Towards Light-Cone Higher-Spin Theories

Aspects of Higher Spin theory

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deep gratitude to Ruslan Metsaev, our light-cone coach and to Lars Brink and Anders Bengtsson for the correspondence

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It is a widespread belief that higher-spin theories have some pathologies in Minkowski space and one cannot avoid AdS;

There are many no-go theorems that seemingly leave no room in flat space;

Recently Ponomarev and Tseytlin argued that the there can be more coupling constants: coming from background and a genuine one l_{pl};

Also there are surprising relations between cubic actions in Minkowski (Metsaev) and AdS (Bekaert et al; Sleight, Taronna)

Light-cone approach is the best to resolve the puzzle as it requires no more than needed to have a well-defined QFT — Poincare invariance;

Pros and Cons of Light-Cones

+ no extra assumptions, just study the interactions of a given set of Wigner particles;
+ manifestly Poincare-invariant S-matrix;
- not manifestly Lorentz-covariant expressions;
+ independent of the description: gauge potentials/dual gauge potentials/curvatures/set of auxiliary fields;
- quantum computations are generally harder than in the covariant methods;
+ allows to overcome some of the no-go results;
- most of the beautiful covariant structures, e.g. diffeomorphisms, get lost;
+ more fundamental is only S-matrix itself
+ manifest unitarity, control over degrees of freedom;

The idea is that QFT is about writing explicitly P^A and J^{AB}

$$\begin{aligned}
[P^A, P^B] &= 0\\
[J^{AB}, P^C] &= P^A\eta^{BC} - P^B\eta^{AC}\\
[J^{AB}, J^{CD}] &= J^{AD}\eta^{BC} - J^{BD}\eta^{AC} - J^{AC}\eta^{BD} + J^{BC}\eta^{AD}
\end{aligned}$$

The stability group of the Cauchy surface depends on its type and is for $x^0 - x^3 = 0$ it is the biggest.

Splitting Lorentz indices as $A = +, -, i$ we find many more generators and relations whose total number is the same.

There are few generators that we care:

$$P^- = H\,, J^{a-} \qquad \Longleftrightarrow \qquad [H, J^{a-}] = 0 \qquad [J^{a-}, J^{b-}] = 0$$

Living on Light Front

$4d$ simplification: everything is equal to two scalars, except for a scalar field, which is one scalar:

$$\Phi^{\lambda}_{q} \qquad\qquad \delta_{J_2}\Phi^{\lambda} = \left(-H_2\frac{\partial}{\partial\bar{q}} - q\frac{\partial}{\partial q^+} + \lambda\frac{q}{q^+}\right)\Phi^{\lambda}$$

The Dirac bracket of two fields of helicities $\lambda_{1,2}$ is

$$[\Phi^{\lambda_1}_{q_1}, \Phi^{\lambda_2}_{q_2}] = \frac{\delta^{\lambda_1+\lambda_2}\delta(q_1+q_2)}{(q_1^+ - q_2^+)}$$

and the problem is to construct a nonlinear completion of dynamical generators modulo kinematical constraints:

$$Q_{\xi} = \int \Phi^{-\lambda}_{-q}\delta_G\Phi^{\lambda}_{q} + \begin{cases} 0, & \text{if kinematical,} \\ O(\Phi^3), & \text{if dynamical,}\ H, J, \bar{J} \end{cases}$$

All cubic vertices in $4d$ were found by Brink, Bengtsson[2], Linden; see also Metsaev. In momentum space

$$H+ = \int \delta\left(\sum p_i\right) \Phi\Phi\Phi \times h$$

where the vertex has a very simple form

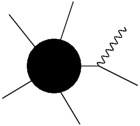
$$h_{\lambda_1,\lambda_2,\lambda_3} = C^{\lambda_1,\lambda_2,\lambda_3} \frac{\bar{\mathbb{P}}_{12}^{\lambda_1+\lambda_2+\lambda_3}}{\beta_1^{\lambda_1}\beta_2^{\lambda_2}\beta_3^{\lambda_3}} + c.c.$$

where $p^a = p, \bar{p}, p^+ \equiv \beta$ and $\bar{\mathbb{P}}_{km} = \bar{p}_k \beta_m - \bar{p}_m \beta_k$.

Structure constants $C^{\lambda_1,\lambda_2,\lambda_3}$ remain free at the cubic level.

