On higher spin scattering in flat space

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D. Ponomarev and AT [arXiv:1603.06273]

R. Roiban and AT (in progress)

To better understand AdS/CFT and HS theories
study quantum theories of infinite towers of spins:
partition functions, simplest scattering amplitudes

• define quantum theory (summation over spins, etc.)
in a way consistent with symmetries

• uncover possible simplicity due to large symmetry:
e.g. zero effective number of dynamical d.o.f.,
partition function $Z = 1$,   "trivial" S-matrix, etc.

start with simpler flat-space HS theory

Free massless HS theory in flat space

$S = \int d^4x\, \partial^n \phi^{m_1 \ldots m_s} \partial_n \phi_{m_1 \ldots m_s} + \ldots$

$\delta \phi_{m_1 \ldots m_s} = \partial_{(m_1} \epsilon_{m_2 \ldots m_s)} + \ldots, \qquad s = 0, 1, 2, \ldots$

$s > 0$: massless vector, graviton, etc. have 2 d.o.f. in 4d

Curious observation: [Beccaria, AT 2015]

- total no. of d.o.f. is zero if each spin enters just once

$$1 + \sum_{s=1}^{\infty} 2 = 1 + 2\zeta(0) = 0$$

- Total partition function:

$$Z_{\rm MHS} = \prod_{s=0}^{\infty} Z_{{\rm MHS},s}$$

$$= \Big[\frac{1}{\det \Delta_0}\Big]^{1/2} \Big[\frac{\det \Delta_0}{\det \Delta_{1\perp}}\Big]^{1/2} \Big[\frac{\det \Delta_{1\perp}}{\det \Delta_{2\perp}}\Big]^{1/2} \Big[\frac{\det \Delta_{2\perp}}{\det \Delta_{3\perp}}\Big]^{1/2} \ldots = 1$$

• cancellation of physical spin s and ghost det for spin $s+1$:
∞ gauge symmetry of the theory
• $Z = 1$ in purely bosonic theory as in topological theory
• cancellation is formal (cf. 1-1+1-1+...=0 vs 1-1+1-1+...= 1/2)
assumes regularization consistent with symmetry
• similar in AdS: 1-loop $Z = 1$ [Giombi, Klebanov 13]

• Interacting massless HS theory in flat space?
HS symmetry: ∞ tower of higher spin conserved charges:
hidden simplicity? fixing S-matrix uniquely? (cf. integrability)
UV finiteness?
"flat limit" of Vasiliev's theory in AdS?
leading Regge trajectory "truncation" of $\alpha' \to 0$ string?

Interacting massless higher spins in flat $d \geqslant 4$ space:

- free theory $\int d^4x\, \partial\phi_s \partial\phi_s, \quad \delta\phi_s = \partial\epsilon_{s-1}$ [Fronsdal]
- interacting theory? various $s > 2$ "no-go theorems"

no minimal interactions – no long-range forces

[Weinberg; Cachazo, Benincasa, ,...

review: Bekaert, Boulanger, Sundell 1007.0435]

- consistent theory may still exist if contains

(i) infinite tower of spins $s = 0, 1, 2, 3, ..., \infty$

(ii) higher derivative (non-minimal) cubic interactions

$$\partial^n \phi_{s_1}\phi_{s_2}\phi_{s_3}, \quad s_2 + s_3 - s_1 \leqslant n \leqslant s_2 + s_3 + s_1 \quad (s_1 \leqslant s_2 \leqslant s_3)$$

e.g. 2-2-2 vertex has $\partial^2, \partial^4, \partial^6$ and 2-3-3 vertex has $\partial^4, ..., \partial^6$

[light-cone: Bengtsson, Bengtsson, Brink; Metsaev;

covariant: Fotopoulos, Tsulaia; Boulanger, Leclerc, Sundell;

Manvelyan, Mkrtchyan, Ruhl; Sagnotti, Taronna; Joung,Taronna]

• Noether procedure: deform $\delta\phi_s = \partial\epsilon_{s-1} + ...$, add 4-vertex,...
fixes 3-point coupling constants [l.c. - Poincare: Metsaev]

$$g_{s_1 s_2 s_3} = g\,\frac{\ell^{s_1+s_2+s_3-1}}{(s_1+s_2+s_3-1)!}$$

• two constants (cf. string th.): g= dimensionless and ℓ= length

$$\frac{1}{g^2}\int d^4x\Big[\sum_s \partial\phi_s\partial\phi_s+\sum \ell^{n-1}\partial^n\phi_{s_1}\phi_{s_2}\phi_{s_3}+\sum \ell^{k-2}\partial^k\phi^4+...\Big]$$

– ϕ^3 terms: two covariant structures $\partial^{s_1+s_2+s_3}$ and $\partial^{s_2+s_3-s_1}$
– ϕ^4 remains to be fixed (local?)
– effectively non-local theory: no. ∂ grows with s and n of ϕ^n

• ℓ eliminated by $x=\ell x'$, $\phi_s=\ell^{-1}\phi'_2$, $[x']=[\phi'_s]=0$, g left
• Einstein theory: $(x, h=\phi'_2)$, $[x]=1$, $[h]=0$,
single $\kappa=g^2\ell^2$, $[\kappa]=2$, $S=\frac{1}{\kappa}\int d^4x(h\partial^2h+h\partial h\partial h+....)$

HS action for Fronsdal fields in AdS

- $\int d^4x \sum_s \phi_s(\nabla^2 + \mu_s)\phi_s$ + 3-point + Noether procedure
- or reconstruct from CFT data assuming vectorial AdS/CFT

e.g. cubic vertices: attach bulk-to-boundary propagators $\to$ match correlators of currents $\langle J_{s_1} J_{s_2} J_{s_3}\rangle$ in CFT [Bekaert,Erdmenger,Ponomarev,Sleight 15; Taronna, Sleight 16]

