

On nonequilibrium quarkonium evolution in the QGP fireball

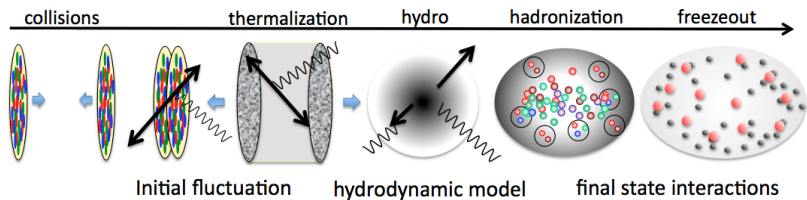
Yan Zhu

Technische Universität München

with N. Brambilla, A. Vairo, P. Vander Griend

7 Sep. 2018, Garching, Exploring the Perfect Liquid

Transport process in Heavy-Ion collisions



Nonaka et al. 2012

The system in Heavy-Ion collisions is non-equilibrium.

Non-equilibrium evolution of the system:

- ▶ Transport of soft particles: Classic Yang-Mills (CGC) evolution, Effective kinetic theory, Hydrodynamic evolution
- ▶ Transport of hard particles: Jet, Electromagnetic probes, Heavy flavor

Ingredients for the transport process:

- ▶ Transport coefficients
- ▶ Transport equations

Heavy quarkonium in the QGP

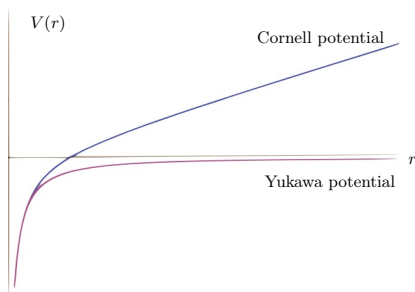
The quarkonium suppression is expected to serve as a signal for the formation of a quark-gluon plasma (QGP). Matsui and Satz 1986

- ▶ Heavy quark produced at the beginning of heavy-ion collisions.
- ▶ It is sensitive to the properties and evolution of QGP.
- ▶ Final state dilepton doesn't interact with other hadrons.

The explanation of Matsui and Satz 1986 is that

- ▶ Cornell potential $V(r) = -\frac{\alpha_s}{r} + \sigma r$ at $T = 0$
- ▶ Yukawa potential $V(r) = -\frac{\alpha_s e^{-r/m_D}}{r}$ and $\sigma = 0$ for $T > T_c$

Screening is stronger at high T , as the Debye radius $r_D \sim 1/m_D$ decreases with increasing T .

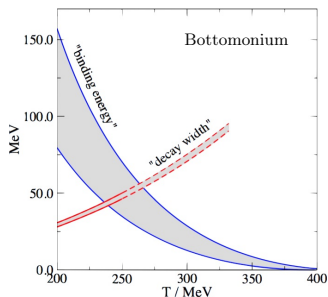


Heavy quarkonium in the QGP

Static potential of heavy quarkonium in real time at finite temperature for $1/r \sim gT$ is found to have an imaginary part. Laine et al. 2007; Brambilla et al. 2008; Beraudo et al. 2008; Rothkopf et al. 2012; Burnier and Rothkopf 2012; Burnier et al. 2014.

$$V(r) = -\frac{\alpha_s e^{-rm_D}}{r} - \alpha_s m_D - i\alpha_s T \phi(rm_D)$$

with $\phi(x)$ a monotonic function with $\phi(0) = 0$ and $\phi(\infty) = 1$, which is related to the decay width of the heavy quark.



Laine et al. 2007

Heavy quarkonium in the QGP

Scales of Heavy quarkonium $M \gg Mv \gg Mv^2$. $v \ll 1$ makes heavy quarkonium a **non-relativistic** bound state.

- ▶ M : hard scale
- ▶ $Mv \sim p \sim 1/r$: soft scale, momentum transfer between $Q\bar{Q}$
- ▶ $Mv^2 \sim E$: ultrasoft(US) scale, binding energy

Thermodynamic scales of the QGP: $T, m_D \sim gT, \dots$

QCD scale: Λ_{QCD}

Mechanisms for quarkonium evolution in QGP includes **regeneration, and dissociation** (color screening and scattering, and gluon absorption).

The different scales for the quarkonium evolution in a thermal medium calls for an effective field theory description of the system.

Effective field theories

The non-relativistic QCD (NRQCD) is carried out by integrating out the hard scale M . [Caswell and Lepage 1986](#); [Bodwin et al. 1995](#)

- ▶ $M \gg p, E, \Lambda_{QCD}$ ($m_{c,b} = 1.27, 4.2$ GeV, $\Lambda_{QCD} = 0.2$ GeV), the matching can be done perturbatively.
- ▶ The Lagrangian is organised as an expansion of $\frac{1}{M}$.

