Coupling matter to the higher spin square

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- J.R., *"On matter coupled to the higher spin square,"* arXiv:1603.07845 [hep-th].

Overview

- Intro: the higher spin square (*HSS*)
- Vasiliev's matter equations and particle spectrum
- Matter coupled to the *HSS*
- Outlook

Introduction

- Longstanding idea: string theory has a phase of enhanced higher spin gauge symmetry, spontaneously broken in Minkowksi vacuum.
- Gaberdiel-Gopakumar's concrete candidate: tensionless limit of strings on

$$AdS_3 \times S^3 \times T^4$$

- can study through dual CFT, the $\mathcal{N} = (4,4)$ symmetric orbifold

$$Sym^N(T^4) \equiv \frac{(T^4)^N}{S^N}$$

in large N limit

The higher spin square

- Single-particle chiral symmetry generators of $Sym^N(T^4)$ at large N are in 1-1 correspondence with chiral operators on T^4:

$$\mathcal{O}(z) \qquad \leftrightarrow \qquad \sum_{i=1}^{N} \mathcal{O}_i(z)$$

- Theory on a single T^4 is free $\mathcal{N} = (4,4)$ SCFT with 2 complex bosons and 2 complex fermions
  $\rightarrow$ huge 'stringy' chiral algebra
- Consider 'wedge subalgebra' annihilating the vacuum: the Higher Spin Square (*HSS*). Plays role of of bulk gauge algebra.

THE HIGHER SPIN SQUARE

Structures within single-particle chiral algebra:

$$\begin{array}{lccccc} \text{CFT}: & \mathcal{N}=4\ SCA & \subset & \mathcal{W}_\infty^{\mathcal{N}=4}[0] & \subset & \text{stringy algebra} \\ & \downarrow & & \downarrow & & \downarrow \\ \text{bulk}: & su(1,1|2) & \subset & shs_2[0] & \subset & HSS \end{array}$$

In addition, HSS (and the stringy algebra) contain a 'horizontal' higher spin ($\mathcal{W}_\infty$) algebra

The untwisted sector

Interpret untwisted sector in terms of HSS representations

(Gaberdiel, Gopakumar 2015)

- Chiral partition function for single T^4 in NS sector

$$Z_{T^4} \equiv \mathrm{tr}\, q^{L_0} y^{2J_0^3} = \prod_{n=1}^{\infty} \frac{(1+yq^{n-\frac{1}{2}})^2(1+y^{-1}q^{n-\frac{1}{2}})^2}{(1-q^n)^4}$$

$$\equiv \sum_{r\in\mathbb{N}/2, l\in\mathbb{Z}} d(r,l) q^r y^l$$

- Untwisted sector partition function on $Sym^N(T^4)$ = N-particle sector of second quantized T^4
- Statmech trick: introduce generating function

$$\mathcal{Z} = \sum z^N Z_N, \qquad \text{e.g. } \mathcal{Z} = \begin{cases} (1-zq^h)^{-1} & \text{for boson} \\ (1+zq^h)^{+1} & \text{for fermion} \end{cases}$$

THE UNTWISTED SECTOR

- Generating function for untwisted sector partition function

$$Z^U \equiv \sum_N z^N Z_N^U = \prod_{r,\bar{r},l,\bar{l}} \left(1 - (-1)^{r+\bar{r}} z q^{r-\frac{1}{4}} \bar{q}^{r-\frac{1}{4}} y^l \bar{y}^{\bar{l}}\right)^{(-1)^{l+\bar{l}+1} d(r,l) d(\bar{r},\bar{l})}$$

- Large N behaviour

$$Z_N^U \sim (q\bar{q})^{-\frac{N}{4}} \prod_{r,\bar{r},l,\bar{l}}{}' \left(1 - (-1)^{r+\bar{r}} q^r \bar{q}^{\bar{r}} y^l \bar{y}^{\bar{l}}\right)^{(-1)^{l+\bar{l}+1} d(r,l) d(\bar{r},\bar{l})}$$

