Frobenius-Chern-Simons Gauge Theory

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Introductory remarks

- Desirable to have an action principle for Vasiliev HS theory for computing quantum corrections and holography.

- Previous work on actions:

  – Fradkin & Vasiliev'87: Curvature square giving terms up to cubics.

  Useful but is the full curvature expansion feasible? Relation to Vasiliev perturbative expansion?

  – Doroud & Smolin'11: Hamilton type in 4D.

  HS invariants? Amplitudes?

  – Boulanger & Sundell'11: Hamilton type, aux dimension, higher forms.

  HS invariants used to compute tree level amplitudes but each invariant has undeteremined overall coefficient.

Frobenius-Chern-Simons action

Boulanger, Sezgin and Sundell, arXiv:1505.04957

Extension of Boulanger-Sundell with the following key new ingredients:

- Enlargement of the HS gauge symmetry $hs(4)$ to $hs(4) \times hs(4) \times \mathcal{F}$ where $\mathcal{F}$ is an 8-dimensional graded Frobenius algebra.
- Replacement of the rigid symplectic structure on the twistor space by a dynamical and bifundamental even form.
- Emergence of a Chern-Simons action for a superconnection.

Aim of this talk: Review the FCS model. Describe most general cubic action, show emergence of FCS gauge theory with generalized (quasi) Frobenius algebra. Describe also higher order interactions.

Vasiliev's original equations:

$$dW + W \star W + \Phi \star (J - \bar{J}) = 0 \ ,$$
$$d\Phi + W \star \Phi - \Phi \star \pi(W) = 0 \ .$$

- One-form $W(x, Z; Y)$ and zero-form $\Phi(x, Z; Y)$ on $\mathcal{M}_4 \times \mathcal{Z}_4$.
- Z and Y are 4-component Majorana spinors. Exterior derivative given by:

$$d = d_X + d_Z$$

- Star product is Moyal-like, implementing normal ordering of (Z, Y).
- π-twist defined by $\pi(y, \bar{y}, z, \bar{z}) = (-y, \bar{y}, -z, \bar{z})$

Twisted central two-form crucial for the theory:

$$J = -\frac{i}{4}\left(dz^{\alpha}dz_{\alpha}\;\kappa + d\bar{z}^{\dot{\alpha}}d\bar{z}_{\dot{\alpha}}\;\bar{\kappa}\right)\ ,$$

defined in terms of the inner Klein operator

$$\kappa = e^{iy^{\alpha}z_{\alpha}}\ , \qquad \kappa\star\kappa = 1\ .$$

The following relations hold:

$$dJ = 0\ , \qquad J\star W = \pi(W)\star J\ , \qquad J\star\Phi = \pi(\Phi)\star J$$

Introducing extra dimension and dynamical two-form

- In addition to $\mathcal{M}_4 \times \mathcal{Z}_4$, introduce an auxiliary dimension, such that the total manifold on which the fields live is

$$\mathcal{M}_9 = \mathcal{X}_5 \times \mathcal{Z}_4$$

where $\partial \mathcal{Z}_4 = \emptyset$ and $\partial \mathcal{X}_4$ contains $\mathcal{M}_4$. [Boulanger and Sundell, 2011].

- Take fields in associative higher spin algebra extended by outer Klein operators as well.

- Let $J - \bar{J} \;\rightarrow\; \widetilde{B}$ where $\widetilde{B}$ is a dynamical two-form.

Obstacle:

$$F = B \star \widetilde{B} \,, \qquad DB = 0 \qquad \Longrightarrow \qquad [B, B \star \widetilde{B}]_\star = 0$$

Way out: Introduce an additional master one-form $\widetilde{A}$, let $B, \widetilde{B}$ transform under bifundamental representation of the enlarged HS algebra and postulate

$$dA + A \star A - B \star \widetilde{B} = 0 \ ,$$

$$d\widetilde{A} + \widetilde{A} \star \widetilde{A} - \widetilde{B} \star B = 0 \ ,$$

$$dB + A \star B - B \star \widetilde{A} = 0 \ ,$$

$$d\widetilde{B} + \widetilde{A} \star \widetilde{B} - \widetilde{B} \star A = 0 \ .$$

