INDIANA UNIVERSITY
BLOOMINGTON

Jefferson Lab

Hadronic Tau decays

Emilie Passemar*
Indiana University/Jefferson Laboratory

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In memory of Mike Pennington

Outline :

1. Introduction and Motivation
   Tau lepton as a laboratory to explore the Standard Model and possible extensions

2. 2-body Tau decays
   - $\tau \rightarrow K\pi\nu_\tau$: Ex: V_{us} determination, CP violating asymmetry
   - Lepton Flavour Violation: $\tau \rightarrow \pi\pi\mu$

3. 3-body Tau decays: $\tau \rightarrow 3\pi\nu_\tau$

4. Conclusion and outlook

1.1 Hadronic τ-decays

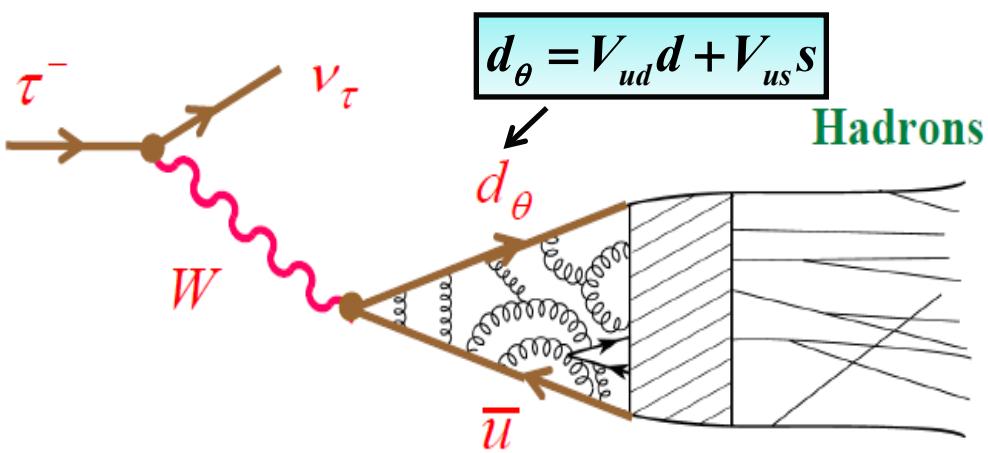
- τ lepton discovered in 1976 by M. Perl et al. (SLAC-LBL group)

  PDG'16

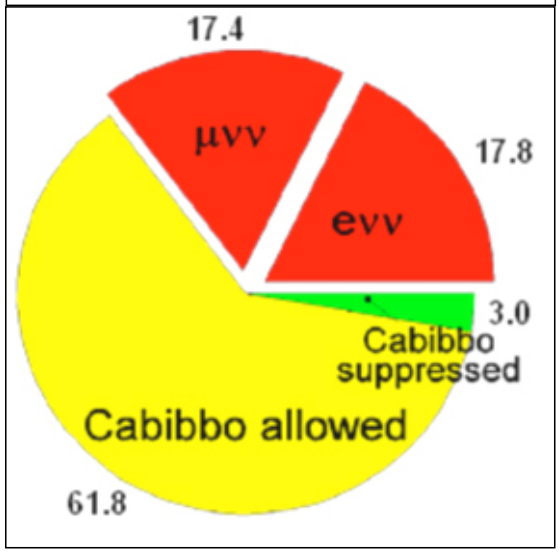
  – Mass : $m_\tau = 1.77682(16)\ \text{GeV}$

  – Lifetime : $\tau_\tau = 2.096(10)\cdot 10^{-13}\ s$

- The only lepton heavy enough to decay into hadrons : lots of semileptonic decays !



Including QED & QCD corrections:



1.1 Hadronic τ-decays

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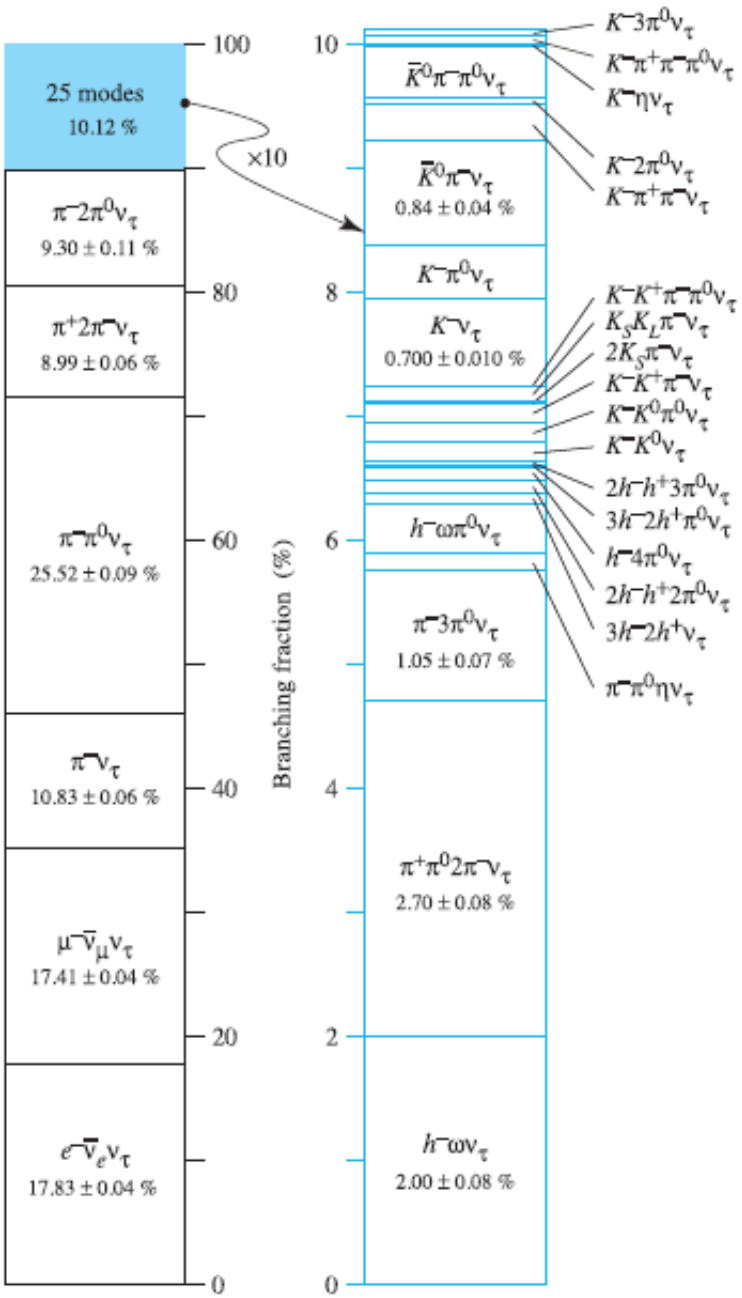
- The only lepton heavy enough to decay into hadrons : lots of semileptonic decays !

  ➡ Very rich phenomenology

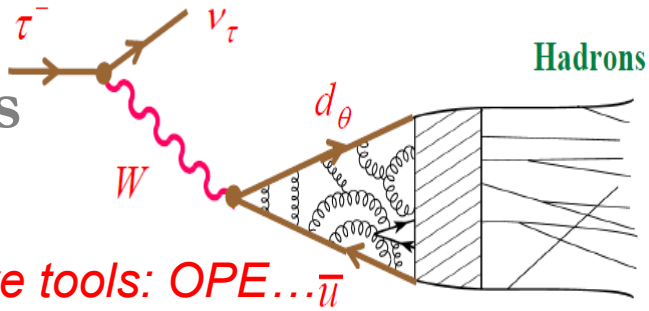
  Test of QCD and EW interactions

- For the tests:
  - Precise measurements needed
  - Hadronic uncertainties under control

PDG'14



1.2 Test of QCD and EW interactions



- Inclusive τ-decays : full hadron spectra, *perturbative tools: OPE…*

$$\tau \to (\bar{u}d, \bar{u}s)\nu_\tau$$

⇒ fundamental SM parameters: $\alpha_s(m_\tau)$, $|V_{us}|$, m_s
QCD studies

- Exclusive τ-decays : specific hadron spectrum, *non perturbative tools*

$$\tau \to (PP, PPP, \ldots)\nu_\tau$$

⇒ Study of ffs, resonance parameters (M_R, Γ_R)
Hadronization of QCD currents

- τ decays: tool to search for New Physics in inclusive and exclusive decays :

⇒ Unitarity test, CPV, LFV, EDMs, etc.

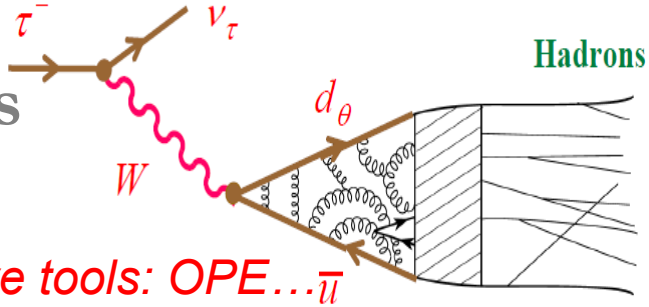
Test of unitarity $$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \overset{?}{=} 1$$

$0^+ \to 0^+$ β decays

K_{l3} decays or τ decays

Negligible (B decays)

1.2 Test of QCD and EW interactions



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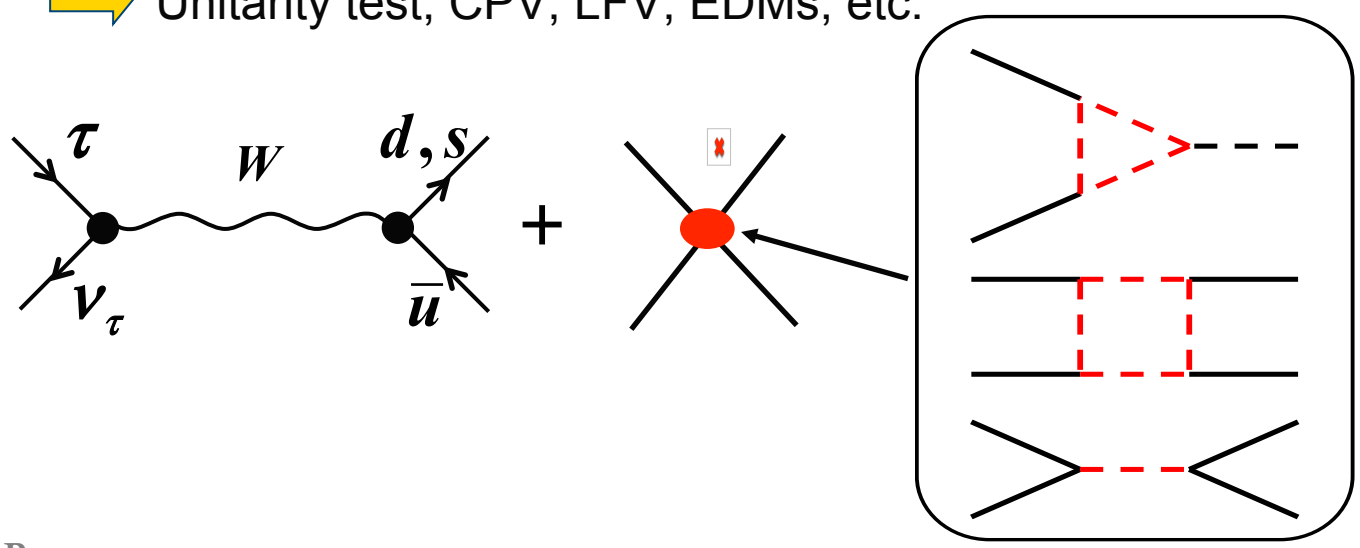


SUSY loops,
Leptoquarks,
Z', Charged Higgs,
Right-Handed
Currents,….

1.3 Exclusive hadronic processes

- For the exclusive hadronic processes $\tau \rightarrow H\nu_\tau$:

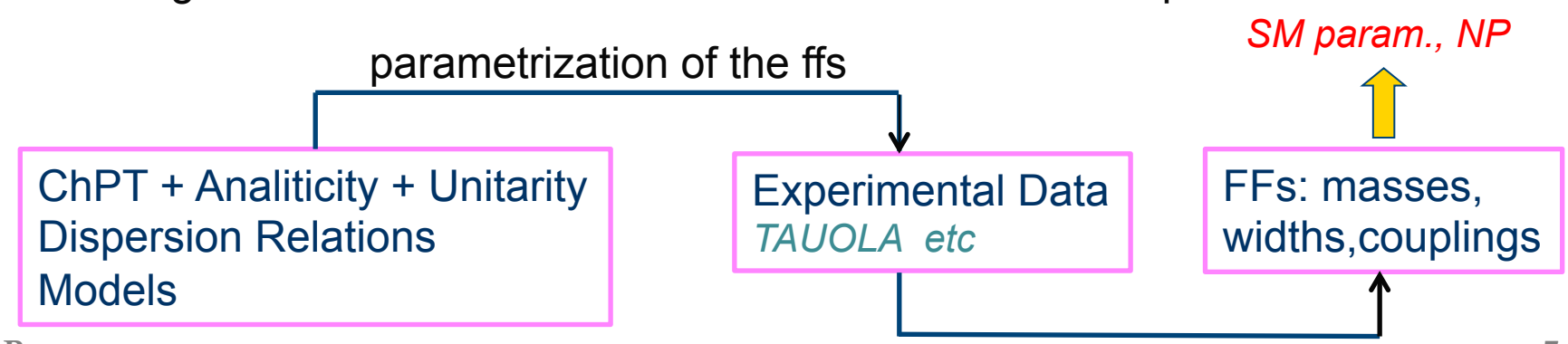
$$M(\tau \rightarrow H\nu_\tau) = \frac{G_F}{\sqrt{2}} V_{CKM} \bar{u}_{\nu_\tau} \gamma^\mu (1-\gamma^5) u_\tau H_\mu$$

- The hadronic matrix element : $H_\mu = \langle H | (V_\mu - A_\mu) e^{iL_{QCD}} | 0 \rangle = (Lorentz\ \text{struct.})^i_\mu F_i(q^2)$

- Experimental measurement : decay rate

$$d\Gamma(\tau \rightarrow H\nu_\tau) = \frac{G_F^2}{4m_\tau} |V_{CKM}|^2 L_{\mu\nu} H^{\mu\nu} dP_S$$

- Challenge : determination of the form factors to extract SM parameters or NP



1.3 Exclusive hadronic processes

Experimental situation :

- $\tau \to PP\nu_\tau$

$$\begin{cases} \pi^-\pi^0, K^-K^0 \\ K^-\pi^0, \bar{K}^0\pi^- \\ \eta \text{ modes} \end{cases}$$

Branching fractions
Spectrum

Branching fractions

ALEPH, CLEOIII, OPAL
Belle, BaBar

- $\tau \to PPP\nu_\tau$

$$\begin{cases} \pi\ \pi\ \pi \\ KK\pi \\ K\pi\pi \\ \eta \text{ modes} \\ \text{KKK} \end{cases}$$

Branching fractions
Spectrum

Branching fractions

ALEPH, CLEOIII, OPAL
Belle, BaBar

- $\tau \to> 3P\nu_\tau$

Theoretical situation

Parametrization using

ChPT + Analiticity + Unitarity
Dispersion relations on the market

⇨ Reasonably good control

Parametrization using
ChPT + Analiticity + Unitarity+
Resonances

⇨ Much more difficult and model dependent

⇨ Poor knowledge

1.4 Experimental situation

- A lot of progress in tau physics since its discovery on all the items described before ⇨ important experimental efforts from *LEP*, *CLEO, B factories: Babar, Belle,*

  ⇨ More to come from *LHCb, BES, Belle II, CMS, ATLAS*

- But τ physics has still potential "*unexplored frontiers*"

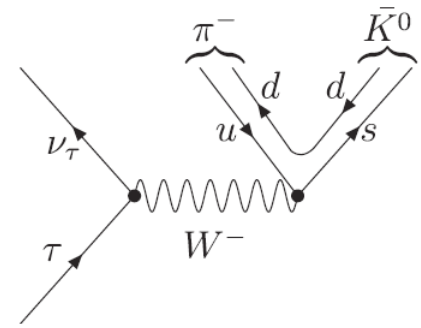
  ⇨ deserve future exp. & th. efforts

- In the following, some selected examples

Experiment	Number of τ pairs
LEP	~3×10^5
CLEO	~1×10^7
BaBar	~5×10^8
Belle	~9×10^8
Belle II	~10^{12}

2. 2 body τ-decays: $\tau \rightarrow K\pi\nu_\tau$

2.1 Introduction: $\tau \to K\pi\nu_\tau$ CP violating asymmetry



- $$A_Q = \frac{\Gamma\left(\tau^+ \to \pi^+ K_S^0 \bar{\nu}_\tau\right) - \Gamma\left(\tau^- \to \pi^- K_S^0 \nu_\tau\right)}{\Gamma\left(\tau^+ \to \pi^+ K_S^0 \bar{\nu}_\tau\right) + \Gamma\left(\tau^- \to \pi^- K_S^0 \nu_\tau\right)}$$

$$= |p|^2 - |q|^2 \approx (0.36 \pm 0.01)\%$$ in the SM

Bigi & Sanda'05
Grossman & Nir'11

$$\left|K_S^0\right\rangle = p\left|K^0\right\rangle + q\left|\bar{K}^0\right\rangle$$
$$\left|K_L^0\right\rangle = p\left|K^0\right\rangle - q\left|\bar{K}^0\right\rangle$$

$$\left\langle K_L \middle| K_S \right\rangle = |p|^2 - |q|^2 \simeq 2\,\mathrm{Re}\left(\varepsilon_K\right)$$

- Experimental measurement : *BaBar'11*

$$A_{Q\,\mathrm{exp}} = \left(-0.36 \pm 0.23_{\mathrm{stat}} \pm 0.11_{\mathrm{syst}}\right)\%$$ ⇨ 2.8σ from the SM!

