

Charm-loop effect in $B \rightarrow K^{(*)} ll$ decays beyond leading twist



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In collaboration

with D. van Dyk and J. Virto

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DFG Deutsche
Forschungsgemeinschaft



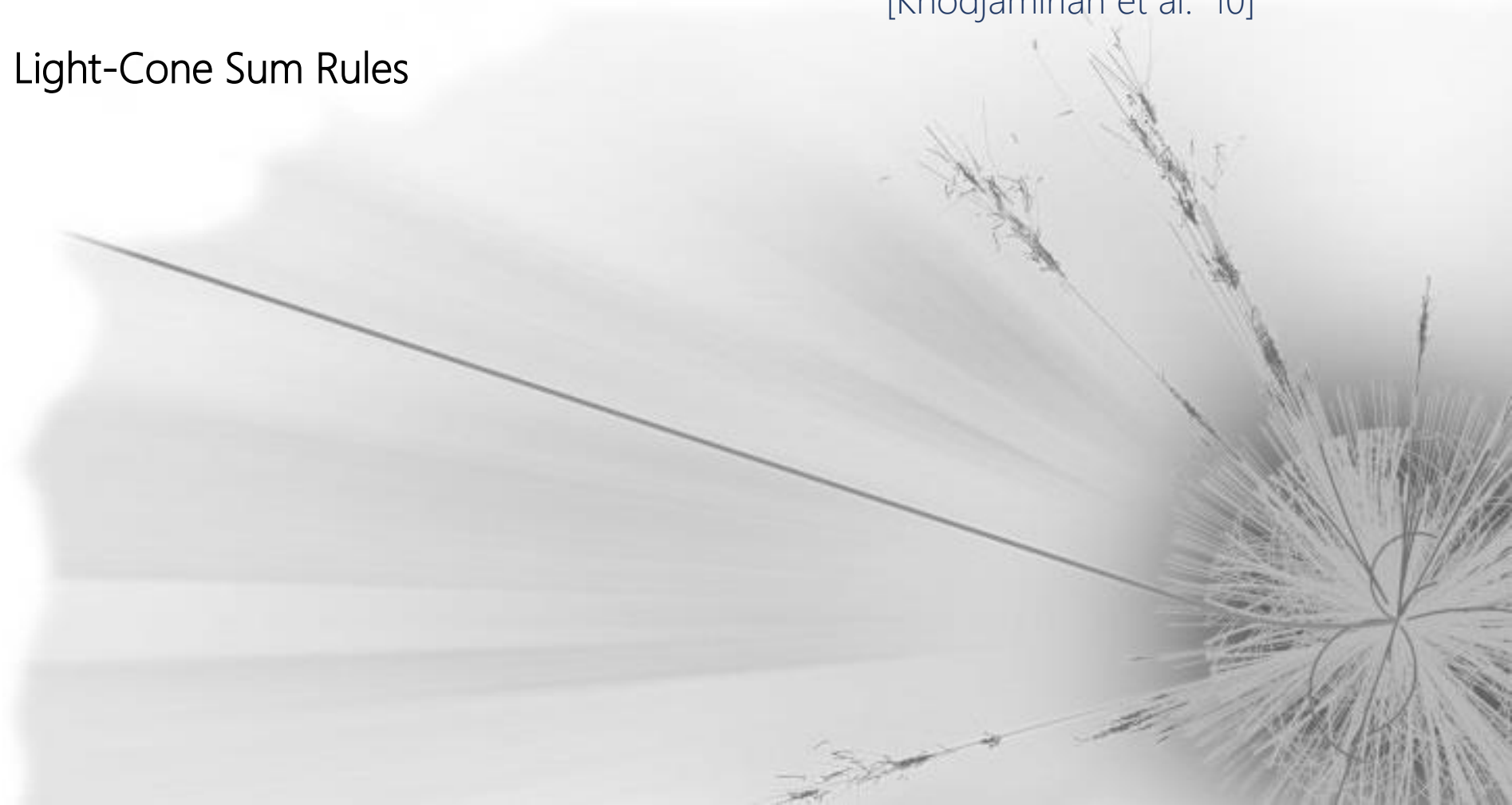
What's new?

1/19

- revisit **soft gluon contributions to the "charm-loop"** effect in rare $B \rightarrow K^{(*)}ll$ decays

[Khodjamirian et al. '10]

- method of B -meson Light-Cone Sum Rules



What's new?

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- revisit **soft gluon contributions to the "charm-loop"** effect in rare $B \rightarrow K^{(*)}ll$ decays
[Khodjamirian et al. '10]
- method of B -meson Light-Cone Sum Rules
- **inclusion of higher-twist corrections** and complete set of B -meson light-cone distribution amplitudes (**B -LCDAs**)
- preliminary results show a **smaller soft gluon contributions** to the charm-loop comparing with the previous calculation

Introduction

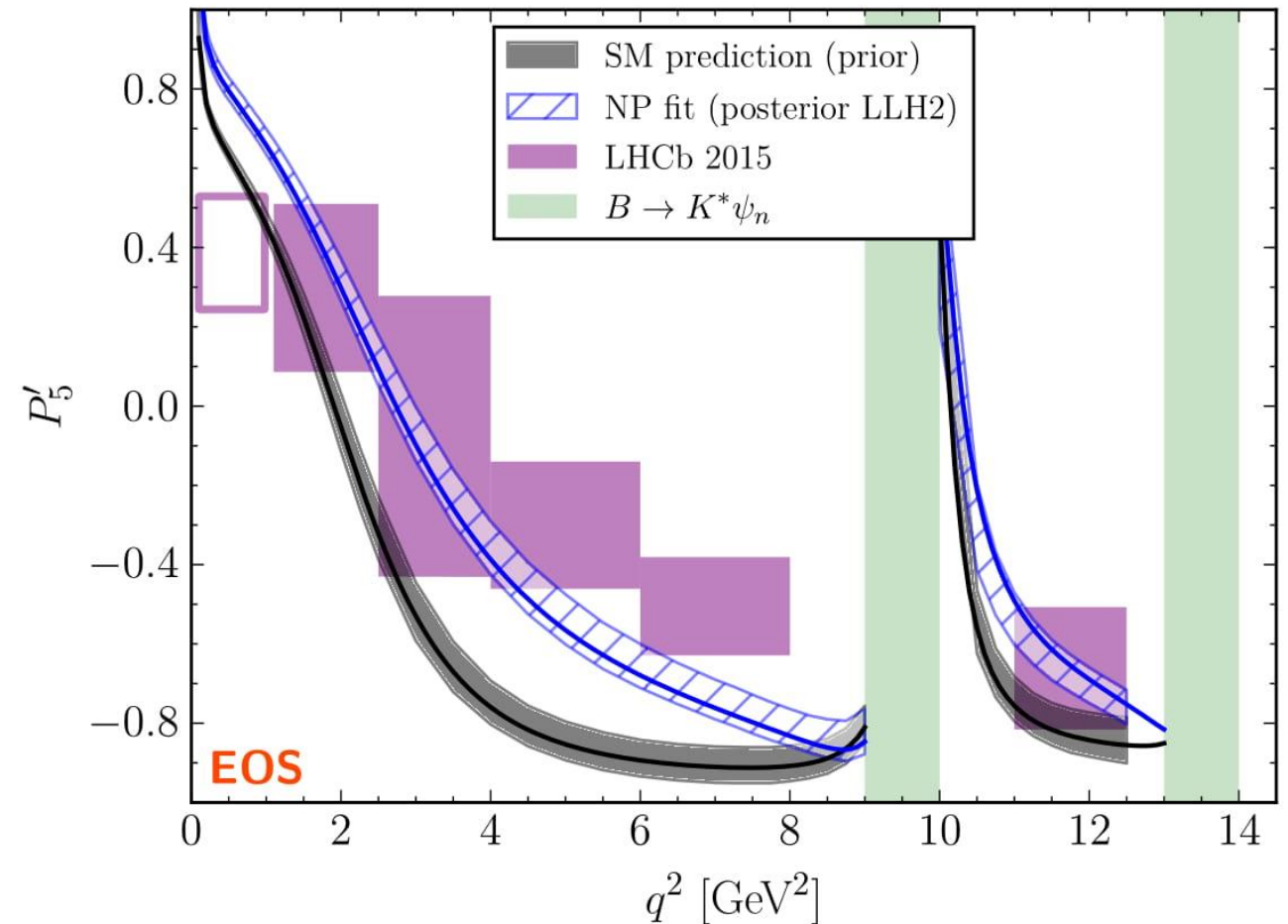
Anomalies in $b \rightarrow sll$

FCNC are loop, GIM and CKM suppressed in the SM.