Can be written as 'off-shell' amplitudes (e.g. Bengtsson, Ananth) $[12]^{s_1+s_2-s_3}[23]^{s_2+s_3-s_1}[13]^{s_1+s_3-s_2} + c.c.$

Weinberg's low energy theorem constrains S-matrix by checking gauge-invariance of the process with a low-energy spin-s particle attached



$$q_\nu \sum g_i \frac{p_i^\nu p_i^\mu \ldots p_i^\mu}{q\cdot p} = \sum g_i p_i^\mu \ldots p_i^\mu = 0$$

may not apply due to:

- assumes Poincare invariant cubic vertices;
- HS S-matrix may eventually be of δ-type like in CHS (Joung, Nakach, Tseytlin);
- spin-changing vertices not included (only $s-s-s'$);

Not-so-no-Go Theorems

Coleman-Mandula theorem constrains the symmetries of nontrivial S-matrix to be a direct product of Poincare and others.

may not apply due to:

- full algebra might be=Poincare$\times$internal HS symmetry, like in YM = Poincare $\times SU(N)$. Also, gravity is just a theory of two scalars without any trace of diffeomorphisms. Consistent with Bekaert, Boulanger, Leclerq;
- infinite number of fields;
- massless HS fields;

BCFW may not apply (Dempster, Tsulaia; Benincassa, Cachazo; Ponomarev, Tseytlin; Bengtsson) due too many restrictions on the amplitude and the summation over infinite number of spins.

Gravitational Couplings of HS Fields

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$+s+s-2$. Top-vertex, $2s-2$-derivative vertex, non-abelian=interesting;

$+s-s+2$. **Gravitational minimal coupling**, 2-derivative. cf. Deser and Aragone vs. Benincasa, Cachazo, Conde; Dempster, Tsulaia and Bengtsson^2, Linden; Metsaev. At $s=2$ this is Einstein-Hilbert!

The very fact of existence of Lorentz-covariant vertices crucially depends on the set of (auxiliary fields) and other assumptions. For example, $F_{\mu\nu}$ vs. A_μ; ∞ of auxiliary fields Berkovits, Hull for self-dual fields; finite number of compensator fields for self-dual fields or breaking of Lorentz symmetry (Henneaux, Bunster; ...; Pasti et al)

Examples

All covariant theories can be translated into the light-cone language

$$H = \Phi \frac{\partial\bar{\partial}}{\partial^+}\Phi + "\Phi^3" + \ldots$$

Phi-cube. There is no difference for Φ^3

$$H_3 = \Phi^0\Phi^0\Phi^0 C^{0,0,0}$$

Yang-Mills can be recognized from

$$H_3 = f^{abc}\Phi^1_a\Phi^1_b\Phi^{-1}_c \left[\frac{\bar{\mathbb{P}}_{12} C^{1,1,-1}\beta_3}{\beta_1\beta_2} + \text{c.c.}\right]$$

Gravity starts from the two-derivative E-H vertex

$$H_3 = \Phi^2\Phi^2\Phi^{-2} \left[\frac{\bar{\mathbb{P}}^2_{12}\beta^2_3 C^{2,2,-2}}{\beta^2_1\beta^2_2} + \text{c.c.}\right]$$

Some of the couplings gets related by the quartic analysis

Yang-Mills plus Scalar. Covariantly we know that $gA\partial\Phi\Phi$ and $g\partial A^3$ have the same g; $C^{0,0,1} = C^{1,1,-1}$:

$$H_3 = \Phi^0\Phi^0\Phi^1\bar{\mathbb{P}}C^{0,0,1} + \Phi^1\Phi^1\Phi^{-1}\bar{\mathbb{P}}C^{1,1,-1}$$

Gravity plus Scalar. Scalar should couple universally to gravity, which implies $C^{0,0,2} = C^{2,2,-2}$:

$$H_3 = \Phi^2\Phi^2\Phi^{-2}\bar{\mathbb{P}}^2C^{2,2,-2} + \Phi^0\Phi^0\Phi^2\bar{\mathbb{P}}^2C^{0,0,2}$$

Gravity plus Yang-Mills. Spin-one should also couple with the same strength, $C^{1,-1,2} = C^{2,2,-2}$:

$$H_3 = \Phi^2\Phi^2\Phi^{-2}\bar{\mathbb{P}}^2C^{2,2,-2} + \Phi^1\Phi^{-1}\Phi^2\bar{\mathbb{P}}^2C^{1,-1,2}$$

Not very surprising given that covariant actions are known.

Reincarnation of Weinberg's Theorem

Weinberg theorem does not apply to light-cone vertices, but the universality conclusion is still true!

Generalizing the $0-1-2$ examples all higher-spin fields couple universally to spin-two $g_{\mu\nu}$:

$$s-s-2: \qquad\qquad C^{s,-s,2}=C^{2,2,-2}=gl_{pl}$$

Generalizing the $0-1$ example all higher-spin fields couple universally to spin-one A_μ:

$$s-s-1: \qquad\qquad C^{s,-s,1}=C^{1,1-1}=g$$

Metsaev

Metsaev studied the closure of the algebra at the quartic order

$$[J_3, H_3] + [J_2, H_4] = [H_2, J_4]$$

Surprisingly, the equations split into two sets

$$\sum_{\omega} C^{s_1,s_2,\omega} C^{s_3,s_4,-\omega} \times \text{kinematics} = 0$$

$$\sum_{\omega} C^{s_1,s_2,\omega} \bar{C}^{s_3,s_4,\omega} \times \text{kinematics} + J_2 H_4 = H_2 J_4$$

The former admits many solutions. The latter has not been studied yet in detail.

