TRR110 Workshop, Munich July 11-13, 2018

Université Paris Sud
Comprendre le monde, construire l'avenir®

IPN
Institut de Physique Nucléaire Orsay

Isospin and chiral symmetries in $D \to \bar{K}\pi\pi$ amplitudes in the Khuri-Treiman formalism

Bachir Moussallam

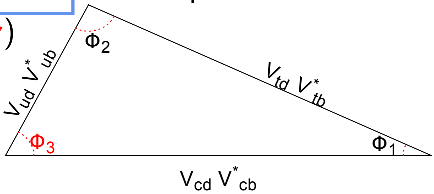
with:
Emi Kou (LAL, Orsay)
Tetiana Moskalets (LAL, Orsay)

Introduction:

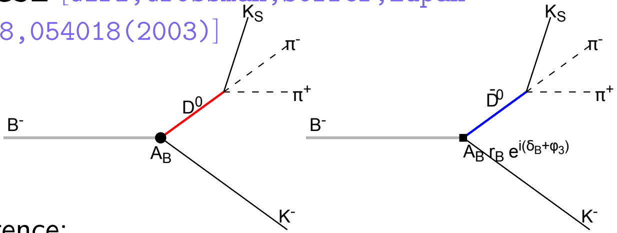
- $D \to \bar{K}\pi\pi$ nice laboratory for 3-body final-state interactions
  Previous work: [Franz Niecknig, Bastian Kubis, JHEP 1510,142 (2015), P.L. B780,471 (2018)]:Khuri-Treiman
  Also: [S.X. Nakamura, P.R. D93,014005 (2015), P.C. Magalhães et al. P.R. D84,094001 (2011)]
- Khuri-Treiman [PR 119,1115 (1960)] renewed interest for $\eta \to 3\pi$
- Experimentally: evidence for exotic κ meson in $D^+ \to K^-\pi^+\pi^+$ [E791, PRL 89,121801 (2001)]

  → But: κ not seen in D^0 decays ?

- Interest in $D^0 \to K_S\pi^-\pi^+$ mode: precision measurement of CKM angle ϕ_3 (or γ)



- Idea GGSZ [Giri,Grossman,Soffer,Zupan P.R.D68,054018(2003)]



Interference:

$$|\mathcal{A}_{B\mp}|^2 \sim \cdots + 2r_B \cos\left(\phi_{D^0}(s,t,u) - \phi_{D^0}(u,t,s) - \delta_B \mp \phi_3\right)$$

$$\phi_{D^0}(s,t,u) = Arg[\mathcal{A}(D^0 \to K_S\pi^-\pi^+)]$$

Outline:

- Introduction
- Isospin symmetry in $D \to \bar{K}\pi\pi$ (general)
- Isospin symmetry + S, P-waves dominance
- Khuri-Treiman equations for D^+, D^0 amplitudes
- Soft pions relations
- KT solutions and some results (for D^+)
- Summary/outlook

Isospin symmetry in $D \to \bar{K}\pi\pi$ (general)

- **Key point:** H_W has definite isospin transformation property

$$H_W = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \left[C_1\, O_1 + C_2\, O_2 \right] + h.c.$$

with

$$O_1 = \bar{s}_i \gamma^\mu (1-\gamma^5) c_j \, \bar{u}_j \gamma_\mu (1-\gamma^5) d_i$$
$$O_2 = \bar{s}_i \gamma^\mu (1-\gamma^5) c_i \, \bar{u}_j \gamma_\mu (1-\gamma^5) d_j$$

are pure $I = 1$ operators

→ Use Wigner-Eckart theorem

$$\langle I_1 m_1 | T_{I,I_z} | I_2 m_2 \rangle = C^{I\, I_2\, I_1}_{I_z\, m_2\, m_1} \, \mathcal{F}^{I_1\, I_2}_{I}$$

- Consider $\langle D\pi|H_W|\bar{K}\pi\rangle$, $\langle DK|H_W|\pi\pi\rangle$, Isospin amplitudes:

$$\vec{\mathcal{F}}(s,t,u) = \begin{pmatrix} \mathcal{F}^{\frac{3}{2}\frac{3}{2}}(s,t,u) \\ \mathcal{F}^{\frac{3}{2}\frac{1}{2}}(s,t,u) \\ \mathcal{F}^{\frac{1}{2}\frac{3}{2}}(s,t,u) \\ \mathcal{F}^{\frac{1}{2}\frac{1}{2}}(s,t,u) \end{pmatrix} \quad \vec{\mathcal{G}}(s,t,u) = \begin{pmatrix} \mathcal{G}^{10}(s,t,u) \\ \mathcal{G}^{12}(s,t,u) \\ \mathcal{G}^{01}(s,t,u) \\ \mathcal{G}^{11}(s,t,u) \end{pmatrix}$$