The potential non-relativistic QCD (pNRQCD) is derived by further integrating out the soft scale mv from NRQCD. [Pineda and Soto 1998](#); [Brambilla et al. 2000](#)

- ▶ $M \gg p \gg E, \Lambda_{QCD}$, heavy quarks and anti-quarks are described with bound states of color-singlet ($S = S/\sqrt{N_c}$) and octet ($O = O^a T^a / \sqrt{T_F}$) with Coulomb potential ($h_{s,o} = \frac{\mathbf{p}_1^2}{2M} + \frac{\mathbf{p}_2^2}{2M} + V_{s,o}$).
- ▶ The Lagrangian is organised as an expansion of $\frac{1}{M}$ and r .
- ▶ To NLO in multipole expansion:

$$\begin{aligned} L_{pNRQCD} = & L_{q+g} + \int d^3r \left\{ \text{Tr} [S^\dagger (i\partial_0 - h_s) S + O^\dagger (iD_0 - h_s) O] \right. \\ & \left. + \text{Tr} [O^\dagger \mathbf{r} \cdot g\mathbf{E} S + S^\dagger \mathbf{r} \cdot g\mathbf{E} O + \frac{1}{2} (O^\dagger \mathbf{r} \cdot g\mathbf{E} O + O^\dagger O \mathbf{r} \cdot g\mathbf{E})] \right\}. \end{aligned}$$

Transport equations

Quantum transport equations

- ▶ Evolution of correlators (or density matrices): Schwinger-Dyson equation, Keldysh-Baym equation [Akamatsu 2015](#), [Brambilla et al. 2017](#)

(Semi-)Classical transport equations

- ▶ Langevin equation, Fokker-Plank equation [Blaizot et al. 2015](#) : **Momentum drag/diffusion coefficient**

$$\dot{\mathbf{r}} = \frac{\mathbf{P}}{M}, \quad M\ddot{\mathbf{r}}_i = -F_i - \eta_{ij}\mathbf{p}_j + \xi_i$$

$$\langle \xi_i(t)\xi_i(t') \rangle = \delta(t-t')\lambda_{ij} \text{ with } \lambda_{ij} = 2MT\eta_{ij}$$

$$\left(\frac{\partial}{\partial t} + \frac{\mathbf{p} \cdot \nabla_{\mathbf{r}}}{M} \right) P(t, \mathbf{r}, \mathbf{p}) = \nabla_{\mathbf{p}_i} [(\eta_{ij}\mathbf{p}_j + F_i)P] + \frac{1}{2} \nabla_{\mathbf{p}_i} \nabla_{\mathbf{p}_j} [\lambda_{ij}P]$$

- ▶ Boltzmann equation [Muller et al. 2017](#): **Loss/gain term**

$$\left(\frac{d}{dt} + \frac{\mathbf{p} \cdot \nabla_{\mathbf{r}}}{M} \right) f(t, \mathbf{r}, \mathbf{p}) = -C_- + C_+$$

Relations from quantum to classical transport equations was discussed by e.g. [Zurek 1991](#); [Akamatsu 2015](#); [Blaizot and Escobedo 2017](#); [De Boni 2017](#)

The relation between different transport equations is not fully understood. It would be important to derive those equations from QCD.

Open Quantum System

Wikipedia: In physics, an **open quantum system** is a quantum-mechanical system which interacts with an external quantum system, the environment.

For Heavy quarkonium in high energy heavy-ion collisions: the QGP + heavy quark is considered as a large closed system.

$$H = H_Q + H_{QGP} + H_I$$

The Heavy quarkonium (HQ) evolution is described by

$$\rho_Q(t) = \text{Tr}_{QGP}[\rho(t)]$$

- ▶ Interaction between the HQ system and the QGP is weak:

$$\rho(t) = \rho_Q \otimes \rho_{QGP}$$

Evolution of the HQ system:

$$i \frac{d\rho_Q(t)}{dt} = [H_Q, \rho_Q(t)] + iD\rho_Q(t)$$

- ▶ $iD\rho_Q(t)$ describes the dissipation of the HQ system due to interaction with the QGP.

Density evolution with pNRQCD

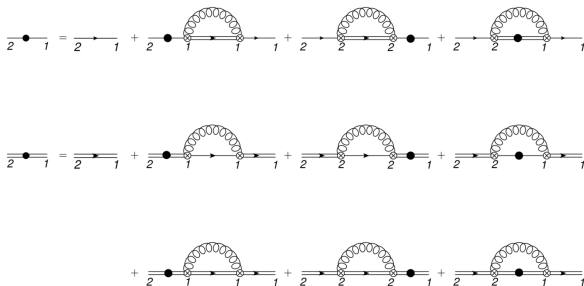
Brambilla, Escobedo et al. 2017

The density matrices for color-singlet and octet are

$$\langle \mathbf{r}_1, \mathbf{r}_2 | \rho_s(t, t) | \mathbf{r}'_1, \mathbf{r}'_2 \rangle \equiv \rho_s(t, \mathbf{r}_1, \mathbf{r}_2; \mathbf{r}'_1, \mathbf{r}'_2) = \left\langle S_1^\dagger(t, \mathbf{r}_1, \mathbf{r}_2) S_2(t, \mathbf{r}'_1, \mathbf{r}'_2) \right\rangle$$

$$\frac{\delta^{ab}}{N_c^2 - 1} \langle \mathbf{r}_1, \mathbf{r}_2 | \rho_o(t, t) | \mathbf{r}'_1, \mathbf{r}'_2 \rangle \equiv \frac{\delta^{ab}}{N_c^2 - 1} \rho_o(t, \mathbf{r}_1, \mathbf{r}_2; \mathbf{r}'_1, \mathbf{r}'_2) = \left\langle O_1^{a\dagger}(t, \mathbf{r}_1, \mathbf{r}_2) O_2^b(t, \mathbf{r}'_1, \mathbf{r}'_2) \right\rangle$$