- CP violation in the tau decays should be of opposite sign compared to the one in D decays in the SM

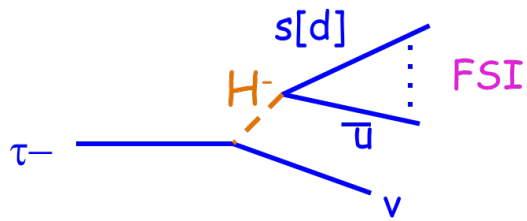
Grossman & Nir'11

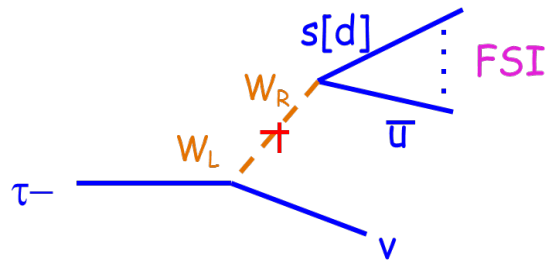
$$A_D = \frac{\Gamma\left(D^+ \to \pi^+ K_S^0\right) - \Gamma\left(D^- \to \pi^- K_S^0\right)}{\Gamma\left(D^+ \to \pi^+ K_S^0\right) + \Gamma\left(D^- \to \pi^- K_S^0\right)} = (-0.54 \pm 0.14)\%$$

Belle, Babar, CLEO, FOCUS

2.1 $\tau \to K\pi\nu_\tau$ CP violating asymmetry

- New physics? Charged Higgs, W_L-W_R mixings, leptoquarks, tensor interactions (*Devi, Dhargyal, Sinha'14, Cirigliano, Crivellin, Hoferichter'17*)?





Bigi'Tau12

Very difficult to explain!

- Need to investigate how large can be the prediction in realistic new physics models: it looks like *a tensor interaction* can explain the effect but in conflict with bounds from neutron EDM and $D\bar{D}$ mixing

Cirigliano, Crivellin, Hoferichter'17

➡ light BSM physics?

2.1 $\tau \to K\pi\nu_\tau$ CP violating asymmetry

- In this measurement, need to know hadronic part ⇨ form factors

$$\langle K\pi | \bar{s}\gamma_\mu u | 0 \rangle = \left[(p_K - p_\pi)_\mu + \frac{\Delta_{K\pi}}{s}(p_K + p_\pi)_\mu \right] f_+(s) - \frac{\Delta_{K\pi}}{s}(p_K + p_\pi)_\mu f_0(s)$$

vector scalar

with $s = Q^2 = (p_K + p_\pi)^2$

$$\Delta_{K\pi} = \left(M_K^2 - M_\pi^2\right)$$

2.2 $\tau \to K\pi\nu_\tau$ angular CP violating asymmetry

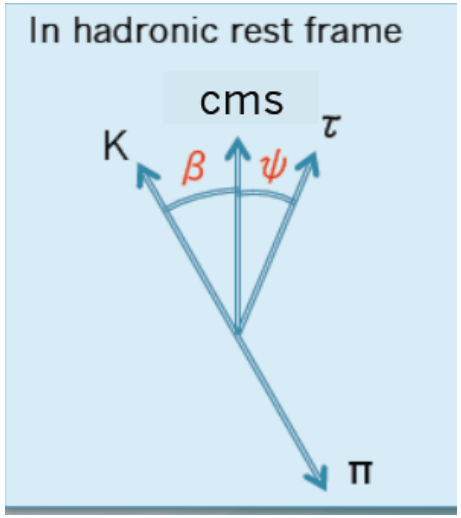
- Measurement of the angular CP asymmetry from Belle:

$$\frac{d\Gamma\left(\tau^- \to K\pi^-\nu_\tau\right)}{d\sqrt{Q^2}\,d\cos\theta\;d\cos\beta} = \left[A(Q^2) - B(Q^2)\left(3\cos^2\psi - 1\right)\left(3\cos^2\beta - 1\right)\right]\left|f_+(s)\right|^2 + m_\tau^2\left|\tilde{f}_0(s)\right|^2 - C(Q^2)\cos\psi\cos\beta\,\mathrm{Re}\left(f_+(s)\tilde{f}_0^*(s)\right)$$

CP violating term
S-P interference

  - $A(Q^2)$, $B(Q^2)$, $C(Q^2)$: kinematic factors
  - Angles:
    in $K\pi$ rest frame
    - β: angle between kaon and e^+e^- CMS frame
    - Ψ: angle between τ and CMS frame

    in τ rest frame
    - θ: angle between τ direction in CMS and direction of $K\pi$ system (dependence with Ψ)



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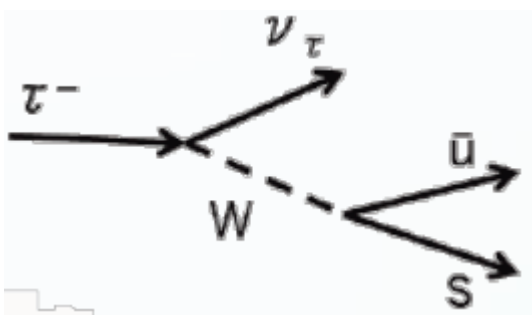
CP violating term
S-P interference

– $A(Q^2)$, $B(Q^2)$, $C(Q^2)$: kinematic factors

Charged Higgs contribution



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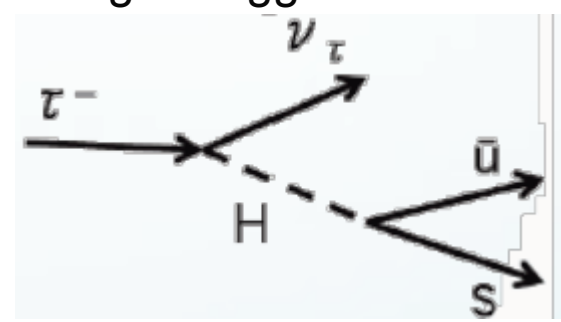


Khün & Mirkes'05

$$\tilde{f}_0(s) = f_0(s) + \frac{\eta^2}{m_\tau^2} f_H(s)$$

with $$f_H(s) = \frac{s}{m_u - m_s} f_0(s)$$

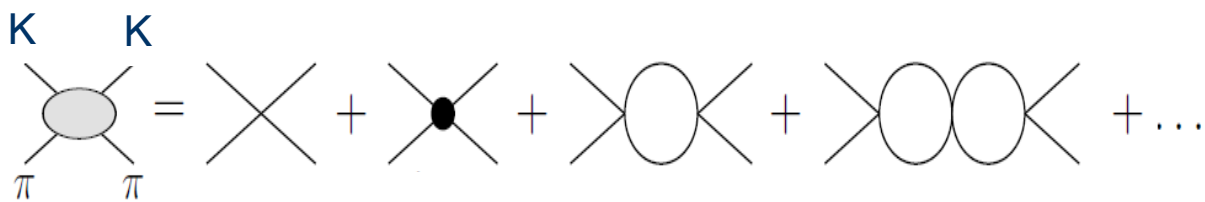
2.3 Kπ form factors

- Belle uses sum of BWs to fit the invariant mass distribution *Belle'08*

$$F_V = \frac{1}{1+\beta+\chi}\left[BW_{K^*(892)}(s) + \beta BW_{K^*(1410)}(s) + \chi BW_{K^*(1680)}(s)\right]$$

$$F_S = \varkappa \frac{s}{M^2_{K_0^*(800)}} BW_{K_0^*(800)}(s) + \gamma \frac{s}{M^2_{K_0^*(1430)}} BW_{K_0^*(1430)}(s)$$

- Can be justified for the vector but not for the scalar!
  ➡ Use a parametrization relying on dispersion relations instead:
  - Resum all final state Kπ rescattering



  - Allow to combine with $K \to \pi l \nu_l$ precise measurements

- Several theoretical parametrizations proposed: All rely on analyticity and unitarity and crossing symmetry *Jamin, Pich, Portolés'06,'08, Moussallam'08, Boito, Escribano, Jamin'09,'10, Bernard, Boito, E.P.'11, Bernard'14*

2.4 Dispersive representation for the form factors

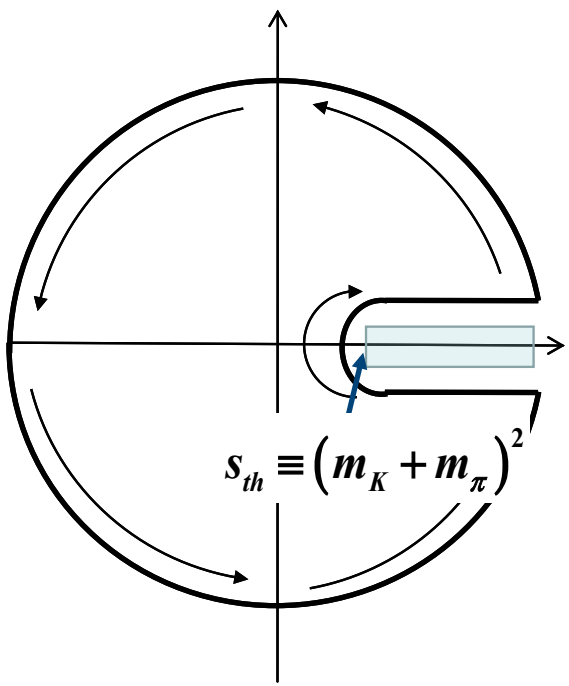
- Parametrization to analyse both K_{l3} and $\tau \longrightarrow K\pi\nu_\tau$ ⇨ Use dispersion relations

Unitarity:

$$disc\left[\bar{f}_{0,+}(s)\right] \propto t_\ell^{I*}(s)\,\bar{f}_{0,+}(s)$$

- Omnès representation: ⇨

$$\boxed{\bar{f}_{+,0}(s) = \exp\left[\frac{s}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{s'}\frac{\phi_{+,0}(s')}{s'-s-i\varepsilon}\right]}$$



$\phi_{+,0}(s)$: phase of the form factor

- $s < s_{in}$: $\phi_{+,0}(s) = \delta_{K\pi}(s)$

  Kπ scattering phase

- $s \geq s_{in}$: $\phi_{+,0}(s)$ unknown

  ⇨ $\phi_{+,0}(s) = \phi_{+,0as}(s) = \pi \pm \pi \quad \left(\bar{f}_{+,0}(s) \to 1/s\right)$

Brodsky & Lepage

- Subtract dispersion relation to weaken the high energy contribution of the phase. Improve the convergence but sum rules to be satisfied!

2.4 Dispersive representation for the FFs

Bernard, Boito, E.P.'11

Bernard'14

- Dispersion relation with n subtractions in $\bar{s}$:

$$\bar{f}_{+,0}(s) = \exp\left[P_{n-1}(s) + \frac{(s-\bar{s})^n}{\pi} \int_{s_{th}}^{\infty} \frac{ds'}{(s'-\bar{s})^n} \frac{\phi_{+,0}(s')}{s'-s-i\varepsilon} \right]$$

➢ $\bar{f}_0(s)$ ⇨ dispersion relation with 3 subtractions: 2 in s=0 and 1 in $s=\Delta_{K\pi}$

Callan-Treiman

$$\bar{f}_0(s) = \exp\left[\frac{s}{\Delta_{K\pi}} \left(\ln C + (s-\Delta_{K\pi}) \left(\frac{\ln C}{\Delta_{K\pi}} - \frac{\lambda_0'}{m_\pi^2} \right) + \frac{\Delta_{K\pi} s (s-\Delta_{K\pi})}{\pi} \int_{(m_K+m_\pi)^2}^{\infty} \frac{ds'}{s'^2} \frac{\phi_0(s')}{(s'-\Delta_{K\pi})(s'-s-i\varepsilon)} \right) \right]$$

➢ $\bar{f}_+(s)$ ⇨ dispersion relation with 3 subtractions in s=0

Boito, Escribano, Jamin'09,'10

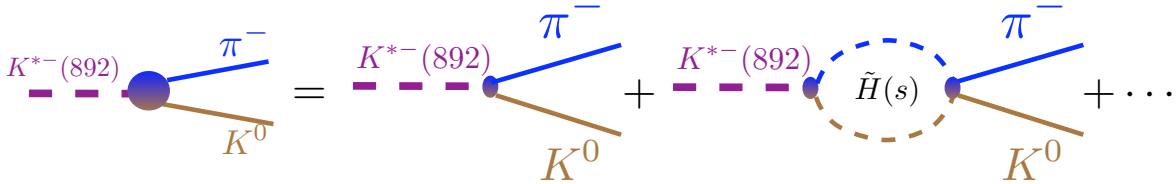
$$\bar{f}_+(s) = \exp\left[\lambda_+' \frac{s}{m_\pi^2} + \frac{1}{2}\left(\lambda_+'' - \lambda_+'^2\right)\left(\frac{s}{m_\pi^2}\right)^2 + \frac{s^3}{\pi} \int_{(m_K+m_\pi)^2}^{\infty} \frac{ds'}{s'^3} \frac{\phi_+(s')}{(s'-s-i\varepsilon)} \right]$$

Jamin, Pich, Portolés'08

Extracted from a model including
2 resonances K*(892) and K*(1414)

Modeling of the phase

- Model for $\phi_+(s)$:



$$\tilde{f}_+(s) = \left[\frac{m_{K^*}^2 - \kappa_{K^*}\left(\mathrm{Re}\,\tilde{H}_{K\pi}(0) + \mathrm{Re}\,\tilde{H}_{K\eta}(0)\right) + \beta s}{D\left(m_{K^*}, \Gamma_{K^*}\right)} - \frac{\beta s}{D\left(m_{K^{*\prime}}, \Gamma_{K^{*\prime}}\right)} \right]$$

$K^*(892)$ $K^*(1410)$

with $D\left(m_n, \Gamma_n\right) = m_n^2 - s - \kappa_n \sum \mathrm{Re}\,\tilde{H} - i m_n \Gamma_n(s)$

Boito, Escribano, Jamin'09,'10

Jamin, Pich, Portolés'08

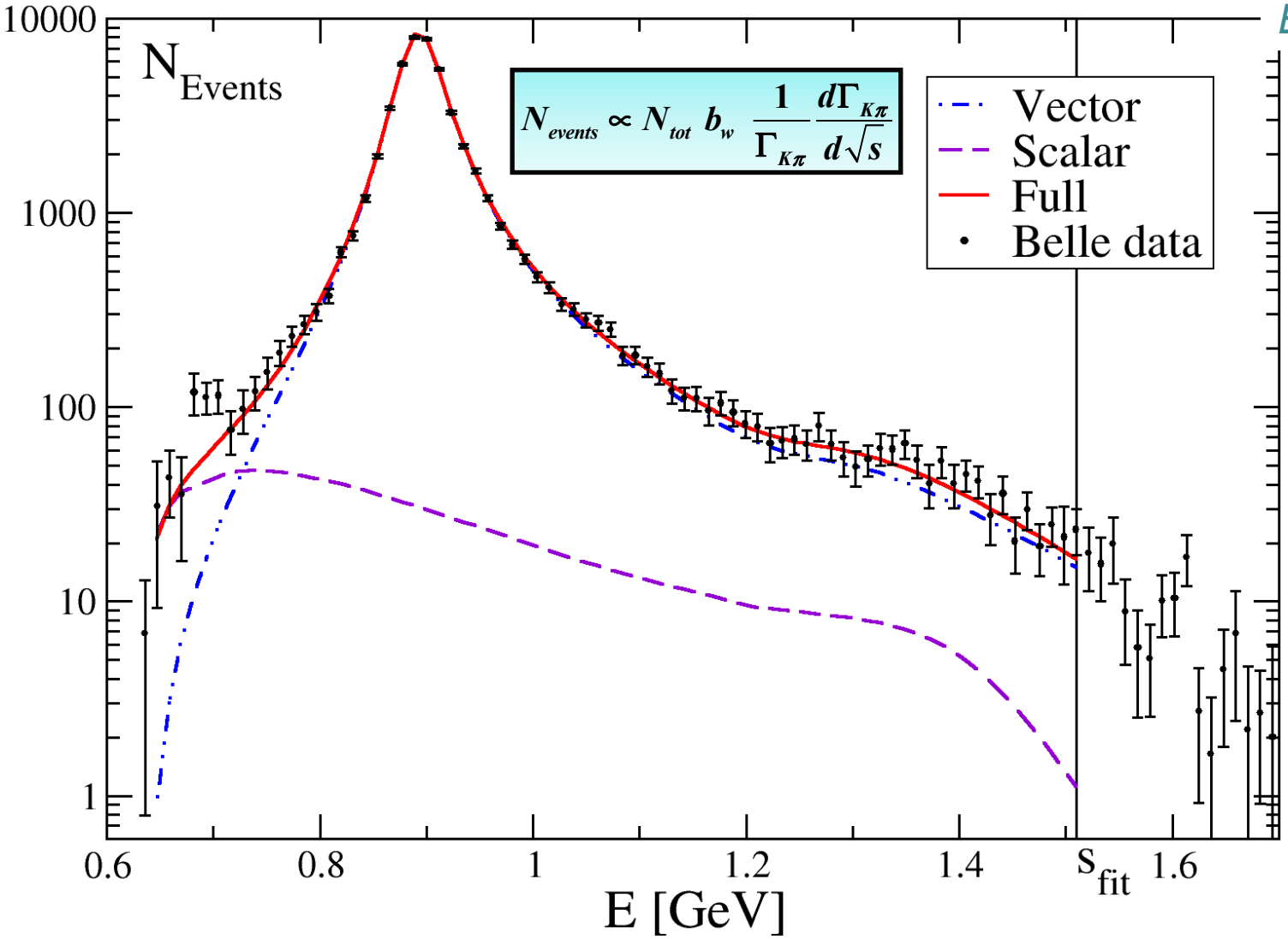
Bernard'14

$$\tan \delta_{K\pi}^{P,1/2} = \frac{\mathrm{Im}\,\tilde{f}_+(s)}{\mathrm{Re}\,\tilde{f}_+(s)}$$

Fit to the $\tau \to K\pi\nu_\tau$ decay data + K_{l3} constraints

Bernard, Boito, E.P.'11

Bernard'14



2.5 Determination of the form factors

Bernard, Boito, E.P.'11
Antonelli, Cirigliano Lusiani, E.P.'13

- Results of the fits:

	$\tau \to K\pi\nu_\tau$ & $K_{\ell 3}$ Belle	$\tau \to K\pi\nu_\tau$ & $K_{\ell 3}$ SuperB
$\ln C$	0.20193 ± 0.00892	0.20034 ± 0.00557
$\lambda_0' \times 10^3$	13.139 ± 0.965	13.851 ± 0.592
m_{K^*}[MeV]	892.09 ± 0.22	892.01 ± 0.21
Γ_{K^*}[MeV]	46.287 ± 0.417	46.494 ± 0.436
$m_{K^{*\prime}}$[MeV]	1292.5 ± 47.2	1259.8 ± 27.2
$\Gamma_{K^{*\prime}}$[MeV]	171.64 ± 234.65	205.41 ± 10.27
β	-0.0204 ± 0.0289	-0.0350 ± 0.0229
$\lambda_+' \times 10^3$	25.714 ± 0.332	25.655 ± 0.268
$\lambda_+'' \times 10^3$	1.1988 ± 0.0313	1.2176 ± 0.0089
$\chi^2/d.o.f$	$59.7/67$	$56.5/67$
I_K^τ	0.7655 ± 0.0416	0.7857 ± 0.0105
$f_+(0)V_{us}$	0.2134 ± 0.0061	0.21103 ± 0.0037

Very accurate determination of K*(892)!