This system is Cartan integrable and implies the gauge transformations:

$$\delta A = d\epsilon + [A, \epsilon]_\star + \eta \star \widetilde{B} + B \star \widetilde{\eta} \ ,$$

$$\delta \widetilde{A} = d\widetilde{\epsilon} + [\widetilde{A}, \widetilde{\epsilon}]_\star + \widetilde{\eta} \star B + \widetilde{B} \star \eta \ ,$$

$$\delta B = d\eta + A \star \eta + \eta \star \widetilde{A} - \epsilon \star B + B \star \widetilde{\epsilon} \ ,$$

$$\delta \widetilde{B} = d\widetilde{\eta} + \widetilde{A} \star \widetilde{\eta} + \widetilde{\eta} \star A - \widetilde{\epsilon} \star \widetilde{B} + \widetilde{B} \star \epsilon \ .$$

Note the new symmetry parameters $\widetilde{\epsilon}, \eta, \widetilde{\eta}$.

Building Frobenius-Chern-Simons action

- Introduce an auxiliary dimensions.
- Introduce Lagrange multiplier fields.
- Extend field content to include set of odd and even forms.
- Employ Frobenius algebra to arrange all the fields into one superconnection.
- Write a Chern-Simons type action for the superconnection.

Frobenius algebra: A finite-dimensional, unital, associative algebra A defined over a field k, and equipped with a non-degenerate bilinear form $\sigma : A \times A \to k$ that satisfies the equation $\sigma(a \cdot b, c) = \sigma(a, b \cdot c)$.

Example: Any matrix algebra with the bilinear form $\sigma(a, b) = \mathrm{tr}\,(a \cdot b)$.

- We employ the the 3-graded 8-dimensional Frobenius algebra

$$\mathcal{F} = \bigoplus_{i,j=1,2} e_{ij} \oplus (h\, e_{ij}) = \mathcal{F}^{(-1)} \oplus \mathcal{F}^{(0)} \oplus \mathcal{F}^{(+1)}$$

$$e_{ij},\, h\, e_{ij} \in \mathcal{F}^{(j-i)}$$

with product rule:

$$e_{ij}e_{kl} = \delta_{jk}e_{il} \ , \qquad he_{ij} = (-1)^{i-j}e_{ij}h \ , \qquad h^2 = 1$$

- Master fields:

$$X = \sum_{i,j} X^{ij}e_{ij} = \begin{pmatrix} A & B \\ \widetilde{B} & \widetilde{A} \end{pmatrix} \ , \qquad P = \sum_{i,j} P^{ij}e_{ij} = \begin{pmatrix} V & U \\ \widetilde{U} & \widetilde{V} \end{pmatrix}$$

$(A, \widetilde{A}, V, \widetilde{V})$ are odd forms, $(B, \widetilde{B}, U, \widetilde{U})$ are even forms.

Top odd forms, and $(B_8, U_8, \widetilde{B}_0, \widetilde{U}_0)$ are truncated out consistently (thanks to 3-grading), to avoid algebraic constraints on master fields.

Superconnection and Frobenius-Chern-Simons action

Assemble X and P into superconnection

$$Z = hX + P$$

Use the trace operation $\mathrm{Tr}_{\mathcal{A}}$ (more on this below) to construct the action

$$S = \int_{\mathcal{M}_9} \mathrm{Tr}_{\mathcal{A}} \left(\tfrac{1}{2}\, Z \star qZ + \tfrac{1}{3}\, Z \star Z \star Z \right) - \frac{1}{4} \int_{\partial \mathcal{M}_9} \mathrm{Tr}_{\mathcal{A}} \left[h \pi_h(Z) \star Z \right]$$

where $q := hd$ and $\pi_h(h) = -h$. Equivalently

$$S = \int_{\mathcal{M}_9} \mathrm{Tr}_{\mathcal{A}} \left(P \star F^X + \tfrac{1}{3}\, P \star P \star P \right)$$

Invariant under $\delta Z = qZ + [Z, \theta]_\star$.

The component form of the action:

$$S = \int_{\mathcal{M}_9} \mathrm{Tr}_{\mathcal{A}} \Big[V \star \Big(F + B \star \widetilde{B} \Big) + \widetilde{U} \star DB + \tfrac{1}{3} V^{\star 3} - V \star U \star \widetilde{U}$$
$$+ \widetilde{V} \star \Big(\widetilde{F} + \widetilde{B} \star B \Big) + U \star \widetilde{D}\widetilde{B} + \tfrac{1}{3} \widetilde{V}^{\star 3} - \widetilde{V} \star \widetilde{U} \star U \Big] \, ,$$

- The terms cubic in Lagrange multipliers require the duality extended field content. This has significant effects on the Feynman rules.

