Theory progress in inclusive $\bar{B} \rightarrow X_{s/d} \ell^+ \ell^-$ decays

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Outline

- Observables
- Complementarity between incl. and excl. decays
- Five-particle contributions
- Resonances
- Input parameters
- Preliminary results
- Outlook

[Qin,Vos,TH'18]
Double differential decay width \( z = \cos \theta_\ell \)

\[
\frac{d^2\Gamma}{dq^2 dz} = \frac{3}{8} \left[ (1 + z^2) H_T(q^2) + 2 z H_A(q^2) + 2 (1 - z^2) H_L(q^2) \right]
\]

Note: \( \frac{d\Gamma}{dq^2} = H_T(q^2) + H_L(q^2) \), \( \frac{dA_{FB}}{dq^2} = 3/4 H_A(q^2) \)

Dependence of the \( H_i \) on WCs

\[
H_T(q^2) \propto 2s(1 - s)^2 \left[ |C_9 + \frac{2}{s} C_7|^2 + |C_{10}|^2 \right]
\]

\[
H_A(q^2) \propto -4s(1 - s)^2 \text{Re} \left[ C_{10} \left( C_9 + \frac{2}{s} C_7 \right) \right]
\]

\[
H_L(q^2) \propto (1 - s)^2 \left[ |C_9 + 2 C_7|^2 + |C_{10}|^2 \right]
\]

Perturbative corrections at quark level are known to NNLO QCD + NLO QED

[Ali,Hiller,Handoko,Morozumi’96; Bauer,Burrell’99; Buchalla,Isidori,Rey’97]

1/\( m_b^2 \), 1/\( m_b^3 \) and 1/\( m_c^2 \) non-perturbative corrections also known

Observables in $\bar{B} \to X_s(d) \ell^+\ell^-$

- In high-$q^2$ region, ratio
  \[ \mathcal{R}(s_0) = \frac{\int_{s_0}^1 d\hat{s} \, d\Gamma(\bar{B} \to X_s(d)\ell^+\ell^-)/d\hat{s}}{\int_{s_0}^1 d\hat{s} \, d\Gamma(\bar{B} \to X_u\ell\nu)/d\hat{s}} \]  
  \[[\text{Ligeti,Tackmann'07}]\]

- Normalize to semileptonic $\bar{B}^0 \to X_u\ell\nu$ rate with the same cut
  Need differential semi-leptonic $b \to u$ rate

- In $b \to d$ transitions, matrix elements of $P_{1/2}^{(u)}$ are not CKM-suppressed.
  \[ P_1^u = (\bar{d}_L \gamma_\mu T^a u_L)(\bar{u}_L \gamma^\mu T^a b_L) \]
  \[ P_2^u = (\bar{d}_L \gamma_\mu u_L)(\bar{u}_L \gamma^\mu b_L) \]

- All sides of the $b \to d$ UT are democratic in size, $O(\lambda^3)$
- Expect “sizable” CP asymmetries
  \[ A_{CP} = \frac{\Gamma(\bar{B} \to X_d \ell^+\ell^-) - \Gamma(B \to X_d \bar{\ell}^+\ell^-)}{\Gamma(\bar{B} \to X_d \ell^+\ell^-) + \Gamma(B \to X_d \bar{\ell}^+\ell^-)} \]

- However, potentially large power-corrections from $u$-loops \[[\text{Benzke,Lee,Neubert,Paz'10}]\]
Inclusive $\bar{B} \rightarrow X_s \ell^+ \ell^-$: WC sensitivity

- Study ratios $R_i = \frac{C_i(\mu_0)}{C_i^{SM}(\mu_0)}$ in different bins of $q^2$

- Model-independent constraints on high-scale WCs through angular observables $H_{T,A,L}(q^2)$

- Extrapolation to the full Belle-II statistics ($50 \text{ ab}^{-1}$)

[See also Lee,Ligeti,Stewart,Tackmann’06; K. Flood]
Complementarity study

- Include BR and $A_{FB}$ in $\bar{B} \to X_s \ell^+ \ell^-$ with 50/ab.
- derive model-independent constraints on $C_9$ and $C_{10}$

Q: If the true values for the NP contributions are $C_{9}^{\text{NP}}$ and $C_{10}^{\text{NP}}$, with which significance will the Belle II measurements exclude the SM ($C_{9}^{\text{NP}} = C_{10}^{\text{NP}} = 0$)?

