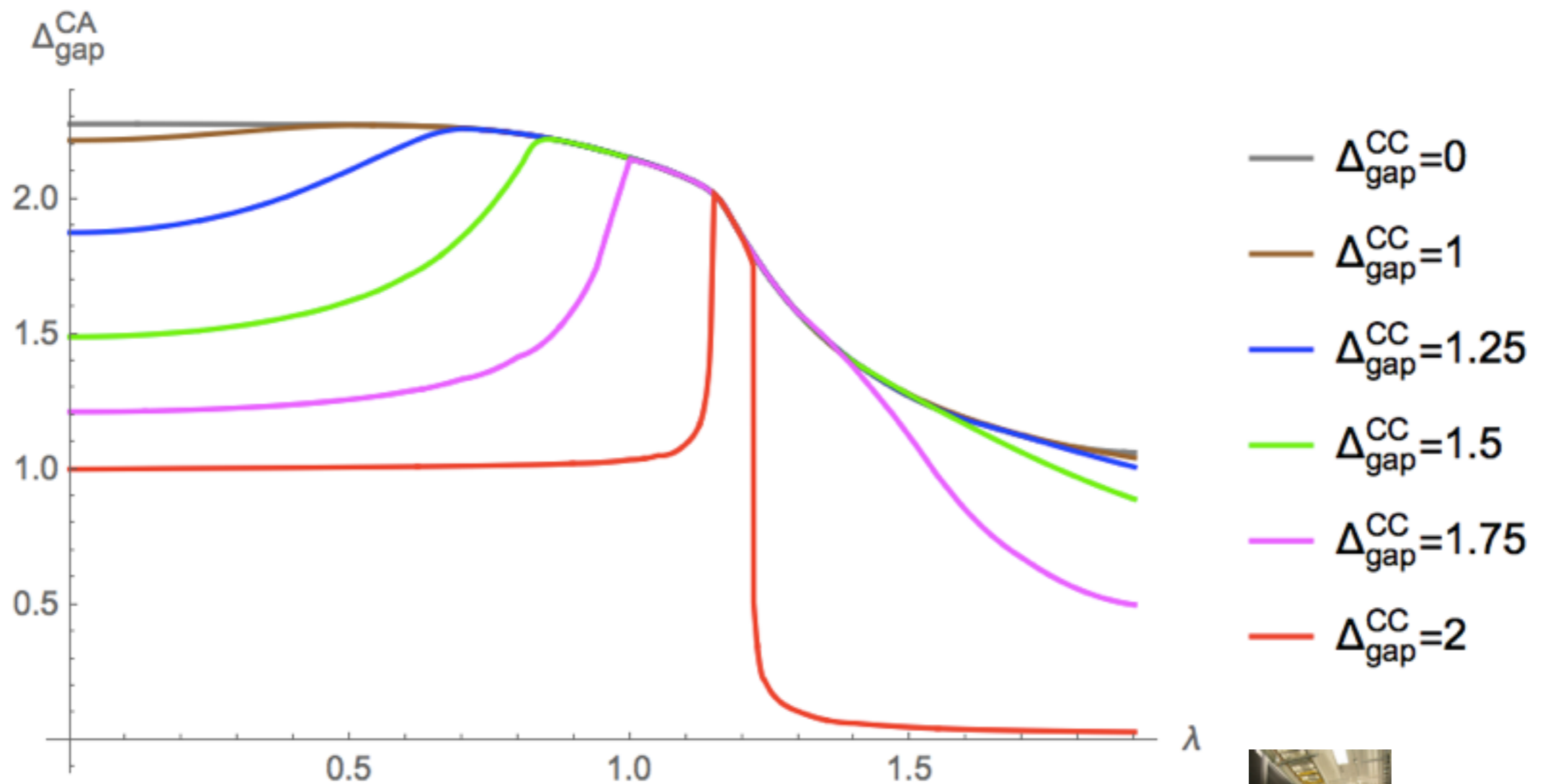
$N=(2,2)$ Bootstrap

$c=9$, chiral-anti-chiral OPE, $q=1$



$N=(2,2)$ Bootstrap

