Aspects of χral perturbation theory, an on-shell perspective

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[work in progress]

χral perturbation th.

- Effective theory, used to describe strong physics at low energies
- Observables are systematically expanded in powers of momenta
- Example: $A_{2\to 2}$ for pions and/or kaons

$A_{2\to 2}$ up to $\mathcal{O}(p^4)$

- Complete (and complicated) expression derived long ago, function of masses, couplings, Mandelstam invariants

- Take the derivative $\partial/\partial \ln \mu^2$ of the renormalised expression

$$\frac{\partial}{\partial \ln \mu^2} A(\mu) \sim A_{div}$$

- A_{div} looks *very* simple

$$\delta_s = \delta_{IJ}\delta_{KL}$$

kinematics **flavour**

$$A_{div}[N=2] = C\ (s^2 + t^2 + u^2)\ (\delta_s + \delta_t + \delta_u)$$

$$A_{div}[N=3] = \frac{9}{8}C\ (s^2 + t^2 + u^2)\ (\delta_s + \delta_t + \delta_u)$$

These simple results raise some questions:

A. Is this structure maintained at any N?

B. What are those coefficients? (kaons provide a +1/8…?)

C. Is there an underlying principle, dictating this structure?

“Where” should we look for the answers?

- One-loop divergent part of A ≡ sum of <u>bubble coefficients</u> in the Passarino-Veltman expansion of A

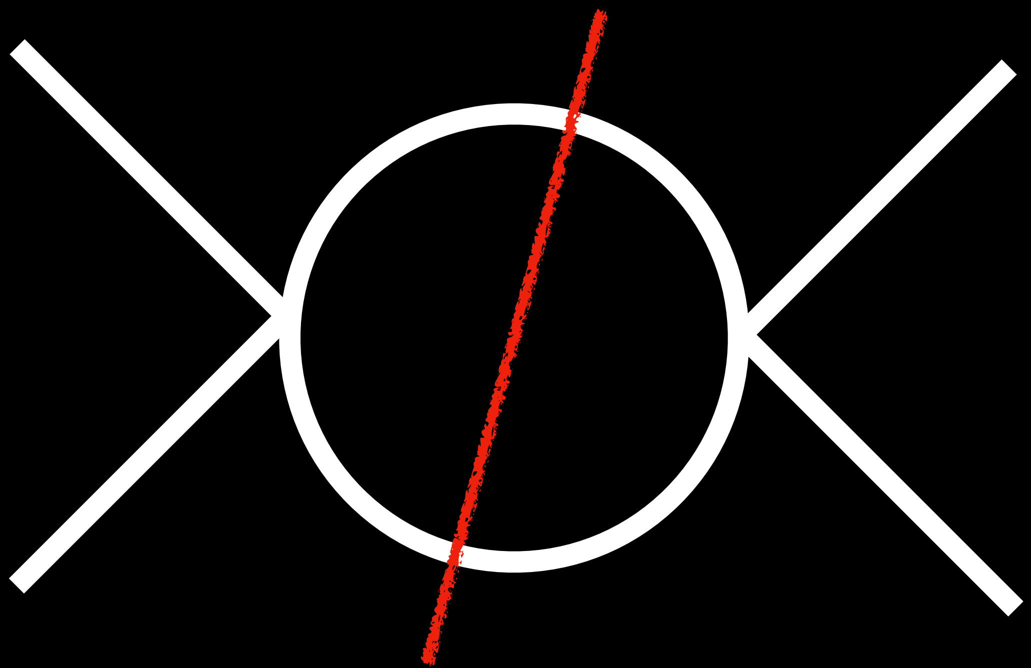
$$A_{1-loop} = \sum_i C_i I_i + R \quad \rightarrow \quad \boxed{A_{div} = \# \sum_{i \in bub.} C_i}$$