- parameters in action: $g^2 = \hbar = \frac{1}{N}$, $\quad L$ =AdS radius, and ℓ (?)

$$S_{\text{AdS}} = \frac{1}{g^2}\int d^4x\Big[\sum_s \phi_s(\nabla^2 + \mu_s)\phi_s + \sum L^{-2m}\ell^{n-2m-1}\nabla^n\phi_{s_1}\phi_{s_2}\phi_{s_3} + \sum L^{-2m}\ell^{k-2m-2}\nabla^k\phi^4 + \ldots\Big]$$

L also in AdS metric $ds^2 = du^2 + e^{-u/L}dx_m dx_m$

- ℓ is a scale, L = vacuum/background parameter, $[L/\ell] = 0$

Lower-derivative terms in AdS and flat space limit

• assume flat action has curved analog:

$\partial \to \nabla, \quad [\nabla, \nabla] \sim R \sim L^{-2}$

$\partial^n \to \sum R^m \nabla^{n-2m}$, i.e. get terms $L^{-2m}\partial^{n-2m}$

[cf. string theory: $\ell \to \sqrt{\alpha'}$ and ℓ enters also mass terms]

mass parameters $\mu_s \sim R \sim L^{-2}$

• having both ℓ and L in AdS action consistent with CFT ?

[freedom in relating bulk and boundary scales]

AdS action may admit flat-space limit: $L \to \infty$

with ℓ, coordinates and fields fixed ?

then low-derivative terms go away in flat-space action

Simpler flat space theory may shed light on AdS theory:

• UV limit (controlled by highest-derivative terms)
in AdS and in flat space may be similar

• despite ∂^n vertices and scale ℓ theory may be UV finite
[cf. string theory: under prescription of summation
implied by underlying world-sheet formulation]

• if reconstruct tree AdS action from CFT
then crucial test of duality – quantum corrections to
"on-shell" value of the effective action should vanish:
correlators of conserved currents in free boundary CFT
do not receive $1/N$ corrections

• simplest non-trivial test:
vanishing of 1-loop correction to 2-point function

“self-energy” correction: two types of one-loop Witten diagrams

(i) –O– “bubble” (2 bulk-to-bndry propagators and two 3-vertices)

(ii) O< “tadpole” (with 4vertex)

Aim: compute similar self-energy diagrams in flat HS theory

Simplest case: spin 0 member of HS tower on external lines

• tree-level exchange and 1-loop bubble graph

determined by 3-point vertex 0-s_2-s_3 with $\partial^n, \ n = s_2 + s_3$

• use explicit value of its coefficient $\sim \frac{\ell^{s_2+s_3-1}}{(s_2+s_3-1)!}$

• full g^2 self-energy: add also ghost-loop contribution

• tadpole graph contribution: need guess for 4-vertex 0-0-s-s

Plan

I. tree-level 0-0-0-0 scattering amplitude:
– exchange part (following [Bekaert, Joung, Mourad 09])
– possible contribution of 4-scalar "contact" vertex

II. bubble diagram contribution to 1-loop spin 0 self-energy
– possibility of UV divergence cancellation
against tadpole graph contribution

III. tree-level 0-0-0-s amplitude:
gauge invariance and locality constraint on 4-vertex

Free higher spin action

symmetric higher spin tensors

$$\phi_s(x,u) = \phi^{a_1\dots a_s}(x)\, u_{a_1}\dots u_{a_s}$$

Fronsdal action

$$S^{(2)}[\phi_s] = \tfrac{1}{2}\int d^dx \left[\phi_s(x,\partial_u)\,\widehat{T}\widehat{\mathcal{F}}\;\phi_s(x,u)\right]_{u=0}$$

$$\widehat{T} = 1-\tfrac{1}{4}u^2\partial_u^2, \quad \widehat{\mathcal{F}} \equiv \partial_x^2-(u\cdot\partial_x)\,\widehat{D}, \quad \widehat{D} \equiv (\partial_x\cdot\partial_u)-\tfrac{1}{2}(u\cdot\partial_x)\partial_u^2$$

off-shell field ϕ_s double-traceless

$$(\partial_u^2)^2\phi_s(x,u) = 0$$

$\widehat{T}$ is invertible: $\quad \widehat{T}^{-1} \equiv 1 - \frac{1}{2d+4u\cdot\partial_u-12}u^2\partial_u^2$

free equations

$$\widehat{\mathcal{F}}\;\phi(x,u) = 0$$

gauge transformations

$$\delta_s^{(0)}\phi_s(x,u) = (u\cdot\partial_x)\varepsilon_{s-1}(x,u)$$

with traceless parameter $\quad \partial_u^2\,\varepsilon_{s-1}(x,u) = 0$

de Donder gauge: $\quad \partial^{a_1}\phi_{a_1\ldots a_s} + \ldots = 0$

$$\widehat{D}\,\phi_s \equiv \Big[(\partial_x\cdot\partial_u) - \tfrac{1}{2}(u\cdot\partial_x)\partial_u^2\Big]\phi_s(x,u) = 0$$

$$S^{(2)}[\phi_s] = \tfrac{s!}{2}\int d^dx\,\Big[\phi_s(x,\partial_u)\,\widehat{T}\,\partial_x^2\,\phi_s(x,u)\Big]_{u=0}$$

equations of motion

$$\Box\phi_s(x,u) = 0\ , \qquad\qquad \Box = \partial_x^2$$

Cubic interaction vertices

• scattering of spin 0 particles:

need cubic vertices with $s_1 = 0, s_2, s_3$

– traceless-transverse vertices give consistent vertices in de Donder gauge [Manvelyan, Mkrchtchan, Ruhl]

• found adding cubic part $S^{(3)}$ to free action $S^{(2)}$ and requiring gauge invariance of combined action