Sensitive to new physics contributions.

Tension between experiment and SM in several observables:

- lepton flavour universality ratios $R_{K^{(*)}}^{\mu e}$
- angular observables $B \rightarrow K^* \mu\mu$ (P'_5, \dots)
- ...

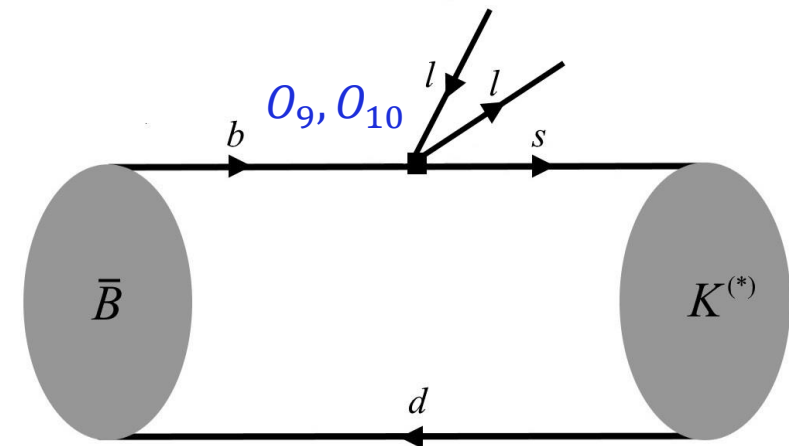
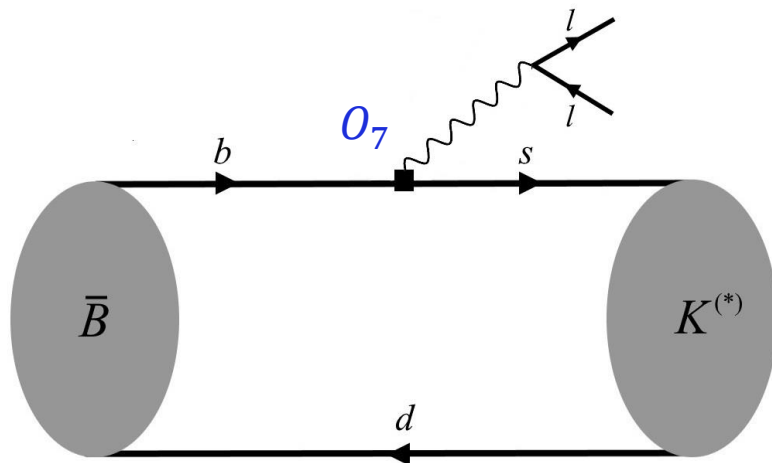


$b \rightarrow sl\bar{l}$ effective Hamiltonian

transitions described by the effective Hamiltonian

$$H(b \rightarrow sl^+l^-) = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu) \quad \mu = m_b$$

main contribution to $B \rightarrow K^{(*)}ll$ in the SM given by local operators O_7, O_9, O_{10}



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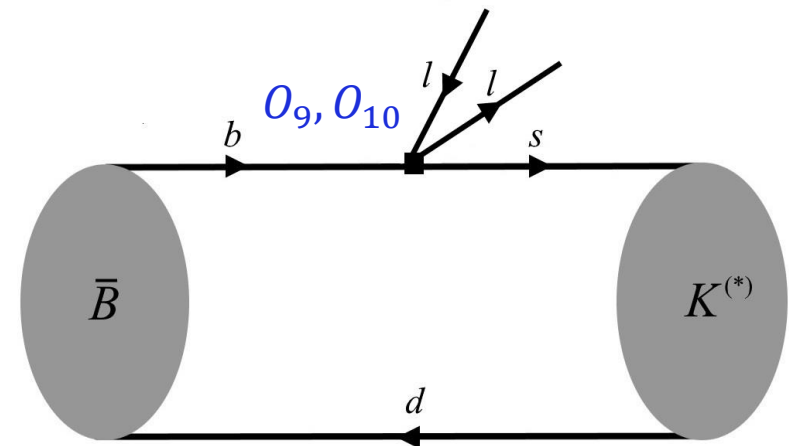
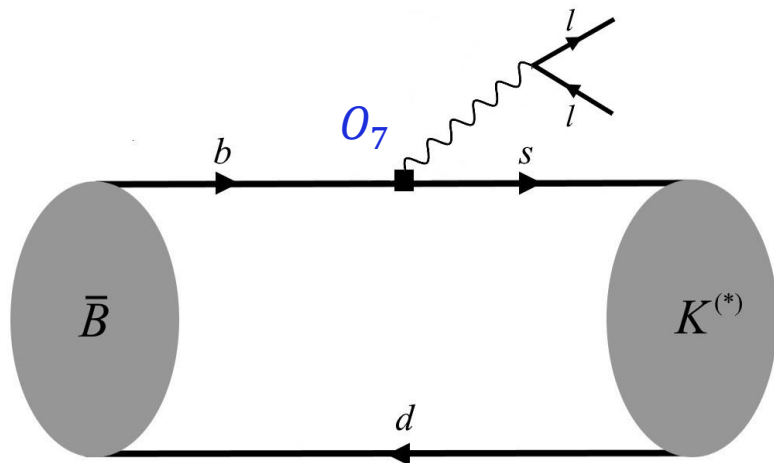
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main contribution to $B \rightarrow K^{(*)}ll$ in the SM given by local operators O_7, O_9, O_{10}

$$O_7 = \frac{e}{16\pi^2} m_b (\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu} \quad O_9 = \frac{e}{16\pi^2} (\bar{s}_L \gamma^\mu b_L) \sum_l (\bar{l} \gamma_\mu l) \quad O_{10} = \frac{e}{16\pi^2} (\bar{s}_L \gamma^\mu b_L) \sum_l (\bar{l} \gamma_\mu \gamma_5 l)$$

hadronic matrix elements computed in a previous work ($B \rightarrow K^{(*)}$ form factors) [NG/Kokulu/van Dyk '18]



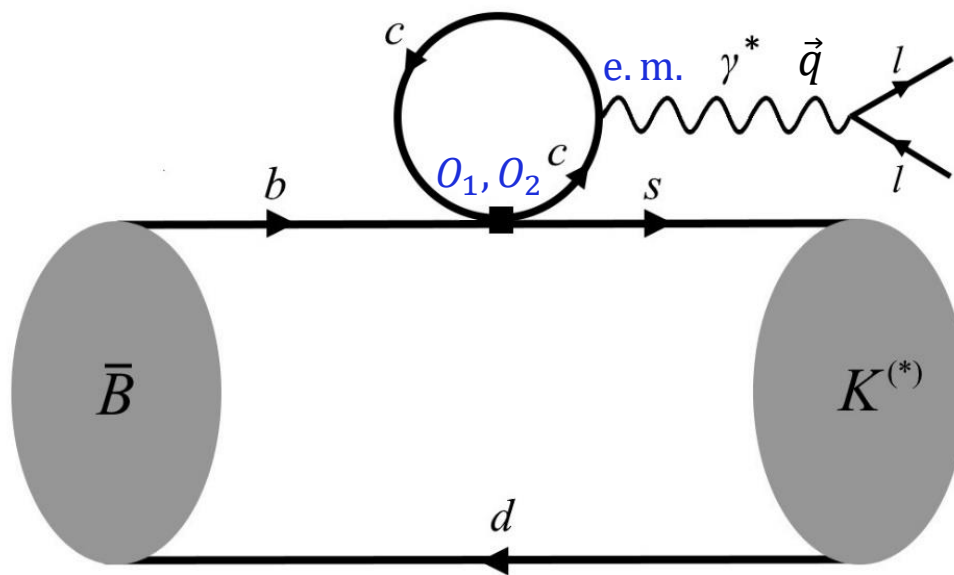
Charm-loop in $B \rightarrow K^{(*)} ll$