Physical amplitudes (all orderings)

$$\begin{aligned} \mathcal{A}_1 &: \quad D^+\pi^- \to K^-\pi^+ & \mathcal{A}_4 &: \quad D^0\pi^- \to K^-\pi^0 \\ \mathcal{A}_2 &: \quad D^+\pi^- \to \overline{K}^0\pi^0 & \mathcal{A}_5 &: \quad D^0\pi^0 \to K^-\pi^+ \\ \mathcal{A}_3 &: \quad D^+\pi^0 \to \overline{K}^0\pi^+ & \mathcal{A}_6 &: \quad D^0\pi^- \to \overline{K}^0\pi^- \\ & & \mathcal{A}_7 &: \quad D^0\pi^0 \to \overline{K}^0\pi^0 \\ & & \mathcal{A}_8 &: \quad D^0\pi^+ \to \overline{K}^0\pi^+ \end{aligned}$$

express with $\mathcal{F}^{KK'}$ (or $G^{II'}$). Derive crossing matrices

$$\vec{\mathcal{F}}(s,t,u) = C_{us}\vec{\mathcal{F}}(u,t,s), \ \vec{\mathcal{G}}(s,t,u) = C_{st}\vec{\mathcal{F}}(t,s,u)$$

Four (general) isospin relations

Symmetric:

$$\begin{aligned}
\mathcal{A}_1(s,t,u) &= -\sqrt{2}\,(\mathcal{A}_2(s,t,u)+\mathcal{A}_2(u,t,s)) \\
\mathcal{A}_1(s,t,u) &= \quad \sqrt{2}\,(\mathcal{A}_4(s,t,u)+\mathcal{A}_4(u,t,s)) \\
\mathcal{A}_1(s,t,u) &= \mathcal{A}_6(s,t,u)+\mathcal{A}_6(u,t,s)-2\mathcal{A}_7(s,t,u)
\end{aligned}$$

Antisymmetric:

$$\begin{aligned}
\mathcal{A}_2(s,t,u)-\mathcal{A}_2(u,t,s) = &\quad \mathcal{A}_4(s,t,u)-\mathcal{A}_4(u,t,s) \\
&-\sqrt{2}\quad (\mathcal{A}_6(s,t,u)-\mathcal{A}_6(u,t,s))
\end{aligned}$$

Isospin symmetry+ S, P dominance

- Dominance of S, P-waves is a general feature of D, B decay amplitudes. Expression with one-variable functions

$$\begin{aligned}\mathcal{A}(s,t,u) = \quad & F_0(s) + Z_s F_1(s) \\ & + \tilde{F}_0(u) + Z_u \tilde{F}_1(u) \\ & + G_0(t) + (s-u) G_1(t)\end{aligned}$$

with

$$Z_s = s(t-u) + \Delta, \quad Z_u = u(t-s) + \Delta$$

→ Holds in isobar model

- Derivation from dispersion relations [Khuri, Treiman, (1960), Fuchs, Sazdjian, Stern (1993)]

  Dispersive integrals over $J \geqslant 2$ waves approximated by polynomials $P_n(s,t,u)$

  → One-variable functions are analytic with right-hand cut

  → They can absorb P_n if $n \leqslant 2$

  → Since $s + t + u = m_D^2 + m_K^2 + 2m_\pi^2$, a family of polynomial redefinitions leave the physical amplitude invariant

  Fixed by imposing $z = 0$ conditions

Application to $D \to K\pi\pi$ isospin amplitudes

- With S, P dominance:

$$
\begin{aligned}
\vec{\mathfrak{F}}(s,t,u) = \;& \vec{F}_0(s) + Z_s\,\vec{F}_1(s) \\
& + \mathbf{C_{us}} \left(\vec{F}_0(u) + Z_u\,\vec{F}_1(u) \right) \\
& + \mathbf{C_{st}^{-1}} \left(\vec{G}_0(t) + (s-u)\,\vec{G}_1(t) \right)
\end{aligned}
$$