$c=9$, chiral-anti-chiral OPE, $q=1$

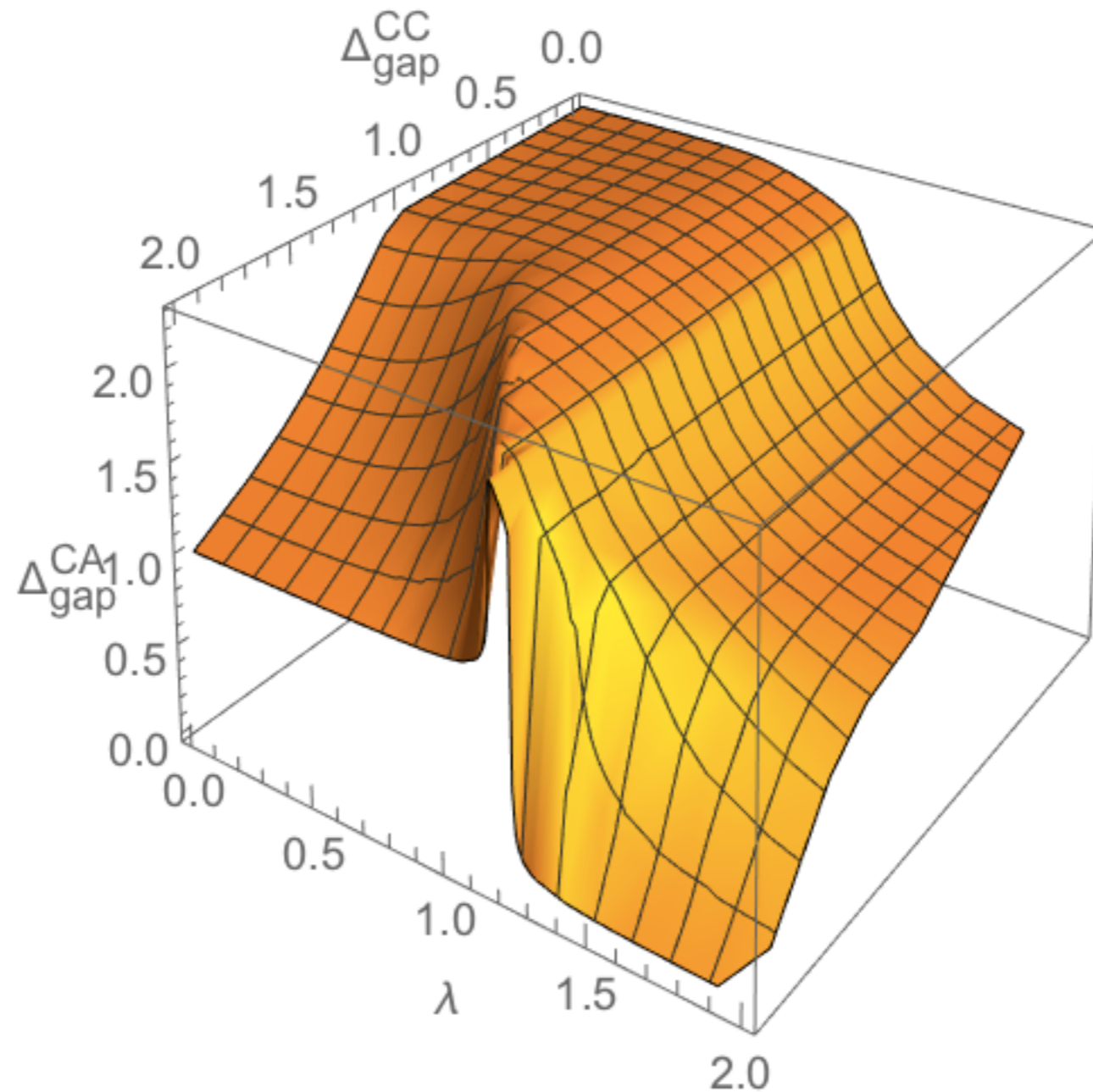


Still waiting for improved precision results ...



