$B \to M_1 M_2$ Form Factors : Beyond the Narrow-Width Limit

(Based on 1701.01633 and 1709.00173)

Javier Virto

Massachussets Institute of Technology

and

Technische Universität München

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- **Correlation function**

$$F_\mu(k,q) = i\int d^4x e^{ik\cdot x}\langle 0|\mathrm{T}\{\bar{d}(x)\gamma_\mu u(x), \bar{u}(0) i m_b \gamma_5 b(0)\}|\bar{B}^0(q+k)\rangle$$

- **Unitarity relation**

$$\begin{aligned} 2\,\mathrm{Im} F_\mu(k,q) &= \sum_\lambda (2\pi)\delta(k^2 - m_\rho^2) \underbrace{\langle 0|\bar{d}\gamma_\mu u|\rho_\lambda(k)\rangle}_{m_\rho f_\rho\, \varepsilon(\lambda)_\mu} \underbrace{\langle \rho_\lambda(k)|\bar{u} i m_b \gamma_5 b|\bar{B}^0(q+k)\rangle}_{(\varepsilon(\lambda)^\star\cdot q)\ A_0^{B\rho}(q^2)} + \cdots \\ &= q_\mu\ 4\pi\, m_\rho\, f_\rho\, A_0^{B\rho}(q^2) + \cdots \end{aligned}$$

- **Dispersion relation + LCOPE + Borel + duality**

$$2 m_\rho\, f_\rho\, A_0^{B\rho}(q^2)\, e^{-m_\rho^2/M^2} = F_{OPE}(M^2, q^2, s_0)$$

$F_{OPE}(M^2, q^2, s_0)$ = Some expression derived from $F_\mu(k,q)$, independent of hadronic states.

$B \to \pi\pi$ form factors from B LCDAs

Cheng, Khodjamirian, JV, 1701.01633

- **Correlation function**

$$F_\mu(k,q) = i\int d^4x\, e^{ik\cdot x}\langle 0|\mathrm{T}\{\bar{d}(x)\gamma_\mu u(x), \bar{u}(0) i m_b \gamma_5 b(0)\}|\bar{B}^0(q+k)\rangle$$

- **Unitarity relation**

$$2\mathrm{Im}F_\mu(k,q) = m_b \int d\tau_{2\pi}\, \underbrace{\langle 0|\bar{d}\gamma_\mu u|\pi(k_1)\pi(k_2)\rangle}_{F_\pi^\star(k^2)} \underbrace{\langle \pi(k_1)\pi(k_2)|\bar{u}\gamma_5 b|\bar{B}^0(q+k)\rangle}_{F_t(k^2,q^2,\cos\theta_\pi)} + \cdots$$

$$= q_\mu \frac{s\sqrt{q^2}[\beta_\pi(s)]^2}{4\sqrt{6}\pi\sqrt{\lambda}} F_\pi^\star(k^2)\, F_t^{(\ell=1)}(k^2,q^2) + \cdots$$

- **Dispersion relation + LCOPE + Borel + duality**

$$-\int_{4m_\pi^2}^{s_0^{2\pi}} ds\, e^{-s/M^2} \frac{s\sqrt{q^2}\,[\beta_\pi(s)]^2}{4\sqrt{6}\pi^2\sqrt{\lambda}} F_\pi^\star(s)\, F_t^{(1)}(s,q^2) = F_{OPE}(M^2,q^2,s_0)$$

$B \to \pi\pi$ form factors from B LCDAs

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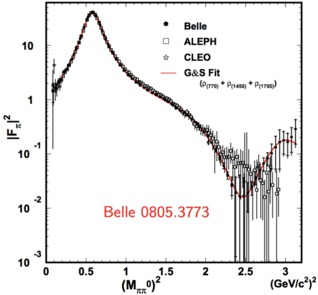
- **Collecting both sum rules:**

$$2 m_\rho\, f_\rho\, A_0^{B\rho}(q^2)\, e^{-m_\rho^2/M^2} = F_{OPE}$$

$$-\int_{4m_\pi^2}^{s_0^{2\pi}} ds\; e^{-s/M^2}\, \frac{s\,\sqrt{q^2}\,[\beta_\pi(s)]^2}{4\sqrt{6}\pi^2\sqrt{\lambda}}\, F_\pi^\star(s)\, F_t^{(1)}(s,q^2) = F_{OPE}$$