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additional non-local contributions come from O_1 and O_2 combined with the e.m. current (charm-loop contribution)

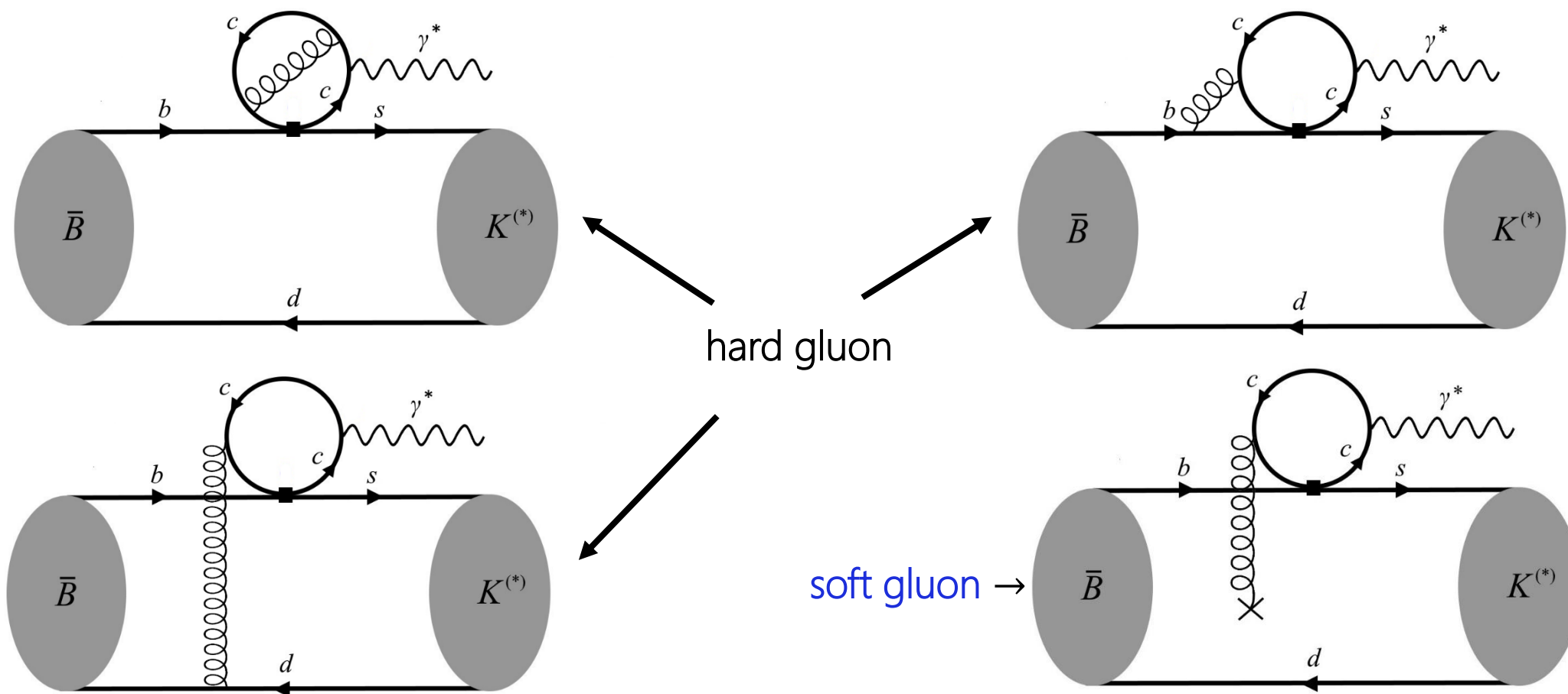
$$O_1 = (\bar{s}_L \gamma^\mu c_L)(\bar{c}_L \gamma_\mu b_L)$$

$$O_2 = (\bar{s}_L^j \gamma^\mu c_L^i)(\bar{c}_L^i \gamma_\mu b_L^j)$$



Charm-loop in $B \rightarrow K^{(*)} ll$

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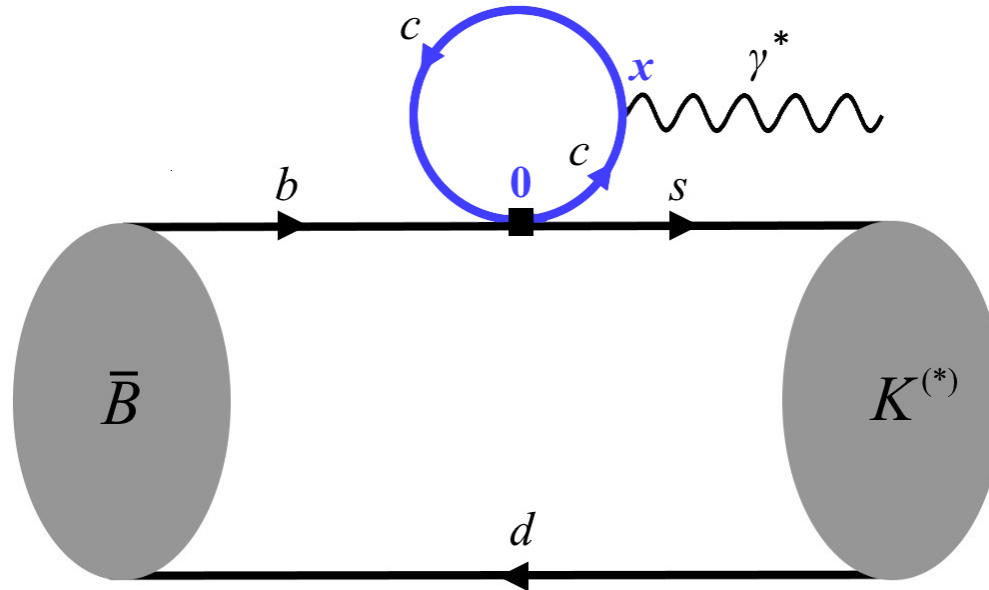
Operator Product Expansion (OPE)