- For each point $(C_{9}^{\text{NP}}, C_{10}^{\text{NP}})$, we consider hypothetical measurements of $BR$ and $A_{FB}$, with central values given by the theory predictions at the corresponding NP point, and errors given by the experimental sensitivity study plus an extra 5% to account for non-perturbative effects.

[Ishikawa, Virto, TH: Belle II Physics Book, 1808.10567, sec. 9.4.5]
Superimpose the one-, two-, and three-sigma regions obtained from the current global fit, which is dominated by the exclusive \(b \rightarrow s\mu\mu\) measurements at LHCb (red regions).

Underlying hadronic uncertainties in inclusive mode are quite different and independent of those in exclusive transitions.

Hence, precision measurements of the \(\bar{B} \rightarrow X_s \ell^+ \ell^-\) channel provide important complementary information in the context of global fits.

Synergy between LHCb and Belle II measurements can play a decisive role in the search for NP via \(b \rightarrow s\ell\ell\) transitions.
Five-particle contributions

- Five-particle $b \to s(d) q\bar{q} \ell^+ \ell^-$ contribution to $\bar{B} \to X_{s(d)} \ell^+ \ell^-$ with $q = u, d, s$
- Expand in $\alpha_s$ and $\kappa = \alpha_e/\alpha_s$ through to $O(\alpha_s^3 \kappa^3)$
- Include tree-level interferences of
  
  $$(P_{1,2}^u, P_{3,...,6})$$
  
  with
  
  $$(P_{1,2}^u, P_{3,...,6})$$

... and $(P_{1,2}^u, P_{3,...,6})$ with $(P_{7,...,10})$. 

\[\text{Theory progress in inclusive } \bar{B} \to X_{s/d} \ell^+ \ell^- \text{ decays}\]
Discard

- one-loop interferences of \((P^{(u)}_{1,2}, P_{3,\ldots,6})\) with \((P^u_{1,2}, P_{3,\ldots,6})\)

Interferences that require fewer-particle cuts involving loops for renormalisation
Five-particle contributions

- Adopt two different phase-space parametrisations

\[
\frac{d\Phi_5}{ds} |M|^2 = \frac{\pi^2 m_b^6}{16(2\pi)^{11}} \int_s^{s_1} ds_1 \int_{s_2/s_1}^{1-s_1+s_2} ds_2 \int_{s_2/s_1}^{u_2^+} du_1 \int_{u_2^-}^{u_1} du_2 \int_{u_2^-}^{u_3^+} du_3 \int_{t_2^-}^{t_2^+} dt_2 \int_{t_3^-}^{t_3^+} dt_3 \\
\times \frac{(s_2 - s)}{s_2 (u_2^+ - u_2^-) (u_3^+ - u_3^-) \sqrt{(t_2^+ - t_2)(t_2 - t_2^-)} \sqrt{(t_3^+ - t_3)(t_3 - t_3^-)}} \left( \frac{1}{2} |M|^2_{s_34 \to s_34^+} + \frac{1}{2} |M|^2_{s_34 \to s_34^-} \right)
\]

\[
\int d\Phi_5 |M|^2 = \frac{m_b^6}{4^8 \pi^9} \int_0^1 dt_2 \ldots dt_6 \ldots dt_{10} \frac{t_2 t_6 t_7^2 (1 - t_6)(1 - t_7) \delta(t_2 t_4 t_6 t_7 - s_{14})}{\sqrt{t_8} \sqrt{1 - t_8} \sqrt{t_{10} \sqrt{1 - t_{10}}} \left( |M|^2_{t_5=0} + |M|^2_{t_5=1} \right)}
\]