$\delta^{(0)} S^{(3)} + \delta^{(1)} S^{(2)} = 0$

[Manvelyan et al 10; Sagnotti,Taronna 10; Joung et al 11]

traceless-transverse part of cubic vertex $\quad (\partial_{x_{ij}} \equiv \partial_{x_i} - \partial_{x_j})$

$$S^{(3)}[\phi_0, \phi_{s_2}, \phi_{s_3}] = g_{0 s_2 s_3} \int d^d x \Big[(\partial_{u_2} \cdot \partial_{x_{31}})^{s_2} (\partial_{u_3} \cdot \partial_{x_{12}})^{s_3} \times \phi_0(x_1) \phi_{s_2}(x_2, u_2) \phi_{s_3}(x_3, u_3) \Big]_{\substack{u_i = 0 \\ x_i = x}}$$

• leading-order deformation analysis leaves $g_{0s_2s_3}$ undetermined

• $\delta^{(0)}S^{(4)} + \delta^{(1)}S^{(3)} + \delta^{(2)}S^{(2)} = 0$

involves quartic vertices $S^{(4)}$ – so far not known

• to fix $g_{0s_2s_3}$ use result in l.c. gauge [Metsaev 91]

from requiring closure of Poincare algebra to g^2 order:

all cubic couplings expressed in terms of single parameter

$$g_{s_1s_2s_3} = g\frac{\ell^{s_1+s_2+s_3-1}}{(s_1+s_2+s_3-1)!}$$

g = dimensionless coupling counting power of fields

ℓ = unique dimensional parameter

• similar $g_{s_1s_2s_3}$ in action for massless HS fields in AdS_4

constructed from AdS/CFT [Skvortsov 15; Sleight,Taronna 16]

Ghost action in de Donder gauge

$$S^{(2)}[\bar{c}_{s-1}, c_{s-1}] = \int d^dx \left[\bar{c}_{s-1}(x, \partial_u)\, \widehat{D}\, \frac{\delta_s^{(0)}\phi_s}{\delta\varepsilon_{s-1}} c_{s-1}(x, u)\right]_{u=0}$$

$$= \int d^dx \left[\bar{c}_{s-1}(x, \partial_u)\partial_x^2 c_{s-1}(x, u)\right]_{u=0}$$

Deformation of gauge transformations

need vertices involving ghost fields;

deform free gauge transf due to presence of cubic vertex

$$\delta_{s_2}^{(1)}\phi_{s_3}(x_3, u_3) = g_{0s_2s_3}\tfrac{s_2}{s_3!}\Big[\,\widehat{T}_3^{-1}\,(u_3\cdot\partial_{x_{12}})^{s_3}$$

$$\times\,\Big(\partial_{u_2}\cdot(-2\partial_{x_1} - \partial_{x_2})\Big)^{s_2-1}\phi_0(x_1)\,\varepsilon_{s_2-1}(x_2, u_2)\Big]_{x_1=x_2=x_3}$$

cubic vertex with ghosts:

$$S^{(3)}[\phi_0, \bar{c}_{s_3-1}, c_{s_2-1}] = -\, g_{0s_2s_3}\, s_2 \int d^dx \Big[(\partial_{u_3} \cdot \partial_{x_{12}})^{s_3-1}$$

$$\times\, (\partial_{u_2} \cdot \partial_{x_{31}})^{s_2-1} (\partial_{x_{12}} \cdot \partial_{x_3}) \bar{c}_{s_3-1}(x_3, u_3) \phi_0(x_1) c_{s_2-1}(x_2, u_2) \Big]_{\substack{u_i=0 \\ x_i=x}}$$

Free HS propagator in de Donder gauge

[Fronsdal 78; Francia 07]

$$\mathcal{D}^d_s(u, u'; p) = -\tfrac{i}{p^2} \left[\mathcal{P}^d_s(u, u') + \tfrac{s(s-1)}{(d+2s-4)(d+2s-6)} \, \tfrac{u^2 u'^2}{s^2(s-1)^2} \, \mathcal{P}^d_{s-2}(u, u') \right]$$

$\mathcal{P}^d_s$ = generating function for rank-s traceless projector

$$\mathcal{P}^d_s(u, u') = \tfrac{1}{(s!)^2} \, \tfrac{s!}{(\frac{d}{2}-1)_s} \left(\tfrac{1}{2} \sqrt{u^2 u'^2} \right)^s C_s^{\frac{d}{2}-1} \left(\tfrac{u \cdot u'}{\sqrt{u^2 u'^2}} \right)$$

$$C^\alpha_s(z) \equiv \sum_{k=0}^{[s/2]} \tfrac{(-1)^k (\alpha)_{s-k}}{k!(s-2k)!} (2z)^{s-2k}$$

C^α_s – Gegenbauer polynomial; $\quad (\alpha)_k = \Gamma(\alpha + k)/\Gamma(\alpha)$

Fronsdal field propagator: $\mathcal{D}_s^d \sim$ traceless projector in $d-2$

$$\mathcal{D}_s^d(u,u';p) = -\frac{i}{p^2}\,\mathcal{P}_s^{d-2}(u,u')$$

in $d=4$: $$\mathcal{P}_s^2(u,u') = \tfrac{2}{(s!)^2}\left(\tfrac{1}{2}\sqrt{u^2u'^2}\right)^s T_s\left(\tfrac{u\cdot u'}{\sqrt{u^2u'^2}}\right)$$

$$\begin{aligned} T_s(z) &\equiv \tfrac{s}{2}\sum_{k=0}^{[s/2]}\tfrac{(-1)^k(s-k-1)!}{k!(s-2k)!}(2z)^{s-2k} \\ &= \tfrac{1}{2}\left[\left(z+\sqrt{z^2-1}\right)^s + \left(z-\sqrt{z^2-1}\right)^s\right] \end{aligned}$$