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expand correlator near the light-cone ($x^2 \simeq 0$)

$$\int d^4y e^{iky} \langle K^{(*)}(k) | T \{ O_{1,2}(0), \bar{c} \gamma_\mu c(x) \} | B(q+k) \rangle$$

expansion yields new, non-local matrix elements, to be computed



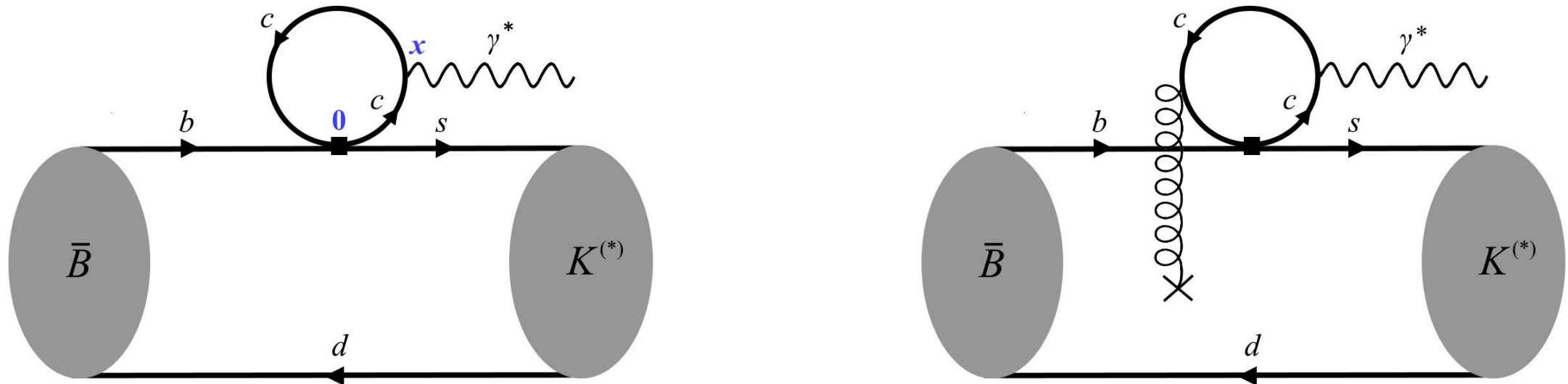
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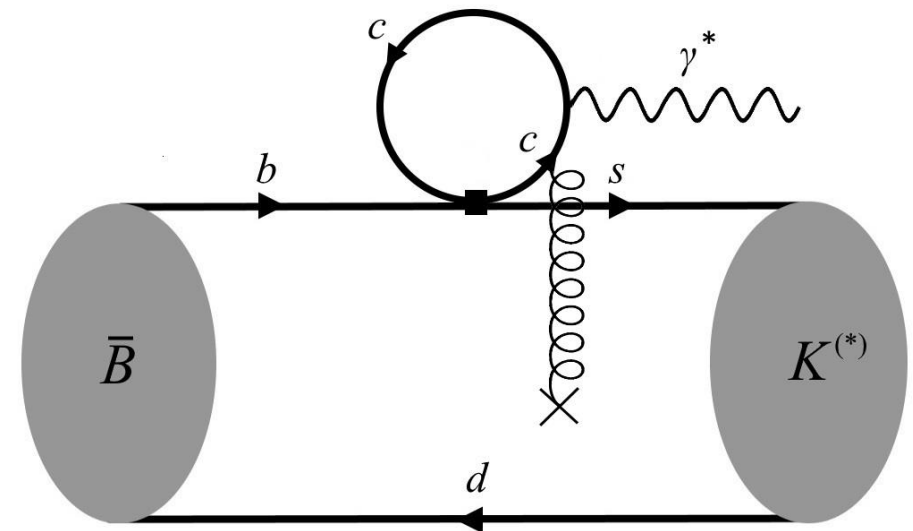
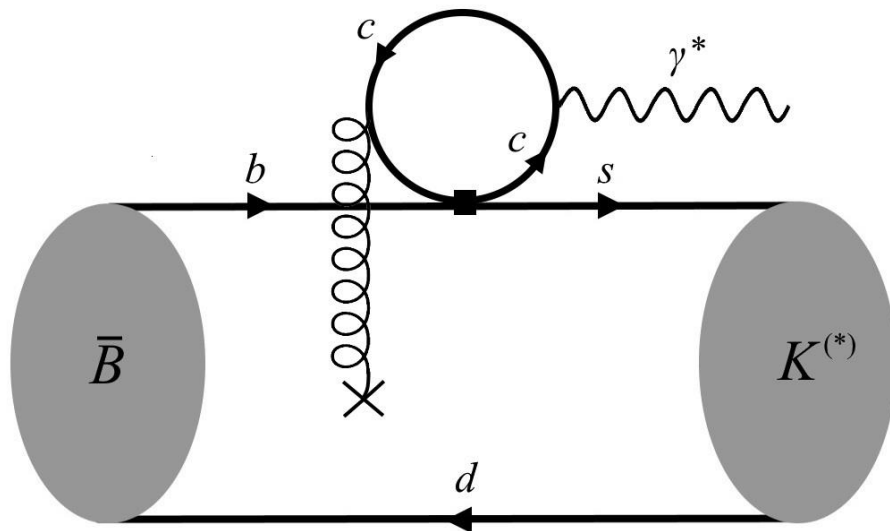
Charm-loop soft gluon emission

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two diagrams with a soft-gluon

effects not included in QCD factorization (soft effects that are not included in the FFs)

non-local hadronic matrix element q^2 dependent (q^2 is the dilepton mass squared)



Our approach
to the calculation

Methods to compute hadronic matrix elements

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QCD perturbation theory breaks down at low energies
non-perturbative techniques are needed
to compute hadronic matrix elements

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Light-cone sum rules (LCSRs)

quark-hadron duality approximation

universal B -meson matrix elements

applicable for both local and non-local
matrix elements

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Lattice QCD

numerical evaluation of correlators
in a finite and discrete space-time

non-local matrix elements still
work in progress

($K \rightarrow \pi\pi$; not yet possible for heavy
mesons)

Light-cone Sum Rules in a nutshell 1

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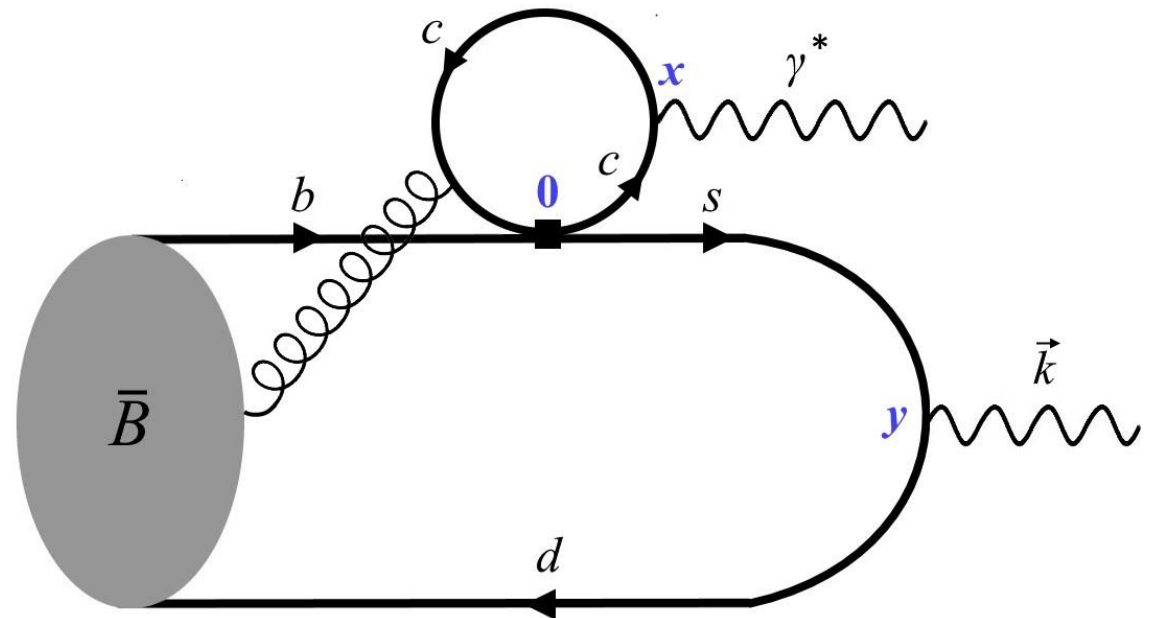
LCSRs are used to determine hadronic matrix elements
from a correlation function $\Pi(k, q)$

$$\Pi(k, q) = i \int d^4y e^{iky} \langle 0 | T \{ J_{int}(y), \tilde{O}_\mu(0, x) \} | B(q+k) \rangle$$

where

$$J_{int} = \bar{d} \gamma_\nu (\gamma_5) s$$

$$\tilde{O}_\mu(0, x) = I_{\mu\rho\alpha\beta}(\tilde{q}) \bar{s}_L \gamma_\rho b_L(0) G(ux)$$