$\vec{F}_0$, $\vec{F}_1$: 8 functions
$\vec{G}_0$, $\vec{G}_1$: 4 functions

- A linear redefinition can be performed

$$\vec{F}_j \to \mathbf{U}_F \vec{F}_j, \quad \vec{G}_j \to \mathbf{U}_G \vec{G}_j,$$

(which mix $D\pi$ or DK isospins) such that

→ D^+ amplitudes involve only $\boxed{6}$ functions [Niecknig, Kubis (2015)]

$[F_0^{3/2}, F_0^{1/2}, F_1^{3/2}, F_1^{1/2}, G_0^2, G_1^1]$

→ D^0 amplitudes involve $\boxed{6+6}$ functions

$+ [H_0^{3/2}, H_0^{1/2}, H_1^{3/2}, H_1^{1/2}, G_0^0, \widetilde{G}_1^1]$

Expressions of D^+ amplitudes

- $D^+ \to K^- \pi^+ \pi^+$

$$\mathcal{A}_1(s,t,u) = -\sqrt{2}\Big[F_0^{3/2}(s) + F_0^{1/2}(s) + Z_s\big(F_1^{3/2}(s) + F_1^{1/2}(s)\big) + (s \leftrightarrow u)\Big] + G_0^2(t)$$

- $D^+ \to \overline{K}^0 \pi^0 \pi^+$

$$\mathcal{A}_2(s,t,u) = -2F_0^{3/2}(s) + F_0^{1/2}(s) + Z_s\big(-2F_1^{3/2}(s) + F_1^{1/2}(s)\big) + 3F_0^{3/2}(u) + 3\,Z_u F_1^{3/2}(u) - \frac{\sqrt{2}}{4}G_0^2(t) + (s-u)G_1^1(t)$$

Same as [Niecknig, Kubis (2015)]

D^0 amplitudes

- $D^0 \to K^-\pi^0\pi^+$

$$\begin{aligned}\mathcal{A}_4(s,t,u) = &-2F_0^{3/2}(s) + F_0^{3/2}(u) - F_0^{1/2}(u) - 2Z_s F_1^{3/2}(s)\\ &+Z_u\left(F_1^{3/2}(u) - F_1^{1/2}(u)\right) + \tfrac{\sqrt{2}}{4}G_0^2(t) + (s-u)G_1^1(t)\\ &+\sqrt{2}\Big[H_0^{3/2}(s) - H_0^{3/2}(u) - \tfrac{1}{2}\left(H_0^{1/2}(s) - H_0^{1/2}(u)\right)\\ &+Z_s H_1^{3/2}(s) - Z_u H_1^{3/2}(u) - \tfrac{1}{2}\left(Z_s H_1^{1/2}(s) - Z_u H_1^{1/2}(u)\right)\Big]\\ &-2(s-u)\widetilde{G}_1^1(t)\end{aligned}$$

- $D^0 \to \overline{K}^0 \pi^- \pi^+$

$$\mathcal{A}_6(s,t,u) = \sqrt{2}\left[F_0^{3/2}(s) + Z_s F_1^{3/2}(s)\right] + \tfrac{1}{6} G_0^2(t) - \left[H_0^{3/2}(s) + H_0^{1/2}(s) + Z_s\left(H_1^{3/2}(s) + H_1^{1/2}(s)\right)\right] - 3\left(H_0^{3/2}(u) + Z_s H_1^{3/2}(u)\right) - G_0^0(t) - (s-u)\widetilde{G}_1^1(t)$$

- $D^0 \to \overline{K}^0 \pi^0 \pi^0$

$$\mathcal{A}_7(s,t,u) = \sqrt{2}\left[F_0^{3/2}(s) + \tfrac{1}{2} F_0^{1/2}(s) + Z_s\left(F_1^{3/2}(s) + \tfrac{1}{2} F_1^{1/2}(s)\right) + (s \leftrightarrow u)\right] - \tfrac{1}{3} G_0^2(t) - \left[2H_0^{3/2}(s) + \tfrac{1}{2} H_0^{1/2}(s) + Z_s\left(2H_1^{3/2}(s) + \tfrac{1}{2} H_1^{1/2}(s)\right) + (s \leftrightarrow u)\right] - G_0^0(t)$$