T_s is the Chebyshev polynomial of first kind

Ghost propagator

$$\mathcal{G}_{s-1}^d(u,u';p) = -\frac{i}{p^2}\cdot\mathcal{P}_{s-1}^d(u,u')$$

Cubic vertex for physical fields $(p_{ij} \equiv p_i - p_j)$

$$\mathcal{V}(\partial_{u_2}, \partial_{u_3}; p_1, p_2, p_3) = 2ig_{0s_2s_3}(-ip_{31} \cdot \partial_{u_2})^{s_2}(-ip_{12} \cdot \partial_{u_3})^{s_3}$$

ghost vertex:

$$\mathcal{W}(\partial_{u_2}, \partial_{u_3}; p_1, p_2, p_3) = ig_{0s_2s_3}s_2(-ip_{31}\cdot\partial_{u_2})^{s_2-1}(-ip_{12}\cdot\partial_{u_3})^{s_3-1}(-p_{12}\cdot p_3)$$

Tree-level 4-scalar scattering amplitude

• exchange of tower of higher spin fields:

for complex external scalars and arbitrary coupling constants computed in [Bekaert, Joung, Mourad 09]

• here: real scalar is $s = 0$ member of HS tower

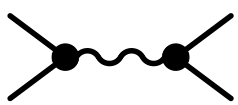
(i) use of explicit values of coupling constants of HS theory

(ii) add contribution of contact 4-vertex

• in real scalar case all interactions with odd spins vanish and all exchanges add up with same sign (no UV improvement)

Exchange contribution

s-channel exchange of spin j field



$$\equiv \quad \mathcal{A}^j_{exch}(\mathrm{s}, \mathrm{t}, \mathrm{u})$$

Mandelstam variables $\quad (p_i^2 = p_i'^2 = 0, \quad \mathrm{s} + \mathrm{t} + \mathrm{u} = 0)$

$$\mathrm{s} \equiv -(p_1 + p_2)^2, \quad \mathrm{t} \equiv -(p_1 + p_1')^2, \quad \mathrm{u} \equiv -(p_1 + p_2')^2$$

$$\mathcal{A}^j_{exch}(\mathrm{s},\mathrm{t},\mathrm{u}) = -\frac{i g^2_{00j}}{\mathrm{s}} \frac{2^{-j} j!}{(\frac{d}{2}-2)_j} \big(t+u\big)^j \, C_j^{\frac{d}{2}-2}\big(\tfrac{\mathrm{t}-\mathrm{u}}{\mathrm{t}+\mathrm{u}}\big)$$

$$d=4: \qquad \mathcal{A}^j_{exch}(\mathrm{s},\mathrm{t},\mathrm{u}) = -\frac{i g^2_{00j}}{\mathrm{s}} 2^{-j+1} \, (\mathrm{t}+\mathrm{u})^j \;\; T_j\big(\tfrac{\mathrm{t}-\mathrm{u}}{\mathrm{t}+\mathrm{u}}\big)$$

$$\mathcal{A}_{exch}(\mathrm{s},\mathrm{t},\mathrm{u}) = \sum_{j=0,2,4,\ldots}^{\infty} \mathcal{A}^j_{exch}(\mathrm{s},\mathrm{t})$$

$$F(z) \equiv \sum_{k=0}^{\infty} g^2_{00\,2k} \, (\tfrac{z^2}{4})^{2k} = g^2 \sum_{k=0}^{\infty} \tfrac{1}{[(2k-1)!]^2} \, (\tfrac{\ell^2 z^2}{4})^{2k}$$
$$= \tfrac{1}{8} g^2 \, (\ell z)^2 \big[I_0(\ell z) - J_0(\ell z) \big]$$

$$J_\alpha(z) = \left(\tfrac{z}{2}\right)^\alpha \textstyle\sum_{k=0}^\infty \frac{(-1)^k}{\Gamma(k+1)\Gamma(\alpha+1+k)} \, \left(\tfrac{z}{2}\right)^{2k}$$
$$I_\alpha(z) = \left(\tfrac{z}{2}\right)^\alpha \textstyle\sum_{k=0}^\infty \frac{1}{\Gamma(k+1)\Gamma(\alpha+1+k)} \, \left(\tfrac{z}{2}\right)^{2k}$$

• sum over spins is convergent

• non-trivial dependence on Mandelstam variables and ℓ

$$\mathcal{A}_{exch}(\mathrm{s},\mathrm{t},\mathrm{u}) = -\frac{i}{\mathrm{s}}\Big[F\big(\sqrt{\mathrm{s}+\mathrm{t}}+\sqrt{\mathrm{t}}\,\big) + F\big(\sqrt{\mathrm{s}+\mathrm{t}}-\sqrt{\mathrm{t}}\,\big)\Big]$$

Full exchange amplitude

$$\widehat{\mathcal{A}}_{exch}(\mathrm{s},\mathrm{t},\mathrm{u}) = \mathcal{A}_{exch}(\mathrm{s},\mathrm{t},\mathrm{u}) + \mathcal{A}_{exch}(\mathrm{t},\mathrm{s},\mathrm{u}) + \mathcal{A}_{exch}(\mathrm{u},\mathrm{t},\mathrm{s})$$

• Regge limit: $\mathrm{t}\to\infty$, s=fixed

$$\widehat{\mathcal{A}}_{exch}(\mathrm{s},\mathrm{t},\mathrm{u}) \sim -\frac{ig^2}{\mathrm{s}}\ell^2\mathrm{t}\, I_0(\ell\sqrt{8\mathrm{t}}) \sim -\frac{ig^2}{\mathrm{s}}\frac{(\ell^2\mathrm{t})^{3/4}}{2^{5/4}\pi^{1/2}}\, e^{\ell\sqrt{8\mathrm{t}}}$$

• despite $\sqrt{\mathrm{s}}$ and $\sqrt{\mathrm{t}}$ in F exchange amplitude is analytic in s,t ($e^{a\sqrt{\mathrm{t}}}$ is an artefact of large t limit)