2.5 Kπ form factors from $\tau \to K\pi\nu_\tau$ and K_{l3} decays

- Precise extraction of Kπ scattering phase and good determination of K*

$m_{K^*} = 892.02 \pm 0.21$ MeV and $\Gamma_{K^*} = 46.300 \pm 0.426$ MeV

PDG : $m_{K^*} = 891.66 \pm 0.26$ MeV and $\Gamma_{K^*} = 50.8 \pm 0.9$ MeV

⇨ Tau-Charm: $m_{K^*} = 892.02 \pm 0.02$ MeV and $\Gamma_{K^*} = 46.300 \pm 0.044$ MeV

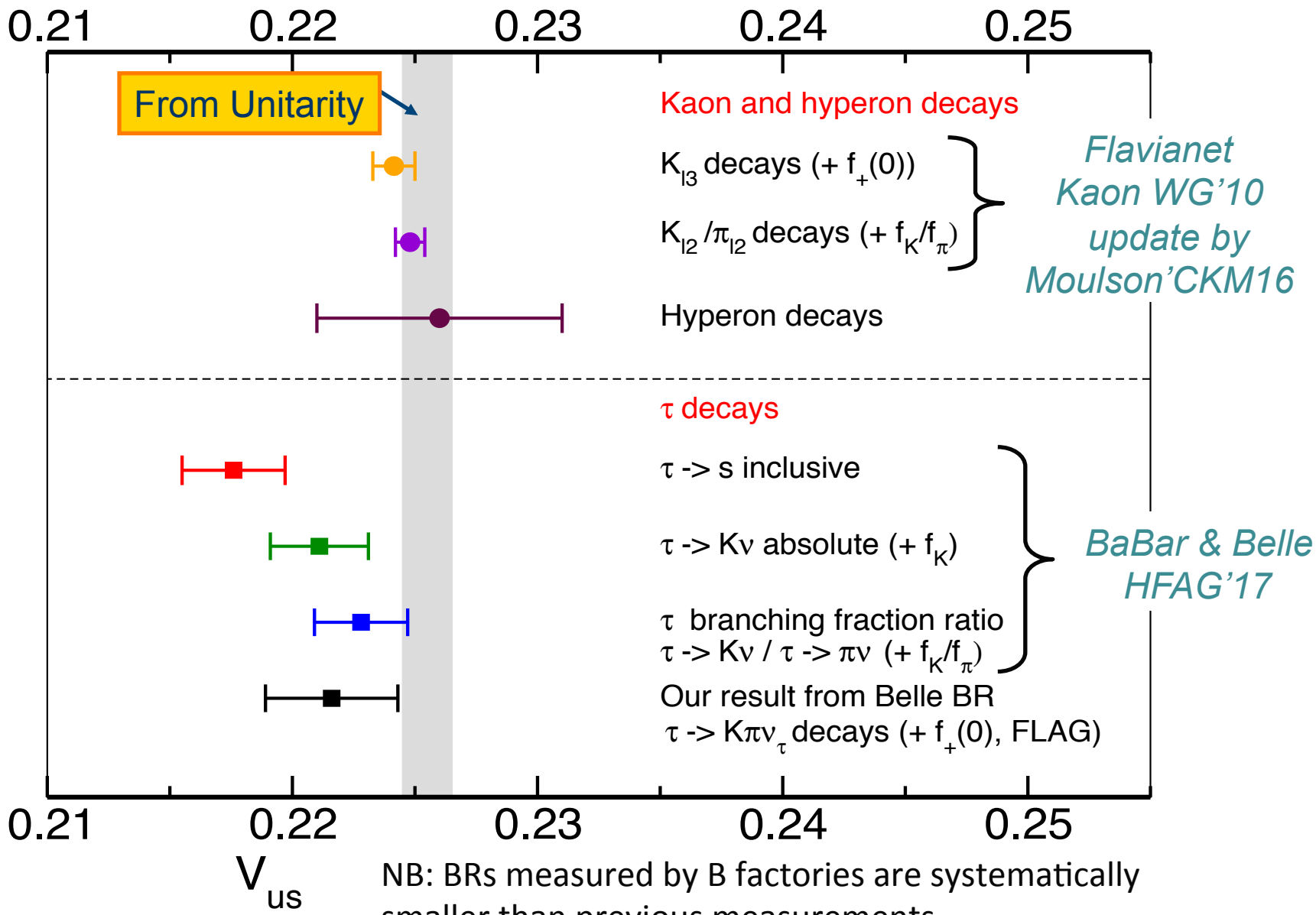
- Callan-Treiman test or lattice QCD test (F_K/F_π and $f_+(0)$)

- V_{us} from $\tau \to K\pi\nu_\tau$: $\Gamma_{\tau\to K\pi\nu_\tau} = N \left| f_+(0) V_{us} \right|^2 I_K^\tau$ with $I_K^\tau = \int ds\, F\left(s, \bar{f}_+(s), \bar{f}_0(s)\right)$

- Prediction of the strange Brs and V_{us}

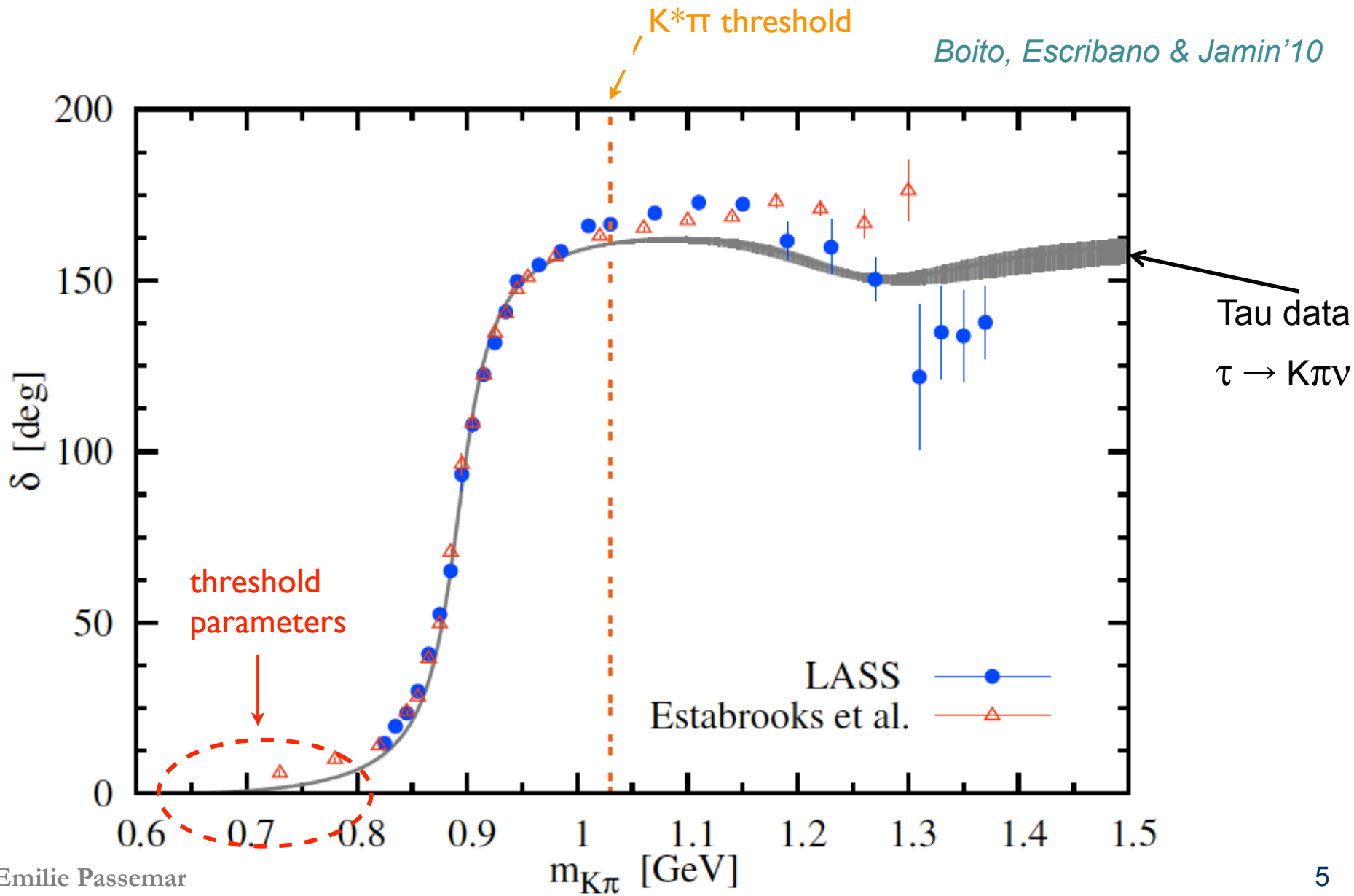
- Use of the form factors for CPV tests, etc.

2.6 V_{us} determination



NB: BRs measured by B factories are systematically smaller than previous measurements

2.6 Kπ phase shift



2.7 Separating scalar from vector form factors

- In this measurement, need to know hadronic part ⇨ form factors

$$\langle K\pi|\ \bar{s}\gamma_\mu u\ |0\rangle = \left[(p_K - p_\pi)_\mu + \frac{\Delta_{K\pi}}{s}(p_K + p_\pi)_\mu\right] f_+(s) - \frac{\Delta_{K\pi}}{s}(p_K + p_\pi)_\mu\, f_0(s)$$

with $s = Q^2 = (p_K + p_\pi)^2$

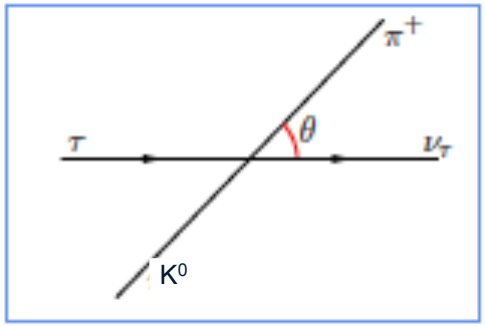
vector

scalar

- Up to now know from decay spectrum but difficult to disentangle scalar and vector form factor ⇨ consider the FB asymmetry instead

$$A_{FB} = \frac{d\Gamma(\cos\theta) - d\Gamma(-\cos\theta)}{d\Gamma(\cos\theta) + d\Gamma(-\cos\theta)}$$

Beldjoudi & Truong'94
Moussallam, B2TIP



- Formula: can disentangle scalar and vector FF easily

$$A_{FB}(s) = \frac{3\Delta_{\pi^+K^0}\sqrt{\lambda_{\pi^+K^0}(s)}|f_V^{K\pi}(s)||f_0^{K\pi}(s)|\cos\left(\delta_1^{1/2} - \delta_0^{1/2}\right)}{\underbrace{|f_V^{K\pi}(s)|^2\lambda_{\pi^+K^0}(s)}_{\text{vanishes at threshold}}(1 + 2s/m_\tau^2) + 3|f_0^{K\pi}(s)|^2\Delta^2_{\pi^+K^0}}.$$

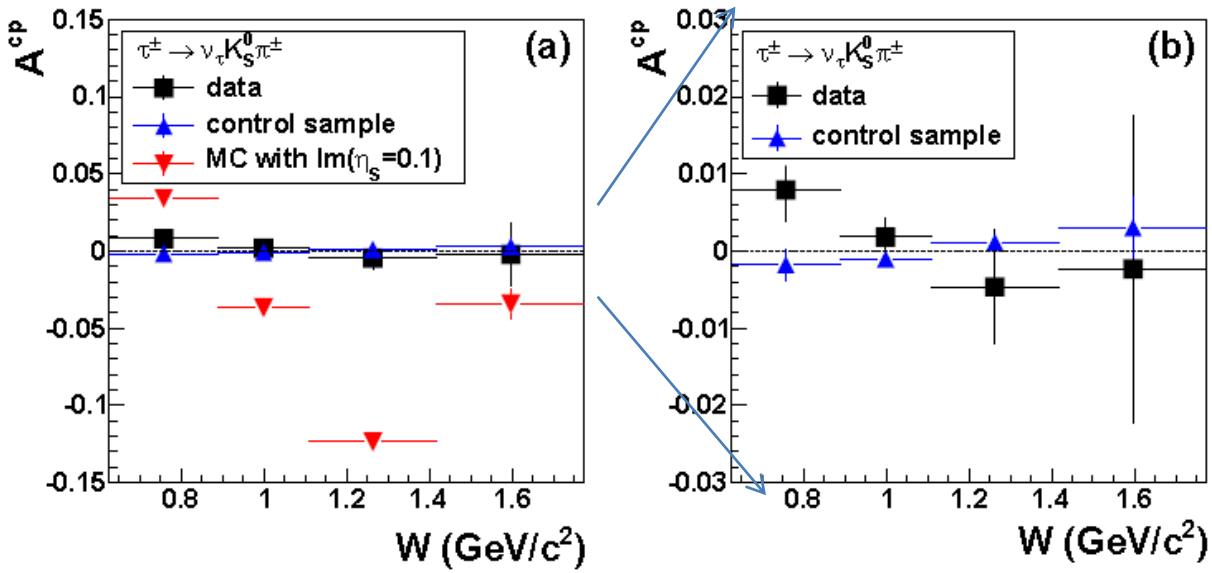
⇨ Never done before!

2.8 $\tau \to K\pi\nu_\tau$ CP violating asymmetry

- Measurement of the direct contribution of NP in the angular CP violating asymmetry done by *CLEO* and *Belle*

  ➡ Belle does not see any asymmetry at the *0.2 - 0.3% level*

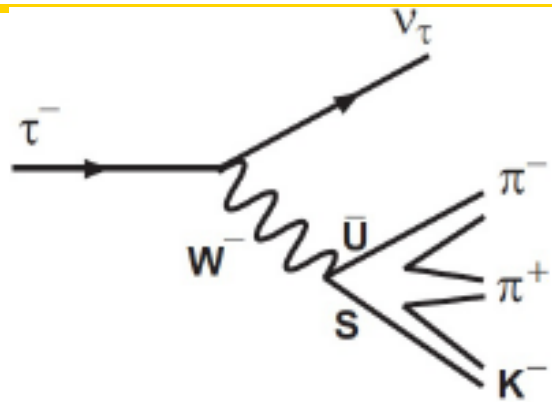
Belle'11



- Problem with this measurement? ➡ It would be great to have other experimental measurements from *Belle II*

2.9 Three body CP asymmetries

- Ex: $\tau \to K\pi\pi\nu_\tau$



e.g., Choi, Hagiwara and Tanabashi'98
Kiers, Little, Datta, London et al.,'08
Mileo, Kiers and, Szynkman'14

- A variety of CPV observables can be studied : $\tau \to K\pi\pi\nu_\tau$, $\tau \to \pi\pi\pi\nu_\tau$ rate, angular asymmetries, triple products,….