Khuri-Treiman equations

- Consider partial-waves ($j = 0, 1$)

$$\mathcal{F}_j^K(s) \equiv \int dz_s P_j(z_s) \langle D\pi | H_W | \bar{K}\pi \rangle^K$$
$$\mathcal{G}_j^I(t) \equiv \int dz_t P_j(z_t) \langle DK | H_W | \pi\pi \rangle^I$$

- Impose elastic unitarity relations

$$\begin{aligned} \text{disc}[\mathcal{F}_j^K(s)] &= \exp(-i\delta_j^K(s)) \sin(\delta_j^K(s)) \mathcal{F}_j^K(s) \\ \text{disc}[\mathcal{G}_j^I(t)] &= \exp(-i\delta_j^I(t)) \sin(\delta_j^I(t)) \mathcal{G}_j^I(t) \end{aligned}$$

$\delta_j^K(s)$: πK phase shifts
$\delta_j^K(t)$: $\pi\pi$ phase shifts

- Combine unitarity with dispersion relations for one-variable functions: (linear asymptotic behaviour) e.g.

$$F_0^{1/2}(s) = \alpha + \beta s + \frac{s^2}{\pi}\int_{{m_K+m_\pi}^2}^{\infty} \frac{\exp(-i\delta_0^{1/2}(s'))\sin(\delta_0^{1/2}(s'))}{(s')^2(s'-s)}\, ds' \times \left(F_0^{1/2}(s') + \widehat{F}_0^{1/2}(s')\right)$$

$\widehat{F}_0^{1/2}(s)$: linear combination of angular integrals over F, G functions.

→ Use Muskhelishvili-Omnès to transform into nonsingular equations [Neveu, Scherk, Ann. Phys.57,39 (1970)]

KT equations for D^+

$$
\begin{aligned}
F_0^{3/2}(s) &= \Omega_0^{3/2}(s)\left[C_0 s + s^2 \hat{I}_{0F}^{3/2}(s)\right] \\
F_0^{1/2}(s) &= \Omega_0^{1/2}(s)\left[C_1 s + C_2 s^2 + s^3 \hat{I}_{0F}^{1/2}(s)\right] \\
F_1^{3/2}(s) &= \Omega_1^{3/2}(s)\hat{I}_{1F}^{3/2}(s) \\
F_1^{1/2}(s) &= \Omega_1^{1/2}(s)\left[C_3 + s\hat{I}_{1F}^{1/2}(s)\right] \\
G_0^2(t) &= \Omega_0^2(t)\left[C_4 t + t^2 \hat{I}_{0G}^2(t)\right] \\
G_1^1(t) &= \Omega_1^1(t)\left[C_5 t + t^2 \hat{I}_{1G}^1(t)\right]
\end{aligned}
$$

→ Omnès functions, e.g.

$$
\Omega_0^{1/2}(s) = \exp\left[\frac{s}{\pi}\int_{(m_\pi+m_K)^2}^{\infty} ds' \frac{\delta_0^{1/2}(s')}{s'(s'-s)}\right]
$$

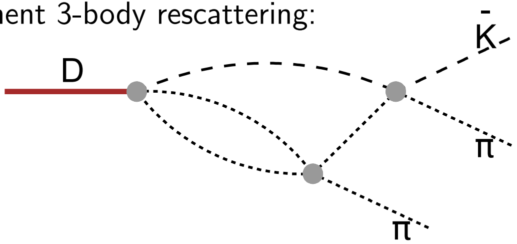
→ Asymptotic phase shifts assumed

$$\delta_0^{3/2}(\infty) = \delta_1^{3/2}(\infty) = \delta_0^2(\infty) = 0$$
$$\delta_0^{1/2}(\infty) = \delta_1^{1/2}(\infty) = \delta_0^0(\infty) = \delta_1^1(\infty) = \pi$$

→ $\hat{I}$ integrals

$$\hat{I}_{0F}^{1/2}(s) = \frac{1}{\pi}\int_{(m_\pi+m_K)^2}^{\infty} \frac{ds' \sin\delta_0^{1/2}(s')\hat{F}_0^{1/2}(s')}{(s')^3(s'-s)|\Omega_0^{1/2}(s')|}$$

Implement 3-body rescattering:



→ Unicity conditions here:

$$G_0^2(0) = G_1^1(0) = 0, \quad F_0^{3/2}(0) = F_0^{1/2}(0) = 0$$

Easy comparison with isobar-type model

Alternatively, e.g.

$$G_0^2(0) = \dot{G}_0^2(0) = 0, \quad F_0^{3/2}(0) = \dot{F}_0^{3/2}(0) = 0$$

Used by [Niecknig, Kubis (1999)]