Density evolution with pNRQCD



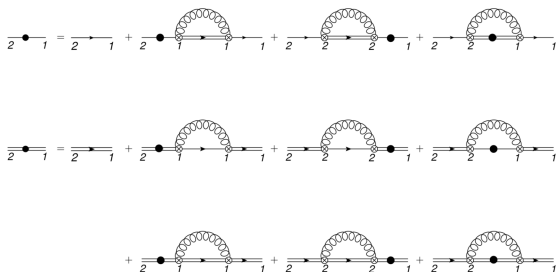
Brambilla, Escobedo et al. 2017

Evolution equations for the density matrix of color-singlet and octet states¹:

$$\begin{aligned} \frac{d\rho_s(t, \mathbf{r}; \mathbf{r}')}{dt} &= \left[-i(h_s(\mathbf{r}) - h_s(\mathbf{r}')) - (\Sigma_s^\dagger(t, \mathbf{r}') + \Sigma_s(t, \mathbf{r})) \right] \rho_s(t, \mathbf{r}; \mathbf{r}') \\ &\quad + \Xi_{so}(t, \rho_o, \mathbf{r}; \mathbf{r}') , \\ \frac{d\rho_o(t, \mathbf{r}; \mathbf{r}')}{dt} &= \left[-i(h_o(\mathbf{r}) - h_o(\mathbf{r}')) - (\Sigma_o^\dagger(t, \mathbf{r}') + \Sigma_o(t, \mathbf{r})) \right] \rho_o(t, \mathbf{r}; \mathbf{r}') \\ &\quad + \Xi_{os}(t, \rho_s, \mathbf{r}; \mathbf{r}') + \Xi_{oo}(t, \rho_o, \mathbf{r}; \mathbf{r}') . \end{aligned}$$

¹Higher order correction has been included on the right hand side:
 $e^{-ih_{s(o)}(t-t_0)} \rho_{s(o)}(t_0, t_0) e^{-ih_{s(o)}(t-t_0)} \rightarrow \rho_{s(o)}(t, t)$, which makes the evolution a **Markovian** process.

Self-energies in real time formalism



The self-energies are expressed as²

$$\Sigma_s(t, \mathbf{r}_1, \mathbf{r}_2) = \frac{2g^2}{N_c} \int_0^\infty dt_1 \left\langle \text{Tr} \{I_g(t_1) T^c\} e^{-ih_o(\mathbf{r})t_1} \text{Tr} \{T^c I_g(0)\} e^{ih_s(\mathbf{r})t_1} \right\rangle$$

Gluon field in multipole expansion (with field redefinition):

$$I_g = A_0^a(\mathbf{r}_1) T^a - A_0^a(\mathbf{r}_2) \tilde{T}^a \approx A_0^a(\mathbf{0}) (T^a - \tilde{T}^a) - \frac{1}{2} \mathbf{r} \cdot \mathbf{E} (T^a + \tilde{T}^a).$$

- **Lindblad equation** has been derived in weak and strong coupling plasma.
Brambilla, Escobedo, Soto, and Vairo 2017

² $\int_0^{t-t_0} dt_1 \rightarrow \int_0^\infty dt_1$, and neglecting density matrix in 11 and 22 correlators

Quarkonium evolution in a strongly coupled plasma

Scales for strongly coupled plasma: $M \gg p \sim Mv \gg T \sim m_D \sim gT \gg E \sim Mv^2$.

- ▶ We can expand the exponentials $e^{\pm ih_{s,o}(t-t_0)} \approx 1 \pm ih_{s,o}(t-t_0)$ with $h_{s,o} = -\frac{\nabla_{\mathbf{r}}^2}{M} + V_{s,o}$. ($h_{s,o}(t-t_0) \ll 1$ as the correlation time $t-t_0 \sim \frac{1}{T}$.)
- ▶ The LO expansion is used for deriving **Lindblad equation**.
- ▶ The LO and the NLO correspond to the **momentum diffusion** and **momentum drag** in the Fokker-Plank equation and Langevin equation in a semi-classical approximation.

$$\Sigma_s(t, \mathbf{r}) = \frac{g^2}{2N_c} \int_0^\infty dt' \left\{ r_i r_j \langle E_i^a(t') E_j^a(0) \rangle + t' \langle r_i E_i^a(t') (i\delta V(\mathbf{r}) r_j + \dot{r}_j) E_i^a(0) \rangle \right\}$$

- ▶ $\delta V(\mathbf{r}) = V_s(\mathbf{r}) - V_o(\mathbf{r})$ and $\dot{\mathbf{r}} = i[\nabla_{\mathbf{r}}^2, \mathbf{r}]/M$.

Defining the real and imaginary parts of the correlator through

$$\frac{g^2}{2N_c} \int_0^\infty dt' \langle E_i^a(t) E_j^a(0) \rangle = \frac{1}{2} [\kappa_{ij}(t) + i\gamma_{ij}(t)] \text{ and}$$
$$\frac{g^2}{2N_c} \int_0^\infty dt' \langle E_i^a(0) E_j^a(t) \rangle = \frac{1}{2} [\kappa_{ij}(t) - i\gamma_{ij}(t)], \text{ the singlet self-energy reads}$$