- Using the boundary condition $P|_{\partial\mathcal{M}_9} = 0$, the field equation reduce to $dA + A \star A + B \star \widetilde{B} = 0$, etc but note that these are duality extended.

- We choose $\mathcal{Z}$ to be topologically $S^2 \times S^2$ obtained from $\mathbb{R}^4$ by adding suitable points at infinity to $\mathbb{R}^4$.

- For $\mathcal{X}_5$, one simple alternative is the semi-infinite cylinder

$$\mathcal{X}_5 = \mathcal{X}_4 \times [0, \infty) \, , \qquad \partial\mathcal{X}_4 = \emptyset \, .$$

Higher spin algebra and trace operations

$$\mathcal{A} = \mathcal{F} \otimes \mathcal{W} \otimes \mathcal{K}$$

Here $\mathcal{F}$ is the Frobenius algebra, $\mathcal{W} := \mathcal{W}(k_y, \bar{k}_{\bar{y}})|_{k_y=0=\bar{k}_{\bar{y}}}$, the extended Weyl algebra is defined as

$$\mathcal{W}(k_y, \bar{k}_{\bar{y}}) = \bigoplus_{m,\bar{m}=0,1} \mathrm{Aq}(2; k_y, \bar{k}_{\bar{y}}) \star (\kappa_y)^{\star m} \star (\bar{\kappa}_{\bar{y}})^{\star \bar{m}}$$

The Weyl algebra $\mathrm{Aq}(2; k_y, \bar{k}_{\bar{y}})$ consists of star polynomials of $(y^\alpha, \bar{y}^{\dot{\alpha}})$ obeying the algebra stated before and outer Klein operators $(k_y, \bar{k}_{\bar{y}})$ obey

$$\{k_y, y^\alpha\}_\star = 0 = \{\bar{k}_{\bar{y}}, \bar{y}^{\dot{\alpha}}\}_\star \ , \qquad k_y \star k_y = \bar{k}_{\bar{y}} \star \bar{k}_{\bar{y}} = 1 \ ,$$

$$[\bar{k}_{\bar{y}}, y^\alpha]_\star = 0 = [k_y, \bar{y}^{\dot{\alpha}}]_\star \ , \qquad [k_y, \bar{k}_{\bar{y}}]_\star = 0 \ .$$

Inner Klein operators

$$\kappa_y = 2\pi\delta^2(y) \qquad \bar{\kappa}_{\bar{y}} = 2\pi\delta^2(\bar{y})$$

required to generate the vacuum expectation values for $\widetilde{B}$

$$\mathcal{K} = \{1, k, \bar{k}, k \star \bar{k}\} \ , \qquad k = k_y \star k_z \ , \quad \bar{k} = \bar{k}_{\bar{y}} \star \bar{k}_{\bar{z}}$$

where $(k_z \ , \bar{k}_{\bar{z}})$ obey relations similar to those of $(k_z \ , \bar{k}_{\bar{z}})$

- The trace operation in the action sets $(k, \bar{k}, \kappa_y, \bar{\kappa}_{\bar{y}})$ to zero and takes the trace of 2×2 matrix.

- In performing the star products, the Y and Z dependence is factorized (call this separation of variables) but then the ensuing star product can be performed in normal order.

- The central off-mass shell theorem has been verified but higher order perturbative analysis is under study.

- General variation of the action requires $P|_{\partial\mathcal{M}_9} = 0$ and invariance under ϵ^P transformation requires $\epsilon^P|_{\partial\mathcal{M}_9} = 0$.

Reduction of the system

Letting $A = \widetilde{A} \equiv W$ and choosing a suitable ansatz for $\widetilde{B}$ we obtain from the FCS field equations $D_W B = 0$ and

$$F_W + \mathcal{V} \star J - \overline{\mathcal{V}} \star \overline{J} + \mathcal{U}_0 \star J \star \overline{J} + \mathcal{U}_1 \star J_{[2]} + \mathcal{U}_2 \star J_{[4]} = 0$$

where $\mathcal{V}$ and U are functions of B vanishing at $B = 0$, while $J_{[2]}$ and $J_{[4]}$ are central, closed but not exact forms on $\mathcal{X}_4$. These are objects we choose depending on the topology of $\mathcal{X}_4$.