- Coefficients can be extracted using a generalisation of the “Cutkosky rule”

$$\boxed{cut_i^*[I_j] = \delta_{ij} \rightarrow cut_i^*[A_{1-l}] = C_i}$$

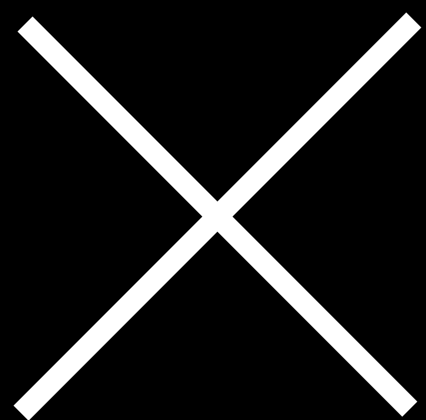
- $cut[A]$ is some convolution of ***<u>tree-level (on-shell) amplitudes</u>***

s-channel double cut

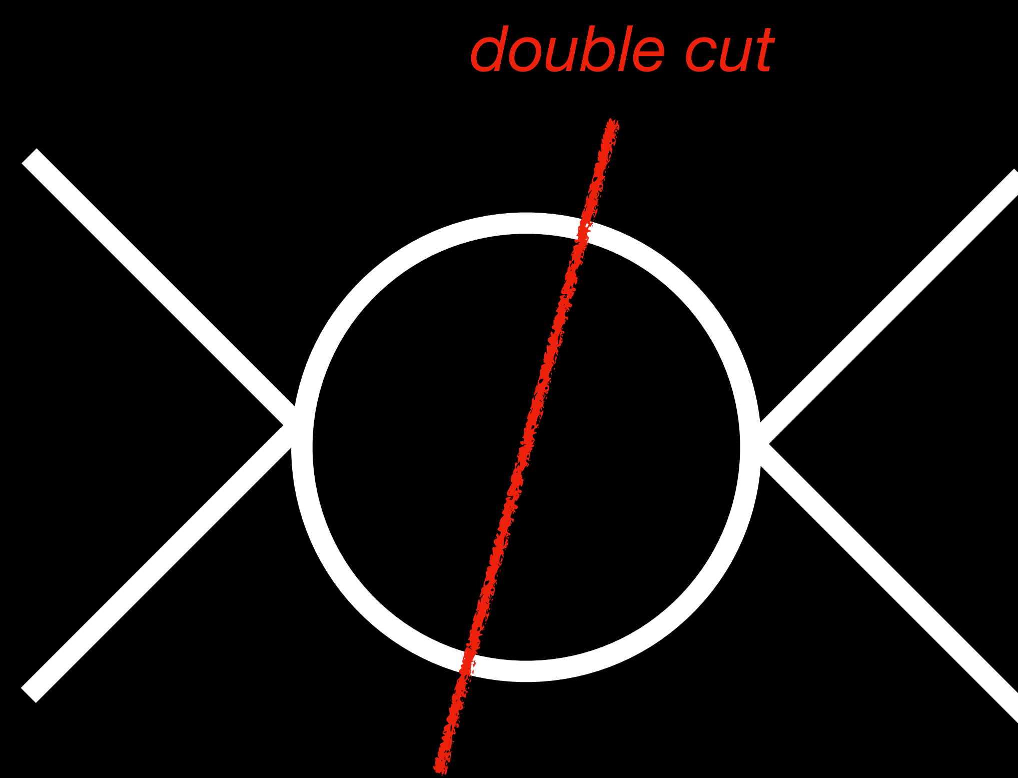


$$C_s \sim A_L \times A_R$$

- Due to the locality of counter-terms, A_{div} is a local 4pt amplitude

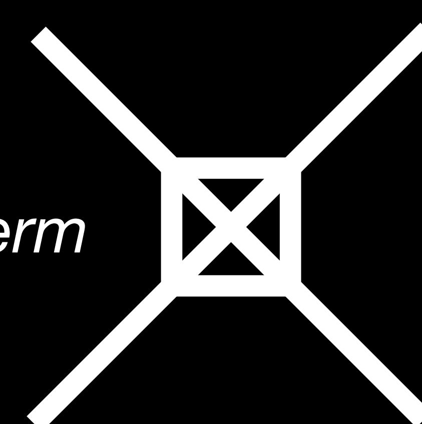


TREE



LOOP

counter-term



Need to talk only about 4pt <u>contact</u> amplitudes, at $O(p^2)$ and $O(p^4)$

Classification of A_{4pt}

- Amplitude for 4pt scattering of derivatively coupled π^I
- Must be a Lorentz scalar → function of Mandelstam invariants

$$A_{IJKL}(s, t, u)$$

- Crossing symmetry implies: A singlet under exchange of any two indices
- Invariance under linear G: A is a G-singlet tensor