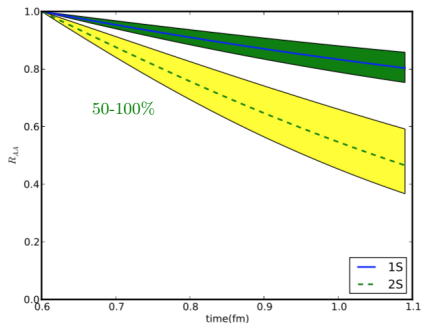
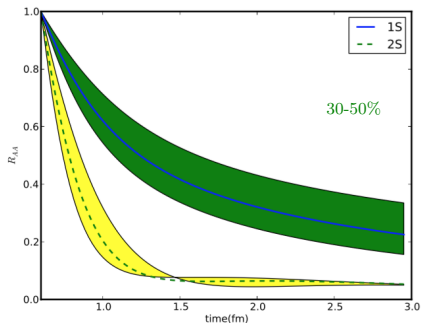
$$\Sigma_s(t, \mathbf{r}) = \frac{1}{2} r_i r_j (\kappa_{ij}(t) + i\gamma_{ij}(t)) + \frac{1}{8T} r_i r_j \delta V(\mathbf{r}) \kappa_{ij}(t) + \frac{1}{4MT} r_i \partial_{r_j} \kappa_{ij}(t)$$

Quantum transport equation in the Lindblad form

The Lindblad equation for the Markovian evolution of the density matrix is

$$\frac{d}{dt}\rho = -i[H, \rho] - \frac{1}{2} \sum_n [C_n^\dagger C_n \rho + \rho C_n^\dagger C_n - 2C_n \rho C_n^\dagger]$$

$$C_i^0 = \sqrt{\frac{\kappa(t)}{N_c^2 - 1}} r^i \begin{pmatrix} 0 & 1 \\ \sqrt{N_c^2 - 1} & 0 \end{pmatrix} \text{ and } C_i^1 = \sqrt{\frac{2(N_c^2 - 4)\kappa(t)}{N_c^2 - 1}} r^i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$



Brambilla et al. 2017

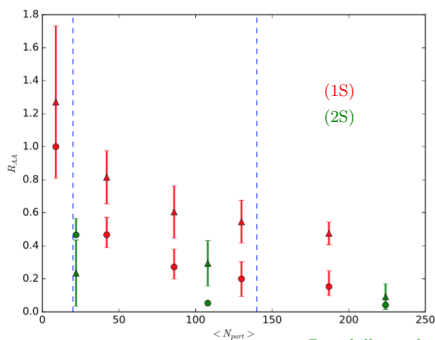
Bottomonium Suppression with $1.8 \lesssim \frac{\kappa}{T^3} \lesssim 3.4$ and $\gamma = 0$.

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Brambilla et al. 2017

Bottomonium Suppression with $1.8 \lesssim \frac{\kappa}{T^3} \lesssim 3.4$ and $\gamma = 0$.

Semi-classical approximation in a strongly coupled plasma

Semi-classical limit

- ▶ In the static ($M \rightarrow \infty$) limit $\mathbf{r} = \mathbf{r}'$.
- ▶ The semi-classical limit is derived through Taylor expansion of small $\mathbf{r}^- = \mathbf{r} - \mathbf{r}'$.

With $\mathbf{r}^- = \mathbf{r} - \mathbf{r}'$ and $\mathbf{r}^+ = \frac{\mathbf{r} + \mathbf{r}'}{2}$, the semi-classical approximation of the singlet self-energy reads

$$\begin{aligned} \Sigma_{ss^\dagger}(t, \mathbf{r}^+, \mathbf{r}^-) &\equiv \Sigma_s(t, \mathbf{r}^+, \mathbf{r}^-) + \Sigma_s^\dagger(t, \mathbf{r}^+, \mathbf{r}^-) \approx (r_i^+ r_j^+ + \frac{1}{4} r_i^- r_j^-) \kappa_{ij}(t) + i r_i^+ r_j^- \gamma_{ij}(t) \\ &+ \left[\frac{1}{8T} (2r_i^+ r_j^- + r_i^+ r_j^+ r_k^- \partial_{r_k^+}) \delta V(\mathbf{r}^+) + \frac{1}{2MT} (r_i^+ \partial_{r_i^+} + r_i^- \partial_{r_i^-}) \right] \kappa_{ij}(t) \end{aligned}$$

Fokker-Plank equation

The Fokker-Plank equation can be worked out by **Wigner transformation** of the semi-classical results.

$$\rho(t, \mathbf{r}, \mathbf{r}') = \int \frac{d^3 p}{(2\pi)^3} e^{i\mathbf{p} \cdot \mathbf{r}^-} \rho(t, \mathbf{r}^+, \mathbf{p})$$

Fokker-Plank equation

In the basis of $\rho_0 = \rho_s + \rho_o$ and $\rho_8 = \rho_s - \rho_o/(N_c^2 - 1)$,

- ▶ Static limit (Similar to [Blaizot and Escobedo 2017](#) with Non-relativistic QCD):

$$\frac{d\rho_0(t, \mathbf{r}^+, \mathbf{p})}{dt} = 0 \quad \leftarrow \text{color equilibrium states } \rho_s = \frac{\rho_o}{N_c^2 - 1}$$

$$\frac{d\rho_8(t, \mathbf{r}^+, \mathbf{p})}{dt} = -\frac{N_c^2}{N_c^2 - 1} \kappa(t)(r^+)^2 \rho_8(t, \mathbf{r}^+, \mathbf{p}) \quad \leftarrow \text{decreases in time}$$