- Staying differential in \( q^2 = (p_{\ell^+} + p_{\ell^-})^2 \) allows to perform the calculation in \( D = 4 \) dimensions. Seven integrations required. Almost all results analytical.

\[
f_1(\hat{s}) = \frac{\pi}{6} - \arctan(x)
\]

\[
f_7(\hat{s}) = 2if_1(\hat{s}) \left[ Li_2 \left( \frac{1 - ix}{1 + ix} \right) - c.c. \right] - \left[ Li_3 \left( \frac{1 - ix}{1 + ix} \right) + c.c. \right] + \frac{2\zeta(3)}{3}, \quad x = \sqrt{\frac{\hat{s}}{4 - \hat{s}}}
\]
**Five-particle contributions**

- Differential branching ratio and forward-backward asymmetry
  
  \[
  \frac{d\mathcal{B}(b \to d\ell^+\ell^- q\bar{q})}{d\hat{s}} = \mathcal{B}(\bar{B} \to X_c e\bar{\nu}) \exp \left| \frac{V_{td}^* V_{tb}}{V_{cb}} \right|^2 \frac{4}{C\Phi_u} \left( \sum_{i, j=1}^{10} R_{ij}^{ij} C_i^* C_j \mathcal{F}_{ij}(\hat{s}) \right)
  \]

  \[
  \frac{dA_{FB}}{d\hat{s}} = \mathcal{B}(\bar{B} \to X_c e\bar{\nu}) \exp \left| \frac{V_{td}^* V_{tb}}{V_{cb}} \right|^2 \frac{4}{C\Phi_u} \sum_{i=1}^{6} (-R_{i10}^{i10} C_i C_{10} A_i(\hat{s}) + c.c.)
  \]

- \(O(1\%)\) correction in case of \(\bar{B} \to X_d \ell^+ \ell^-\)

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<tbody>
<tr>
<td>(\mathcal{B}(b \to d\ell^+\ell^- q\bar{q}) \times 10^{-10})</td>
<td>9.22</td>
<td>0.30</td>
<td>9.52</td>
</tr>
<tr>
<td>(A_{FB}(b \to d\ell^+\ell^- q\bar{q}) \times 10^{-12})</td>
<td>1.48</td>
<td>0.49</td>
<td>1.97</td>
</tr>
</tbody>
</table>

- \(O(0.01\%)\) correction in case of \(\bar{B} \to X_s \ell^+ \ell^-\) due to CKM suppression

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<tr>
<td>(\mathcal{B}(b \to s\ell^+\ell^- q\bar{q}) \times 10^{-10})</td>
<td>2.18</td>
<td>0.05</td>
<td>2.23</td>
</tr>
<tr>
<td>(A_{FB}(b \to s\ell^+\ell^- q\bar{q}) \times 10^{-11})</td>
<td>1.57</td>
<td>0.52</td>
<td>2.10</td>
</tr>
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Resonances

- In the $q^2$ spectrum of $\bar{B} \to X_{s(d)} \ell^+ \ell^-$, $c\bar{c}$ resonances show up as large peaks.

- Large parton-hadron duality-violating effects for the integrated $\bar{B} \to X_{s(d)} \ell^+ \ell^-$ spectrum. 