Fixed angle limit

$$\mathrm{s},\mathrm{t},\mathrm{u}\to\infty,\quad \frac{\mathrm{t}}{\mathrm{s}}=-\sin^2\frac{\theta}{2},\quad \frac{\mathrm{u}}{\mathrm{s}}=-\cos^2\frac{\theta}{2},\quad \theta=\text{fixed}$$

$$\widehat{\mathcal{A}}_{exch}(\mathrm{s},\mathrm{t},\mathrm{u})\sim ig^2|\mathrm{s}|^{3/4}\,e^{\ell\sqrt{|\mathrm{s}|}\;f(\theta)}\to\infty\,,\qquad\qquad f(\theta)>0$$

exponential growth: an indication of UV divergences in loops

cf. string theory: Shapiro-Virasoro amplitude is UV-soft

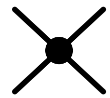
$$A_4=g^2\frac{\Gamma(-1-\frac{1}{4}\alpha'\mathrm{s})\Gamma(-1-\frac{1}{4}\alpha'\mathrm{s})\Gamma(-1-\frac{1}{4}\alpha'\mathrm{s})}{\Gamma(2+\frac{1}{4}\alpha'\mathrm{s})\Gamma(2+\frac{1}{4}\alpha'\mathrm{s})\Gamma(2+\frac{1}{4}\alpha'\mathrm{s})}$$

$$A_4\to\quad g^2|\mathrm{s}|^{-6}(\sin\theta)^{-6}e^{-\alpha'|\mathrm{s}|\,h(\theta)}\to 0$$

$$h(\theta)=-\tfrac{1}{4}\Big(\sin^2\tfrac{\theta}{2}\log\sin^2\tfrac{\theta}{2}-\cos^2\tfrac{\theta}{2}\log\cos^2\tfrac{\theta}{2}\Big)>0$$

4-vertex contribution

full 4-point amplitude: add contribution of 0-0-0-0 vertex



- expected to be effectively "non-local": infinite series in ∂^n
- may "soften" large p behaviour of exchange contribution
- guess 4-scalar vertex in flat-space HS action from

its form in AdS action reconstructed using AdS/CFT [Bekaert, Erdmenger, Ponomarev, Sleight 2015]: $\nabla \to \partial$

$$S^{(4)}[\phi_0] = g^2 \int d^4x \Big[\sum_{j=0}^{\infty} f_{2j}\big(\Delta_{x_{34}}\big) \, \big(\partial_{x_{12}} \cdot \partial_{x_{34}}\big)^{2j} \times \phi_0(x_1)\phi_0(x_2)\phi_0(x_3)\phi_0(x_4)\Big]_{x_i=x}$$

$$\Delta_{x_{34}} \equiv (\partial_{x_3} + \partial_{x_4})^2, \qquad \partial_{x_{12}} \equiv \partial_{x_1} - \partial_{x_2}$$

$f_{2j}(z)$ = infinite series in z, regular at $z = 0$: no poles

choose large z asymptotics same as in AdS 4-scalar vertex:

$$z \to \infty : \qquad f_{2j}(z) \to c_{2j} \frac{\ell^{4j-2}}{z} \,, \qquad c_{2j} = \frac{1}{[(2j-1)!]^2}$$

then asymptotic contribution to 4-scalar amplitude is

$$\sum_{j=0}^{\infty} f_{2j}(\mathrm{s})\, (\mathrm{t}-\mathrm{u})^{2j} = \frac{2\mathrm{t}+\mathrm{s}}{2\mathrm{s}} \left[I_0\!\left(2\ell\sqrt{2\mathrm{t}+\mathrm{s}}\right) - J_0\!\left(2\ell\sqrt{2\mathrm{t}+\mathrm{s}}\right) \right]$$

UV behaviour as in exchange amplitude: cancellation possible?

need to find complete 4-vertex directly in flat-space theory ...

Remarks:

• soft UV behaviour is expected in higher-spin theory in AdS:
4-scalar amplitude = free CFT 4-point correlator ($\phi_0 \to \Phi^*\Phi$)

• Witten diagrams in AdS in Mellin representation
look similar to scatt amplitudes in flat space [Penedones 2010]
[AdS exch – Mellin ampl with poles related to dim's of ops;
contact interactions with $\partial^{2n} \to$ Mellin amps
$\sim n$-order polynomials in $\mathrm{s}, \mathrm{t}, \mathrm{u}$ =Mellin variables]

• total AdS scatt amp similar to Mellin transform (in u, v)
of 4-point correlator of spin 0 operator in free $O(N)$ CFT:
distribution $\delta(\mathrm{s} - \frac{1}{2})\delta(\mathrm{t} - \frac{3}{2}) + ...$ [Taronna; Bekaert et al]

• suggests (?) exch + 4-vertex tree-level 4-scalar amplitude
in flat space may also have trivial large p asymptotics

• similar behaviour for external scalar scattering
in conformal higher-spin theory [Joung,Nakach, AT 15]

Comment on BCFW constructibility condition

• condition requires that amplitudes vanish under infinite
complex shifts of momenta [Benincasa, Cachazo 07]
assumption of analyticity (vanishing at $\infty \rightarrow$
ampl can be reconstructed from poles and residues)

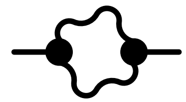
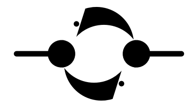
• leads to recurrence relations which allow to express
any tree-level amplitude in terms of on-shell 3-point ones

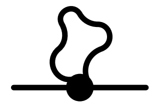
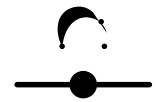
• applied to 4-scalar amplitude would allow to determine
quartic scalar self-coupling in terms of cubic vertices
but BCFW construction can be applied only if cubic vertices
satisfy non-trivial consistency condition - "four-particle" test;
cubic HS vertices fail to satisfy this test
[Fotopoulos, Tsulaia 08; Dempster, Tsulaia 12; Bengtsson 16]

• not clear if condition of BCFW constructibility should apply
to an effectively non-local HS theory containing infinite number of fields
with higher derivatives of any order in vertices
(e.g. assumption of analyticity may fail if sums over spins do not
converge fast and get amplitudes $\sim$ distributions)