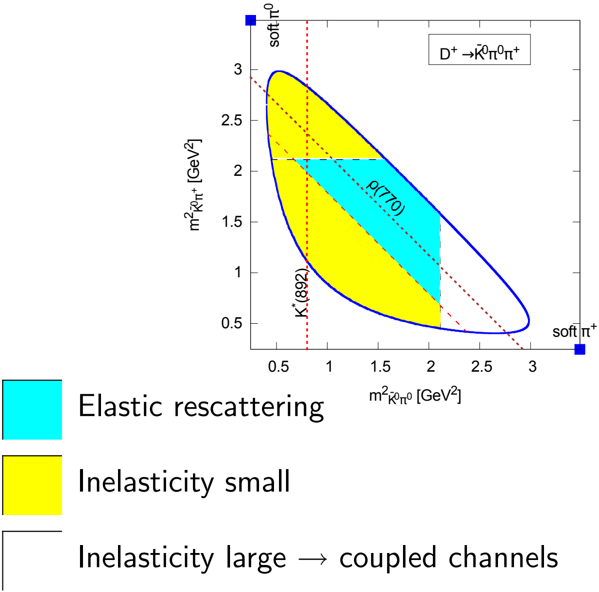
Supplementary KT equations for D^0

$$
\begin{aligned}
H_0^{3/2}(s) &= \Omega_0^{3/2}(s)\left[D_0 + s^2 \hat{I}_{0H}^{3/2}(s)\right]\\
H_0^{1/2}(s) &= \Omega_0^{1/2}(s)\left[D_1 s + D_2 s^2 + s^3 \hat{I}_{0H}^{1/2}(s)\right]\\
H_1^{3/2}(s) &= \Omega_1^{3/2}(s)\hat{I}_{1H}^{3/2}(s)\\
H_1^{1/2}(s) &= \Omega_1^{1/2}(s)\left[D_3 + s \hat{I}_{1H}^{1/2}(s)\right]\\
G_0^{0}(t) &= \Omega_0^{0}(t)\left[D_4 t + D_5 t^2 + t^3 \hat{I}_{0G}^{2}(t)\right]\\
\widetilde{G}_1^{1}(t) &= \Omega_1^{1}(t)\left[D_6 t + t^2 \hat{I}_{1\tilde{G}}^{1}(t)\right]
\end{aligned}
$$

Note: linear dep. on D_i and C_i parameters

Soft pions relations

Elastic/inelastic rescattering in $D^+ \to \overline{K}^0 \pi^0 \pi^+$:



Elastic rescattering

Inelasticity small

Inelasticity large $\longrightarrow$ coupled channels

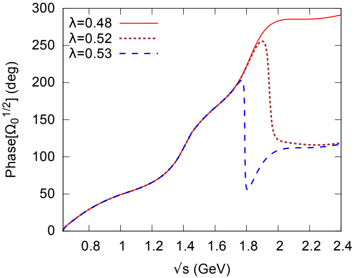
- "Optimal" choice for Omnès phase in inelastic region ?
  - → From two-channel Omnès matrix:

$$\boldsymbol{\Omega} = \begin{pmatrix} \Omega_{11}(s) & \Omega_{12}(s) \\ \Omega_{21}(s) & \Omega_{22}(s) \end{pmatrix}$$

  - → Effective one-channel Omnès function:

$$\Omega^{eff}(s) = \Omega_{11}(s) + \lambda\, \Omega_{12}(s)$$

  - → πK phase as a function of λ



  - → Probe this with soft pion relations ?

Soft pions relations

- Exact in $SU(2)$ chiral limit: should hold up to $O(m_\pi^2/\Lambda^2)$, $\Lambda \simeq 1$ GeV (in general)
- Extra constraints could eliminate unphysical fits (e.g. $\eta \to 3\pi$)
- Relate $D \to K\pi\pi$ to $D \to K\pi$, e.g.

$$\lim_{p\to 0} \langle \overline{K}^0 \pi^+ \pi^0(p) | H_W | D^+ \rangle = -\frac{i}{F_\pi} \langle \overline{K}^0 \pi^+ | [Q_5^3, H_W] | D^+ \rangle = \frac{i}{F_\pi} \langle \overline{K}^0 \pi^+ | H_W | D^+ \rangle$$

[Gronau, PRL 83,4005 (1999)]