  Same principle as in charm, *see Bevan'15*

  Difficulty : Treatement of the hadronic part
  Hadronic final state interactions have to be taken into account!
  ➡ Disentangle weak and strong phases

- More form factors, more asymmetries to build but same principles as for 2 bodies

3. Lepton Flavour Violation: $\tau \to \pi\pi\mu$

Celis, Cirigliano, E.P.'14

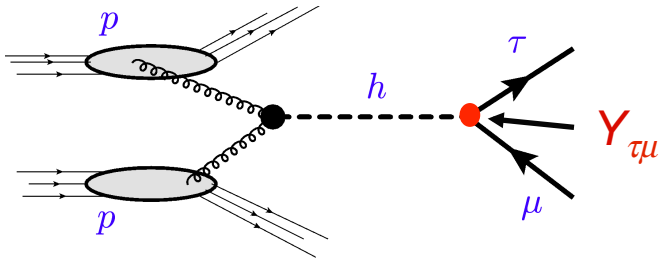
3.1 Introduction: Non standard LFV Higgs coupling

- $$\Delta\mathcal{L}_Y = -\frac{\lambda_{ij}}{\Lambda^2}\left(\bar{f}_L^i f_R^j H\right) H^\dagger H \Rightarrow -Y_{ij}\left(\bar{f}_L^i f_R^j\right) h$$

Goudelis, Lebedev, Park'11
Davidson, Grenier'10
Harnik, Kopp, Zupan'12
Blankenburg, Ellis, Isidori'12
McKeen, Pospelov, Ritz'12
Arhrib, Cheng, Kong'12

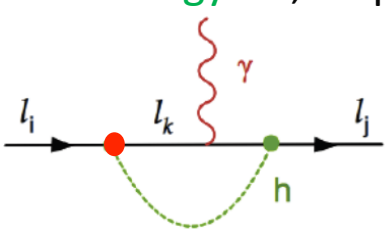
In the SM: $Y_{ij}^{h_{SM}} = \frac{m_i}{v}\delta_{ij}$

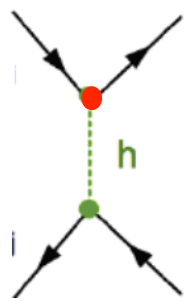
- High energy : LHC

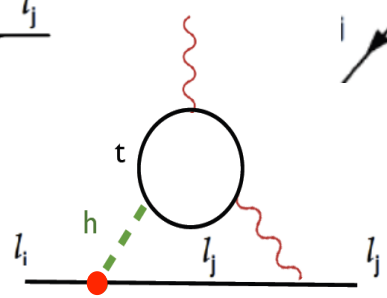


Hadronic part treated with perturbative QCD

- Low energy : D, S operators







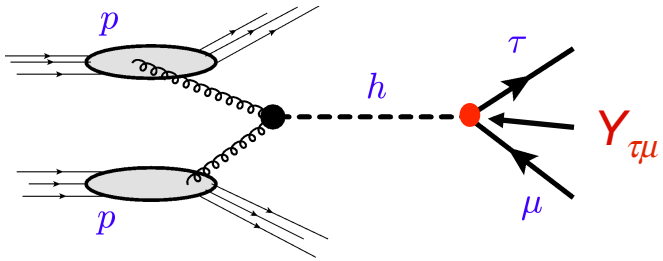
3.1 Non standard LFV Higgs coupling

- $$\Delta\mathcal{L}_Y = -\frac{\lambda_{ij}}{\Lambda^2}\left(\bar{f}_L^i f_R^j H\right) H^\dagger H \Rightarrow -Y_{ij}\left(\bar{f}_L^i f_R^j\right) h$$

Goudelis, Lebedev, Park'11
Davidson, Grenier'10
Harnik, Kopp, Zupan'12
Blankenburg, Ellis, Isidori'12
McKeen, Pospelov, Ritz'12
Arhrib, Cheng, Kong'12

In the SM: $Y_{ij}^{h_{SM}} = \frac{m_i}{v}\delta_{ij}$

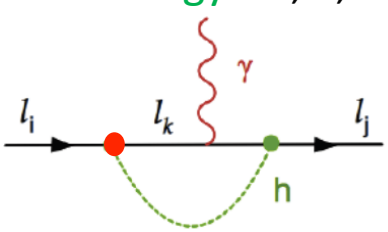
- High energy : LHC

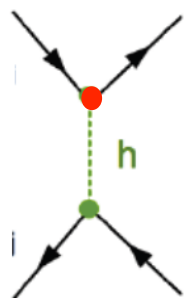


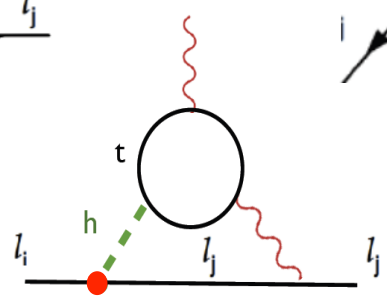
Hadronic part treated with perturbative QCD

Reverse the process

- Low energy : D, S, G operators







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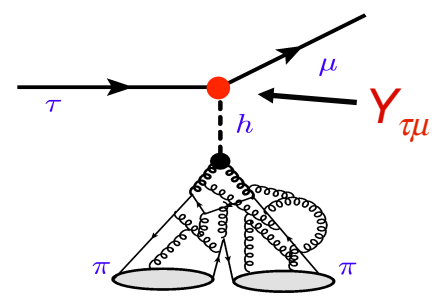
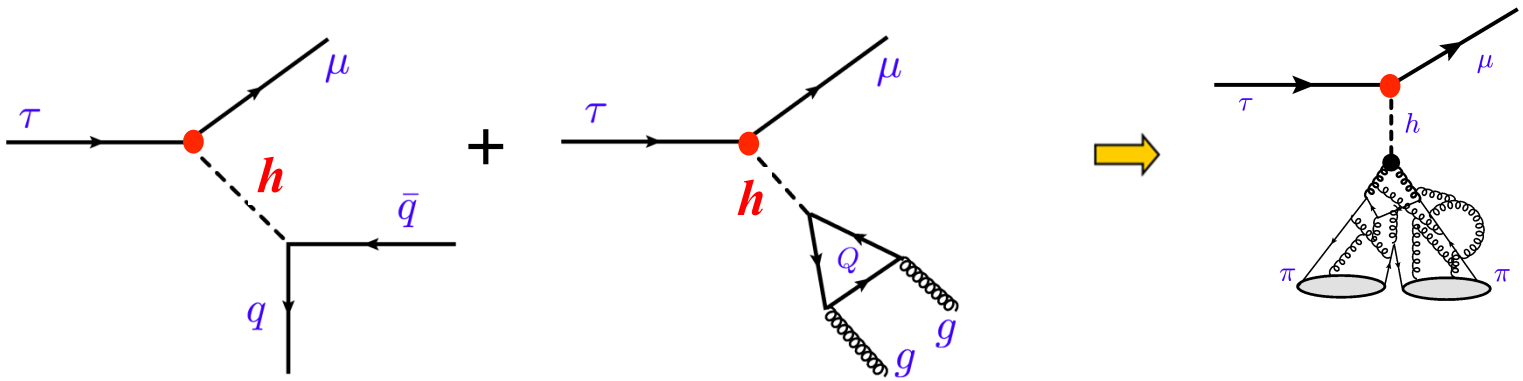


Hadronic part treated with non-perturbative QCD

3.2 Constraints from $\tau \to \mu\pi\pi$

- Tree level Higgs exchange



- Problem : Have the hadronic part under control, ChPT not valid at these energies!

  ➡ Use *form factors* determined with *dispersion relations* matched at low energy to *CHPT*

  Daub, Dreiner, Hanhart, Kubis, Meissner'13

  Celis, Cirigliano, E.P.'14

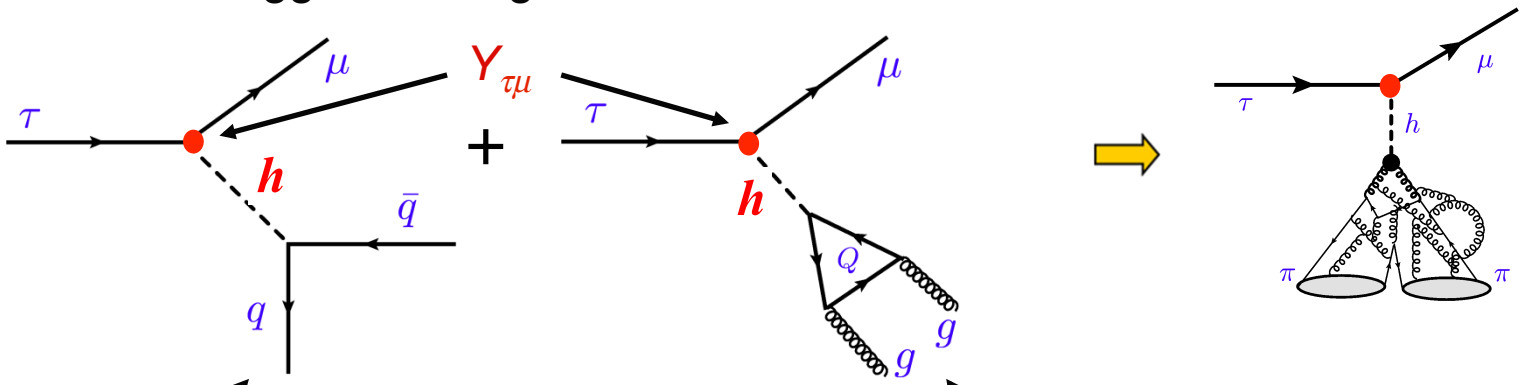
- Dispersion relations: based on unitarity, analyticity and crossing symmetry

  ➡ Take *all rescattering* effects into account

  $\pi\pi$ final state interactions important

3.2 Constraints from $\tau \to \mu\pi\pi$

- Tree level Higgs exchange



$$s = \left(p_{\pi^+} + p_{\pi^-}\right)^2$$

$$\langle \pi^+\pi^- | m_u \bar{u}u + m_d \bar{d}d | 0 \rangle \equiv \Gamma_\pi(s) \qquad \langle \pi^+\pi^- | \theta^\mu_\mu | 0 \rangle \equiv \theta_\pi(s)$$

$$\langle \pi^+\pi^- | m_s \bar{s}s | 0 \rangle \equiv \Delta_\pi(s)$$

$$\boxed{\theta^\mu_\mu = -9\frac{\alpha_s}{8\pi} G^a_{\mu\nu} G^{\mu\nu}_a + \sum_{q=u,d,s} m_q \bar{q}q}$$

$$\boxed{\frac{d\Gamma(\tau \to \mu\pi^+\pi^-)}{d\sqrt{s}} = \frac{(m_\tau^2 - s)^2 \sqrt{s - 4m_\pi^2}}{256\pi^3 m_\tau^3} \frac{\left(|Y^h_{\tau\mu}|^2 + |Y^h_{\mu\tau}|^2\right)}{M_h^4 v^2} |\mathcal{K}_\Delta \Delta_\pi(s) + \mathcal{K}_\Gamma \Gamma_\pi(s) + \mathcal{K}_\theta \theta_\pi(s)|^2}$$

$$f\left(y^h_q\right)$$

3.3 Unitarity

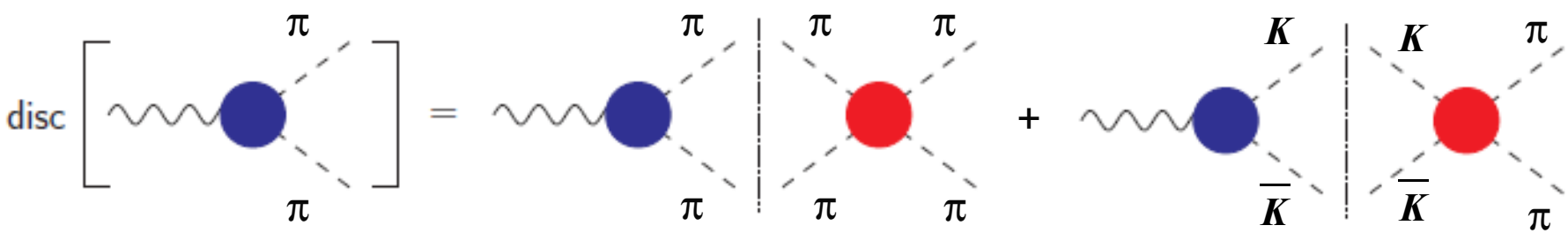
- *Coupled channel analysis* up to $\sqrt{s}$~1.4 GeV: *Mushkhelishvili-Omnès* approach

  Inputs: I=0, S-wave ππ and KK data

  Donoghue, Gasser, Leutwyler'90
  Moussallam'99
  Daub, Dreiner, Hanhart, Kubis, Meissner'13
  Celis, Cirigliano, E.P.'14

  See also *Osset & Oller'98*
  Lahde & Meissner'06

- Unitarity ⇒ the discontinuity of the form factor is known



⇒

$$\mathrm{Im}F_n(s) = \sum_{m=1}^{2} T^*_{nm}(s)\sigma_m(s)F_m(s)$$

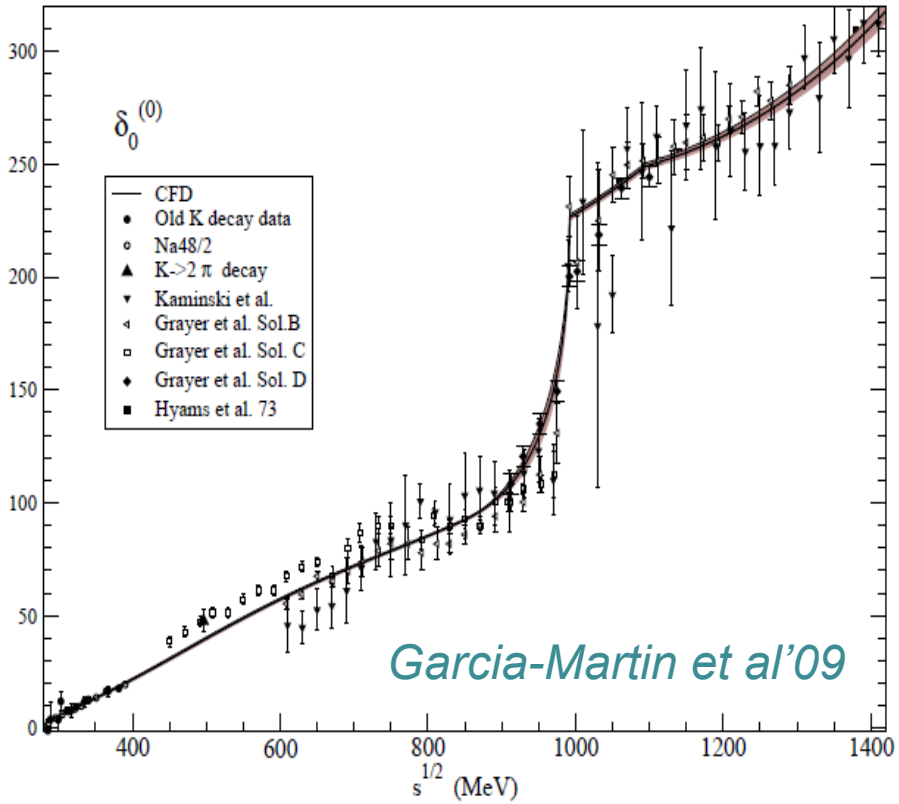
$n = \pi\pi, K\bar{K}$

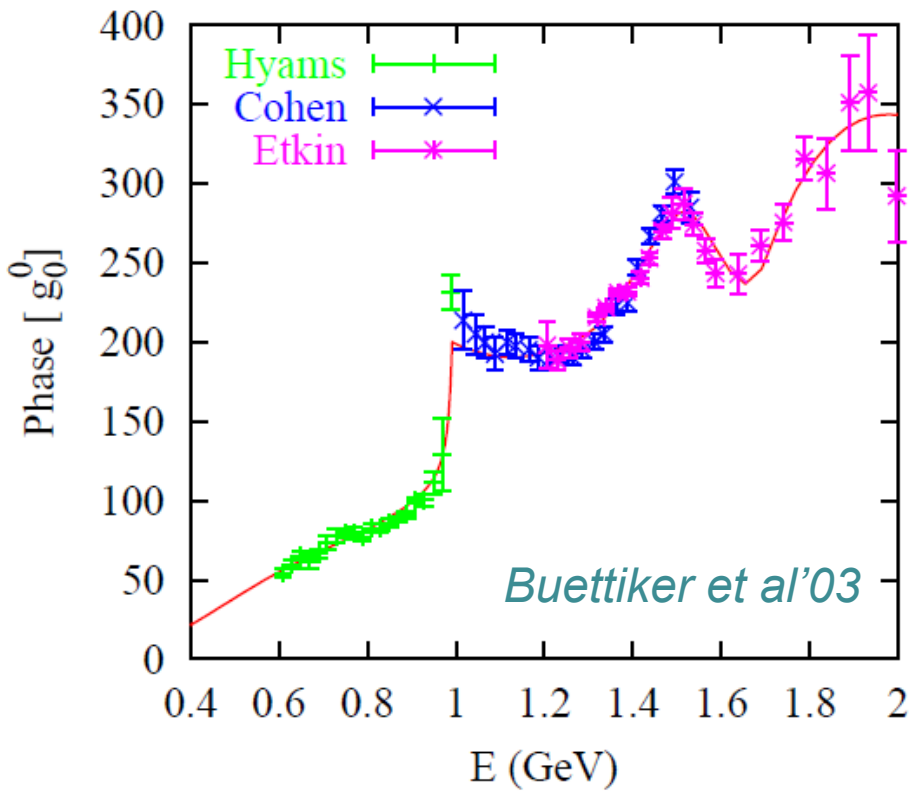
Scattering matrix:

$$\begin{pmatrix} \pi\pi \to \pi\pi, & \pi\pi \to K\bar{K} \\ K\bar{K} \to \pi\pi, & K\bar{K} \to K\bar{K} \end{pmatrix}$$

3.4 Inputs for the coupled channel analysis

- Inputs : $\pi\pi \to \pi\pi, K\bar{K}$





- A large number of theoretical analyses *Descotes-Genon et al'01, Kaminsky et al'01, Buettiker et al'03, Garcia-Martin et al'09, Colangelo et al.'11* and all agree
- 3 inputs: $\delta_\pi(s)$, $\delta_K(s)$, η from *B. Moussallam* ➡ *reconstruct T matrix*

3.4 Dispersion relations

- General solution to *Mushkhelishvili-Omnès* problem:

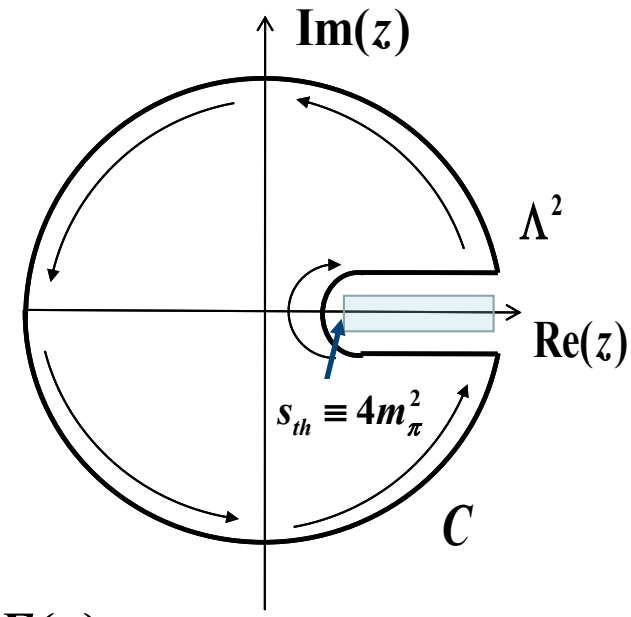
$$\boxed{\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}}$$

Canonical solution falling as 1/s for large s (obey unsubtracted dispersion relations)

Polynomial determined from a matching to ChPT + lattice

Canonical solution $X(s) = C(s), D(s)$:

- Knowing the discontinuity of X(s) ⇨ write a dispersion relation for it
- Analyticity of the FFs: X(z) is
  - real for z < s_{th}
  - has a branch cut for z > s_{th}
  - analytic for complex z
- Cauchy Theorem and Schwarz reflection principle:



$$X(s) = \frac{1}{\pi}\oint_C dz \frac{X(z)}{z-s}$$

$$= \frac{1}{2i\pi}\int_{s_{th}=4M_\pi^2}^{\Lambda^2} dz \frac{disc\left[F(z)\right]}{z-s-i\varepsilon} + \frac{1}{2i\pi}\int_{|z|=\Lambda^2} dz \frac{F(z)}{z-s}$$

$\Lambda \to \infty$ ⇨

$$\boxed{X(s) = \frac{1}{\pi}\int_{4M_\pi^2}^{\infty} dz \frac{\mathrm{Im}\left[X(z)\right]}{z-s-i\varepsilon}}$$

X(s) can be reconstructed everywhere from the knowledge of ImX(s)

3.4 Dispersion relations

- General solution to *Mushkhelishvili-Omnès* problem:

$$\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}$$

Canonical solution falling as 1/s for large s (obey unsubtracted dispersion relations)

Polynomial determined from a matching to ChPT + lattice

- Canonical solution found by solving the dispersive integral equations iteratively starting with Omnès functions

$$\boldsymbol{X(s) = C(s), D(s)}$$

$$\mathrm{Im} X_n^{(N+1)}(s) = \sum_{m=1}^{2} T^*_{mn} \sigma_m(s) X_m^{(N)}(s) \longrightarrow X_n^{(N+1)}(s) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} ds' \frac{\mathrm{Im} X_n^{(N+1)}(s')}{s' - s}$$

3.5 Determination of the polynomial

- Fix the polynomial with requiring $\boxed{F_P(s) \to 1/s}$ + ChPT:

Brodsky & Lepage'80

- Feynman-Hellmann theorem:

$$\boxed{\Gamma_P(0) = \left(m_u \frac{\partial}{\partial m_u} + m_d \frac{\partial}{\partial m_d} \right) M_P^2}$$

$$\boxed{\Delta_P(0) = \left(m_s \frac{\partial}{\partial m_s} \right) M_P^2}$$

- At LO in ChPT:

$$M_{\pi^+}^2 = (m_\mathrm{u} + m_\mathrm{d})\, B_0 + O(m^2)$$
$$M_{K^+}^2 = (m_\mathrm{u} + m_\mathrm{s})\, B_0 + O(m^2)$$
$$M_{K^0}^2 = (m_\mathrm{d} + m_\mathrm{s})\, B_0 + O(m^2)$$

3.5 Determination of the polynomial

- Fix the polynomial with requiring $\boxed{F_P(s) \to 1/s}$ + ChPT:

Brodsky & Lepage'80

- Feynman-Hellmann theorem:

$$\boxed{\Gamma_P(0) = \left(m_u \frac{\partial}{\partial m_u} + m_d \frac{\partial}{\partial m_d} \right) M_P^2}$$

$$\boxed{\Delta_P(0) = \left(m_s \frac{\partial}{\partial m_s} \right) M_P^2}$$

- At LO in ChPT:

$$M_{\pi^+}^2 = (m_u + m_d)\, B_0 + O(m^2)$$
$$M_{K^+}^2 = (m_u + m_s)\, B_0 + O(m^2)$$
$$M_{K^0}^2 = (m_d + m_s)\, B_0 + O(m^2)$$

$$\Rightarrow \boxed{\begin{aligned} P_\Gamma(s) &= \Gamma_\pi(0) = M_\pi^2 + \cdots \\ Q_\Gamma(s) &= \frac{2}{\sqrt{3}} \Gamma_K(0) = \frac{1}{\sqrt{3}} M_\pi^2 + \cdots \\ P_\Delta(s) &= \Delta_\pi(0) = 0 + \cdots \\ Q_\Delta(s) &= \frac{2}{\sqrt{3}} \Delta_K(0) = \frac{2}{\sqrt{3}} \left(M_K^2 - \frac{1}{2} M_\pi^2 \right) + \cdots \end{aligned}}$$

3.5 Determination of the polynomial

- At LO in ChPT:

$$M^2_{\pi^+} = (m_u + m_d)\, B_0 + O(m^2)$$
$$M^2_{K^+} = (m_u + m_s)\, B_0 + O(m^2)$$
$$M^2_{K^0} = (m_d + m_s)\, B_0 + O(m^2)$$

- For the scalar FFs:

$$\begin{aligned} P_\Gamma(s) &= \Gamma_\pi(0) = M_\pi^2 + \cdots \\ Q_\Gamma(s) &= \frac{2}{\sqrt{3}}\Gamma_K(0) = \frac{1}{\sqrt{3}}M_\pi^2 + \cdots \\ P_\Delta(s) &= \Delta_\pi(0) = 0 + \cdots \\ Q_\Delta(s) &= \frac{2}{\sqrt{3}}\Delta_K(0) = \frac{2}{\sqrt{3}}\left(M_K^2 - \frac{1}{2}M_\pi^2\right) + \cdots \end{aligned}$$

- Problem: large corrections in the case of the kaons!
  ➡ Use lattice QCD to determine the SU(3) LECs

$$\Gamma_K(0) = (0.5 \pm 0.1)\; M_\pi^2$$

$$\Delta_K(0) = 1^{+0.15}_{-0.05}\,(M_K^2 - 1/2M_\pi^2)$$

Daub, Dreiner, Hanhart, Kubis, Meissner'13
Bernard, Descotes-Genon, Toucas'12

3.5 Determination of the polynomial

- For θ_P enforcing the asymptotic constraint is not consistent with ChPT
  The unsubtracted DR is not saturated by the 2 states

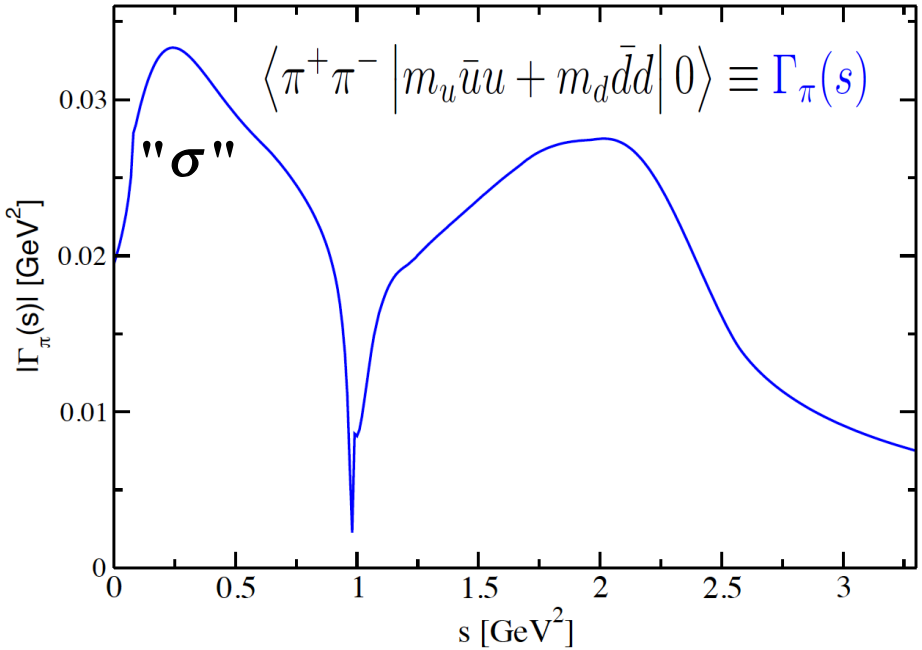
  ➡ Relax the constraints and match to ChPT

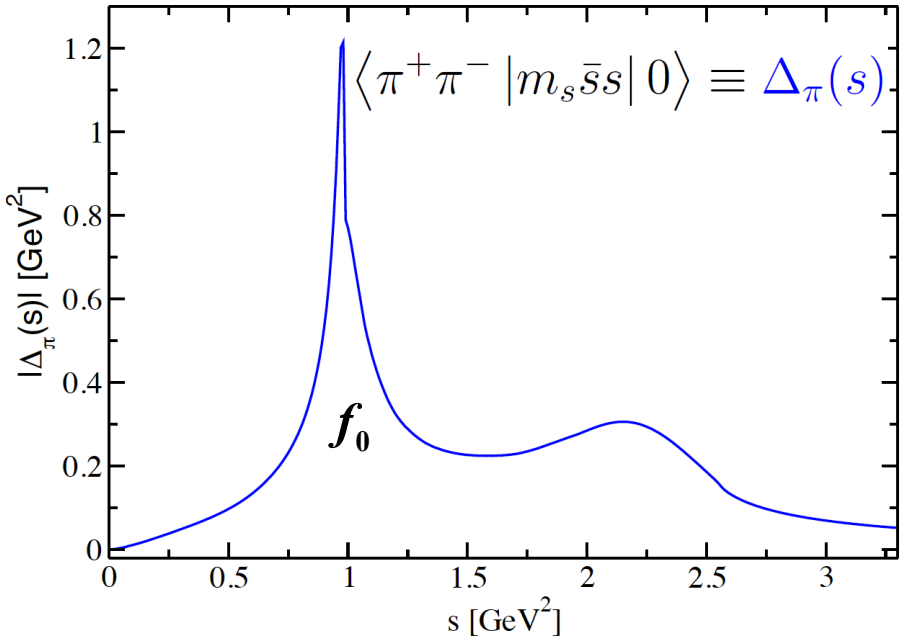
$$\boxed{\begin{aligned} P_\theta(s) &= 2M_\pi^2 + \left(\dot\theta_\pi - 2M_\pi^2\dot C_1 - \frac{4M_K^2}{\sqrt{3}}\dot D_1\right) s \\ Q_\theta(s) &= \frac{4}{\sqrt{3}}M_K^2 + \frac{2}{\sqrt{3}}\left(\dot\theta_K - \sqrt{3}M_\pi^2\dot C_2 - 2M_K^2\dot D_2\right) s \end{aligned}}$$

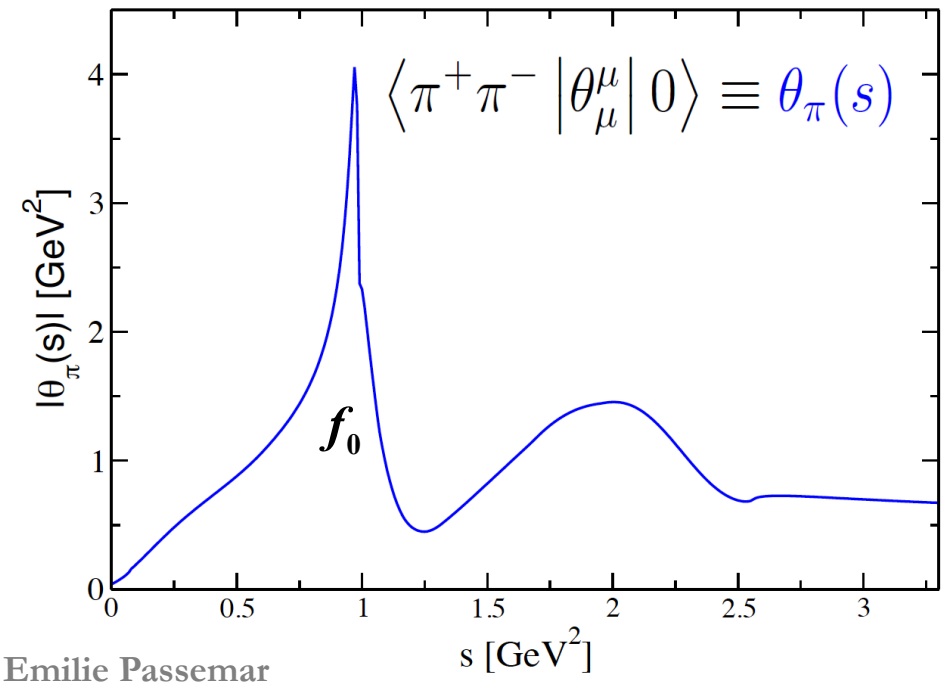
with $\dot f = \left(\frac{df}{ds}\right)_{s=0}$

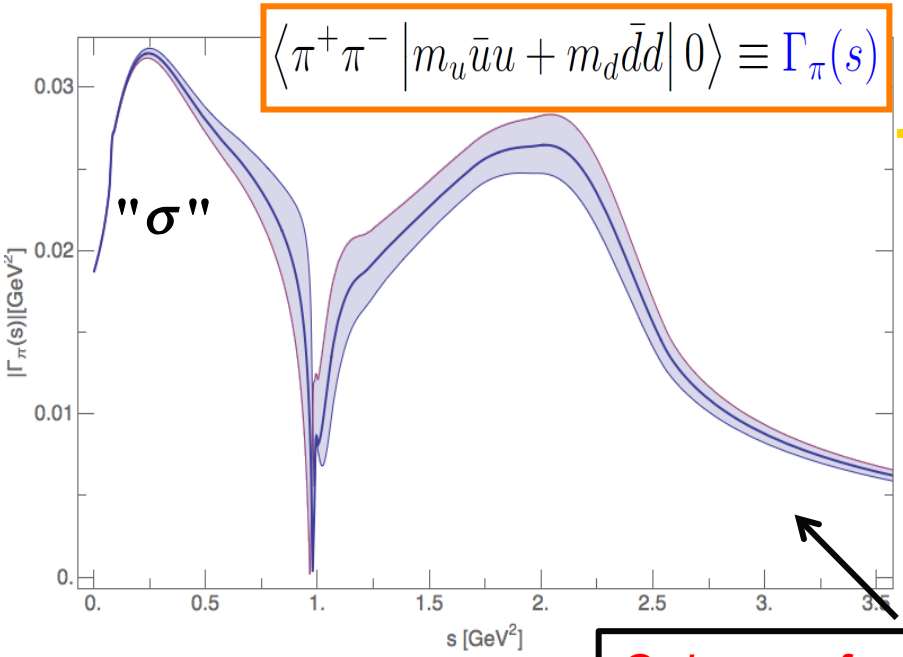
- At LO ChPT: $\dot\theta_{\pi,K} = 1$

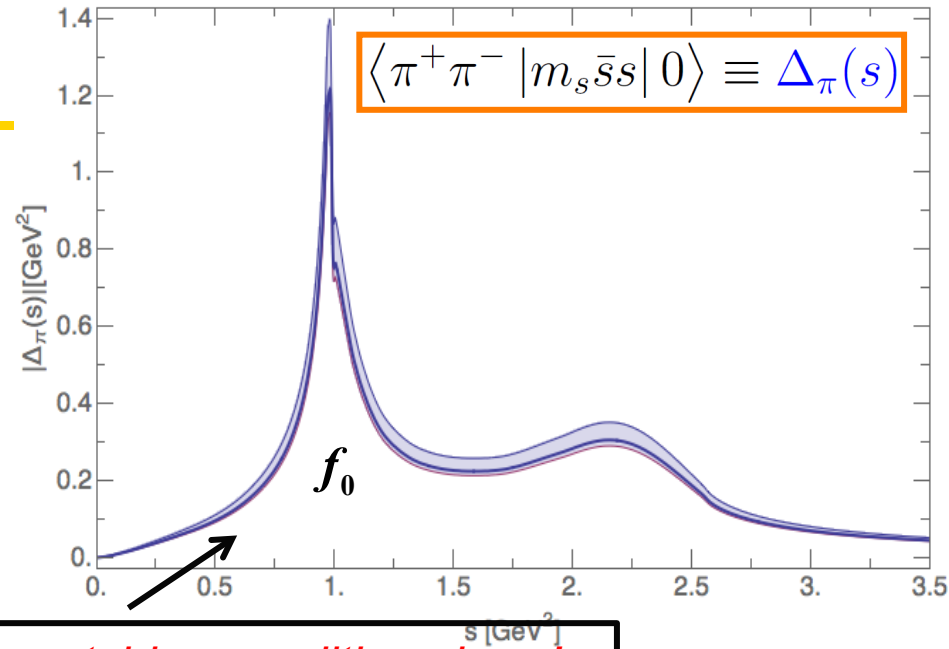
- Higher orders ➡ $\boxed{\dot\theta_K = 1.15 \pm 0.1}$


$\langle \pi^+\pi^- | m_u \bar{u}u + m_d \bar{d}d | 0 \rangle \equiv \Gamma_\pi(s)$
"σ"
$|\Gamma_\pi(s)|$ [GeV^2]
s [GeV^2]