- Similar calculation for the chiral sector gives extended vacuum character

$$Z_N^{vac} \sim q^{-\frac{N}{4}} \prod_{r,l}{}' \left(1 - (-1)^l q^r y^l\right)^{(-1)^{l+1} d(r,l)}$$

The untwisted sector

- Now, can decompose

$$Z_N^U \sim |Z_N^{vac}|^2 \exp \sum_k Z_1\left(q^k, (-1)^{k+1} y^k, \bar{q}^k, (-1)^{k+1} \bar{y}^k\right)$$

- Red factor has form of multi-particle matter excitations in bulk. Single particle partition function is a HSS character:

$$Z_1(q, y, \bar{q}, \bar{y}) = |Z_{T^4} - 1|^2 = |\chi_{min}|^2$$

- Minimal repr. of *HSS* contains similar minimal repr. of $shs_2[0]$ subalgebra. Expectation: comes from a single Vasiliev-like bulk matter field. Goal of this talk: to check this explicitly.

Vasiliev's higher spin-matter system

Vasiliev, 1990

- Higher spin gauge symmetry $H \times \tilde{H}$, contains $SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$ (gravity). Massless higher spin background: flat connections

$$dA - A \wedge A = 0, \qquad d\tilde{A} - \tilde{A} \wedge \tilde{A} = 0$$
$$A \to hAh^{-1} + dhh^{-1}, \qquad \tilde{A} \to \tilde{h}\tilde{A}\tilde{h}^{-1} + d\tilde{h}\tilde{h}^{-1}$$

- Assume higher spin Lie algebra arises from associative 'lone star' algebra $\mathcal{A}$. Take $C, \tilde{C}$ scalars valued in $\mathcal{A}$. Linearized matter equations

$$dC - AC + C\tilde{A} = 0, \qquad d\tilde{C} - \tilde{A}\tilde{C} + \tilde{C}A = 0$$
$$C \to hC\tilde{h}^{-1}, \qquad \tilde{C} \to \tilde{h}\tilde{C}h^{-1}$$

GENERAL MATTER SOLUTION IN ADS

- Global AdS background in pure gauge form

$$A = dgg^{-1}, \qquad \tilde{A} = d\tilde{g}\tilde{g}^{-1}$$
$$g = R(\rho)e^{iL_0 x_+}, \qquad \tilde{g} = \tilde{R}(\rho)e^{-iL_0 x_-}, \qquad x_\pm = t \pm \phi$$

- General solution for the scalar fields

$$C = gC_0\tilde{g}^{-1}, \qquad \tilde{C} = \tilde{g}\tilde{C}_0 g^{-1} \qquad C_0, \tilde{C}_0 \in \mathcal{A}$$

- Suppose we can diagonalize the L_0 action on $\mathcal{A}$ from both right and left. L_0 has positive spectrum $\rightarrow$ C describes positive frequency modes, i.e. single particle states. $\tilde{C}$ describes negative frequency ones.

Single particle representations

- Global symmetry is again $H \times \tilde{H}$, generated by

$$g \rightarrow g h_0 \qquad \tilde{g} \rightarrow \tilde{g}\tilde{h}_0$$

- On scalar solutions in 'zero gauge' C_0, acts by left resp. right multiplication

$$C_0 \rightarrow h_0 C_0 \tilde{h}_0^{-1}$$

- Wish to decompose $\mathcal{A}$ (as a vector space) into $H \times \tilde{H}$ irreps as

$$\mathcal{A} = \oplus_i \left(V_i, W_i^*\right)$$

- Single-particle partition function decomposes into characters of global symmetry as

$$Z^{1-part}(q, \bar{q}) = \sum_i \chi_{V_i}(q)\overline{\chi_{W_i}(q)}$$

Examples

- Let $\mathcal{A} = 5 \times 5$ matrices, $H = SL(2, \mathbb{R})$ with generators embedded in $\mathcal{A}$ as $L_m = L_m^{\mathbf{2}} \oplus L_m^{\mathbf{3}}$. Then

$$\mathcal{A} = (V_{\mathbf{2}}, V_{\mathbf{2}}^*) \oplus (V_{\mathbf{2}}, V_{\mathbf{3}}^*) \oplus (V_{\mathbf{3}}, V_{\mathbf{2}}^*) \oplus (V_{\mathbf{3}}, V_{\mathbf{3}}^*)$$