- From 3 experimental modes

$$D^+ \to \overline{K}^0 \pi^+$$
$$D^0 \to \overline{K}^0 \pi^0$$
$$D^0 \to K^- \pi^+$$

+Isospin symmetry, one derives

$$|\mathcal{A}^{(3/2)}_{D\to K\pi}| = (1.377 \pm 0.024) \cdot 10^{-6}\ \text{GeV}$$

$$|\mathcal{A}^{(1/2)}_{D\to K\pi}| = (3.730 \pm 0.039) \cdot 10^{-6}\ \text{GeV}$$

$$\phi^{(1/2)} - \phi^{(3/2)} = \pm(91.4 \pm 3.0)^\circ$$

- Consider all $D \to K\pi\pi$ amplitudes: 8 soft pion relations

  → 2 compatibility equations

  → 2 relations:

$$\frac{i\,\mathcal{A}^{(3/2)}_{D\to K\pi}}{F_\pi} = 3F_0^{3/2}(m_D^2) - 2F_0^{3/2}(m_K^2) + F_0^{1/2}(m_K^2) - \frac{\sqrt{2}}{4}G_0^2(m_\pi^2) - (m_D^2 - m_K^2)G_1^1(m_\pi^2)$$

$$\frac{i\,\mathcal{A}^{(1/2)}_{D\to K\pi}}{F_\pi} = -3F_0^{1/2}(m_D^2) - 2F_0^{1/2}(m_K^2) - 5F_0^{3/2}(m_K^2) + \frac{5\sqrt{2}}{4}G_0^2(m_\pi^2) - (m_D^2 - m_K^2)G_1^1(m_\pi^2)$$

Note: only D^+ functions involved

Solving KT equations and some results

Numerical resolution

- Discretise integration regions:

$$
\begin{cases}
s_1 = (m_K + m_\pi)^2, s_2, \cdots, s_{N_s} \\
u_1 = (m_K + m_\pi)^2, u_2 = s_2, \cdots, u_{N_s} = s_{N_s} \\
t_1 = 4m_\pi^2, t_2, \cdots, t_{N_t}
\end{cases}
$$

- Values of amplitude functions on grid

$$
\mathbb{F} \equiv \begin{pmatrix} F_0^{3/2}(s_1) \\ \vdots \\ F_0^{3/2}(s_{N_s}) \\ \vdots \\ G_1^1(t_1) \\ \vdots \\ G_1^1(t_{N_t}) \end{pmatrix} \qquad \mathbb{H} \equiv \begin{pmatrix} H_0^{3/2}(s_1) \\ \vdots \\ H_0^{3/2}(s_{N_s}) \\ \vdots \\ \widetilde{G}_1^1(t_1) \\ \vdots \\ \widetilde{G}_1^1(t_{N_t}) \end{pmatrix}
$$

- Linear system of equations for $\mathbb{F}$

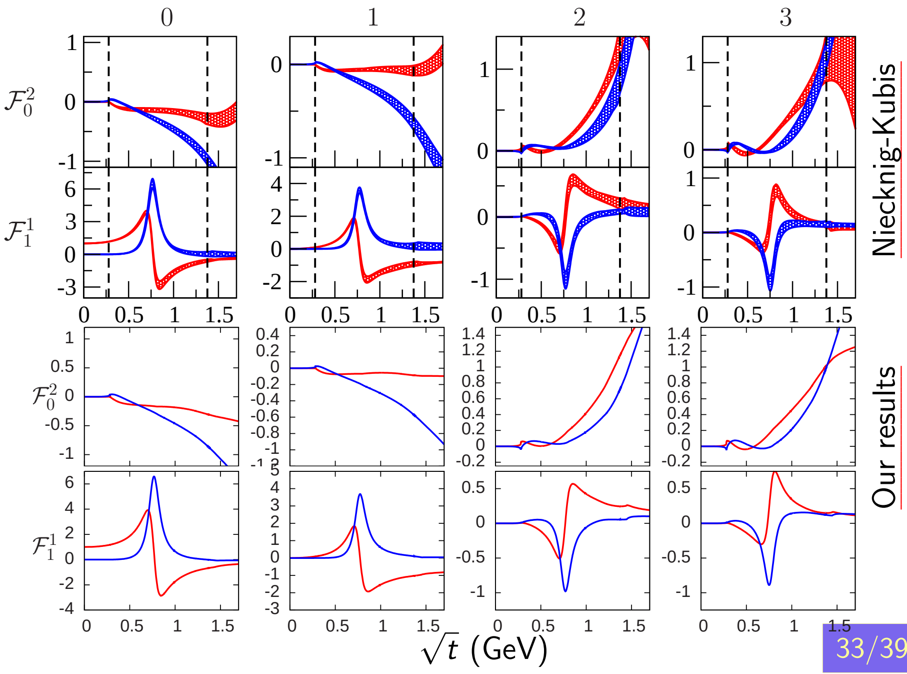
$$\mathbb{F} = \mathbb{F}_{(0)} + \mathcal{W}_I^F \times \widehat{\mathbb{F}}$$
$$\widehat{\mathbb{F}} = \widehat{\mathbb{F}}_{(0)} + \mathcal{W}_K^F \times (\mathbb{F} + \widehat{\mathbb{F}})$$