- In $\bar{B} \to X_d \ell^+ \ell^-$ also light-quark resonances are relevant.
Idea of Krüger and Sehgal:

- Assume factorization of $b \to s(d)$ and $c\bar{c}$ current
- Replace one-loop perturbative function by dispersion integral over $R_{\text{had}}$

\[
\Pi^{q\bar{q}} = \frac{9\alpha}{4\pi} \left( g(\hat{m}_q, \hat{s}) + \frac{4}{9} + \frac{8}{9} \ln \hat{m}_q \right)
\]

\[
\Pi_{\text{had}}^\gamma = \frac{4}{9} \Pi^{c\bar{c}} + \frac{4}{9} \Pi^{u\bar{u}} + \frac{1}{9} \Pi^{d\bar{d}} + \ldots
\]

\[
\text{Im} \Pi_{\text{had}}^\gamma(\hat{s}) = \frac{\alpha}{3} R_{\text{had}}(\hat{s})
\]

\[
\text{Re} \Pi_{\text{had}}^\gamma(\hat{s}) = \frac{\alpha \hat{s}}{3\pi} P \int_0^\infty \frac{R_{\text{had}}(\hat{s}')}{\hat{s}'(\hat{s}' - \hat{s})} d\hat{s}'
\]

\[
\text{Im } g(\hat{m}_c, \hat{s}) = \frac{\pi}{3} R_{\text{had}}^{J/\psi}(\hat{s})
\]

\[
\text{Im } g(\hat{m}_u, \hat{s}) = \text{Im } g(\hat{m}_d, \hat{s}) = \frac{4\pi}{15} (R_{\text{had}}^\rho(\hat{s}) + R_{\text{had}}^\omega(\hat{s}))
\]

\[
\text{Re } g(\hat{m}_c, \hat{s}) = \frac{8}{9} \ln \hat{m}_c - \frac{4}{9} + \frac{\hat{s}}{3} P \int_0^\infty \frac{R_{\text{had}}^{J/\psi}(\hat{s}')}{\hat{s}'(\hat{s}' - \hat{s})} d\hat{s}'
\]

\[
R_{\text{had}}^{J/\psi}(\hat{s}) = R_{\text{cont}}^{c\bar{c}}(\hat{s}) + R_{\text{res}}^{J/\psi}(\hat{s})
\]

\[
R_{\text{res}}^{J/\psi}(\hat{s}) = \sum_{V=J/\psi,\psi',...} \frac{9\hat{s}}{\alpha^2} \frac{\text{Br}(V \to l^+l^-)}{(\hat{s} - \hat{m}_V^2)^2 + \hat{m}_V^2} \frac{\hat{V}}{\hat{V}_{\text{total}}}
\]

Ensures absence of double-counting with non-factorizable $1/m_c^2$ power corrections

[Buchalla,Isidori,Rey’97; Benzke,Hurth,Turczyk’17]
Resonances

What’s new?

- Much more data has become available: BES, BaBar, ALEPH, ...
- besides $q\bar{q}$-currents, we use also charged current $\bar{u}d$ interference

\[
(q^\mu q^\nu - q^2 g^{\mu\nu})\Pi_{\bar{u}d}(q^2) = i \int d^4x \ e^{iqx} \langle 0| TJ_{\bar{u}d}^\mu(0) J_{\bar{u}d}^\nu(\bar{x})|0 \rangle,
\]

- $J_{\bar{u}d}^\mu = \bar{u}\gamma^\mu d$
- $\Pi_{\bar{u}d}$ is related to the vector spectral function $V_{1d} = 2\pi \Im[\Pi_{\bar{u}d}]$

- Investigation of subtraction point of dispersive integral
- Replacement of perturbative function $h$
- study of uncertainties in KS functions
Use data from BES, BaBar, ALEPH in the resonance regions

Use perturbation theory (program \texttt{rhad}) for asymptotics at large \( s \).
In the replacement of the perturbative function,

\[ h_q(s) \rightarrow h_q(s_0) + \frac{s - s_0}{\pi} \int_{s_0}^{\infty} dt \frac{\text{Im}[h_q(t + i\epsilon)]}{(t - s_0)(t - s - i\epsilon)}. \]

we take into account all factorizable pieces up to two loops. Separation of two-loop perturbative function provided by St. de Boer

Choose as subtraction point \( s_0 = -(5 \text{ GeV})^2 \)