Does this splitting have any analog in higher dimensions?

One solution that easily passes the quartic test is to set all interesting couplings to zero while leaving only the maximal derivative abelian couplings, $C^{s_1,s_2,s_3}\neq 0$, which is obvious from the covariant approach:

$$L=\Phi\Box\Phi+(F_{\mu\nu})^3+(R_{\mu\nu,\lambda\rho})^3+...$$

No surprise!

Interesting HS theories should assume $C^{s,-s,2} \neq 0$, $C^{s,-s,s} \neq 0$, i.e. gravitational and more general non-abelian vertices are included. Then there are two options at least:

- the CC-sector of $[J,H]_4 = 0$ implies that (same for $\bar{C}\bar{C}$)

$$C^{s_1,s_2,s_3} = \frac{ge^{i\theta}(l_{pl})^{s_1+s_2+s_3}}{\Gamma[s_1+s_2+s_3]} \qquad \bar{C}^{s_1,s_2,s_3} = \frac{ge^{-i\theta}(l_{pl})^{s_1+s_2+s_3}}{\Gamma[s_1+s_2+s_3]}$$

  which introduces the analog of the θ-angle, otherwise it is Metsaev

Higher-Spin Theories: Options

Interesting HS theories should assume $C^{s,-s,2} \neq 0$, $C^{s,-s,s} \neq 0$, i.e. gravitational and more general non-abelian vertices are included. Then there are two options at least:

- the CC-sector of $[J, H]_4 = 0$ implies that (same for $\bar{C}\bar{C}$)

$$C^{s_1,s_2,s_3} = \frac{g e^{i\theta}(l_{pl})^{s_1+s_2+s_3}}{\Gamma[s_1+s_2+s_3]} \quad \bar{C}^{s_1,s_2,s_3} = \frac{g e^{-i\theta}(l_{pl})^{s_1+s_2+s_3}}{\Gamma[s_1+s_2+s_3]}$$

  which introduces the analog of the θ-angle, otherwise it is Metsaev
- Self-dual limits $\theta \to \pm i\infty$ lead to a **consistent HS theory that has cubic vertices only and violates parity** cf. Chern-Simons Matter Giombi et al;...

The coefficients agree with those of the AdS_4-action (Bekaert et al; Taronna, Sleight) suggesting that there should be a sensible flat limit and an AdS-lift (~~parity~~ theory as well?).

Towards 4d Light-Cone HS Theory

One has to find both local (!) H_4 and local (!) J_4, which may be obstructed and relies on the fine-tuning of the coefficients to be eaten by H_4 and J_4.

For example the quartic self coupling of the scalar field is

$$\int \delta\left(\sum_i q_i\right)\Phi^4\left[0+\left(\frac{\beta_1-\beta_2}{\beta_1+\beta_2}\frac{\beta_3-\beta_4}{\beta_3+\beta_4}\right)+\text{perm}\right]+\ldots$$

The absence of the non-derivative self-couplings agrees with the computation in Vasiliev theory (Sezgin, Sundell). Test against Bekaert, Erdmenger, Ponomarev, Sleight? Also, quartic analysis by Taronna.

Many free CFT_4's: singletons of all spins $j = 0$ (type-A), $\frac{1}{2}$ (type-B), 1 (type-C, Maxwell), ...

All of them should have AdS_5-duals by (re)construction, computing the correlators of J_s, $s \geq 2j$ (Weinberg-Witten). The HS algebras are known (Fradkin, Linetsky; Gunaydin; Boulanger, E.S.; Manvelyan et al)

All of them might be uplifts of some $M_{4,1}$ HS theories. The Wigner little algebra is $so(3)$, which leads to a HS multiplet Φ with values in $hs(\lambda)$, allowing for the same type of truncations.

Conclusions

It seems that all no-go theorems can be avoided by higher-spin theories;

Minimal gravitational and Yang-Mills coupling of higher-spin fields does exist in $4d$ light-cone approach;

Higher-spin fields couple universally to Gravity and Yang-Mills;

There exists a complete parity-violating Higher-Spin Theory in $4d$ Minkowski space (implicitly constructed in 1990 by Metsaev). Lift to AdS_4? Quantization?

Small fraction of H_4 is known and there is no non-derivative Φ^4 vertex.

Thank you for your attention!

May the Higher-Spin Force be with you