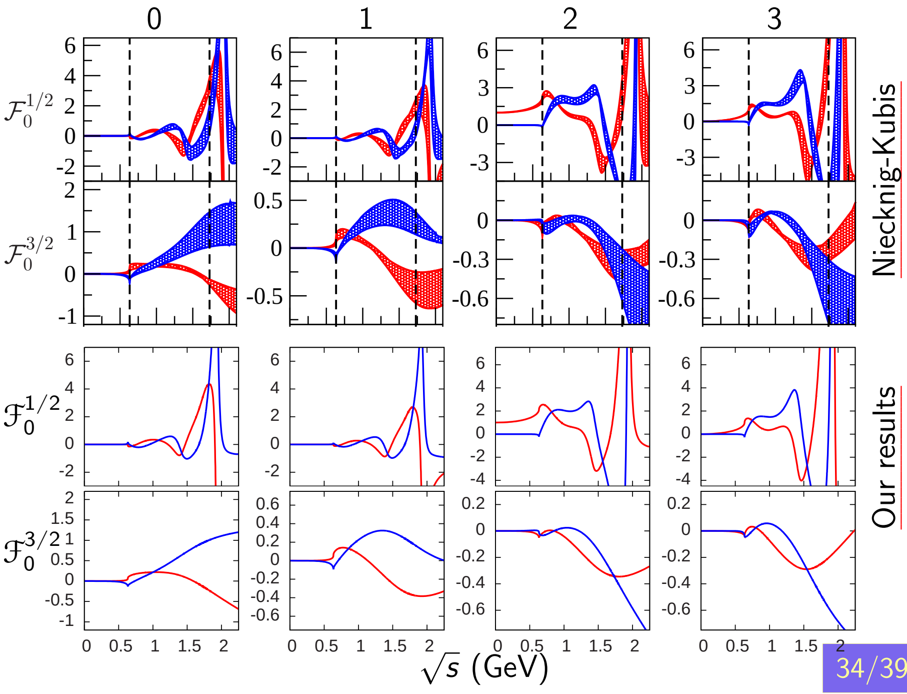
$\mathbb{F}_{(0)}$, $\widehat{\mathbb{F}}_{(0)}$: linear in parameters C_i
$\mathcal{W}_I^F$ matrix: computes rescattering integrals
$\mathcal{W}_K^F$ matrix: computes angular integrations

- System of equations for $\mathbb{H}$ is similar

$$\mathbb{H} = \mathbb{H}_{(0)} + \mathcal{W}_I^H \times \widehat{\mathbb{H}}$$
$$\widehat{\mathbb{H}} = \widehat{\mathbb{H}}_{(0)} + \mathcal{W}_K^{HF} \times (\mathbb{F} + \widehat{\mathbb{F}}) + \mathcal{W}_K^{HH} \times (\mathbb{H} + \widehat{\mathbb{H}}) \ .$$

- First check we correctly solve for $\mathbb{F}$


0
1
2
3
$\mathcal{F}_0^2$
$\mathcal{F}_1^1$
Niecknig-Kubis
Our results
$\sqrt{t}$ (GeV)