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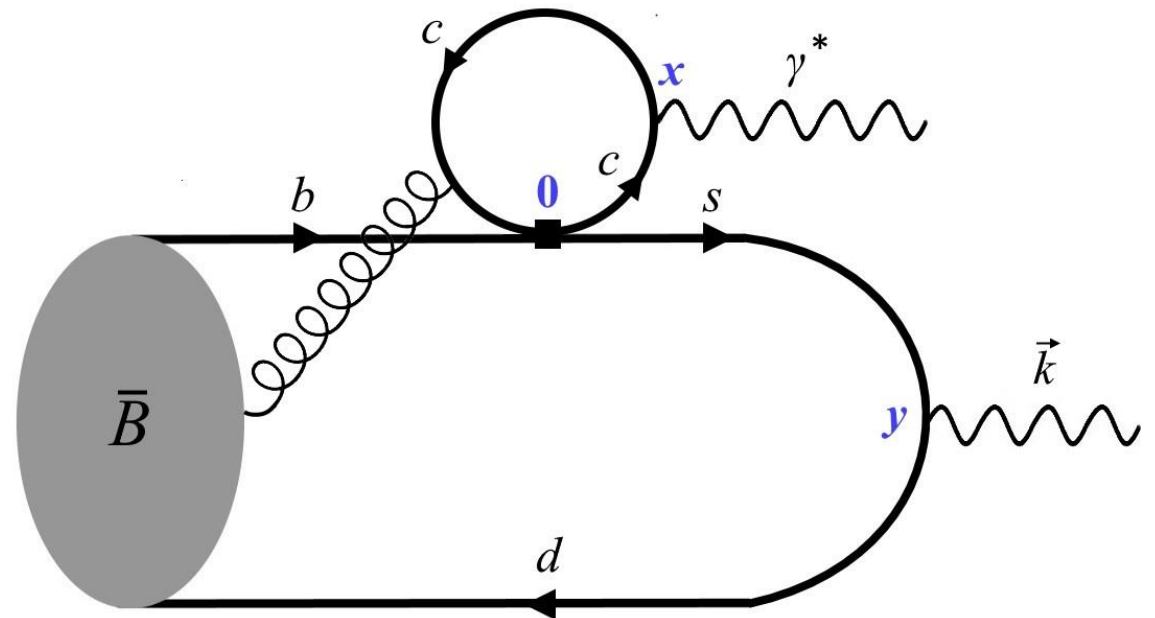
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we want to compute the matrix element

$$\langle K^{(*)}(\mathbf{k}) | \tilde{O}_\mu(\mathbf{0}, \mathbf{x}) | B(\mathbf{q} + \mathbf{k}) \rangle$$

soft gluon emitted by the B -meson (no need
to consider a soft gluon emitted by the Kaon)



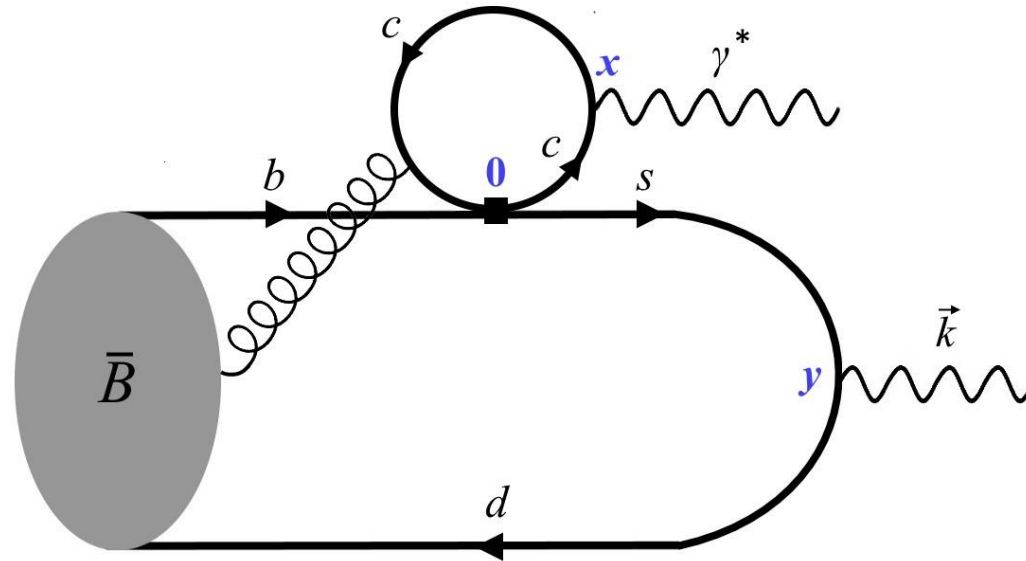
Light-cone Sum Rules in a nutshell 2

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two ways to compute the correlator

$$\Pi(k, q) = i \int d^4x e^{ikx} \langle 0 | T \{ J_{int}(y), \tilde{O}_\mu(0, x) \} | B(q+k) \rangle$$

1
Hadronic
representation
for positive k^2



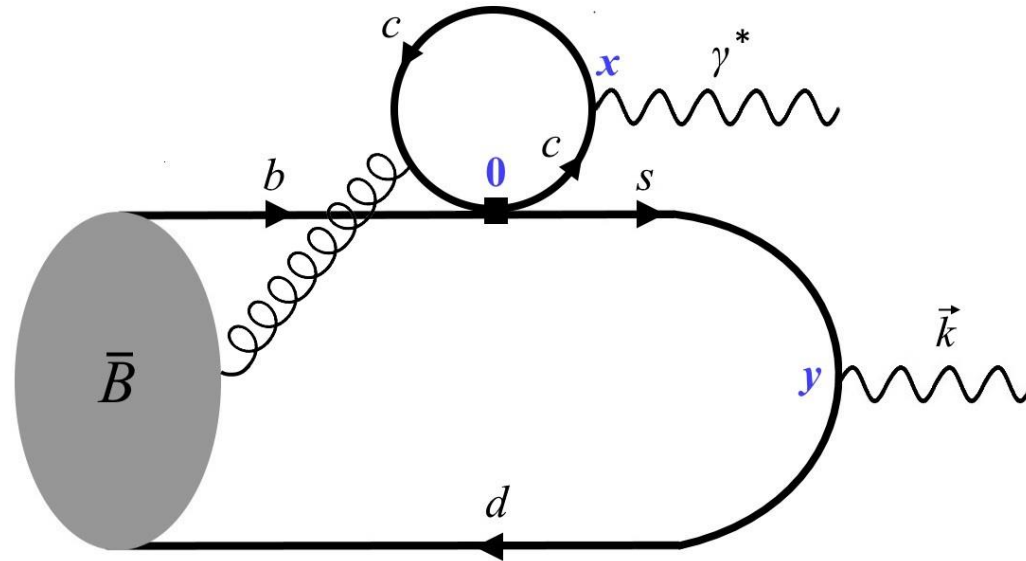
2
OPE for large
negative k^2
and $q^2 \ll 4m_c^2$

Light-cone Sum Rules in a nutshell 2

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2
OPE for large
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the sum rule is obtained matching the result the two different calculations of $\Pi(k, q)$ and using semi-global quark-hadron duality