- ▶ Diagonalising the semi-classical equations of ρ_0 and ρ_8 , the maximum entropy state evolves w.r.t. $(\frac{d}{dt} + \frac{2\mathbf{p} \cdot \nabla_{\mathbf{r}^+}}{M})\rho'_0(t, \mathbf{r}^+, \mathbf{p}) = L'\rho'_0(t, \mathbf{r}^+, \mathbf{p})$

$$\begin{aligned} L' &= \frac{\nabla_{\mathbf{p}_i} [(N_c^2 - 1)\partial_i V'_o(\mathbf{r}^+) + \partial_i V'_s(\mathbf{r}^+)]}{N_c^2} \\ &+ \nabla_{\mathbf{p}_i} \nabla_{\mathbf{p}_j} \left(\frac{1}{4} \kappa_{ij}(t) + \frac{(N_c^2 - 1)^2 \partial_i \delta V'(\mathbf{r}^+) \partial_j \delta V'(\mathbf{r}^+)}{r_{i'}^+ r_{j'}^+ \kappa_{i'j'}(t) N_c^6} \right) + \frac{1}{2MT} \nabla_{\mathbf{p}_i} \mathbf{p}_j \kappa_{ij}(t) \\ &+ \nabla_{\mathbf{p}_i} \nabla_{\mathbf{p}_j} \left\{ \frac{i(N_c^2 - 1) \partial_i \delta V'(\mathbf{r}^+) (2r_k^+ \kappa_{jk}(t) \delta V(\mathbf{r}^+) + r_k^+ r_l^+ \kappa_{kl}(t) \partial_j \delta V(\mathbf{r}^+))}{4TN_c^4 r_{i'}^+ r_{j'}^+ \kappa_{i'j'}(t)} \right. \\ &\left. + \frac{r_n^+ \kappa_{in}(t) \delta V(\mathbf{r}^+) (r_k^+ \kappa_{jk}(t) \delta V(\mathbf{r}^+) + r_k^+ r_l^+ \kappa_{kl}(t) \partial_j \delta V(\mathbf{r}^+))}{16r_{i'}^+ r_{j'}^+ \kappa_{i'j'}(t) N_c^2 T^2} \right\} \end{aligned}$$

with $\partial_i V'_s(\mathbf{r}^+) = \partial_i V_s(\mathbf{r}^+) + r_i^+ \gamma(t)$, $\partial_i V'_o(\mathbf{r}^+) = \partial_i V_o(\mathbf{r}^+) + r_i^+ \frac{N_c^2 - 2}{2(N_c^2 - 1)} \gamma(t)$,

and $\partial_i \delta V' = \partial_i V'_s - \partial_i V'_o$ and $\partial_i \delta V = \partial_i V_s - \partial_i V_o$.

Langevin equation

The Corresponding Langevin equation (strongly coupled pNRQCD with contribution from **ultra-soft** gluon) is

$$\dot{\mathbf{r}} = \frac{2\mathbf{p}}{M}, \quad \frac{M\ddot{\mathbf{r}}_i}{2} = -\mathbf{F}_i - \eta_{ij}\mathbf{p}_j + \xi_i + \Theta_i \quad (1)$$

$\eta_{ij} = \frac{1}{2MT} \kappa_{ij}(t)$, and $\langle \xi_i(t)\xi_j(t') \rangle = \delta(t-t')\lambda_{ij}$ with $\lambda_{ij} = MT\eta_{ij}$.

The random force $\langle \Theta_i(t)\Theta_j(t') \rangle = \delta(t-t') \frac{2(N_c^2-1)^2 \partial_i \delta V'(\mathbf{r}^+) \partial_j \delta V'(\mathbf{r}^+)}{r_{i'}^+ r_{j'}^+ \kappa_{i'j'}(t) N_c^6} + \mathbf{R.F}$ from potential change of bound states.

The external force $\mathbf{F} = \frac{(N_c^2-1)\nabla V_o'(\mathbf{r}^+) + \nabla V_s'(\mathbf{r}^+)}{N_c^2}$ with $V_s = -\frac{C_F}{r}$, $V_o = \frac{1}{2N_c r}$.

Langevin equation

The Langevin equation from non-relativistic QCD with only **Coulomb** gluon field (Blaizot and Escobedo 2017) is

$$\mathbf{F} = 0$$

$$\eta_{ij} = \frac{C_F}{2MT} \partial_i \partial_j W(0), \text{ and } \langle \xi_i(t) \xi_i(t') \rangle = \delta(t - t') \lambda_{ij} \text{ with } \lambda_{ij} = MT \eta_{ij}$$

$$\langle \Theta_i(t) \Theta_j(t') \rangle = \delta(t - t') \frac{C_F}{N_c^2} \frac{\partial_i V(\mathbf{r}^+) \partial_j V(\mathbf{r}^+)}{W(\mathbf{r}^+) - W(0)}, \text{ with}$$

$$-W(\mathbf{r}) = -W(-\mathbf{r}) = \int_0^\infty dt \langle A_0(t, \mathbf{r}) A_0(0, 0) \rangle + \langle A_0(0, \mathbf{r}) A_0(t, 0) \rangle \text{ and}$$

$$V(r) - V(0) = \frac{\alpha_s e^{-rm_D}}{r} + \alpha_s m_D, \quad W(r) - W(0) = \alpha_s T \phi(rm_D) \text{ (for } 1/r \sim m_D).$$

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The Corresponding Langevin equation (strongly coupled pNRQCD with contribution from **ultra-soft** gluon) is

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$\eta_{ij} = \frac{1}{2MT} \kappa_{ij}(t)$, and $\langle \xi_i(t)\xi_j(t') \rangle = \delta(t-t')\lambda_{ij}$ with $\lambda_{ij} = MT\eta_{ij}$.

The random force $\langle \Theta_i(t)\Theta_j(t') \rangle = \delta(t-t') \frac{2(N_c^2-1)^2 \partial_i \delta V'(\mathbf{r}^+) \partial_j \delta V'(\mathbf{r}^+)}{r_{i'}^+ r_{j'}^+ \kappa_{i'j'}(t) N_c^6} + \mathbf{R.F}$ from **potential change of bound states**.