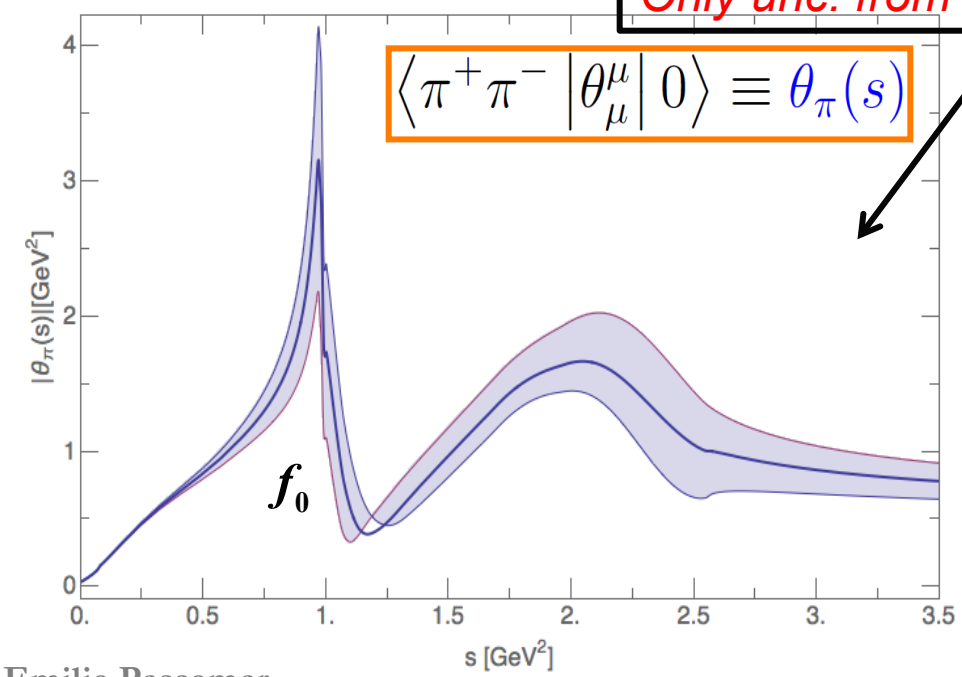

$\langle \pi^+\pi^- | m_s \bar{s}s | 0 \rangle \equiv \Delta_\pi(s)$
f_0
$|\Delta_\pi(s)|$ [GeV^2]
s [GeV^2]


$\langle \pi^+\pi^- | \theta^\mu_\mu | 0 \rangle \equiv \theta_\pi(s)$
f_0
$|\theta_\pi(s)|$ [GeV^2]
s [GeV^2]



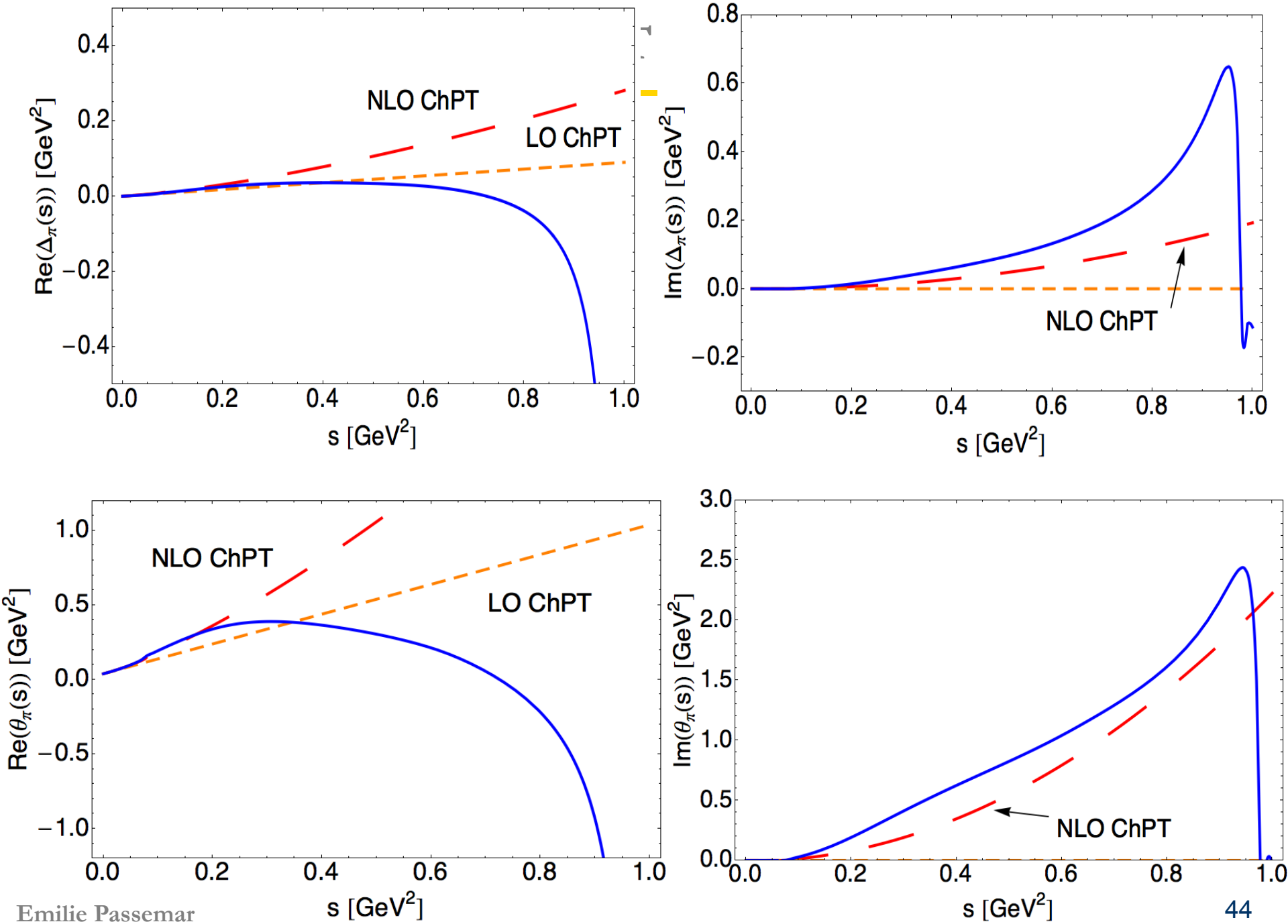


Only unc. from matching conditions here!



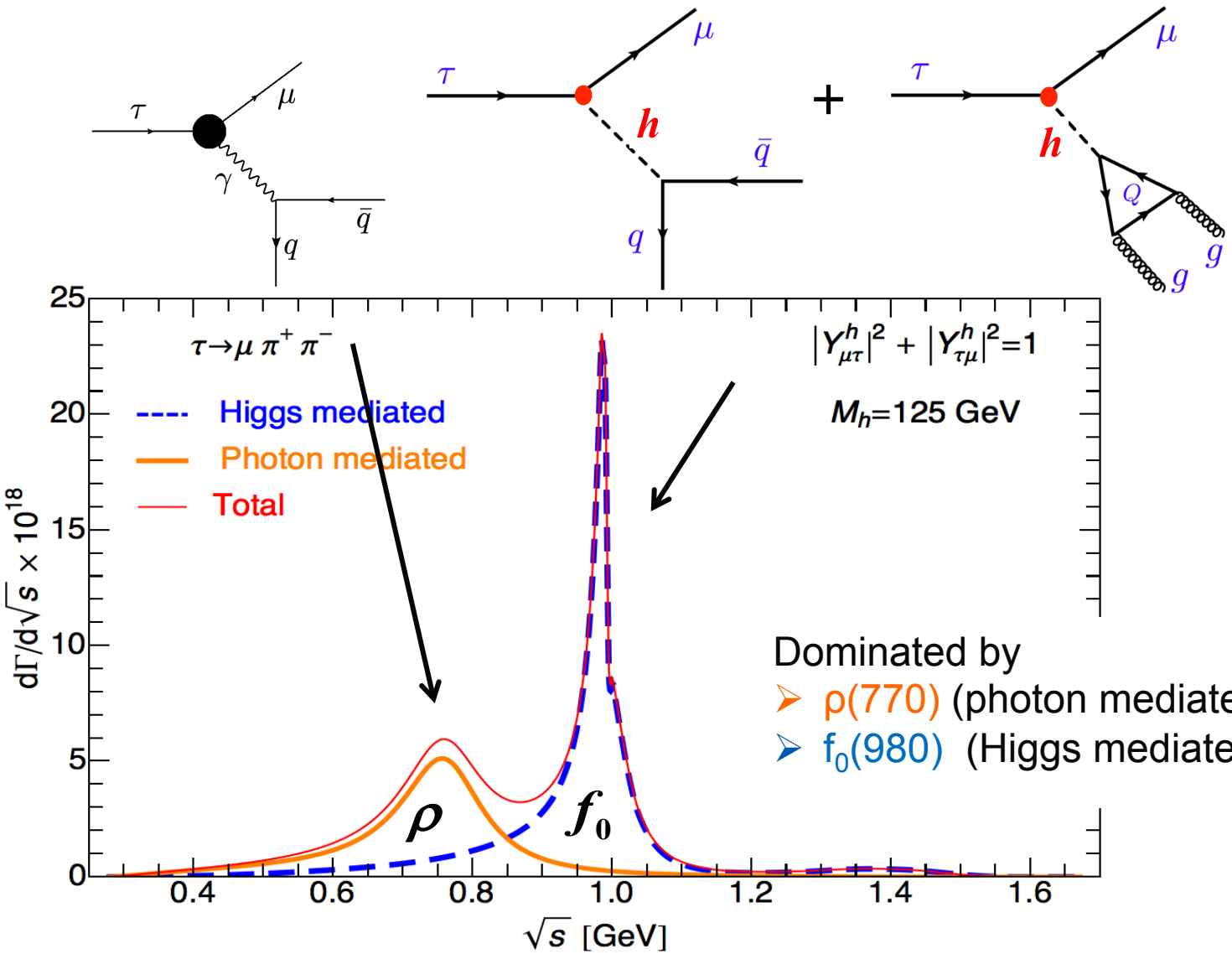
- Uncertainties:
  - Varying s_{cut} (1.4 GeV^2 -1.8 GeV^2)
  - Varying the matching conditions
  - T matrix inputs

See Talk by *S. Ropertz* for precise determination for E >1 GeV


NLO ChPT
LO ChPT
Re(Δ_π(s)) [GeV^2]
s [GeV^2]
Im(Δ_π(s)) [GeV^2]
NLO ChPT
s [GeV^2]
NLO ChPT
LO ChPT
Re(θ_π(s)) [GeV^2]
s [GeV^2]
Im(θ_π(s)) [GeV^2]
NLO ChPT
s [GeV^2]

3.6 Results: Spectrum

Celis, Cirigliano, E.P.'14

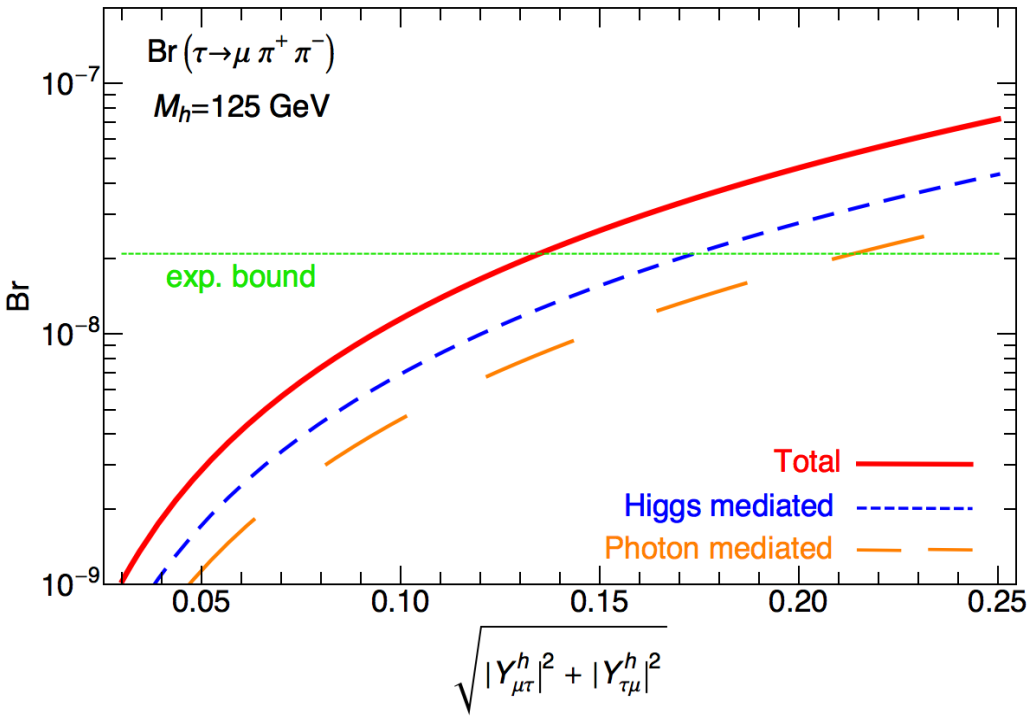


Dominated by

- ρ(770) (photon mediated)
- f_0(980) (Higgs mediated)

3.7 Results: Bounds

Celis, Cirigliano, E.P.'14



Bound:

$$\sqrt{\left|Y^h_{\mu\tau}\right|^2 + \left|Y^h_{\tau\mu}\right|^2} \leq 0.13$$

Process	(BR $\times 10^8$) 90% CL	$\sqrt{\lvert Y^h_{\mu\tau}\rvert^2 + \lvert Y^h_{\tau\mu}\rvert^2}$	Operator(s)
$\tau \to \mu\gamma$	< 4.4 [88]	< 0.016	Dipole
$\tau \to \mu\mu\mu$	< 2.1 [89]	< 0.24	Dipole
$\tau \to \mu\pi^+\pi^-$	< 2.1 [86]	< 0.13	Scalar, Gluon, Dipole
$\tau \to \mu\rho$	< 1.2 [85]	< 0.13	Scalar, Gluon, Dipole
$\tau \to \mu\pi^0\pi^0$	$< 1.4 \times 10^3$ [87]	< 6.3	Scalar, Gluon

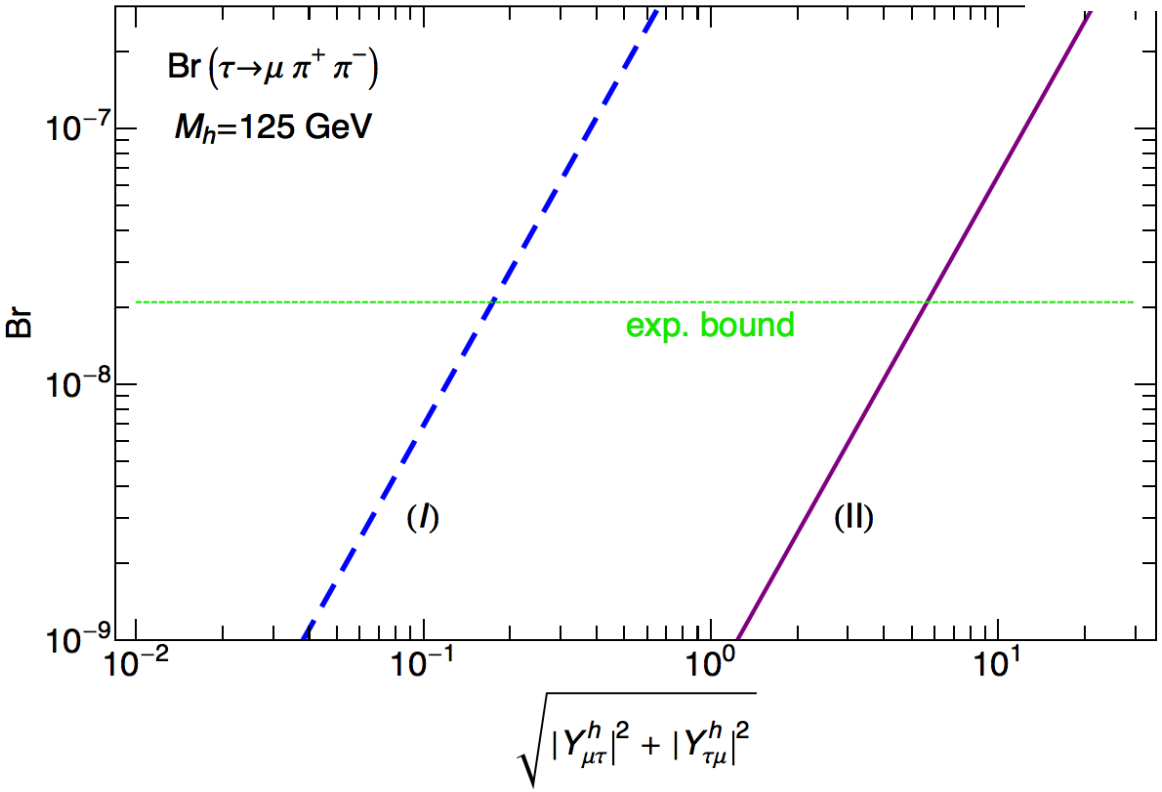
Less stringent but more robust handle on LFV Higgs couplings

?

BaBar'10, Belle'10'11'13 except last from *CLEO'97*

3.8 Impact of our results

Celis, Cirigliano, E.P.'14



- Dispersive treatment of hadronic part ➡ bound reduced by one order of magnitude!

- ChPT, EFT only valid at low energy for $\mathbf{p << \Lambda = 4\pi f_\pi \sim 1\ GeV}$

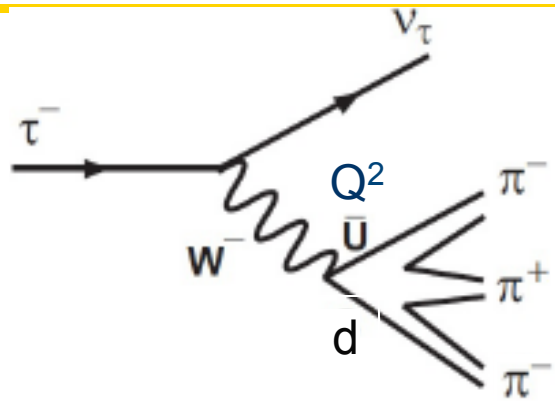
  ➡ *not valid up to* $E = (m_\tau - m_\mu)$!