Compare with Vasiliev's extended system, namely $D_W B = 0$ and

$$F_W + \mathcal{V} \star J - \overline{\mathcal{V}} \star \overline{J} + \left(g^{-2} + \mathcal{U}_0\right) \star J \star \overline{J} + J_{[2]} + J_{[4]} = 0$$

where g is a constant, and $J_{[2]}$ and $J_{[4]}$ are central, closed but not exact forms on $\mathcal{X}_4$. These are objects defined by the above equation.

The integral of $\mathcal{L}_{[4]}$ has been proposed as a candidate for the generating functional of correlators in the AdS_4/CFT$_3$ higher spin holography [Vasiliev'15].

Our approach, instead, is as follows:

Add Chern classes to our FCS action as follows:

$$S_{\text{tot}} = S + \sum_{p,n} \int_{\mathcal{X}_{2p} \times \mathcal{Z}_4} \text{Tr}_{\mathcal{A}} \left(c_n F^{\star n} + \tilde{c}_n \widetilde{F}^{\star n} \right) \ ,$$

where $\mathcal{X}_{2p} \subseteq \mathcal{X}_4$, possibly augmented by Chern-Simons forms at boundary!

On shell these Chern classes reduce to

$$\int_{\mathcal{X}_0 \times \mathcal{Z}_4} \text{Tr}_{\mathcal{A}} (B \star \widetilde{B})^{\star n} \quad \text{for} \quad n = 1, 2$$

The consequences for amplitude computation under investigation.

Generalizations

Consider the general cubic action

$$S = \int_{\mathcal{M}} \mathrm{Tr}_{\mathcal{H}} \Big[\big(\tfrac{1}{2} \, A^I \star dA^J \, \Sigma_{IJ} + \tfrac{1}{2} \, B^P \star dB^Q \, \Omega_{PQ} + \tfrac{1}{3} \, t_{IJK} A^I \star A^J \star A^K + s_{IPQ} \, A^I \star B^P \star B^Q \Big] + \tfrac{1}{4} \oint_{\partial\mathcal{M}} \mathrm{Tr}_{\mathcal{H}} [A^I \star A^J \, \Theta_{IJ} + B^P \star B^Q \, \Xi_{PQ}]$$

$\mathcal{M}$ is a noncommutative manifold of dimension $2n+1$ and

$$A^I = A^I_{[1]} + A^I_{[3]} + \cdots + A^I_{[2n+1]} \ , \qquad I = 1, \dots, N_+$$
$$B^P = B^P_{[0]} + B^P_{[2]} + \cdots + B^P_{[2n]} \ , \qquad P = 1, \dots, N_-$$

are differential forms valued in an associative algebra $\mathcal{H}$ with (cyclic) trace operation $\mathrm{Tr}_{\mathcal{H}}$.

Appropriately cyclic trace operations, holding of Leibniz' rule holds together with Stokes' theorem requires

$$\Sigma_{IJ} = \Sigma_{JI} \ , \qquad \Omega_{PQ} = -\Omega_{QP} \ , \qquad t_{IJK} = t_{JKI} \ ,$$
$$\Theta_{IJ} = -\Theta_{JI} \ , \qquad \Xi_{PQ} = \Xi_{QP} \ .$$

We assume that Σ_{IJ} and Ω_{PQ} are nondegenerate with inverses defined by

$$\Omega^{RP}\Omega_{QP} = \delta^R_Q \ , \qquad \Sigma^{IK}\Sigma_{JK} = \delta^I_J \ .$$

The total variation

$$\delta S = \int_{\mathcal{M}} \mathrm{Tr}_{\mathcal{H}} \left[\delta A^I \star R^J \, \Sigma_{IJ} + \cdots \right]$$

where the generalized curvatures are defined as

$$R^I := dA^I + t^I{}_{JK} \, A^J \star A^K + s^I{}_{PQ} \, B^P \star B^Q \ , \qquad R^P := \cdots$$