Split up \( h_q(s) \) into \( h_{u,d,s}(s) \) in case of the light quarks.
Resonances

Results, low-$q^2$ region

[Hurth, Jenkins, Lunghi, Qin, Vos, TH in prep.]
Results, high-$q^2$ region

\begin{align*}
\text{Im}[h_u(s)] & \quad \text{Im}[h_u(s)] \text{ (PT)} \\
\text{Re}[h_u(s)] - \text{Re}[h_u(-25)] & \quad \text{Re}[h_u(s)] - \text{Re}[h_u(-25)] \text{ (PT)}
\end{align*}

\begin{align*}
\text{Im}[h_c(s)] & \quad \text{Im}[h_c(s)] \text{ (PT)} \\
\text{Re}[h_c(s)] - \text{Re}[h_c(-25)] & \quad \text{Re}[h_c(s)] - \text{Re}[h_c(-25)] \text{ (PT)}
\end{align*}
Checks: Reproduce asymptotic behaviour of perturbative function

- Upper plot: Real part of $h_u(s) - h_u(-25)$
  - Re[$h_u(s) - h_u(-25)$]
  - Blue line: $h_u(s) - h_u(-25)$
  - Orange line: Perturbative – 0
  - Green line: Perturbative – 1 ($\mu = 5$ GeV)

- Lower plot: Imaginary part of $h_u(s)$
  - Im[$h_u(s)$]
  - Blue line: $h_u(s)$
  - Orange line: Perturbative – 0
  - Green line: Perturbative – 1 ($\mu = 5$ GeV)
Cascade decays $B \to X_1 (\psi \to X_2 \ell^+ \ell^-)$ constitute another LD effect

- On-shell resonance $\psi = \eta_c, \eta_c', J/\psi, \psi', \chi_c J, \ldots$
- Leptonic invariant mass can fall in low-$q^2$ region, $4m_\ell^2 < q^2 < (M_\psi - M_{X_2})^2$
- Effect not captured by KS, $\Lambda/m_c$ power corrections or shape-function uncertainty estimates
- Effect taken into account on experimental side?
- Effect of $M_{X_s,d}$-cut remains to be investigated
- Investigation would greatly benefit from data on $\psi \to (\pi, \pi\pi, \eta, \eta') \ell\ell$
Computing power-corrections requires HQET matrix elements of dimension five and six operators

\[ \lambda_1 \equiv \langle B | \bar{h}_v (iD)^2 h_v | B \rangle \]
\[ \lambda_2 \equiv \langle B | \bar{h}_v g_{\mu\nu} G^{\mu\nu} h_v | B \rangle \]

\[ \frac{1}{3} \rho_1 \left( g_{\alpha\beta} - v_{\alpha} v_{\beta} \right) v_{\mu} \equiv \frac{1}{2m_B} \langle B | \bar{h}_v iD_\alpha iD_\mu iD_\beta h_v | B \rangle \]
\[ \frac{1}{2} \rho_2 \ i \epsilon_{\nu \alpha \beta \delta} v^{\nu} v_{\mu} \equiv \frac{1}{2m_B} \langle B | \bar{h}_v iD_\alpha iD_\mu iD_\beta \gamma^{\delta} \gamma_5 h_v | B \rangle \]

\[ f_{q,0,\pm}^0 \equiv \frac{1}{2m_B} \langle B^{0,\pm} | Q_2^q - Q_1^q | B^{0,\pm} \rangle \]

- \( \lambda_{1,2} \) scale as \( m_b^{-2} \) and \( (\rho_{1,2}, f_{q,0,\pm}^0) \) scale as \( m_b^{-3} \)
- In particular, \( \rho_{1,2} \) and \( f_{q,0,\pm}^0 \) introduce sizable uncertainties to the high-\( q^2 \) BR.
• \( \lambda_i \) and \( \rho_i \) were extracted in the kinetic scheme from moments of inclusive
\( B \to X_c \ell \nu \) spectrum