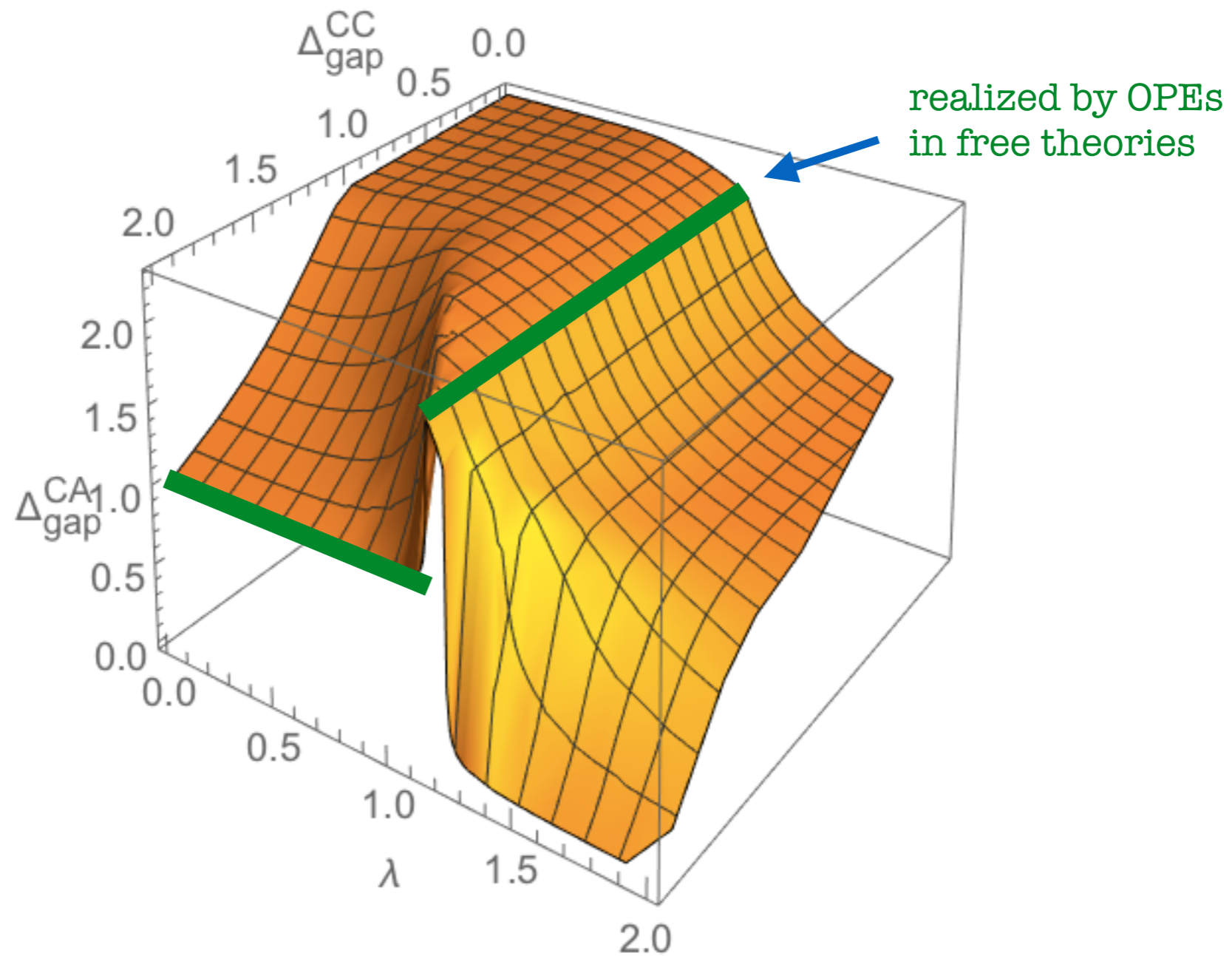
$N=(2,2)$ Bootstrap

$c=9$, gaps in chiral-anti-chiral (CA) vs
chiral-chiral (CC) channels, $q=1$



$\mathcal{N}=(2,2)$ Bootstrap

$c=9$, chiral-anti-chiral OPE, $q=1$



The Beginning

The Beginning

We are just beginning to carve out the landscape of unitary (super-)conformal field theories in two dimensions.