Singlet tensors — SU(N)

- For π^I with indices in the adjoint of *SU(N)*, "group-singlets" are constructed out of traces of generators

$$tr(t_I t_J t_K t_L) \quad \& \quad tr(t_I t_J)\, tr(t_K t_L)$$

single trace **double trace**

- Then combine with kinematics to form a "crossing-singlet"

$$A_{IJKL} \sim \sum_\alpha T^{(\alpha)}_{IJKL} f^{(\alpha)}(s,t,u)$$

Crossing-singlet *A*'s

- Indices exchange is an S_4 action
- Acting on objects belonging to linear spaces (tensor space and kinematic space)
- Natural to use S_4 linear representation theory
- Only *five inequivalent representations*

$$\{1\ ,\ 1_\varepsilon\ ,\ 2\ ,\ 3\ ,\ 3' = 3 \otimes 1_\varepsilon\}$$

Crossing-singlet *A*'s

- The Mandelstam invariants furnish a 2 (in fact $s + t + u = 0$)

  double tr. single tr.

- The tensors split as $1 \oplus 2$ and $1 \oplus 2 \oplus 3$
- Cannot use the *3*'s to form amplitudes
- Degeneracies for *N* = *2, 3*
- At $\mathcal{O}(p^4)$, kinematics furnishes a $1 \oplus 2$

Crossing-singlet *A*’s

- Then contract kinematics and flavour to make S_4 singlets

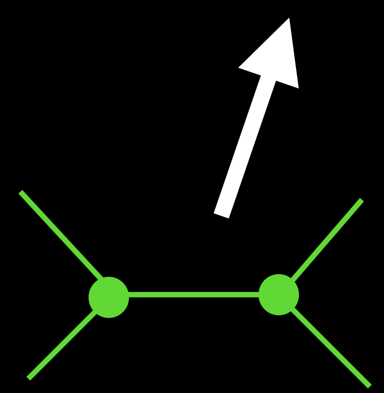
RESULT

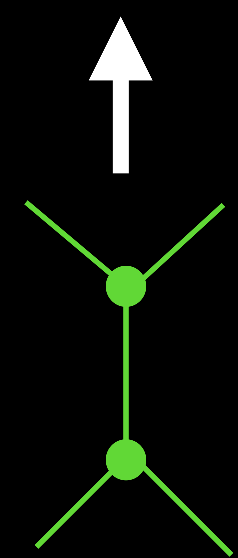
- At $\mathcal{O}(p^2)$, we get ***two*** amplitudes (one for *N*=*2*)
- At $\mathcal{O}(p^4)$, ***four*** (two for *N*=*2* and three for *N*=*3*)
- Compare with counting from χral Lagrangian…

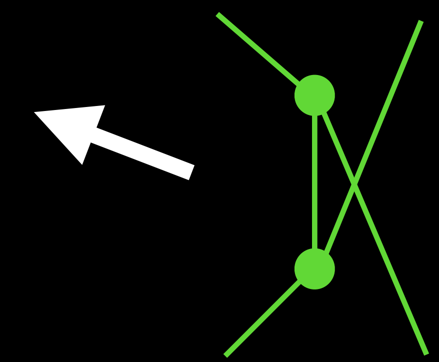
- The coset dictates a unique amplitude at leading order in derivatives
- Consider the *single trace*, at p^2 — appropriate for chiral *SU(N)*

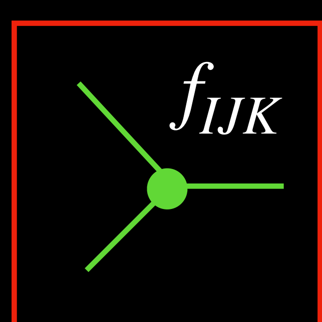
$$A_{tree} = (t-u)f_s + (u-s)f_t + (s-t)f_u$$