4. 3-body τ-decays

4.1 $\tau \to \pi\pi\pi\nu_\tau$

Lorenz, E.P. in progress

- $\tau \to \pi\pi\pi\nu_\tau$



$$s = \left(p_{\pi^-} + p_{\pi_1^+}\right)^2, \quad t = \left(p_{\pi_1^+} + p_{\pi_2^+}\right)^2,$$

$$u = \left(p_{\pi^+} + p_{\pi^+}\right)^2$$

$$s + t + u = Q^2 + 3M^2_{\pi^\pm}$$

- 3-body: form factors function of one variable q^2=s ⇒ amplitude function of s and cosθ or t & u and Q^2
  Structure functions W_X *Kühn, Mirkes'92*

$$\boxed{\mathcal{M} \propto L_\mu H^\mu}$$ with $$\boxed{H_\mu = \langle \pi\pi\pi | V_\mu - A_\mu | 0 \rangle}$$

H_μ: restricted to axial vector current A_μ by G-parity

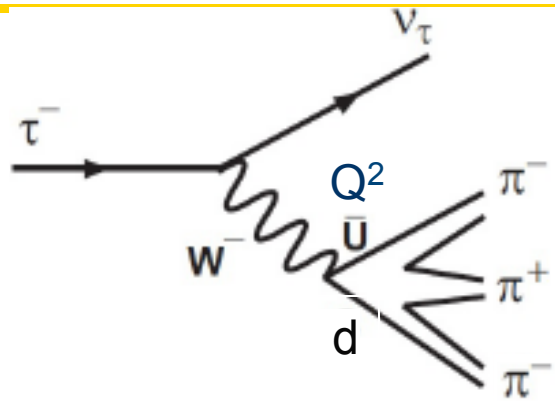
- Consider *helicity amplitudes* $\mathcal{A}_\lambda = \epsilon_\mu(\lambda) H^\mu$ simple partial wave expan.

Polarization vector of final state system with helicity $\lambda = \pm, 0, t$

4.1 $\tau \to \pi\pi\pi\nu_\tau$

Lorenz, E.P. in progress

- $\tau \to \pi\pi\pi\nu_\tau$



$$s = \left(p_{\pi^-} + p_{\pi_1^+}\right)^2, \quad t = \left(p_{\pi_1^+} + p_{\pi_2^+}\right)^2,$$

$$u = \left(p_{\pi^+} + p_{\pi^+}\right)^2$$

$$s + t + u = Q^2 + 3M^2_{\pi^\pm}$$

- 3-body: form factors function of one variable q^2=s of s and cosθ or t & u and Q^2 structure functions W_X → amplitude function

Kühn, Mirkes'92

$$\boxed{\mathcal{M} \propto L_\mu H^\mu}$$ with $$\boxed{H_\mu = \langle \pi\pi\pi | V_\mu - A_\mu | 0 \rangle}$$

H_μ: restricted to axial vector current A_μ by G-parity

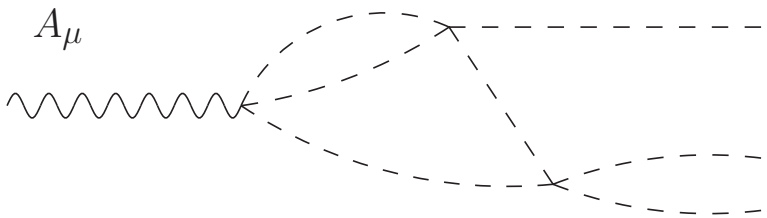
- Consider *helicity amplitudes* $\mathcal{A}_\lambda = \epsilon_\mu(\lambda) H^\mu$ simple partial wave expan.

- W_X: linear combinations of $H^{\lambda\lambda'} = \mathcal{A}_\lambda \mathcal{A}^\dagger_{\lambda'}$

4.2 Decomposition of the amplitude

Lorenz, E.P. in progress

- Analytical continuation of the amplitude and decomposition



$$\mathcal{A}_\lambda^{ijmn}(s,t,u) \propto \sum_I \sum_l \sqrt{2l+1}\, a^l_{I,\lambda}(s)\, d^l_{\lambda 0}(\theta_\pi)\, P_I^{ijmn}$$

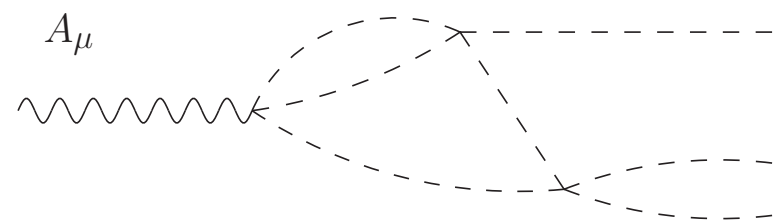
Wigner function

Isospin projection

$$-\sin\theta / \sqrt{l(l+1)}\, P_l'(\cos\theta)$$

4.2 Decomposition of the amplitude

Lorenz, E.P. in progress



- Analytical continuation of the amplitude and decomposition:

$$\boxed{\mathcal{A}_\lambda^{ijmn}(s,t,u) \propto \sum_I \sum_l \sqrt{2l+1}\, a^l_{I,\lambda}(s)\, d^l_{\lambda 0}(\theta_\pi)\, P_I^{ijmn}}$$

Wigner function

Isospin projection

$$-\sin\theta / \sqrt{l(l+1)}\, P_l'(\cos\theta)$$

Khuri-Treiman formalism

see talks this afternoon

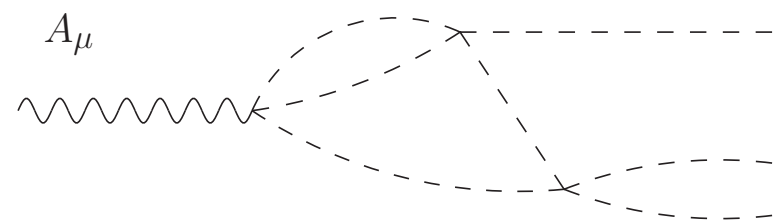
- Bose symmetry: I+l = even
- For the transverse amplitude, P and D-waves dominating:

$$\mathscr{A}_+^{3311}(s,t,u) \propto \sum_{l=0}^{l_{max}} \sum_I (2l+1) \left[d^l_{10}(\theta_s) \left(\frac{K(s)}{4s}\right)^{l-1} P_I^{3311} a^{\text{right}}_{+,Il}(s) + d^l_{10}(\theta_t) \left(\frac{K(t)}{4t}\right)^{l-1} P_I^{3131} a^{\text{right}}_{+,Il}(t) \right.$$
$$\left. + d^l_{10}(\theta_u) \left(\frac{K(u)}{4u}\right)^{l-1} P_I^{1331} a^{\text{right}}_{+,Il}(u) \right],$$

$$K(s) = \frac{t-u}{\cos\theta_s} = \sqrt{\lambda(s, M_\pi^2, M_\pi^2)} \sqrt{\lambda(s, Q^2, M_\pi^2)}.$$

4.2 Decomposition of the amplitude

Lorenz, E.P. in progress



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$$\left. + d^l_{10}(\theta_u) \left(\frac{K(u)}{4u}\right)^{l-1} P_I^{1331} a^{\text{right}}_{+,Il}(u) \right],$$

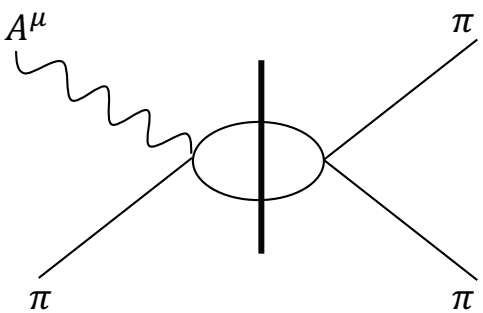
$$K(s) = \frac{t-u}{\cos\theta_s} = \sqrt{\lambda(s, M_\pi^2, M_\pi^2)} \sqrt{\lambda(s, Q^2, M_\pi^2)}.$$

4.3 Dispersion relations

Lorenz, E.P. in progress

- Unitarity : $\text{Disc } a_{Il}^{right}(s) \equiv \text{Disc } a_{Il}(s)$

$$\boxed{\text{Disc } a_{Il}(s) = \rho(s) t_l^*(s) \left(a_{Il}^{right}(s) + a_{Il}^{left}(s) \right)}$$



- Analyticity : Write a dispersion relation for

$$\boxed{a_{Il}^{right}(s) = \frac{1}{\pi} \int_{s_0}^{\infty} ds' \frac{\text{Disc } a_{Il}^{right}(s')}{s' - s}}$$

4.3 Dispersion relations

Lorenz, E.P. in progress

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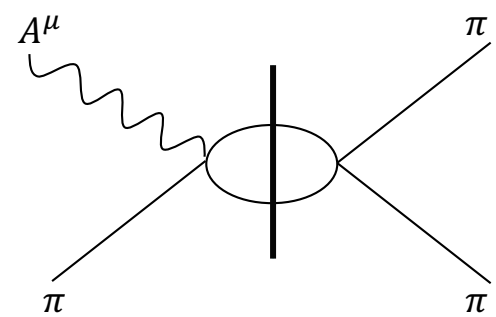


- Analyticity : Write a dispersion relation for

$$\boxed{a_{Il}^{right}(s) = \frac{1}{\pi} \int_{s_0}^{\infty} ds' \frac{\text{Disc } a_{Il}^{right}(s')}{s' - s}}$$

- Neglecting left-hand cut: Omnes solution

$\pi\pi$ phase shift

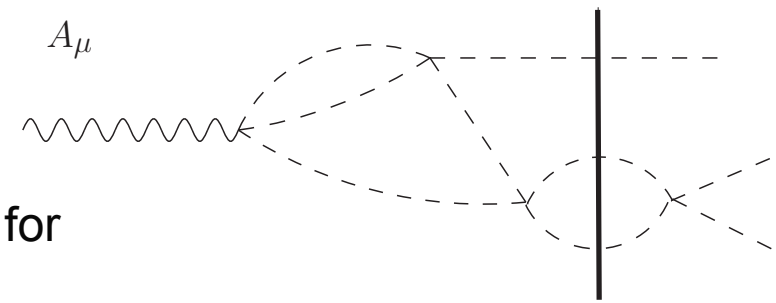
$$\boxed{a_I^{l\,\text{right}}(s) = \Omega_I^l(s) * G(s), \quad \Omega_I^l = \exp\left(\frac{s}{\pi} \int_{s_0}^{\infty} \frac{ds'}{s'} \frac{\delta_I^l(s')}{s' - s} \right)}$$

4.3 Dispersion relations

Lorenz, E.P. in progress

- Unitarity : $\text{Disc } a_{Il}^{right}(s) \equiv \text{Disc } a_{Il}(s)$

$$\boxed{\text{Disc } a_{Il}(s) = \rho(s) t_l^*(s) \left(a_{Il}^{right}(s) + a_{Il}^{left}(s) \right)}$$



- Analyticity : Write a dispersion relation for

$$\boxed{a_{Il}^{right}(s) = \frac{1}{\pi} \int_{s_0}^{\infty} ds' \frac{\text{Disc } a_{Il}^{right}(s')}{s' - s}}$$

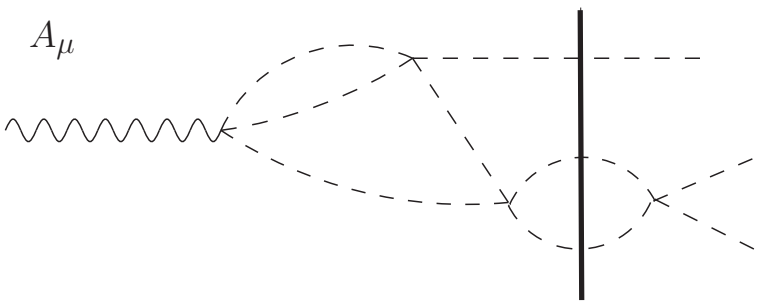
- Solution: Inhomogeneous Omnes solution

$$\boxed{a_{Il}^{right}(s) = \Omega_{Il}(s) \left(\sum_i^{n-1} c_i s^i + \frac{s^n}{\pi} \int_{s_0}^{\infty} \frac{ds'}{s'^n} \frac{\rho(s') t_l^*(s')}{\Omega_{Il}^*(s')} \frac{a_{Il}^{left}(s')}{(s' - s)} \right)}$$

4.3 Dispersion relations

Lorenz, E.P. in progress

- Solution:



$$\boxed{a_{Il}^{right}(s) = \Omega_{Il}(s)\left(\sum_i^{n-1} c_i s^i + \frac{s^n}{\pi}\int_{s_0}^{\infty}\frac{ds'}{s'^n}\frac{\rho(s')t_l^*(s')}{\Omega_{Il}^*(s')}\frac{a_{Il}^{left}(s')}{(s'-s)}\right)}$$

- With

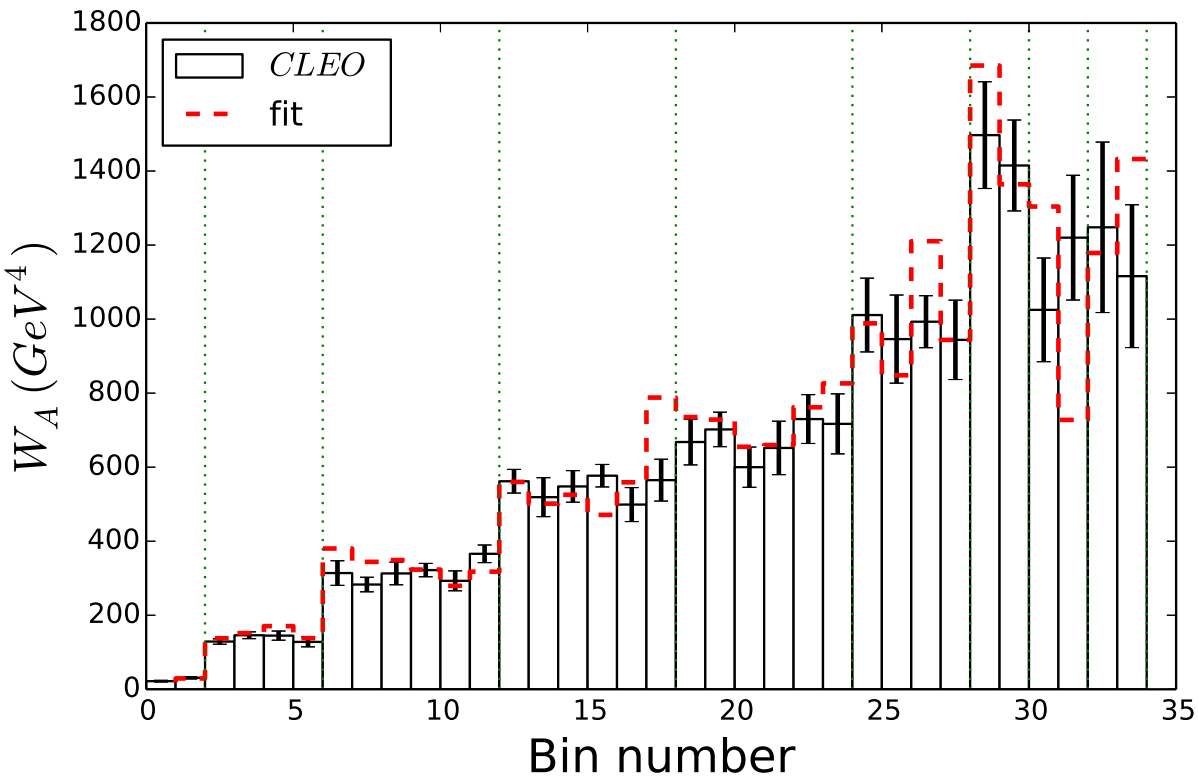
$$a_{Il}^{left}(s) \propto \sum_{I'\,l'}(2l'+1)\int_{-1}^{+1} dz_s(1-z_s^2)P_l'(z_s)\left(P_{l'}'(z_t)C_{st}^{II'}\,a_{I'l'}^{right}(t(s,z_s)) + P_{l'}'(z_u)C_{su}^{II'}\,a_{I'l'}^{right}(u(s,z_s))\right)$$

- Solve by an iterative procedure

4.4 Determination of the amplitude

Lorenz, E.P. in progress

- Analysis in progress : $W_A(s,t,u) \propto |\mathscr{A}_+^{3311}(s,t,u)|^2 + |\mathscr{A}_-^{3311}(s,t,u)|^2$
- Relate the amplitude to the structure functions measured by CLEO

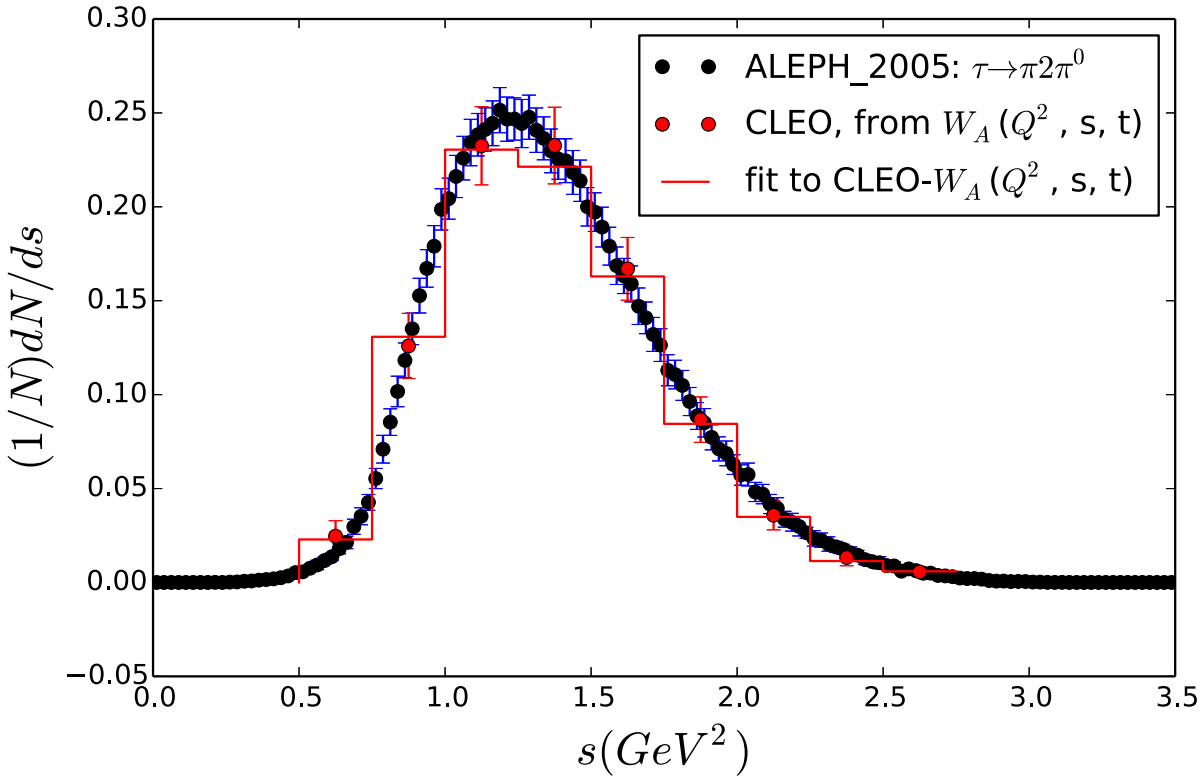


- Problem, low statistics: it would be great to have more measurements!