The **external force** $\mathbf{F} = \frac{(N_c^2-1)\nabla V'_o(\mathbf{r}^+) + \nabla V'_s(\mathbf{r}^+)}{N_c^2}$ with $V_s = -\frac{C_F}{r}$, $V_o = \frac{1}{2N_c r}$.

- ▶ Ultra-soft gluons in QGP introduce extra force in the Langevin equation of ρ'_0 .
- ▶ Potential change of the bound state introduces a random force.
- ▶ When the force is large, it can't be considered as a small random force, thus the Langevin equation is not valid.

Time Evolution of Correlators

The density is related to the correlator through

$$G_s(t, \mathbf{r}, \mathbf{r}') = \langle \mathbf{r} | \rho_s(t, t) | \mathbf{r}' \rangle = \langle S_1^\dagger(t, \mathbf{r}) S_2(t, \mathbf{r}') \rangle,$$

$$\delta^{ab} G_o(t, \mathbf{r}_o, \mathbf{r}'_o) = \frac{\delta^{ab}}{N_c^2 - 1} \langle \mathbf{r}_o | \rho_o(t, t) | \mathbf{r}'_o \rangle = \langle O_1^{a\dagger}(t, \mathbf{r}_1, \mathbf{r}_2) O_2^b(t, \mathbf{r}'_1, \mathbf{r}'_2) \rangle.$$

The evolution of correlators are

$$\begin{aligned} \frac{dG_s(t, \mathbf{r}; \mathbf{r}')}{dt} &= \int d^3 r'' [-i(h_s(\mathbf{r}) - h_s(\mathbf{r}')) - (\Sigma_s^\dagger(t, \mathbf{r}'', \mathbf{r}') + \Sigma_s(t, \mathbf{r}, \mathbf{r}''))] \\ &\quad \times G_s(t, \mathbf{r}'', \mathbf{r}') + \Xi_{so}(t, G_o, \mathbf{r}; \mathbf{r}') \end{aligned}$$

$$\begin{aligned} \frac{dG_o(t, \mathbf{r}; \mathbf{r}')}{dt} &= \int d^3 r'' [-i(h_o(\mathbf{r}) - h_o(\mathbf{r}')) - (\Sigma_o^\dagger(t, \mathbf{r}'', \mathbf{r}') + \Sigma_o(t, \mathbf{r}, \mathbf{r}''))] \\ &\quad \times G_o(t, \mathbf{r}'', \mathbf{r}') + \Xi_{os}(t, G_s, \mathbf{r}; \mathbf{r}') + \Xi_{oo}(t, G_o, \mathbf{r}; \mathbf{r}') \end{aligned}$$

$$h_s(o) = \mathbf{p}^2/M + V_s(o)(\mathbf{r}) \text{ with } V_s(\mathbf{r}) = -C_F \frac{\alpha_s}{r} \text{ and } V_o(\mathbf{r}) = \frac{\alpha_s}{2N_c r}.$$

Assumptions in deriving Boltzmann equation

$$G_s(t, \mathbf{r}, \mathbf{r}') = \int \frac{d^3 p}{(2\pi)^3} e^{i\mathbf{p}\cdot\mathbf{r}^-} G_s(t, \mathbf{r}^+, \mathbf{p})$$
$$\int d^3 r'' \Sigma_s(t, \mathbf{r}, \mathbf{r}'') G_s(t, \mathbf{r}'', \mathbf{r}') = \int \frac{d^3 p}{(2\pi)^3} e^{i\mathbf{p}\cdot\mathbf{r}^-} \Sigma_s(t, \frac{\mathbf{r} + \mathbf{r}''}{2}, \mathbf{p}) G_s(t, \frac{\mathbf{r}'' + \mathbf{r}'}{2}, \mathbf{p})$$
$$= \int \frac{d^3 p}{(2\pi)^3} e^{i\mathbf{p}\cdot\mathbf{r}^-} e^{\frac{i}{2}(\partial_p^{\Sigma_s} \cdot \partial_r^{G_s} - \partial_r^{\Sigma_s} \cdot \partial_p^{G_s})} \Sigma_s(t, \mathbf{r}, \mathbf{p}) G_s(t, \mathbf{r}, \mathbf{p})$$

The system is a weak nonequilibrium system with weak and instantaneous local interactions with the medium (Kandoff-Baym ansatz).

- ▶ (a) The dependence of G_s and Σ_s is slow enough that one can use $\frac{\mathbf{r} + \mathbf{r}''}{2} \approx \frac{\mathbf{r}'' + \mathbf{r}'}{2} \approx \frac{\mathbf{r} + \mathbf{r}'}{2} \approx \mathbf{r}$.
- ▶ (b) The evolution of potential is slow enough to be neglected ($\hbar_{s/o} \approx \mathbf{p}^2/M$).
- ▶ (c) The spectral function of the system is a product of a δ -function and the distribution function ($G_{12} = 2\pi\delta(E - M/2)f$).