Indices raised and lowered as $A^I = \Sigma^{IJ} A_J$ and $B^P = \Omega^{PQ} B_Q$. Curvatures obey Bianchi identities $dR^I + \cdots \equiv 0$ provided that

$$\begin{aligned} t^J{}_{MK} \, t^I{}_{LJ} &= t^J{}_{LM} \, t^I{}_{JK} \ , \\ s^I{}_{PQ} \, s_{IRT} &= -s^I{}_{TP} \, s_{IQR} \ , \\ s_{JP}{}^R \, s_{IRQ} &= -t^K{}_{IJ} \, s_{KPQ} \ , \\ s_I{}^P{}_Q \, s_{JPR} &= s_{JQ}{}^P \, s_{IRP} \end{aligned}$$

Implied Cartan gauge transformations:

$$\delta A^I = d\epsilon^I + \cdots \ , \qquad \delta B^P = d\eta^P + \cdots$$

Constraints on the couplings are equivalent to the existence of a $\mathbb{Z}_2$-graded associative algebra

$$\mathcal{F} = \mathcal{F}^+ \oplus \mathcal{F}^-$$

where

$$\mathcal{F}^+ = \bigoplus_{I=1}^{N_+} \mathbb{C} \otimes e_I \ , \qquad \mathcal{F}^- = \bigoplus_{P=1}^{N_-} \mathbb{C} \otimes f_P$$

with generators obeying the product laws

$$e_I e_J = e_K \, t^K{}_{IJ} \ , \qquad f_P f_Q = -e_I \, s^I{}_{PQ}$$

$$e_I f_R = -f_P s_{IR}{}^P \ , \qquad f_R e_I = f_P \, s_I{}^P{}_R \ ,$$

with a non-degenerate bilinear form

$$(e_I, e_J)_{\mathcal{F}} = \Sigma_{IJ} \ , \qquad (f_P, f_Q)_{\mathcal{F}} = \Omega_{PQ}$$

obeying the invariance condition $(a, bc)_{\mathcal{F}} = (ab, c)_{\mathcal{F}}$ and the graded symmetry property $(a, b^{\pm})_{\mathcal{F}} = \pm(b^{\pm}, a)_{\mathcal{F}}$.

Define the master fields

$$\boldsymbol{A} := \sum_I A^I e_I \ , \qquad \boldsymbol{B} := \sum_P B^P f_P$$

$$\boldsymbol{A}_\pm = \tfrac{1}{2}(1 \pm \Theta)\boldsymbol{A} \ , \qquad \boldsymbol{B}_\pm = \tfrac{1}{2}(1 \pm \Xi)\boldsymbol{B}$$

Analyzing the general variation and gauge invariance of the action, the requirement of globally well defined action leads to

$$\Theta^2 = \mathrm{Id}_{\mathcal{F}+} \ , \qquad \Xi^2 = \mathrm{Id}_{\mathcal{F}-}$$

and that the structure group is gauged by $\boldsymbol{A}_+$ and $\boldsymbol{B}_+$. Defining

$$(\mathcal{A}, \mathcal{B}; \overline{\mathcal{U}}, \overline{\mathcal{V}}) := (\boldsymbol{A}_+, \boldsymbol{B}_+; \boldsymbol{B}_-, \boldsymbol{A}_-)$$

we end up with the action based on quasi-Frobenius algebra

$$\boxed{S = \int_{\mathcal{M}} \mathrm{Tr}_{\mathcal{H}} \Big[\big(\overline{\mathcal{U}}, \mathcal{D}\mathcal{B}\big)_{\mathcal{F}} + \big(\overline{\mathcal{V}}, \mathcal{F} - \mathcal{B} \star \mathcal{B} + \tfrac{1}{3} \overline{\mathcal{V}} \star \overline{\mathcal{V}} - \overline{\mathcal{U}} \star \overline{\mathcal{U}}\big)_{\mathcal{F}} \Big]}$$

Requiring equal number of momenta and coordinate like fields requires that the range of indices I and J are the same.