• We convert these quantities to the pole scheme

• For the weak-annihilation matrix elements \( f_{0,\pm} \) assume flavour symmetries

\[
\begin{align*}
f_V & \equiv f_u^{\pm \text{SU}(2)} = f_d^0 \\
f_{NV} & \equiv f_u^0 \text{SU}(2) = f_d^\pm \text{SU}(3) = f_s^0 \text{SU}(2) = f_s^\pm
\end{align*}
\]

• \( f_V/\text{NV} \): valence and non-valence terms w.r.t. external \( B^{0,\pm} \) states

• Use BR and moments of semileptonic \( D^{0,\pm} \) and \( D_s \) decays

• follow analysis of \([\text{Gambino,Kamenik}'10]\)

• (Largely) uncorrelated combinations are \( f_{NV} \) and \( f_V - f_{NV} \)

• Introduce \( \text{SU}(3) \) and \( \text{SU}(2) \) breaking effects according to \([\text{Ligeti,Tackmann}'07]\)

• Q: Can lattice QCD compute these matrix elements in analogy to the ones from \( B-\bar{B} \)-mixing?
Input parameters

\[
\begin{align*}
\lambda_2^\text{eff} &= \lambda_2 - \frac{\rho_2}{m_b} = 0.130(21) \text{ GeV}^2 \\
\lambda_1 &= -0.267(90) \text{ GeV}^2 \\
\rho_1 &= 0.038(70) \text{ GeV}^3 \\
(f_V + f_{NV})/2 &= (-0.04 \pm 0.17) \text{ GeV}^3 \\
\end{align*}
\]

\[
\begin{align*}
&\quad f_{NV} = (-0.02 \pm 0.16) \text{ GeV}^3 \\
&\quad f_V - f_{NV} = (-0.041 \pm 0.052) \text{ GeV}^3 \\
&\quad [\delta f]_{SU(3)} = (0 \pm 0.04) \text{ GeV}^3 \\
&\quad [\delta f]_{SU(2)} = (0 \pm 0.004) \text{ GeV}^3 \\
\end{align*}
\]

\[
\begin{align*}
\text{BR}(\bar{B} \to X_s \ell^+ \ell^-) &\Rightarrow \left\{ \begin{array}{l}
f_s = f_{NV} \\
f_u = (f_V + f_{NV})/2 \\
\end{array} \right. \\
\mathcal{R}(s_0, \bar{B} \to X_s \ell^+ \ell^-) &\Rightarrow \left\{ \begin{array}{l}
(f_s + f_u^0)/2 = f_{NV} \\
f_s - f_u^0 = [\delta f]_{SU(3)} \\
\end{array} \right. \\
\text{BR}(\bar{B} \to X_d \ell^+ \ell^-) \text{ and } \mathcal{R}(s_0, B \to X_d \ell^+ \ell^-) &\Rightarrow \left\{ \begin{array}{l}
(f_d + f_u)/2 = (f_V + f_{NV})/2 \\
f_d - f_u = [\delta f]_{SU(2)} \\
\end{array} \right. \\
\end{align*}
\]
Preliminary results, low-$q^2$ region

- Include also Log-enhanced QED corrections from $P_{1,2}^u$
- All numbers preliminary

\[ B(\bar{B} \rightarrow X_d e^+ e^-)[1, 6] = (7.79 \pm 0.36_{\text{scale}} \pm 0.14_{\text{CKM}}) \times 10^{-8} \]
\[ B(\bar{B} \rightarrow X_d e^+ e^-)[1, 3.5] = (4.32 \pm 0.17_{\text{scale}} \pm 0.08_{\text{CKM}}) \times 10^{-8} \]
\[ B(\bar{B} \rightarrow X_d e^+ e^-)[3.5, 6] = (3.47 \pm 0.20_{\text{scale}} \pm 0.06_{\text{CKM}}) \times 10^{-8} \]