Time evolution of Correlators

Implementating (a) and (b):

$$\begin{aligned} \left(\frac{d}{dt} + \frac{2\mathbf{p} \cdot \nabla_{\mathbf{r}}}{M} \right) G_s(t, \mathbf{r}, \mathbf{p}) &= -2 \operatorname{Re}\{\Sigma_s(t, \mathbf{r}, \mathbf{p})\} G_s(t, \mathbf{r}, \mathbf{p}) + \Xi_{so}(t, \mathbf{r}, \mathbf{p}) \\ \left(\frac{d}{dt} + \frac{2\mathbf{p} \cdot \nabla_{\mathbf{r}}}{M} \right) G_o(t, \mathbf{r}, \mathbf{p}) &= -2 \operatorname{Re}\{\Sigma_o(t, \mathbf{r}, \mathbf{p})\} G_o(t, \mathbf{r}, \mathbf{p}) + \Xi_{os}(t, \mathbf{r}, \mathbf{p}) \\ &\quad + \Xi_{oo}(t, \mathbf{r}, \mathbf{p}) \end{aligned}$$

For a strongly coupled system with $p \gg T \sim gT \gg E$

$$\begin{aligned} \Sigma_s(t, \mathbf{r}, \mathbf{p}) &= \frac{g^2}{2N_c} \int_0^\infty ds \langle \mathbf{r}, \mathbf{p} | r^i e^{-ih_o s} r^j e^{ih_s s} | \mathbf{r}, \mathbf{p} \rangle \langle E^{a,i}(s, \mathbf{0}) E^{a,j}(0, \mathbf{0}) \rangle \\ &= ig^2 C_F \frac{\int d^3 r_o \int d^3 p_o \int d^4 k}{(2\pi)^3 (2\pi)^4} \frac{\langle \mathbf{r}, \mathbf{p} | r^i | \mathbf{r}_o, \mathbf{p}_o \rangle \langle \mathbf{r}_o, \mathbf{p}_o | r^j | \mathbf{r}, \mathbf{p} \rangle}{E_o - E_s + k_0} k_0^2 D^{>ij}(k) \\ &= ig^2 C_F \int \frac{d^3 p_o}{(2\pi)^3} \int \frac{d^4 k}{(2\pi)^4} \frac{\langle \mathbf{r}, \mathbf{p} | r^i | \mathbf{r}_o, \mathbf{p}_o \rangle \langle \mathbf{r}_o, \mathbf{p}_o | r^j | \mathbf{r}, \mathbf{p} \rangle}{E_o - E_s + k_0} \\ &\quad \times k_0^2 D^{>ij}(k) (2\pi)^3 \delta^3(\mathbf{p} - \mathbf{p}_o - \mathbf{k}) \end{aligned} \quad (2)$$

where we have used

$$\langle E^{a,i}(s, \mathbf{0}) E^{a,j}(0, \mathbf{0}) \rangle = (N_c^2 - 1) \int \frac{d^4 k}{(2\pi)^4} e^{-ik_0 s} k_0^2 D^{>ij}(k)$$

in dimensional regularization.

Time evolution of Correlators

Implementating (a) and (b):

$$\begin{aligned} \left(\frac{d}{dt} + \frac{2\mathbf{p} \cdot \nabla_{\mathbf{r}}}{M} \right) G_s(t, \mathbf{r}, \mathbf{p}) &= -2 \operatorname{Re}\{\Sigma_s(t, \mathbf{r}, \mathbf{p})\} G_s(t, \mathbf{r}, \mathbf{p}) + \Xi_{so}(t, \mathbf{r}, \mathbf{p}) \\ \left(\frac{d}{dt} + \frac{2\mathbf{p} \cdot \nabla_{\mathbf{r}}}{M} \right) G_o(t, \mathbf{r}, \mathbf{p}) &= -2 \operatorname{Re}\{\Sigma_o(t, \mathbf{r}, \mathbf{p})\} G_o(t, \mathbf{r}, \mathbf{p}) + \Xi_{os}(t, \mathbf{r}, \mathbf{p}) \\ &\quad + \Xi_{oo}(t, \mathbf{r}, \mathbf{p}) \end{aligned}$$

With $D_{ij}^>(k) = (\delta_{ij} - k_i k_j / k^2) D^>(k)$, with $D^>(k) = n_B(k_0) + 1$ we get

$$\begin{aligned} \Sigma_s(t, \mathbf{r}, \mathbf{p}) &= ig^2 C_F \int \frac{d^3 p_o}{(2\pi)^3} \int \frac{d^4 k}{(2\pi)^4} \frac{|\langle \mathbf{r}, \mathbf{p} | \epsilon_\lambda^*(\mathbf{k}) \cdot \mathbf{r} | \mathbf{r}_o, \mathbf{p}_o \rangle|^2}{E_o - E_s + k_0} \\ &\quad \times k_0^2 D^>(k) (2\pi)^3 \delta^3(\mathbf{p} - \mathbf{p}_o - \mathbf{k}) \end{aligned}$$

$$\epsilon_\lambda^*(\mathbf{k}) \epsilon_\lambda(\mathbf{k}) = \delta_{ij} - k_i k_j / k^2.$$

The factor $\frac{i}{E_o - E_s + k_0} = G_o^{11}$ is the free octet propagator, which in a thermal medium will be $\frac{i}{E_o - E_s + k_0} + (f_o(E_o, \mathbf{r}_o, \mathbf{p}_o) + \frac{1}{2}) 2\pi \delta(E_o - E_s + k_0)$ (**Kandoff-Baym** ansatz (c)).