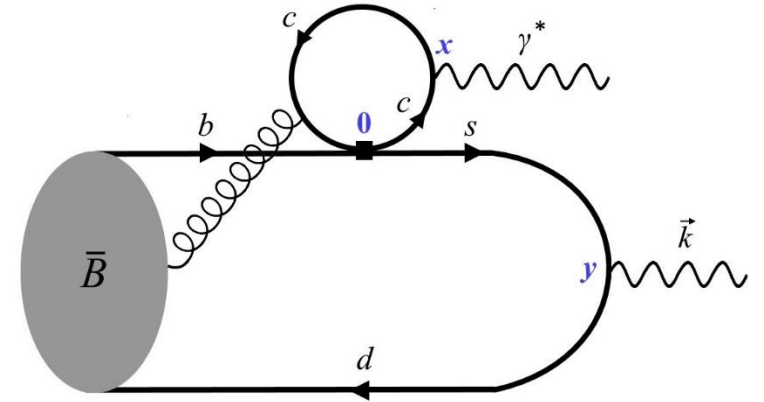
Hadronic calculation

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for positive k^2

$$\Pi(k, q) = i \int d^4y e^{iky} \langle 0 | T \{ J_{int}(y), \tilde{O}_\mu(0, x) \} | B(q+k) \rangle$$

insert a complete set
of hadronic states



Hadronic calculation

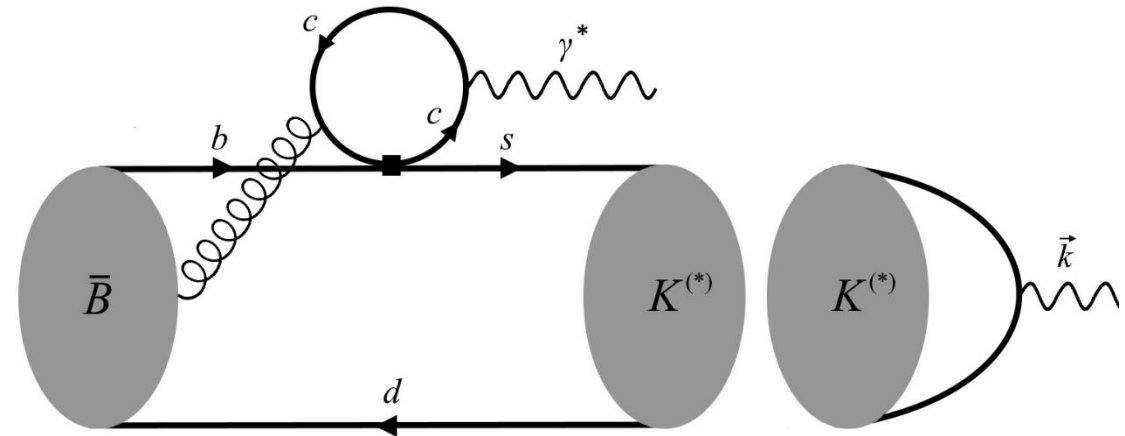
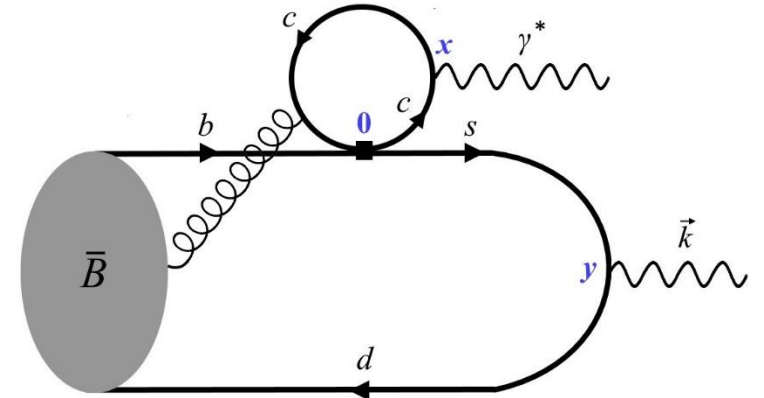
for positive k^2

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insert a complete set of hadronic states

$$\frac{\overbrace{\langle 0 | J_{int} | K^{(*)} \rangle}^{f_{K^{(*)}}}}{k^2 - m_{K^{(*)}}} \langle K^{(*)}(k) | \tilde{O}_\mu(0, x) | B(q+k) \rangle + \text{continuum}$$

hadronic dispersion relation



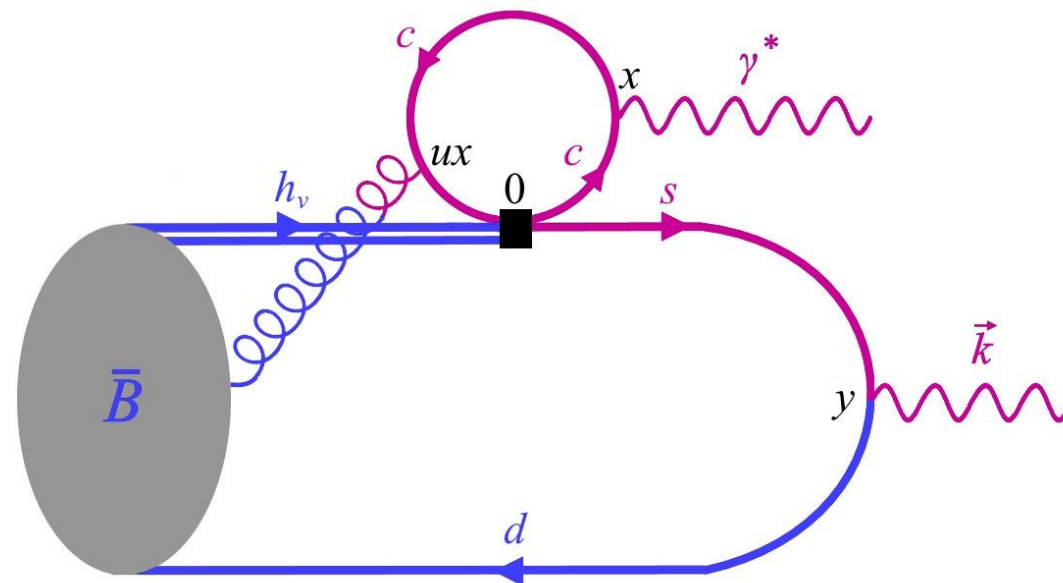
OPE calculation

for large **negative** k^2 and $q^2 \ll 4m_c^2$, the B -meson treated in HQET

$$\Pi(k, q) = i \int d^4y e^{iky} \langle 0 | T \{ J_{int}(y), \tilde{O}_\mu(0, x) \} | B(v) \rangle$$

factorize hard and soft contributions

$$\Pi(k^2, q^2) = \int_0^\infty ds \sum_n \frac{I_n^{\alpha\beta}(s, q^2)}{(s - k^2)^n} \langle 0 | \bar{d}(y) G_{\alpha\beta}(ux) h_\nu(0) | B(v) \rangle$$



OPE calculation

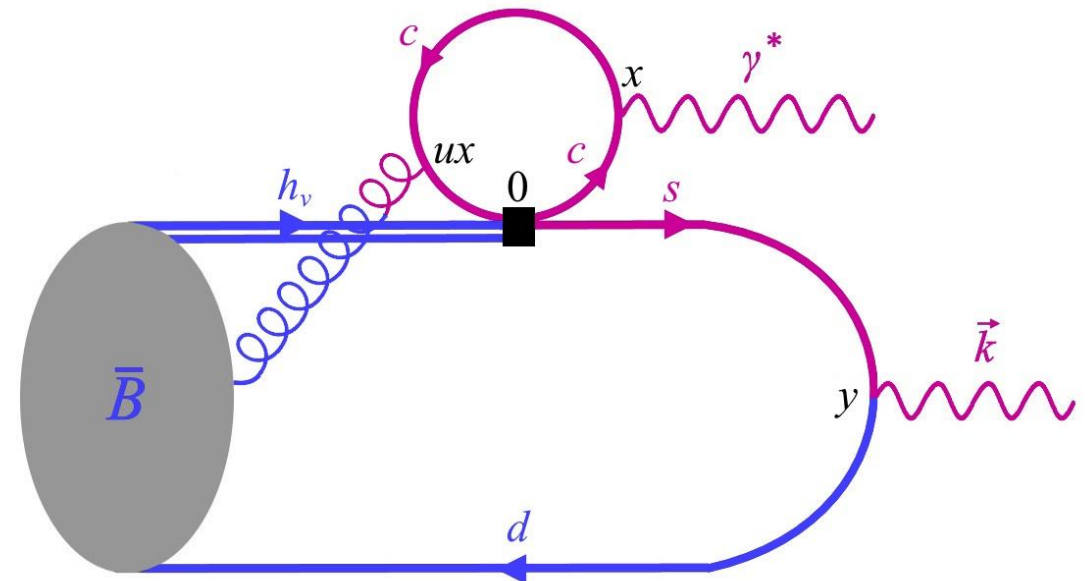
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- compute I_n from a **perturbative** hard scattering kernel
- B -to-vacuum **non-local matrix element** is a necessary **non-perturbative** input