\[ B(\bar{B} \rightarrow X_d \mu^+ \mu^-)[1, 6] = (7.57 \pm 0.34_{\text{scale}} \pm 0.14_{\text{CKM}}) \times 10^{-8} \]
\[ B(\bar{B} \rightarrow X_d \mu^+ \mu^-)[1, 3.5] = (4.16 \pm 0.15_{\text{scale}} \pm 0.08_{\text{CKM}}) \times 10^{-8} \]
\[ B(\bar{B} \rightarrow X_d \mu^+ \mu^-)[3.5, 6] = (3.41 \pm 0.19_{\text{scale}} \pm 0.06_{\text{CKM}}) \times 10^{-8} \]

\[ A_{CP}(\bar{B} \rightarrow X_d e^+ e^-)[1, 6] = (1.4 \pm 0.7_{\text{scale}} \pm 0.03_{\text{CKM}})\% \]
\[ A_{CP}(\bar{B} \rightarrow X_d e^+ e^-)[1, 3.5] = (1.5 \pm 0.3_{\text{scale}} \pm 0.05_{\text{CKM}})\% \]
\[ A_{CP}(\bar{B} \rightarrow X_d e^+ e^-)[3.5, 6] = (1.4 \pm 1.6_{\text{scale}} \pm 0.04_{\text{CKM}})\% \]

\[ A_{CP}(\bar{B} \rightarrow X_d \mu^+ \mu^-)[1, 6] = (1.3 \pm 0.7_{\text{scale}} \pm 0.04_{\text{CKM}})\% \]
\[ A_{CP}(\bar{B} \rightarrow X_d \mu^+ \mu^-)[1, 3.5] = (1.3 \pm 0.4_{\text{scale}} \pm 0.04_{\text{CKM}})\% \]
\[ A_{CP}(\bar{B} \rightarrow X_d \mu^+ \mu^-)[3.5, 6] = (1.4 \pm 1.6_{\text{scale}} \pm 0.04_{\text{CKM}})\% \]
Preliminary results, high-\(q^2\) region

- All numbers **preliminary**

\[ \mathcal{B}(\bar{B} \to X_d e^+ e^-) [> 14.4] = (0.97 \pm 0.12_{\text{scale}} \pm 0.24_{f_{NV}} \pm 0.09_{CKM}) \times 10^{-8} \]
\[ \mathcal{B}(\bar{B} \to X_d \mu^+ \mu^-) [> 14.4] = (1.12 \pm 0.12_{\text{scale}} \pm 0.23_{f_{NV}} \pm 0.09_{CKM}) \times 10^{-8} \]

\[ A_{CP}(\bar{B} \to X_d e^+ e^-) [> 14.4] = (-1.7 \pm 0.1_{\text{scale}} \pm 0.5_{f_{NV}} \pm 7.6_{CKM})\% \]
\[ A_{CP}(\bar{B} \to X_d \mu^+ \mu^-) [> 14.4] = (-1.6 \pm 0.2_{\text{scale}} \pm 0.4_{f_{NV}} \pm 6.7_{CKM})\% \]

\[ R(0)(\bar{B} \to X_d e^+ e^-) [14.4] = (0.95 \pm 0.02_{\text{scale}} \pm 0.05_{f_{NV}}) \times 10^{-4} \]
\[ R(0)(\bar{B} \to X_d \mu^+ \mu^-) [14.4] = (1.11 \pm 0.01_{\text{scale}} \pm 0.02_{f_{NV}}) \times 10^{-4} \]
Inclusive $\bar{B} \rightarrow X_s \ell^+ \ell^-$ is an unsung hero

- Complementarity to $\bar{B} \rightarrow X_s \gamma$ and $\bar{B} \rightarrow K^{(*)} \mu^+ \mu^-$ crucial in the search for NP

- However, not all Belle I data analysed for the BR yet

We update the SM theory predictions in inclusive $\bar{B} \rightarrow X_s \ell^+ \ell^-$ and $\bar{B} \rightarrow X_d \ell^+ \ell^-$ decays