The results for different Feynman diagrams

The results for singlets are

$$\begin{aligned}
 2\Re\{\Sigma_s(t, \mathbf{r}, \mathbf{p})\} &= 2g^2 C_F \int \frac{d^3 p_o}{(2\pi)^3} \int \frac{d^3 k}{2k(2\pi)^3} |\langle \mathbf{r}, \mathbf{p} | \epsilon_\lambda^*(\mathbf{k}) \cdot \mathbf{r} | \mathbf{r}_o, \mathbf{p}_o \rangle|^2 \\
 &\quad \times k^2 \left[\left(f_o(E_o, \mathbf{r}_o, \mathbf{p}_o) + \frac{1}{2} \right) (n_B(k) + 1) \right] \\
 &\quad (2\pi)^4 \delta(E_s - E_o - k) \delta^3(\mathbf{p} - \mathbf{p}_o - \mathbf{k}) \\
 \Xi_{so}(t, \mathbf{r}, \mathbf{p}) &= \frac{g^2 (N_c^2 - 1)}{N_c} \int \frac{d^3 p_o}{(2\pi)^3} \int \frac{d^3 k}{2k(2\pi)^3} |\langle \mathbf{r}, \mathbf{p} | \epsilon_\lambda^*(\mathbf{k}) \cdot \mathbf{r} | \mathbf{r}_o, \mathbf{p}_o \rangle|^2 \\
 &\quad \times k^2 \left[\left(f_s(E_s, \mathbf{r}, \mathbf{p}) + \frac{1}{2} \right) n_B(k) \right] \\
 &\quad (2\pi)^4 \delta(E_s - E_o + k) \delta^3(\mathbf{p} - \mathbf{p}_o + \mathbf{k}) G_o(t, \mathbf{r}_o, \mathbf{p}_o)
 \end{aligned}$$

This two terms can be combined by changing (k_0, \mathbf{k}) to $-(k_0, \mathbf{k})$ and using $n_B(-k) = -(n_B(k) + 1)$.

Similar calculation can be done for octet self-energies.

The Boltzmann equations

$$G_s(t, \mathbf{r}, \mathbf{p}) = f_s(t, \mathbf{r}, \mathbf{p})2\pi\delta(E_s - \frac{M}{2}) \text{ and } G_o(t, \mathbf{r}, \mathbf{p}) = f_o(t, \mathbf{r}_o, \mathbf{p}_o)2\pi\delta(E_o - \frac{M}{2})$$

$$\left(\frac{d}{dt} + \frac{2\mathbf{p} \cdot \nabla_{\mathbf{r}}}{M} \right) f_s(t, \mathbf{r}, \mathbf{p}) = -C_-^s + C_+^s$$

$$\left(\frac{d}{dt} + \frac{2\mathbf{p}_o \cdot \nabla_{\mathbf{r}_o}}{M} \right) f_o(t, \mathbf{r}_o, \mathbf{p}_o) = -C_-^s(s \leftrightarrow o) + C_+^s(s \leftrightarrow o) + C_i$$

$$C_-^s = g^2 C_F \int \frac{d^3 p_o}{(2\pi)^3} \int \frac{d^3 k}{2k(2\pi)^3} |\langle \mathbf{r}, \mathbf{p} | \epsilon_\lambda^*(\mathbf{k}) \cdot \mathbf{r} | \mathbf{r}_o, \mathbf{p}_o \rangle|^2 \Leftarrow \text{Dissociation of Singlets}$$

$$\times k^2 f_s(E_s, \mathbf{r}, \mathbf{p})(n_B(k) + 1)(2\pi)^4 \delta(E_s - E_o - k) \delta^3(\mathbf{p} - \mathbf{p}_o - \mathbf{k})$$

$$C_+^s = g^2 C_F \int \frac{d^3 p_o}{(2\pi)^3} \int \frac{d^3 k}{2k(2\pi)^3} |\langle \mathbf{r}, \mathbf{p} | \epsilon_\lambda^*(\mathbf{k}) \cdot \mathbf{r} | \mathbf{r}_o, \mathbf{p}_o \rangle|^2 \Leftarrow \text{Regeneration of Singlets}$$

$$\times k^2 f_o(t, \mathbf{r}_o, \mathbf{p}_o)(n_B(k) + 1)(2\pi)^4 \delta(E_s - E_o - k) \delta^3(\mathbf{p} - \mathbf{p}_o - \mathbf{k})$$

$$C_i = \frac{g^2(N_c^2 - 4)C_F}{2} \int \frac{d^3 p'_o}{(2\pi)^3} \int \frac{d^3 k}{2k(2\pi)^3} |\langle \mathbf{r}_o, \mathbf{p}_o | \epsilon_\lambda^*(\mathbf{k}) \cdot \mathbf{r} | \mathbf{r}'_o, \mathbf{p}'_o \rangle|^2 \Leftarrow \text{Octets transition}$$

$$\times k^2 (f'_o(E'_o, \mathbf{r}'_o, \mathbf{p}'_o) - f_o(E_o, \mathbf{r}_o, \mathbf{p}_o))(n_B(k) + 1)(2\pi)^4 \delta(E_o - E'_o - k) \delta^3(\mathbf{p}_o - \mathbf{p}'_o - \mathbf{k})$$

Summary

(Semi-)classical description of quarkonia evolution in a strongly coupled plasma has been derived from pNRQCD for a strongly coupled system.

- ▶ Langevin equation and Fokker-Plank equation are derived in a semi-classical approximation.
- ▶ Boltzmann equation is worked out for a weakly nonequilibrium system with weak interaction with the QGP.

In addition,

- ▶ it would be interesting to numerically solve those equations.
- ▶ Similar calculation can be done for a weakly coupled system.
- ▶ Comparing to experimental data (with proper initial condition and medium description) will reveal the properties of QGP and formation of heavy quarkonia.

Thank you!