Three-particle distribution amplitudes

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traditional set three-particle *B*-meson light-cone distribution amplitudes (*B*-LCDA) [Kawamura et al. '01]

$$\langle 0 | \bar{d}(\mathbf{y}) G_{\alpha\beta}(ux) h_v(0) | B(v) \rangle$$
$$= \frac{f_B m_B}{4} \text{Tr} \left\{ \gamma_5 P_+ \left[(v_\alpha \gamma_\beta - v_\beta \gamma_\alpha) (\Psi_A - \Psi_V) - i \sigma_{\alpha\beta} \Psi_V - (y_\alpha v_\beta - y_\beta v_\alpha) \frac{X_A}{v \cdot y} + (y_\alpha \gamma_\beta - y_\beta \gamma_\alpha) \frac{Y_A}{v \cdot y} \right] \right\} (\mathbf{y}, \mathbf{ux})$$

used in the previous calculation Khodjamirian/Mannel/Pivovarov/Wang 2010 (KMWP2010)

Three-particle distribution amplitudes

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basis with all independent Lorentz structures, **8 independent B-LCDAs** [Braun/Ji/Manashov '17]

$$\begin{aligned} & \langle 0 | \bar{d}(y) G_{\alpha\beta}(ux) h_v(0) | B(v) \rangle \\ &= \frac{f_B m_B}{4} \text{Tr} \left\{ \gamma_5 P_+ \left[(v_\alpha \gamma_\beta - v_\beta \gamma_\alpha) (\Psi_A - \Psi_V) - i \sigma_{\alpha\beta} \Psi_V - (y_\alpha v_\beta - y_\beta v_\alpha) \frac{X_A}{v \cdot y} + (y_\alpha \gamma_\beta - y_\beta \gamma_\alpha) \frac{W + Y_A}{v \cdot y} \right. \right. \\ & \left. \left. - i \epsilon_{\alpha\beta\sigma\rho} y^\sigma v^\rho \gamma_5 \frac{\tilde{X}_A}{v \cdot y} + i \epsilon_{\alpha\beta\sigma\rho} y^\sigma \gamma^\rho \gamma_5 \frac{\tilde{Y}_A}{v \cdot y} - (y_\alpha v_\beta - y_\beta v_\alpha) y_\sigma \gamma^\sigma \frac{W}{(v \cdot y)^2} + (y_\alpha \gamma_\beta - y_\beta \gamma_\alpha) y_\sigma \gamma^\sigma \frac{Z}{(v \cdot y)^2} \right] \right\} (\mathbf{y}, \mathbf{ux}) \end{aligned}$$

the LCDAs $\Psi_A, \Psi_V, X_A, Y_A, \dots$ have no definite **twist** (twist = dimension – spin)

higher twists are suppressed by power of Λ_{had}/m_B

express the LCDAs in the traditional form in LCDAs with definite twist

three-particle LCDAs twist basis

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models given for LCDAs up to **twist 4**, twist 5 or higher give corrections of the order $1/m_b^2$

[Braun/Ji/Manashov '17]

$$\Psi_A = \frac{1}{2}(\Phi_3 + \Phi_4)$$

$$\Psi_V = \frac{1}{2}(-\Phi_3 + \Phi_4)$$

$$X_A = \frac{1}{2}(-\Phi_3 - \Phi_4 + 2\Psi_4)$$

$$Y_A = \frac{1}{2}(-\Phi_3 - \Phi_4 + \Psi_4 - \Psi_5)$$

$$\tilde{X}_A = \frac{1}{2}(-\Phi_3 + \Phi_4 - 2\tilde{\Psi}_4)$$

$$\tilde{Y}_A = \frac{1}{2}(-\Phi_3 + \Phi_4 - \tilde{\Psi}_4 + \tilde{\Psi}_5)$$

$$W = \frac{1}{2}(\Phi_4 - \Psi_4 - \tilde{\Psi}_4 + \Psi_5 + \tilde{\Psi}_5 + \tilde{\Phi}_5)$$

$$Z = \frac{1}{4}(-\Phi_3 + \Phi_4 - 2\tilde{\Psi}_4 + 2\tilde{\Psi}_5 + \tilde{\Phi}_5 + \Phi_6)$$

three-particle LCDAs twist basis

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use to compute the sum rule

- all 8 independent Lorentz structures (four of them considered for the first time)
- results using LCDAs up to **twist 4**
- **new models** for the LCDAs

OPE result

obtaining the OPE result

$$\Pi(k^2, q^2) = \int_0^\infty ds \sum_n \frac{I_n^{\alpha\beta}(s, q^2)}{(s - k^2)^n} \langle 0 | \bar{d}(y) G_{\alpha\beta}(ux) h_\nu(0) | B(\nu) \rangle$$

insert the LCDAs in the OPE

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insert the LCDAs in the OPE

$$\Pi(k^2, q^2) = f_B \int_0^\infty ds \sum_{n,t=3,4} \frac{I_{n,t}(s, q^2)}{(s - k^2)^n} \Psi_t(y, ux)$$

integrate by parts
to obtain a dispersive integral

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The Sum Rule

matching of the hadronic representation onto the OPE result

$$\frac{\overbrace{\langle 0 | J_{int} | K^{(*)} \rangle}^{f_{K^{(*)}}} \langle K^{(*)}(k) | \tilde{O}_\mu(0, x) | B(q+k) \rangle}{k^2 - m_{K^{(*)}}} + \text{continuum}$$

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use semi-global quark-hadron duality
 $s_0 =$ effective threshold

$$\frac{\langle K^{(*)}(k) | \tilde{O}_\mu(0, x) | B(q+k) \rangle}{k^2 - m_{K^{(*)}}} = \frac{f_B}{f_{K^{(*)}}} \int_0^{s_0} ds \sum_{t=3,4} \frac{I_t(s, q^2)}{s - k^2} \Psi_t(y, ux)$$

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apply Borel transformation

Sum Rule

$$\langle K^{(*)}(k) | \tilde{O}_\mu(0, x) | B(q+k) \rangle = \frac{f_B}{f_{K^{(*)}}} \int_0^{s_0} ds e^{\frac{m_{K^{(*)}} - s}{M^2}} \sum_{t=3,4} I_t(s, q^2) \Psi_t(y, ux)$$

Numerical Results

Preliminary results and comparison

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$\Delta\mathcal{C9}(q^2)$		KMPW2010	GvDV2019
factorizable contr.		0.27	0.27
$B \rightarrow Kll$	$\tilde{\mathcal{A}}(q^2 = 1)$	$-0.09^{+0.06}_{-0.07}$	0.0004
$B \rightarrow K^*ll$	$\tilde{\mathcal{V}}_1(q^2 = 1)$	$0.6^{+0.7}_{-0.5}$	0.002
	$\tilde{\mathcal{V}}_2(q^2 = 1)$	$0.6^{+0.7}_{-0.5}$	0.003
	$\tilde{\mathcal{V}}_3(q^2 = 1)$	$1.0^{+1.6}_{-0.8}$	0.005

results represented as a q^2 dependent correction to $\mathcal{C9}$
we fully reproduce the results given in KMWP2010