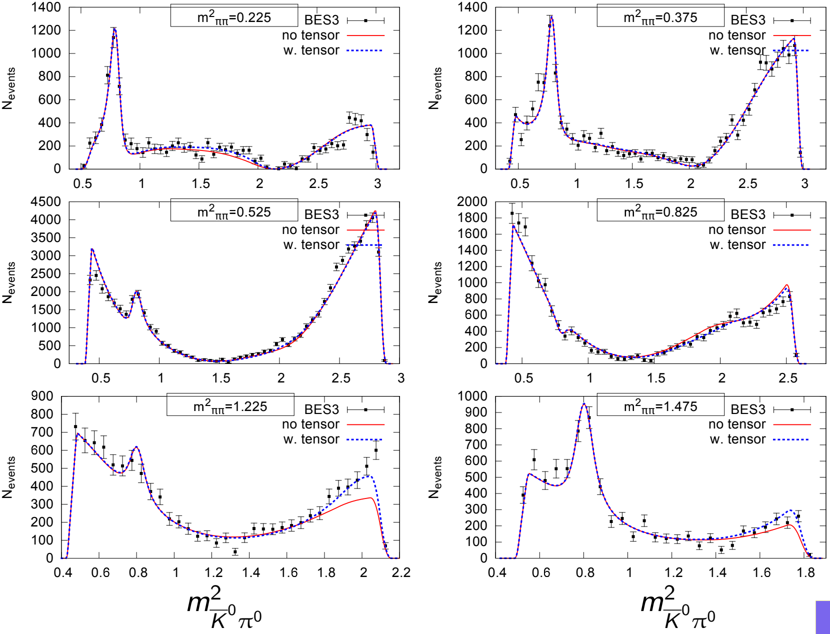


0
1
2
3
$\mathcal{F}_0^{1/2}$
$\mathcal{F}_0^{3/2}$
Niecknig-Kubis
Our results
$\sqrt{s}$ (GeV)

Comparison with experimental data

- $D^+ \to K_S\pi^0\pi^+$ from [BESIII, P.Rev.D89,052001 (2014)]
  with $\mathcal{A}(D^+ \to K_S\pi^0\pi^+) \simeq \frac{1}{\sqrt{2}}\mathcal{A}(D^+ \to \overline{K}^0\pi^0\pi^+)$ (Cabbibo suppressed part neglected)

  → Fit with 11 parameters with $N_{bins} = 1183$
  (Drop bins which overlap border)



m²ππ=0.225
m²ππ=0.375
m²ππ=0.525
m²ππ=0.825
m²ππ=1.225
m²ππ=1.475
BES3
no tensor
w. tensor
N_events
m²_{K̄⁰π⁰}

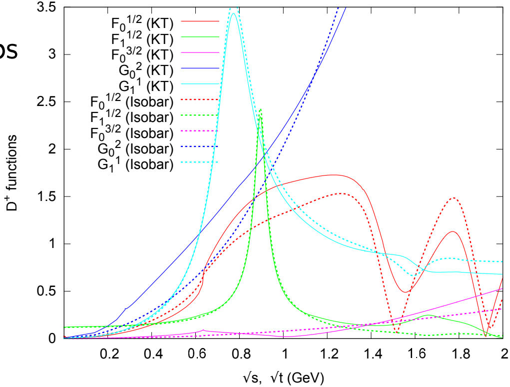
Comparison with isobar-type approach

$(\hat{I}(z) \equiv 0)$

- χ^2/N_{dof}:

	S,P	$S,P+D$
KT solutions	1.92	1.62
Isobar	2.49	2.02

- KT: strong cusps in G_0^2 and $F_0^{1/2}$



- Soft pion relations:

	Exp.	Soft π ($\lambda = 0.52$)	Soft π ($\lambda = 0.53$)
$\frac{10^5}{F_\pi} A^{3/2}_{D\to K\pi}$	1.49 ± 0.03	1.63	1.91
$\frac{10^5}{F_\pi} A^{1/2}_{D\to K\pi}$	4.05 ± 0.04	1.70	4.34
$\phi^{1/2} - \phi^{3/2}$	$\pm(91.4 \pm 0.3)^\circ$	$+130^\circ$	-77°

→ Can be satisfied with some tuning

Summary/outlook

- Implications of isospin symmetry+S,P-wave dominance for $D \to \bar{K}\pi\pi$ amplitudes
  - → D^+: 6 functions
  - → D^0: 6+6 functions
- With Khuri-Treiman equations: good description of D^+
  - → Effectiveness of rescattering integrals $\hat{I}$
- What about $D^0 \to K_S \pi^- \pi^+$? [Not so easy: Belle[PR D73,112009 (2006)], 36 parameters, $\chi 2/N_{dof} = 2.73$]
  - → Data from Belle are being prepared in "easy to use" (for theorists) form