- Let $h = hs[1/2]$ and $\mathcal{A}$ the underlying 'lone-star' algebra, realized by even Weyl-ordered monomials in a single oscillator $a, a^\dagger$. Can show that

$$\mathcal{A} = (V_+, V_+^*) \oplus (V_-, V_-^*) \Rightarrow Z_{1-part} = |\chi_{V_+}|^2 + |\chi_{V_-}|^2$$

with $V_\pm$ the 'minimal' reps of $hs[1/2]$. Contains both standard boundary condition (1st term) and 'alternate' boundary condition (2nd term).

Matter coupled to higher spin square

- Consider Vasiliev's matter system with
  - $h = HSS$ Lie algebra
  - $\mathcal{A} =$ underlying associative algebra
- $\mathcal{A}$ has an oscillator realization in terms of NS-sector modes of chiral theory on T^4: ($\alpha = 1, 2, m \in \mathbb{N}_0, r \in \mathbb{N} + \frac{1}{2}$)

$$\left[a_m^\alpha, \left(a_n^\beta\right)^\dagger\right] = \delta^{\alpha\beta}\delta_{mn}1, \qquad \left[\bar{a}_m^\alpha, \left(\bar{a}_n^\beta\right)^\dagger\right] = \delta^{\alpha\beta}\delta_{mn}1$$

$$\left\{\psi_r^\alpha, \left(\psi_s^\beta\right)^\dagger\right\} = \delta^{\alpha\beta}\delta_{rs}1, \qquad \left\{\bar{\psi}_r^\alpha, \left(\bar{\psi}_s^\beta\right)^\dagger\right\} = \delta^{\alpha\beta}\delta_{rs}1$$

- Basis for T^4 chiral algebra: normal ordered monomials in oscillators
- Basis for *HSS*: normal ordered monomials in oscillators which annihilate in- and out- vacuum, i.e. containing at least one creation and one annihilation mode

SOME SUBALGEBRAS OF *HSS*

- Global $\mathcal{N} = 4$ susy algebra $su(1,1|2)$, with generators $L_0, L_{\pm 1}, J_0^i, G^\alpha_{\pm\frac{1}{2}}, \bar{G}^\alpha_{\pm\frac{1}{2}}$, contained in sector quadratic in oscillators. E.g.

$$L_0 = \sum_{n=1}^{\infty}\left[n\left(\left(a_n^\alpha\right)^\dagger a_n^\alpha + \left(\bar{a}_n^\alpha\right)^\dagger \bar{a}_n^\alpha\right) + \left(n-\frac{1}{2}\right)\left(\left(\psi^\alpha_{n-\frac{1}{2}}\right)^\dagger \psi^\alpha_{n-\frac{1}{2}} + \left(\bar{\psi}^\alpha_{n-\frac{1}{2}}\right)^\dagger \bar{\psi}^\alpha_{n-\frac{1}{2}}\right)\right]$$

$$J_0^3 = -\frac{1}{2}\sum_{n=1}^{\infty}\left(\left(\psi^\alpha_{n-\frac{1}{2}}\right)^\dagger \psi^\alpha_{n-\frac{1}{2}} - \left(\bar{\psi}^\alpha_{n-\frac{1}{2}}\right)^\dagger \bar{\psi}^\alpha_{n-\frac{1}{2}}\right)$$

- Further quadratic operators generate ‘vertical’ higher spin algebra $shs_2[0]$
- Note: L_0, J_0^3 don’t act diagonally on basis of normal-ordered monomials, either from left or right

Fock basis for single oscillator

- For a single bosonic oscillator $a, a^\dagger$, can also work in basis of Fock operators of form:

$$|m\rangle\langle n|$$

- Relation to the normal-ordered basis:

$$|m\rangle\langle n| \quad = \quad = \sum_{p\in\mathbb{N}} \frac{(-1)^p}{p!} \frac{(a^\dagger)^{m+p}}{\sqrt{m!}} \frac{a^{n+p}}{\sqrt{n!}}$$

$$(a^\dagger)^m a^n \quad = \quad \sum_{p\in\mathbb{N}} \frac{\sqrt{(p+m)!(p+n)!}}{p!} |p+m\rangle\langle p+n|$$

- Similarly for single fermion.