One-loop scalar self-energy correction

sum of bubble and tadpole diagrams: physical and ghost

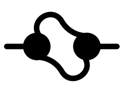
Bubbles :  $+$ 

Tadpoles :  $+$ 

usually give distinct contributions:

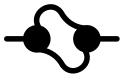
bubble – branch cuts in external p; tadpole – rational functions

Individual bubble diagrams

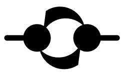


$$= 2g_{0s_2s_3}^2 \int \frac{d^d p_2}{p_2^2 p_3^2} \frac{s_2!\, C_{s_2}^{\frac{d}{2}-2}(1)}{2^{s_2}\left(\frac{d}{2}-2\right)_{s_2}} \frac{s_3!\, C_{s_2}^{\frac{d}{2}-2}(1)}{2^{s_3}\left(\frac{d}{2}-2\right)_{s_3}} (p_{31}^2)^{s_2} (p_{12}^2)^{s_3}$$

$$g_{0s_2s_3} = g\frac{\ell^{s_2+s_3-1}}{(s_2+s_3-1)!}\,, \qquad C_s^{\alpha}(1) = \frac{\Gamma(2\alpha+s)}{\Gamma(2\alpha)\Gamma(s+1)}$$



$$= 2g^2_{0s_2s_3}\,\frac{(d-4)_{s_2}}{2^{s_2}(\frac{d}{2}-2)_{s_2}}\,\frac{(d-4)_{s_3}}{2^{s_3}(\frac{d}{2}-2)_{s_3}}\int\frac{d^dp_2}{p_2^2\,p_3^2}\,(p_{31}^2)^{s_2}\,(p_{12}^2)^{s_3}$$



$$= g^2_{0s_2s_3}\,\frac{s_2\,(d-2)_{s_2-1}}{2^{s_2-1}(\frac{d}{2}-1)_{s_2-1}}\,\frac{s_3\,(d-2)_{s_3-1}}{2^{s_3-1}(\frac{d}{2}-1)_{s_3-1}}$$
$$\times\int\frac{d^dp_2}{p_2^2\,p_3^2}\,(p_{31}^2)^{s_2-1}\,(p_{12}^2)^{s_3-1}(p_{31}\cdot p_2)\,(p_3\cdot p_{12})$$

Summing over spins

physical+ghost loop in $d=4$ $\quad(p_1=k,\ p_2=-p,\ p_3=p-k)$

$$\Sigma(k^2)=g^2\int\frac{d^4p}{p^2(p-k)^2}\sum_{\substack{s_2,s_3=0\\ s_2+s_3=\text{even}}}^{\infty}\frac{\ell^{2(s_2+s_3-1)}}{2^{s_2-1}2^{s_3-1}\big[(s_2+s_3-1)!\big]^2}$$
$$\times\Big[2(2k-p)^{2s_2}(k+p)^{2s_3}$$
$$+\,s_2^2s_3^2(2k-p)^{2(s_2-1)}(k+p)^{2(s_3-1)}(2k\cdot p-p^2)(p^2-k^2)\Big]$$

$$\Sigma(k^2) = g^2 \int \frac{d^4p}{p^2(p-k)^2}\ \Phi(p,k)$$

$$\Phi = \left(\tfrac{4x^2}{x-y} + \tfrac{2(2kp-p^2)(p^2-k^2)}{(2k-p)^2(k+p)^2} \tfrac{4x^2y^2(x^2+4xy+y^2)}{(x-y)^5} \right) \left[I_0(2\sqrt{x}) - J_0(2\sqrt{x}) \right]$$

$$+ \tfrac{2(2kp-p^2)(p^2-k^2)}{(2k-p)^2(k+p)^2} \left(\tfrac{x^3y(x+y)}{(x-y)^3} \left[I_2(2\sqrt{x}) - J_2(2\sqrt{x}) \right] \right.$$

$$\left. + \tfrac{2x^2y(x^2-8xy-5y^2)\sqrt{x}}{(x-y)^4} \left[I_1(2\sqrt{x}) + J_1(2\sqrt{x}) \right] \right) + (x \leftrightarrow y)$$

$$x \equiv \tfrac{1}{2}\ell^2(2k-p)^2, \quad y \equiv \tfrac{1}{2}\ell^2(k+p)^2$$

UV divergences: two possible approaches

(i) use explicit UV cutoff Λ: first sum over spins, then $\Lambda \to \infty$

(ii) use dim reg: first drop all power div in each loop using dimensional regularization, then sum over spins

Explicit cutoff

$$\Sigma = g^2 \int^{\Lambda} \frac{d^4p}{p^4}\Big(- \tfrac{1}{15}\big(\tfrac{\ell p}{\sqrt{2}}\big)^7 \big[I_5(\sqrt{2}\ell p) + J_5(\sqrt{2}\ell p)\big]$$

$$- \big(\tfrac{\ell p}{\sqrt{2}}\big)^6 \big[I_4(\sqrt{2}\ell p) - J_4(\sqrt{2}\ell p)\big] + ...\Big) \to g^2(\ell\Lambda)^{11/2}\, e^{\sqrt{2}\ell\Lambda}$$

asymptotics: $I_\alpha(z) \to \frac{1}{\sqrt{2\pi z}}\, e^z\,, \quad J_\alpha(z) \to \sqrt{\frac{2}{\pi z}} \cos\big(z - \frac{\alpha\pi}{2} - \frac{\pi}{4}\big)$