- Include effects of:
  - multi-parton contributions
  - QED corrections (collinear photons)

- Thorough investigation of:
  - resonances
  - cascade decays
  - input parameters
Outlook

- To-do list
  - Extend pheno to $\bar{B} \rightarrow X_s \ell^+ \ell^-$:
    All angular observables, NP formulas, RH currents . . .
  - Effects beyond KS and resolved contributions? Re-introduces issue of double-counting.
  - Further investigate the effects of an $M_{Xs(d)}$-cut at low $q^2$
    [Lee,Ligeti,Stewart,Tackmann’06; Lee,Tackmann’08]
    - Puts kinematics in “shape-function region” $\Rightarrow$ SCET application
    - Resolved contributions of other operators (e.g. $P_9$) [à la Hurth,Fickinger,Turczyk,Benzke’17]
    - Phenomenological implications for observables other than zero-crossing of FBA
      [à la Lee,Tackmann’08; Bell,Beneke,Li,TH’10]
  - Study effects of lepton-flavour universality violation, e.g. in observable
    $$R_{Xs} = \frac{\mathcal{B}(\bar{B} \rightarrow X_s \mu^+ \mu^-)}{\mathcal{B}(\bar{B} \rightarrow X_s e^+ e^-)}$$
  - Further exploit synergy and complementarity between inclusive and exclusive modes, e.g. take into account full set of angular observables
Backup slides
Cuts on $M_{X_s}$

- The suppression of background from $b \to c \to s \ell \nu \ell \nu$ requires a cut on $M_{X_s}$. Have $M_{X_s} < 1.8 \ (2.0)$ GeV at BaBar (Belle).

- Usually taken into account on experimental side

- This puts kinematics at low-$q^2$ into the shape function region

  ⇒ SCET applicable, define

  \[ p_{X}^\pm = E_X \mp |\vec{p}_X| \]  

  [Lee, Ligeti, Stewart, Tackmann'06]

- High-$q^2$ region hardly affected by the cut

\[ m_{X} \leq 2.0 \text{ GeV} \]
Cuts on $M_{X_s}$

- Compute non-perturbative corrections of leading and subleading order in $\Lambda_{QCD}/m_b$

- Use different models for subleading SF

- Effect on $H_i$ and $\Gamma$ is $\sim -5$ to $-10\%$

- Shift of zero of FBA is $\sim -0.05$ to $-0.10$ GeV$^2$

- Add NNLO QCD-corrections to heavy-light currents in shape function region

- Zero of FBA

$$q_0^2 = \left[ (3.34 \ldots 3.40)^{+0.22}_{-0.25} \right] \text{GeV}^2 \quad \text{for} \quad m_{X_s}^{\text{cut}} = (2.0 \ldots 1.8) \text{GeV}$$

- In same region as inclusive result
- Significantly smaller than exclusive result

[Lee,Tackmann'08]

$\Delta \Gamma(1, 6; m_{X_s}^{\text{cut}})$

$\Delta \Gamma$ [%]

$m_{X_s}^{\text{cut}}$ [GeV]

[Bell,Beneke,Li,TH'10]

Recent analysis of factorization to subleading power in $\bar{B} \to X_s \ell^+ \ell^-$ in presence of a cut on $M_{X_s}$

Systematic analysis of resolved power corrections at $\mathcal{O}(1/m_b)$

Compute so-called resolved contributions, explore numerical impact

Numerical impact

$$\mathcal{F}_{17} \in [-0.5, +3.4] \%, \quad \mathcal{F}_{78} \in [-0.2, -0.1] \%, \quad \mathcal{F}_{88} \in [0, 0.5] \%$$

(normalized to OPE result)

$$\mathcal{F}_{1/m_b} \in [-0.7, +3.8]$$

Resolved contributions stay nonlocal when the hadronic mass cut is released

- Represents irreducible uncertainty independent of the hadronic mass cut