- **ρ-dominance + zero-width limit:**

$$F_\pi^\star(s) \simeq \frac{f_\rho g_{\rho\pi\pi} m_\rho/\sqrt{2}}{m_\rho^2 - s + i\sqrt{2}\Gamma_\rho(s)} \quad , \quad F_t^{(1)}(s,q^2) \simeq -\frac{\beta_\pi(s)\sqrt{\lambda}}{\sqrt{3q^2}}\, \frac{m_\rho g_{\rho\pi\pi} A_0^{B\rho}(q^2)}{m_\rho^2 - s - i\sqrt{2}\Gamma_\rho(s)}$$

$$LHS = 2 f_\rho m_\rho A_0^{B\rho}(q^2) \int_{4m_\pi^2}^{s_0^{2\pi}} ds\, e^{-s/M^2} \underbrace{\left[\frac{\sqrt{s}\,\Gamma_\rho(s)/\pi}{(m_\rho^2 - s)^2 + s\Gamma_\rho^2(s)}\right]}_{\xrightarrow{\Gamma_\rho \to 0}\ \delta(s - m_\rho^2)} \xrightarrow{\Gamma_\rho \to 0} 2 f_\rho m_\rho A_0^{B\rho}(q^2)\, e^{-m_\rho^2/M^2}$$

hep-ph/0611193 ✓

(Similar for all other FFs)

$B \to \pi\pi$ form factors from B LCDAs

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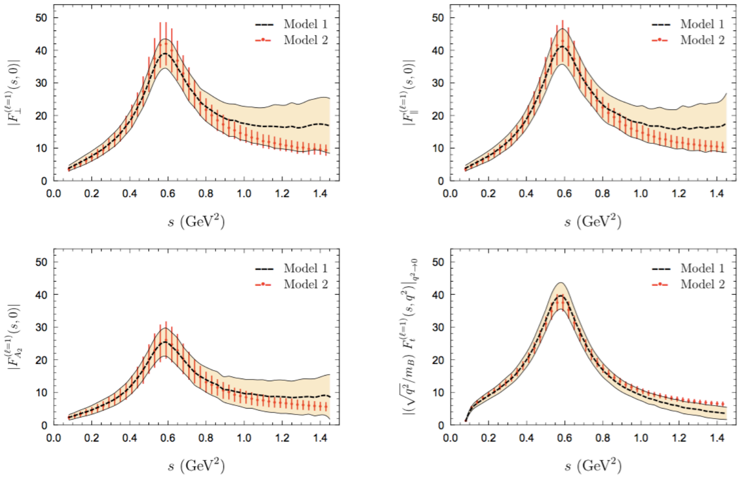
- **One-resonance model : Finite-width effects in $B \to \rho$ form factors**

	$V^{B\rho}(0)$	$A_1^{B\rho}(0)$	$A_2^{B\rho}(0)$	$A_0^{B\rho}(0)$
Inputs of KMO'06	0.31	0.23	0.19	0.26
Updated inputs	0.34	0.26	0.21	0.30
Gaussian scan	0.36 ± 0.17	0.27 ± 0.13	0.22 ± 0.15	0.30 ± 0.06
BSZ'15 (ρ-DAs)	0.33 ± 0.03	0.26 ± 0.03	0.23 ± 0.04	0.36 ± 0.04
Full F_π, $M^2 = 1\,\mathrm{GeV}^2$	0.40 ± 0.19	0.30 ± 0.14	0.24 ± 0.16	0.33 ± 0.07
Final results for ρ-model	0.41 ± 0.11	0.31 ± 0.08	0.25 ± 0.10	0.34 ± 0.04

$\Rightarrow$ Finite-width effects at the level of $\sim 10\%$

☐ **Same analysis for $B \to K\pi$ in progress.** S. Descotes-Genon, A. Khodjamirian, JV

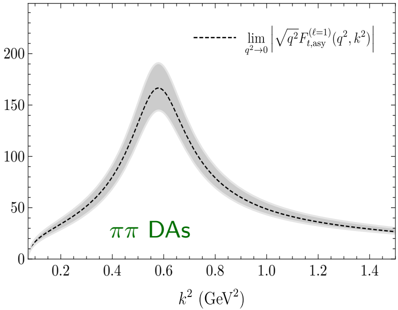
- **Other models with two or three resonances :**



The suppression of F_π outside the ρ hinders sensitivity to ρ', ρ''.

$B \to \pi\pi$ form factor ($F_t^{\ell=1}$)

Cheng, Khodjamirian, JV, 1709.00173



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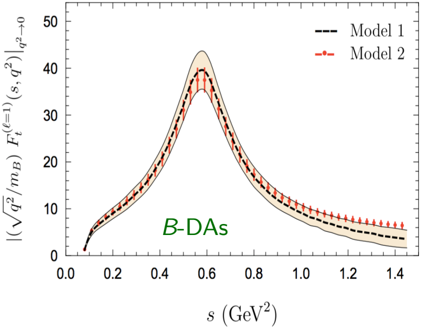

- Both approaches give consistent results
- The results on the left complement Hambrock, Khodjamirian 2015 for $F_\perp$, $F_\parallel$