Keeping subleading k^2 term in large p expansion

$$\Sigma(k^2) = g^2\ell^{-2} \int^{\Lambda} \frac{d^4p}{p^4}\Big(\big[- \tfrac{1}{15}\big(\tfrac{\ell p}{\sqrt{2}}\big)^7 + \tfrac{1}{15}\big(\tfrac{\ell p}{\sqrt{2}}\big)^8 \tfrac{kp}{p^2} - \tfrac{8}{105}\big(\tfrac{\ell p}{\sqrt{2}}\big)^9 \big(\tfrac{kp}{p^2}\big)^2$$

$$- \tfrac{1}{6}\big(\tfrac{\ell p}{\sqrt{2}}\big)^8 \tfrac{k^2}{p^2}\big] \frac{e^{\sqrt{2}\ell p}}{2^{3/4}\sqrt{\pi \ell p}} + ...\Big) \sim g^2\big(1 + \tfrac{1}{7}\ell^2 k^2\big)\,(\ell\Lambda)^{11/2}\, e^{\sqrt{2}\ell\Lambda} + ...$$

Dimensional regularization

first define integrals in dim reg and then sum over s

$$\int d^4p\,(p^2)^n = 0, \qquad\qquad n = -1, 0, 1, ...$$

$$\Sigma(k^2) = g^2 \int \frac{d^4p}{p^2(p-k)^2}\,\Phi\big(p^2, (p-k)^2, k^2\big)$$

only $\Phi(0,0,k^2) \equiv \Phi(k^2)$ contributes:

$$\Sigma(k^2) = g^2\,\Phi(k^2) \int \frac{d^4p}{p^2(k-p)^2} \;\to\; g^2\,\Phi(k^2)\,\log\frac{k^2}{\mu^2}$$

$$\Phi(k^2) = \sum_{\substack{s_2,s_3=0\\ s_2+s_3=\text{even}}}^{\infty} \frac{(8-s_2^2\,s_3^2)\,(\ell^2k^2)^{s_2+s_3}}{[(s_2+s_3-1)!]^2}$$

$$= -\frac{1}{60}\,(\ell k)^7\,\big[I_5(2\ell k) + J_5(2\ell k)\big] - \frac{1}{4}\,(\ell k)^6\,\big[I_4(2\ell k) - J_4(2\ell k)\big]$$
$$+ \frac{7}{2}\,(\ell k)^3\,\big[I_1(2\ell k) + J_1(2\ell k)\big] + 8\,(\ell k)^2\,\big[I_0(2\ell k) - J_0(2\ell k)\big]$$

log div (wave function renorm ?); $\quad \Sigma(0) = 0,\ \Delta m = 0$

Comment on tadpole diagram contribution

- 4-scalar tree-level amplitude: 4-vertex gives similar exponential large p behaviour as exchange diagram
- expect 4-vertex contribution to tadpole diagram will also have exponential UV behaviour
- consider just single virtual $s = 0$ contribution to tadpole: then knowledge of 0-0-0-0 vertex is sufficient
- an estimate for its large virtual momentum behaviour

$$\Sigma_{\rm tp}(k^2) \sim \; g^2 \int \frac{d^4p}{p^2} \sum_{j=0} \Big[f_{2j}\big(-(k-p)^2\big) \; (k+p)^{4j} + f_{2j}\big(-(k+p)^2\big) \; (k-p)^{4j} \Big]$$

leading UV asymptotics

$$\Sigma_{\rm tp} \sim g^2 \int^{\Lambda} \frac{d^4p}{p^2} \sum_{j=0} a_{2j}(-p^2)\left(p^2\right)^{2j}$$

$$= \tfrac{1}{4} g^2 \int^{\Lambda} \frac{d^4p}{p^2}\left[I_0(\sqrt{2}\ell p) - J_0(\sqrt{2}\ell p)\right]$$

similar $\sim e^{\sqrt{2}\ell\Lambda}$ UV behaviour as in bubble diagram

• UV divergence of bubble diagram may get cancelled once all tadpole contributions are included?

• sum of bubble and tadpole diagrams may vanish for any spin s field on external lines?

• expected for special higher-spin theory in AdS dual to $U(N)$ or $O(N)$ scalar CFT: 1-loop or $1/N$ correction to 2-point function of conserved currents is absent in theory with unbroken higher spin symmetry

0-0-0-s tree-level scattering amplitude

gauge-invariance constraints on higher-spin vertices:
impose linearized gauge invariance on on-shell amplitude
more efficient than off-shell Lagrangian approach

Conditions:

- linearized gauge invariance $\delta\phi_{m_1...m_s} \sim \partial_{(m_1}\epsilon_{m_2...m_s)}$ of full amplitude $\mathcal{A}_4 = \mathcal{A}_{\text{exch}} + \mathcal{A}_{cont}$
- locality of 4-point vertex V_{000s} (no $1/p^2$ poles)

Strategy:

- solve non-trivial ("inhomogeneous") gauge-inv cond
- add solution of "homogeneous" eq.: invariant 4-vertex
- choose minimal solution consistent with locality of 4-vertex

Example: scalar electrodynamics

$L = \partial^m \phi^* \partial_m \phi + i A^m (\phi^* \partial_m \phi - \phi \partial_m \phi^*) + A^m A_m \phi^* \phi$

$\delta A_m = \partial_m \epsilon, \;\; \delta \phi = i \phi \epsilon$

$A(1)\phi(2)\phi(3)A(4)$ scattering amplitude:

$A_m \to \zeta_m(p) e^{ip\cdot x}, \;\; p \cdot \zeta = 0$

$$\mathcal{A}_{\rm exch} = \frac{1}{p_{12}^2} \zeta_1 \cdot p_2 \, \zeta_4 \cdot p_3 + \frac{1}{p_{13}^2} \zeta_1 \cdot p_3 \, \zeta_4 \cdot p_2$$

• gauge transformation in leg 1: $\delta \zeta_1 = p_1 \epsilon_1, \;\; \delta \phi = 0$

$\delta \mathcal{A}_{\rm exch} = (\zeta_4 \cdot p_3 + \zeta_4 \cdot p_2) \epsilon_1 = -\zeta_4 \cdot p_1 \, \epsilon_1$