FOCK BASIS FOR $\mathcal{A}$

- Similarly, propose to use a Fock basis for chiral algebra on T^4, with elements

$$|F_1\rangle \langle F_2| \,, \qquad F_1, F_2, \in \text{Fock pace } \mathcal{F}$$

- basis elements for $\mathcal{A}$ must also annihilate in and out-vacua:

$$\left|F_1'\right\rangle \left\langle F_2'\right| \,, \qquad F_1', F_2' \in \mathcal{F}'$$

  with $\mathcal{F}'$ Fock space states containing *at least one particle*

- Note

$$\mathcal{F} = \mathbb{C}\,|0\rangle \oplus \mathcal{F}'$$

FOCK BASIS FOR $\mathcal{A}$

In Fock basis

- L_0, J_0^3 act diagonally from left and right

$$L_0 \left|F\right\rangle \left\langle F'\right| = h_F \left|F\right\rangle \left\langle F'\right| , \qquad \left|F\right\rangle \left\langle F'\right| L_0 = h_{F'} \left|F\right\rangle \left\langle F'\right|$$
$$J_0^3 \left|F\right\rangle \left\langle F'\right| = l_F \left|F\right\rangle \left\langle F'\right| , \qquad \left|F\right\rangle \left\langle F'\right| J_0^3 = l_{F'} \left|F\right\rangle \left\langle F'\right|$$

- Multiplication rule ('star-multiplication') is simple:

$$\left|F_1'\right\rangle \left\langle F_2'\middle|F_3'\right\rangle \left\langle F_4'\right| = \delta_{F_2', F_3'} \left|F_1'\right\rangle \left\langle F_4'\right|$$

Single particle partition function

- Have established that

$$\mathcal{A} = \mathcal{F}' \otimes \mathcal{F}'^*$$

- $\mathcal{F}'$ carries a representation of HSS with character

$$\chi = \chi_{\mathcal{F}'} \equiv \mathrm{tr}_{\mathcal{F}'}\, q^{L_0} y^{2J_0^3} = \mathrm{tr}_{\mathcal{F}}\, q^{L_0} y^{2J_0^3} - 1 = Z_{T^4} - 1$$

This is precisely the minimal representation, $\mathcal{F}' = V_{min}$

- Earlier argument gives single-particle partition function of HSS matter

$$Z_1 = |\chi_{min}|^2$$

QED

Reverse-engineering twisted sectors

- Content of twisted sectors of $Sym^N(T^4)$ in terms of HSS representations is not well-understood.
- Suppose twisted sector contains (multiparticling of) a repr. (V, W^*). Can write down Vasiliev-like matter equations with this single-particle spectrum:

$$\begin{aligned} dC - A_V C + C\tilde{A}_W &= 0 \\ d\tilde{C} - \tilde{A}_W \tilde{C} + \tilde{C} A_V &= 0 \end{aligned}$$

with

- C taking values in (V, W^*)
- $\tilde{C}$ taking values in (W, V^*)

Outlook

Showed that single Vasiliev-like matter field coupled to *HSS* accounts for full untwisted sector of $Sym^N(T^4)$. Many issues to explore!

- Spectrum:
    - AdS multiplet content of matter master fields $C, \tilde{C}$
    - Massless sector: boundary conditions and asymptotic symmetries, vacuum character
    - Understand twisted sectors
- Interactions
    - How does *HSS* constrain higher order terms in eom?
    - Complete interacting theory a la Vasiliev?