4.4 Determination of the amplitude

Lorenz, E.P. in progress

- Analysis in progress
- Decay spectrum measured by ALEPH:



- It would be very good to have in addition the Dalitz plot distribution in Q^2!

5. Conclusion and outlook

Conclusion and outlook

- Hadronic tau decays are very important ⇨ test of
  - QCD
  - EW effects: CPV in $\tau \to K\pi\nu_\tau$, V_{us}, Higgs LFV etc.
- We have looked at:
  - 2 body: FFs for CPV in $\tau \to K\pi\nu_\tau$ and search for LFV in in $\tau \to \pi\pi\mu$
  - 3 body effects in $\tau \to \pi\pi\pi\nu_\tau$ in progress
- Experimental activities: CLEO, Belle, BaBar, LHCb
- Intense theoretical activities : QCD, new physics
- A lot of *very interesting physics* remains to be done in the tau sector!
- But we need more experimental measurements and accurate theoretical prediction until energies of $m_\tau \sim 1.8$ GeV

6. Back-up

3.3 $\tau \to K\pi\nu_\tau$ CP violating asymmetry

- The angular CP asymmetry from Belle:

$$\frac{d\Gamma\left(\tau^- \to K\pi^-\nu_\tau\right)}{d\sqrt{Q^2}\,d\cos\theta\;d\cos\beta} = \left[A(Q^2) - B(Q^2)\left(3\cos^2\psi - 1\right)\left(3\cos^2\beta - 1\right)\right]\left|f_+(s)\right|^2 + m_\tau^2\left|\tilde{f}_0(s)\right|^2 - C(Q^2)\cos\psi\cos\beta\,\mathrm{Re}\left(f_+(s)\tilde{f}_0^*(s)\right)$$

CP violating term
S-P interference

- When integrating on the angle the interference term between scalar and vector vanishes

$$\frac{d\Gamma}{d\sqrt{Q^2}} = \frac{G_F^2\sin^2\theta_c m_\tau^3}{3\times 2^5\times\pi^3 Q^2}\left(1-\frac{Q^2}{m_\tau^2}\right)^2\left(1+\frac{2Q^2}{m_\tau^2}\right)$$

$$\times q_1(Q^2)\left\{q_1(Q^2)^2\,|F_V|^2 + \frac{3}{4}\frac{Q^2}{\left(1+2Q^2/m_\tau^2\right)}|F_S|^2\right\}$$

3.3 $\tau \to K\pi\nu_\tau$ CP violating asymmetry

Devi, Dhargyal, Sinha'14
Cirigliano, Crivellin, Hoferichter'17

- We need a tensor interaction to get some interference:

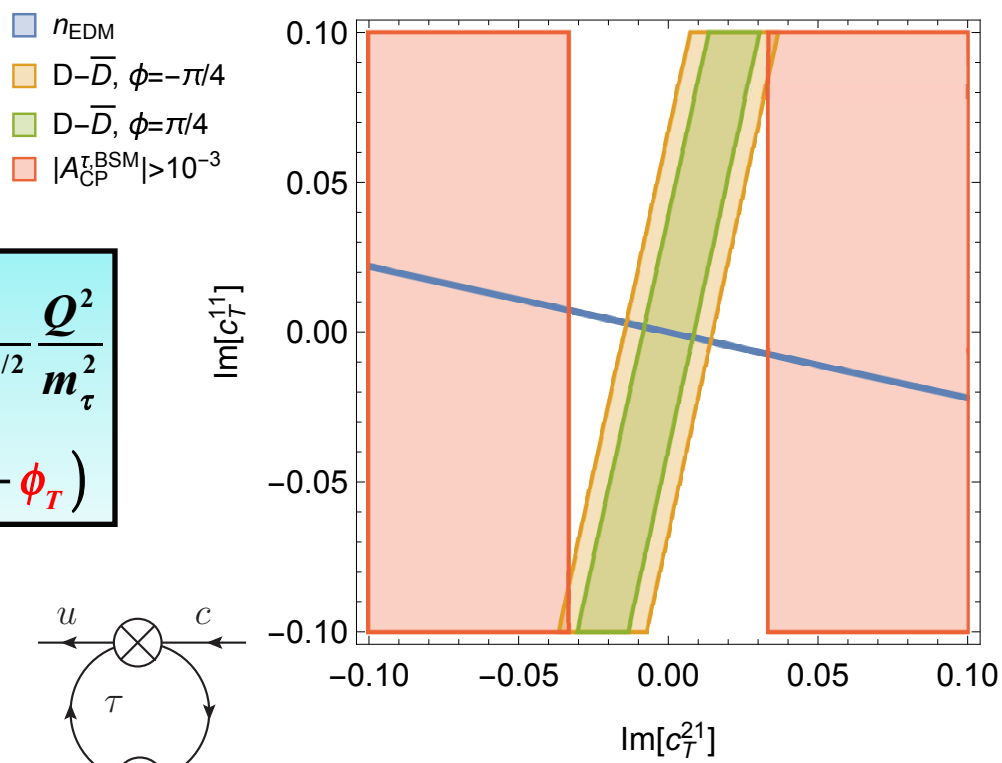
$$\mathcal{H}_T^{eff} \equiv G'(\bar{s}\sigma_{\mu\nu}u)(\bar{\nu}_\tau(1+\gamma_5)\sigma^{\mu\nu}\tau)$$

with $G' = \frac{G_F}{\sqrt{2}} C_T, \quad C_T = |C_T| e^{i\phi_T}$

- When integrating the interference term between vector and tenson does not vanish:

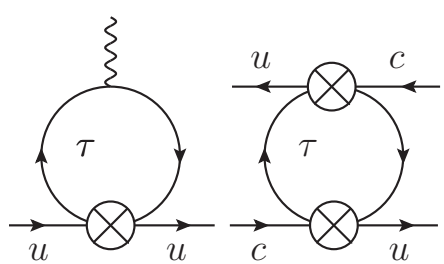
$$\frac{d\Gamma}{dQ^2} = \frac{d\Gamma_{SM}}{dQ^2} + \frac{d\Gamma_T}{dQ^2} + \frac{d\Gamma_{V-T}}{dQ^2}$$

$$\frac{d\Gamma_{V-T}}{dQ^2} = G_F^2 \sin^2\theta_C \frac{m_\tau^3}{32\pi^3}\left(\frac{m_\tau^2 - Q^2}{m_\tau^2}\right)^2 \frac{q_1^3}{(Q^2)^{3/2}} \frac{Q^2}{m_\tau^2} \times |C_T||F_V(s)||F_T(s)|\cos(\delta_T(s) - \delta_V(s) + \phi_T)$$



In conflict with bounds from neutron EDM and D$\bar{D}$ mixing

Cirigliano, Crivellin, Hoferichter'17



3.1 Application 1: Kπ form factors and V_{us}

- Master formula for $\tau \to K\pi\nu_\tau$:

$$\Gamma\left(\tau \to \overline{K}\pi\nu_\tau[\gamma]\right) = \frac{G_F^2 m_\tau^5}{96\pi^3} C_K^2 S_{EW}^\tau \left|V_{us}\right|^2 \left|f_+^{K^0\pi^-}(0)\right|^2 I_K^\tau \left(1 + \delta_{\mathrm{EM}}^{K\tau} + \tilde{\delta}_{\mathrm{SU(2)}}^{K\pi}\right)^2$$

$$I_K^\tau = \int ds\, F\left(s, \overline{f}_+(s), \overline{f}_0(s)\right)$$

Hadronic matrix element: Crossed channel from $K \to \pi l \nu_l$

$$\langle K\pi | \bar{s}\gamma_\mu u | 0\rangle = \left[\left(p_K - p_\pi\right)_\mu + \frac{\Delta_{K\pi}}{s}\left(p_K + p_\pi\right)_\mu\right] f_+(s) - \frac{\Delta_{K\pi}}{s}\left(p_K + p_\pi\right)_\mu f_0(s)$$

vector scalar

with $s = q^2 = (p_K + p_\pi)^2$, $\overline{f}_{0,+}(t) = \dfrac{f_{0,+}(t)}{f_+(0)}$

➡ Use a *dispersive parametrization* to combine with K_{l3} analysis

Extraction of V_{us}

- Decay rate master formula

Antonelli, Cirigliano, Lusiani, E.P.'13

$$\Gamma\left(\tau \to \overline{K}\pi\nu_\tau[\gamma]\right) = \frac{G_F^2 m_\tau^5}{96\pi^3} C_K^2 S_{EW}^\tau \left|V_{us}\right|^2 \left|f_+^{K^0\pi^-}(0)\right|^2 I_K^\tau \left(1 + \delta_{EM}^{K\tau} + \cancel{\tilde{\delta}_{SU(2)}^{K\pi}}\right)^2$$

$$BR\left(\tau \to \overline{K^0}\pi^-\nu_\tau\right) = (0.416 \pm 0.008)\%$$

Belle'14

$$S_{ew} = 1.0201$$

Marciano & Sirlin'88, Braaten & Li'90, Erler'04

$$\delta_{EM}^{\bar{K}^0\tau} = (-0.15 \pm 0.2)\%$$

$$I_{K^0}^\tau = 0.50432 \pm 0.01721$$

$$f_+(0) = 0.9661(32)$$

$$f_+(0)\left|V_{us}\right| = 0.2141 \pm 0.0014_{I_K} \pm 0.0021_{exp}$$

$$\left|V_{us}\right| = 0.2216 \pm 0.0027$$

Extraction of V_{us}

- Decay rate master formula

Antonelli, Cirigliano, Lusiani, E.P.'13

$$\Gamma\left(\tau \to \overline{K}\pi\nu_\tau[\gamma]\right) = \frac{G_F^2 m_\tau^5}{96\pi^3} C_K^2 S_{EW}^\tau \left|V_{us}\right|^2 \left|f_+^{K^0\pi^-}(0)\right|^2 I_K^\tau \left(1 + \delta_{\mathrm{EM}}^{K\tau} + \cancel{\tilde{\delta}_{SU(2)}^{K\pi}}\right)^2$$

$$f_+(0) = 0.9661(32)$$

FLAG'13

$$f_+(0)\left|V_{us}\right| = 0.2141 \pm 0.0014_{I_K} \pm 0.0021_{\exp}$$

$$\left|V_{us}\right| = 0.2216 \pm 0.0027$$

- Result of fit to $K_{l3} + \tau \longrightarrow K\pi\nu_\tau$ and $K\pi$ scattering data including inelasticities in the dispersive FFs

$$f_+(0)\left|V_{us}\right| = 0.2163 \pm 0.0014$$

Bernard'14

4.3 Effective Field Theory approach

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \ldots$$

See e.g.
Black, Han, He, Sher'02
Brignole & Rossi'04
Dassinger et al.'07
Matsuzaki & Sanda'08
Giffels et al.'08
Crivellin, Najjari, Rosiek'13
Petrov & Zhuridov'14
Cirigliano, Celis, E.P.'14

- Build all D>5 LFV operators:
  - Dipole: $\mathcal{L}_{eff}^{D} \supset -\frac{C_D}{\Lambda^2} m_\tau \bar{\mu} \sigma^{\mu\nu} P_{L,R} \tau F_{\mu\nu}$
  - Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector): $\mathcal{L}_{eff}^{S} \supset -\frac{C_{S,V}}{\Lambda^2} m_\tau m_q G_F \bar{\mu}\, \Gamma P_{L,R} \tau\, \bar{q} \Gamma q$
  - Lepton-gluon (Scalar, Pseudo-scalar): $\mathcal{L}_{eff}^{G} \supset -\frac{C_G}{\Lambda^2} m_\tau G_F \bar{\mu} P_{L,R} \tau\, G_{\mu\nu}^a G_a^{\mu\nu}$
  - 4 leptons (Scalar, Pseudo-scalar, Vector, Axial-vector): $\mathcal{L}_{eff}^{4\ell} \supset -\frac{C_{S,V}^{4\ell}}{\Lambda^2} \bar{\mu}\, \Gamma P_{L,R} \tau\, \bar{\mu}\, \Gamma P_{L,R} \mu$
- Each UV model generates a *specific pattern* of them

$\Gamma \equiv 1, \gamma^\mu$

4.4 Model discriminating power of Tau processes

Celis, Cirigliano, E.P.'14

- Summary table:

	$\tau \to 3\mu$	$\tau \to \mu\gamma$	$\tau \to \mu\pi^+\pi^-$	$\tau \to \mu K\bar{K}$	$\tau \to \mu\pi$	$\tau \to \mu\eta^{(\prime)}$
$O^{4\ell}_{S,V}$	✓	–	–	–	–	–
O_D	✓	✓	✓	✓	–	–
O^q_V	–	–	✓ (I=1)	✓(I=0,1)	–	–
O^q_S	–	–	✓ (I=0)	✓(I=0,1)	–	–
O_{GG}	–	–	✓	✓	–	–
O^q_A	–	–	–	–	✓ (I=1)	✓ (I=0)
O^q_P	–	–	–	–	✓ (I=1)	✓ (I=0)
$O_{G\tilde{G}}$	–	–	–	–	–	✓

- The notion of "*best probe*" (process with largest decay rate) is *model dependent*

- If observed, compare rate of processes ⇨ key handle on *relative strength* between operators and hence on the *underlying mechanism*

4.4 Model discriminating power of Tau processes

Celis, Cirigliano, E.P.'14

- Summary table:

	$\tau \to 3\mu$	$\tau \to \mu\gamma$	$\tau \to \mu\pi^+\pi^-$	$\tau \to \mu K\bar{K}$	$\tau \to \mu\pi$	$\tau \to \mu\eta^{(\prime)}$
$O^{4\ell}_{S,V}$	✓	–	–	–	–	–
O_D	✓	✓	✓	✓	–	–
O^q_V	–	–	✓ (I=1)	✓(I=0,1)	–	–
O^q_S	–	–	✓ (I=0)	✓(I=0,1)	–	–
O_{GG}	–	–	✓	✓	–	–
O^q_A	–	–	–	–	✓ (I=1)	✓ (I=0)
O^q_P	–	–	–	–	✓ (I=1)	✓ (I=0)
$O_{G\tilde{G}}$	–	–	–	–	–	✓

- In addition to leptonic and radiative decays, *hadronic decays* are very important sensitive to large number of operators!

- But need reliable determinations of the hadronic part:
  form factors and *decay constants* (e.g. f_η, $f_{\eta'}$)

4.4 Model discriminating power of Tau processes

- Summary table: *Celis, Cirigliano, E.P.'14*

	$\tau \to 3\mu$	$\tau \to \mu\gamma$	$\tau \to \mu\pi^+\pi^-$	$\tau \to \mu K\bar{K}$	$\tau \to \mu\pi$	$\tau \to \mu\eta^{(\prime)}$
$O^{4\ell}_{S,V}$	✓	–	–	–	–	–
O_D	✓	✓	✓	✓	–	–
O^q_V	–	–	✓ (I=1)	✓(I=0,1)	–	–
O^q_S	–	–	✓ (I=0)	✓(I=0,1)	–	–
O_{GG}	–	–	✓	✓	–	–
O^q_A	–	–	–	–	✓ (I=1)	✓ (I=0)
O^q_P	–	–	–	–	✓ (I=1)	✓ (I=0)
$O_{G\tilde{G}}$	–	–	–	–	–	✓

- Recent progress in $\tau \to \mu(e)\pi\pi$ using *dispersive techniques* *Daub et al'13* *Celis, Cirigliano, E.P.'14*

- Hadronic part: $\boxed{H_\mu = \langle \pi\pi | \left(V_\mu - A_\mu\right) e^{iL_{QCD}} | 0 \rangle = \left(\textit{Lorentz struct.}\right)^i_\mu F_i(s)}$ with $s = \left(p_{\pi^+} + p_{\pi^-}\right)^2$

- Form factors determined by solving 2-channel unitarity condition, with I=0 s-wave ππ and KK scattering data as input

$$\boxed{\mathrm{Im}F_n(s) = \sum_{m=1}^{2} T^*_{nm}(s)\sigma_m(s)F_m(s)}$$

$$n = \pi\pi, K\bar{K}$$