• can be cancelled by adding contact $A^m A_m \phi^* \phi$ vertex

$\mathcal{A}_{\rm cont} = \zeta_1 \cdot \zeta_4 \to \;\; \delta \mathcal{A}_{\rm cont} = p_1 \cdot \zeta_4 \, \epsilon_1$

• thus 4-point vertex can be found from condition
of linearized gauge invariance of on-shell amplitude

0-0-0-s exchange amplitude:

0-0-s' and 0-s'-s vertices in de Donder gauge: $\phi_s \to \zeta_s(p)\, e^{ip\cdot x}$

$\zeta_s(p, q^s) \equiv \zeta_{m_1 \dots m_s}(p)\, q^{m_1} \dots q^{m_s}, \quad p_{ij} = p_i \cdot p_j, \quad p_i^2 = 0$

S-channel:

$$\mathcal{A}_{\text{exch}} = -\frac{ig^2}{p_{12}^2} \sum_{s'} \frac{\ell^{2s'+s-2}}{(s'-1)!(s+s'-1)!} (p_{12}^2)^{s'} T_{s'}\big(\tfrac{p_{13}^2 - p_{23}^2}{p_{12}^2}\big)\, \zeta_s(p_4, p_3^s)$$

$$T_s(z) = \tfrac{1}{2}\Big[(z + \sqrt{z^2-1})^s + (z - \sqrt{z^2-1})^s\Big]$$

$$\mathcal{A}_{\text{exch}} = -\frac{2ig^2}{p_{12}^2}\Big[F_s(z_+) + F_s(z_-)\Big]\, \zeta_s(p_4, p_3^s)$$

$$F_s(z) = z^{2-s}\Big[I_s(z) - J_s(z)\Big]\,, \qquad z_{\pm} = \ell\big(\sqrt{p_{13}^2} \pm \sqrt{p_{12}^2 + p_{13}^2}\big)$$

add t and u channels, apply $\delta\zeta_{m_1 \dots m_s}(p) = p_{(m_1}\epsilon_{m_2 \dots m_s)}$

$1/p^2$ poles go away in the variation

$$\delta\mathcal{A}_{\text{exch}} = -2sg^2\left[F_s(z_+) + F_s(z_-)\right]\,\epsilon_{s-1}(p_4, p_3^{s-1}) + ...$$

cancel this against variation of contribution of 0-0-0-s vertex

$$\sum_{k=0}^{s/2} V_{sk}(p_1,p_2,p_3)\,\phi_0(p_1)(p_2{\cdot}\partial_u)^k\phi_0(p_2)(p_3{\cdot}\partial_u)^{s-k}\phi_0(p_3)\,\zeta_s(p_4,u)$$

$$\delta\mathcal{A}_{\text{cont}} = sV_{s0}(p_1,p_2,p_3)\;p_{24}^2\,\zeta_{s-1}(p_4,p_2^{s-1}) + ...$$

gauge-invariance: relation of V_{sk} to Bessel functions in $\mathcal{A}_{\text{exch}}$

local solution for 4-vertex exists for $s=2$ and $s=4$

$s = 2$:

$$V_{20} = \frac{g^2}{p_{12}^2}\Big[F_2(z_+) + F_2(z_-)$$
$$-\tfrac{1}{2}\big[p_{13}^2 R_2(p_{13}^2) + p_{23}^2 R_2(p_{23}^2) + p_{12}^2 R_2(p_{12}^2)\big]\Big]$$

$$R_s(x) \equiv \tfrac{1}{2x}\big[I_s(\sqrt{-x}) - J_s(\sqrt{-x})\big]$$

is $x \to 0$ residue of $F_2(x)$

V_{20} is regular in $p_{12}^2 \to 0$ limit

complete 0-0-0-2 amplitude: simpler than exchange one

$$\mathcal{A} = g^2\big[p_{13}^2 R_2(p_{13}^2) + p_{23}^2 R_2(p_{23}^2) + p_{12}^2 R_2(p_{12}^2)\big]$$
$$\times\Big(\tfrac{\zeta_2(p_4,p_3^2)}{p_{12}^2} + \tfrac{\zeta_2(p_4,p_2^2)}{p_{13}^2} + \tfrac{\zeta_2(p_4,p_1^2)}{p_{23}^2}\Big)$$

$s = 4$:

4-vertex in terms of $R_4 \sim$ Bessels
regular at small p

complete 0-0-0-2 amplitude:

$$\mathcal{A} = U(p_1, p_2, p_3)\, \zeta_4\Big(p_4, (p_{12}^2 p_2 - p_{13}^2 p_3)^4\Big) - \frac{ip_{12}^2}{15 p_{13}^2}\, \zeta_4(p_4, p_2^4) + \dots$$

$$U = \left(\frac{1}{p_{13}^2} + \frac{1}{p_{23}^2}\right) R_4(p_{12}^2) + \text{ cycle}$$

existence of local solution in $s > 4$ case is unclear ...

Conclusions / Open questions

- beginning to learn how to do quantum computations in theories with infinite number of massless higher spin fields
- importance of proper definition of quantum theory (sum over spins, UV regularization) consistent with symmetries
- remarkable large symmetry of higher spin theories: (1-loop $Z = 1$, zero effective number of d.o.f.) massless HS theories have hidden simplicity? ("free-field" representation? "integrable"?)
- motivation to study flat space HS theory:
  – summation over all spins – UV finite theory?
  – viewed as simplified version of massless HS theory in AdS: loop corrections should be simple or vanish?
- flat-space HS theory remains to be constructed (4-vertex, etc)

- HS theory in AdS as reconstructed from CFT:

full 4-point vertex?

uniqueness?

correspondence with Vasiliev's equations?

consistent flat-space limit?

- compute self-energy in AdS_4:

demonstrate vanishing if HS symmetry is unbroken

find one-loop correction due to difference in spin 0 loop

with different b.c.'s – match $1/N$ correction

in interacting 3d CFT (W-F fixed point)