Trace operation and outer Klein operator

Assuming that $\mathcal{F}$ is unital implies that the inner product is equivalent to the nondegenerate graded cyclic supertrace operation

$$\mathrm{STr}_{\mathcal{F}}(a) := (1, a)_{\mathcal{F}} \ , \qquad a \in \mathcal{F}$$

We furthermore assume that $\mathcal{F}$ contains an idempotent element h, referred to as the Klein operator of the $\mathbb{Z}_2$-graded algebra, such that

$$h a^{\pm} h = \pm a^{\pm} \ , \qquad h^2 = 1 \ , \qquad a^{\pm} \in \mathcal{F}^{\pm} \ .$$

Thus the Frobenius algebra decomposes as: $\mathcal{F} = \mathcal{F}_0 \oplus h\mathcal{F}_0$. Defining

$$Z = hX + P \ , \qquad X = \mathcal{A} + \mathcal{B} \ , \qquad P = h(\overline{\mathcal{U}} + \overline{\mathcal{V}})$$

the action becomes

$$S \quad = \quad \int_{\mathcal{M}} \mathrm{Tr}_{\mathcal{H}\otimes\mathcal{F}} \left(\tfrac{1}{2}\, Z \star qZ + \tfrac{1}{3}\, Z \star Z \star Z \right) - \tfrac{1}{4} \oint_{\partial\mathcal{M}} \mathrm{Tr}_{\mathcal{W}\otimes\mathcal{K}\otimes\mathcal{F}} \left[h\pi_h(Z) \star Z \right]$$

Same form as our original FCS action but with general Frobenius algebra with general trace operation it is equipped with.

3-grading from inner Klein operator of $\mathcal{F}_0$

The system can be constrained algebraically off-shell as to remove the top-forms and hence the zero-form constraints on-shell, provided that the algebra admits a three grading defined by

$$\mathcal{F}^{(0)} := \mathcal{F}^{+} , \qquad \mathcal{F}^{(-1)} \oplus \mathcal{F}^{(+1)} := \mathcal{F}^{-}$$

and $\mathcal{F}^{(k)} \equiv 0$ for $k = \pm 2, \pm 3, \ldots$, such that

$$\mathcal{F}^{(k)} \star \mathcal{F}^{(k')} \subseteq \mathcal{F}^{(k+k')}$$

Interesting examples to consider:

Twisted group algebras and Hopf algebras.

An example

Consider the Clifford algebra generated by $2n$ elements γ_i $(i = 1, \ldots, 2n)$ obeying $\{\gamma_i, \gamma_j\} = 2\delta_{ij}$ and take $\mathcal{F}_0 = \mathcal{C}\ell_{2n}$.

The trace operation can be defined as the projection onto the identity: $\mathrm{Tr}_{\mathcal{C}\ell_{2n}} \gamma^{i_1 \cdots i_p} = \delta_{p,0}$. The 3-grading is achieved by the inner Klein operator $\gamma = i^n \gamma_1 \cdots \gamma_{2n}$.

The resulting model consists of:

- odd forms, not containing top-forms valued, in $\frac{1}{4}(1+\gamma)\mathcal{C}\ell_{2n}(1+\gamma)$ and $\frac{1}{4}(1-\gamma)\mathcal{C}\ell_{2n}(1-\gamma)$, both isomorphic to the associative algebras $\mathrm{mat}_{2^{n-1}}(\mathbb{C})$
- even forms, with constrained zero-form and $2n$-form content, valued in $\frac{1}{4}(1+\gamma)\mathcal{C}\ell_{2n}(1-\gamma)$ and $\frac{1}{4}(1-\gamma)\mathcal{C}\ell_{2n}(1+\gamma)$, both forming $2^{n-1} \otimes 2^{n-1}$ matrices.
- This furnishes an FCS generalization Vasiliev equations with Chan-Paton factors.

Higher order corrections

Assuming that the superconnection is globally defined, our FCS action admits the following generalization by higher order terms:

$$S = \int_{\mathcal{M}} \mathrm{Tr}_{\mathcal{H}\otimes\mathcal{F}} \left[\tfrac{1}{2} Z \star qZ + \int_0^1 dt\, Z \star G(tZ) \right] - \tfrac{1}{4} \oint_{\partial\mathcal{M}} \mathrm{Tr}_{\mathcal{H}\otimes\mathcal{F}}\, h\, \pi_h(Z) \star Z \ ,$$

where $G(Z) = \sum_{p=1}^{\infty} G_{2p} Z^{\star 2p}$. The resulting field equations are:

$$qZ + G(Z) = 0$$

Select open problems:

- Obtain worldsheet description via topological open string in whose target the FCS gauge theory lives. (Under study by Per Sundell and collaborators).
- Going back to the basic FCS model, study its perturbative expansion.
- Add the CS boundary terms, quantize the model and compute the amplitudes.