[q^2 is the dilepton mass square]

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$B \rightarrow K^*ll$	$\tilde{\mathcal{V}}_1(q^2 = 1)$	$0.6^{+0.7}_{-0.5}$	0.002
	$\tilde{\mathcal{V}}_2(q^2 = 1)$	$0.6^{+0.7}_{-0.5}$	0.003
	$\tilde{\mathcal{V}}_3(q^2 = 1)$	$1.0^{+1.6}_{-0.8}$	0.005

results represented as a q^2 dependent correction to $\mathcal{C9}$
 we fully reproduce the results given in KMWP2010

[q^2 is the dilepton mass square]

matrix elements parametrized analogously to the form factors:

$$\langle K(k) | \tilde{\mathcal{O}}_\mu(0, x) | B(q+k) \rangle = ((k \cdot q)q_\mu - q^2 k_\nu) \tilde{\mathcal{A}}(q^2) + \dots$$

$$\begin{aligned} \langle K^*(k, \eta) | \tilde{\mathcal{O}}_\mu(0, x) | B(q+k) \rangle = & \epsilon_{\mu\alpha\beta\gamma} \eta^{*\alpha} q^\beta k^\gamma \tilde{\mathcal{V}}_1(q^2) + i \left((m_B^2 - m_{K^*}^2) \eta_\mu^* - (\eta^* \cdot k)(2k + q)_\mu \right) \tilde{\mathcal{V}}_2(q^2) \\ & + i(\eta^* \cdot q) \left(q_\mu - \frac{q^2}{m_B^2 - m_{K^*}^2} (2k + q)_\mu \right) \tilde{\mathcal{V}}_3(q^2) + \dots \end{aligned}$$

Preliminary results and comparison

$\Delta\mathcal{C9}(q^2)$		KMPW2010	GvDV2019
factorizable contr.		0.27	0.27
$B \rightarrow Kll$	$\tilde{\mathcal{A}}(q^2 = 1)$	$-0.09^{+0.06}_{-0.07}$	0.0004
$B \rightarrow K^*ll$	$\tilde{\mathcal{V}}_1(q^2 = 1)$	$0.6^{+0.7}_{-0.5}$	0.002
	$\tilde{\mathcal{V}}_2(q^2 = 1)$	$0.6^{+0.7}_{-0.5}$	0.003
	$\tilde{\mathcal{V}}_3(q^2 = 1)$	$1.0^{+1.6}_{-0.8}$	0.005
$B_s \rightarrow \phi ll$...	-	???

[q^2 is the dilepton mass square]

results represented as a q^2 dependent correction to $\mathcal{C9}$
 we fully reproduce the results given in KMWP2010

matrix elements parametrized analogously to the form factors:

$$\langle K(k) | \tilde{\mathcal{O}}_\mu(0, x) | B(q+k) \rangle = ((k \cdot q)q_\mu - q^2 k_\nu) \tilde{\mathcal{A}}(q^2) + \dots$$

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Why such different results?

different inputs: B -LCDAs models depend on λ_H^2, λ_E^2



KMPW10:

$$\lambda_H^2 = \lambda_E^2 = 0.31 \pm 0.15 \text{ GeV}^2$$

⇒ twist 3 does not contribute

we use $\lambda_E^2 = 0.03 \pm 0.02 \text{ GeV}^2$

$$\lambda_H^2 = 0.06 \pm 0.03 \text{ GeV}^2$$

⇒ ~10 times smaller [Nishikawa/Tanaka '14]

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18/19

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3-particle B -LCDAs heavy quark (twist) expansion

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KMPW10: the 3-pt B -LCDAs twist expansion was not known

we use Braun/Ji/Manashov '17

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Lorentz structures considered in $\langle 0 | \bar{d}(y) G_{\alpha\beta}(ux) h_v(0) | B(v) \rangle$

→

all 8 independent Lorentz structures
⇒ partial cancelation (new structures come with an opposite sign)

KMPW10: 4 Lorentz structures

Summary

- update the previous calculation to the **soft gluon contributions to the charm-loop** effect in $B \rightarrow K^{(*)}ll$ decays
- add results for $B_s \rightarrow \phi ll$
- **numerical ~100 smaller comparing with KMPW2010** (inputs, Lorentz structures, twist expansion, new B -LCDAs models)
- results will be easily accessible in the open source software **EOS** (<https://github.com/eos/eos>)



Thank you!

Three-particle LCDAs models and $\lambda_{H,E}^2$

20/19

KMPW2010

Braun/Ji/Manashov

$$\Psi_A(\omega_1, \omega_2) = \Psi_V(\omega_1, \omega_2) = \frac{\lambda_E^2}{6\lambda_B^4} \omega_2^2 e^{-\frac{\omega_1 + \omega_2}{\lambda_B}}$$

$$X_A = \frac{\lambda_E^2}{6\lambda_B^4} \omega_2 (2\omega_1 - \omega_2) e^{-\frac{\omega_1 + \omega_2}{\lambda_B}}$$

$$Y_A = -\frac{\lambda_E^2}{24\lambda_B^4} \omega_2 (7\lambda_B - 13\omega_1 + 3\omega_2) e^{-\frac{\omega_1 + \omega_2}{\lambda_B}}$$

$$\Phi_3(\omega_1, \omega_2) = \frac{\lambda_E^2 - \lambda_H^2}{6\lambda_B^4} \omega_1 \omega_2^2 e^{-\frac{\omega_1 + \omega_2}{\lambda_B}}$$

$$\Phi_4(\omega_1, \omega_2) = \frac{\lambda_E^2 + \lambda_H^2}{6\lambda_B^4} \omega_2^2 e^{-\frac{\omega_1 + \omega_2}{\lambda_B}}$$

$$\Psi_4(\omega_1, \omega_2) = \frac{\lambda_E^2}{3\lambda_B^4} \omega_1 \omega_2 e^{-\frac{\omega_1 + \omega_2}{\lambda_B}}$$

$$\tilde{\Psi}_4(\omega_1, \omega_2) = \frac{\lambda_H^2}{3\lambda_B^4} \omega_1 \omega_2 e^{-\frac{\omega_1 + \omega_2}{\lambda_B}}$$

$\lambda_{H,E}^2$ definition

$$\langle 0 | \bar{d}(0) G_{\alpha\beta}(0) h_\nu(0) | B(\nu) \rangle = -\frac{i}{6} f_B \lambda_H^2 \text{Tr}[\gamma_5 \Gamma P_+ \sigma_{\alpha\beta}] - \frac{i}{6} f_B (\lambda_H^2 - \lambda_E^2) \text{Tr}[\gamma_5 \Gamma P_+ (v_\alpha \gamma_\beta - v_\beta \gamma_\alpha)]$$

Threshold s_0 determination

$$f_{K^{(*)}} e^{-\frac{m_{K^{(*)}}}{M^2}} \langle K^{(*)}(k) | \tilde{O}_\mu(0, x) | B(q+k) \rangle = f_B \int_0^{s_0} ds e^{-\frac{s}{M^2}} \sum_{t=3,4} I_t(s, q^2) \Psi_t(y, ux) \quad (1)$$



derive with respect to $1/M^2$
and divide by (1)

$$m_{K^{(*)}} = \frac{\int_0^{s_0} ds s e^{-\frac{s}{M^2}} \sum_{t=3,4} I_t(s, q^2) \Psi_t(y, ux)}{\int_0^{s_0} ds e^{-\frac{s}{M^2}} \sum_{t=3,4} I_t(s, q^2) \Psi_t(y, ux)}$$

daughter sum rule to extract s_0

Preliminary results and comparison

22/19

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$B_s \rightarrow \phi ll$...	-	???