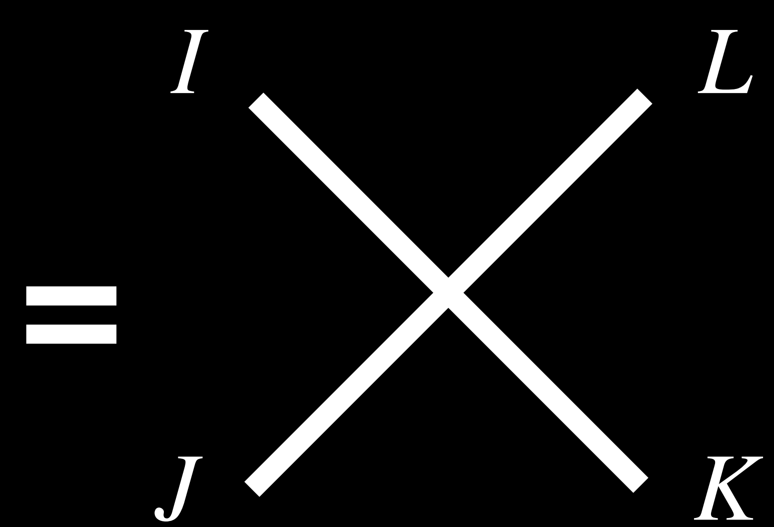
$$f_s \equiv f_{IJM} f_{KLM} \ , \ f_t \equiv f_{ILM} f_{JKM} \ , \ f_u \equiv f_{IKM} f_{LJM}$$





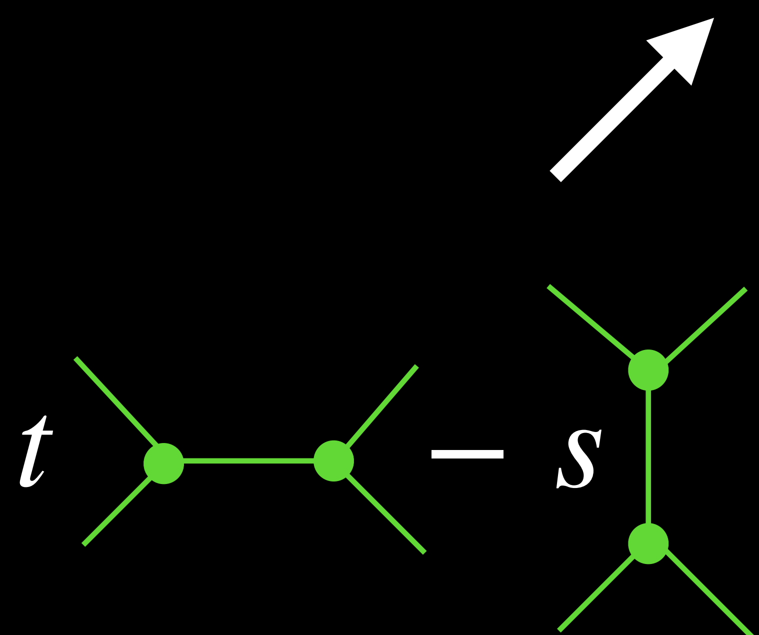


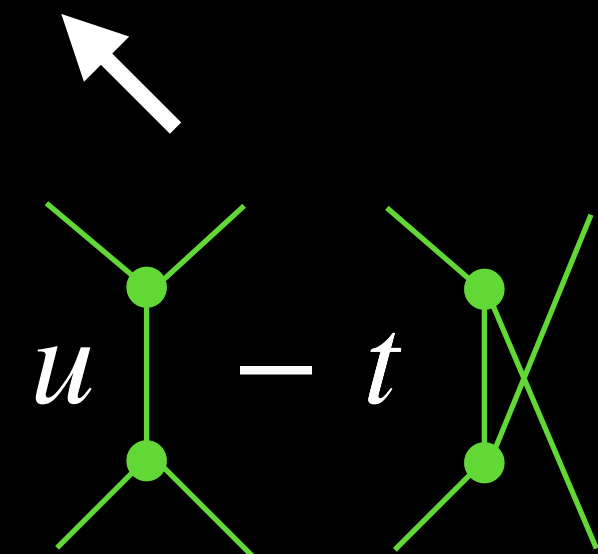




Many equivalent forms for A_{tree} – *a consequence of* $f_s + f_t + f_u = 0$

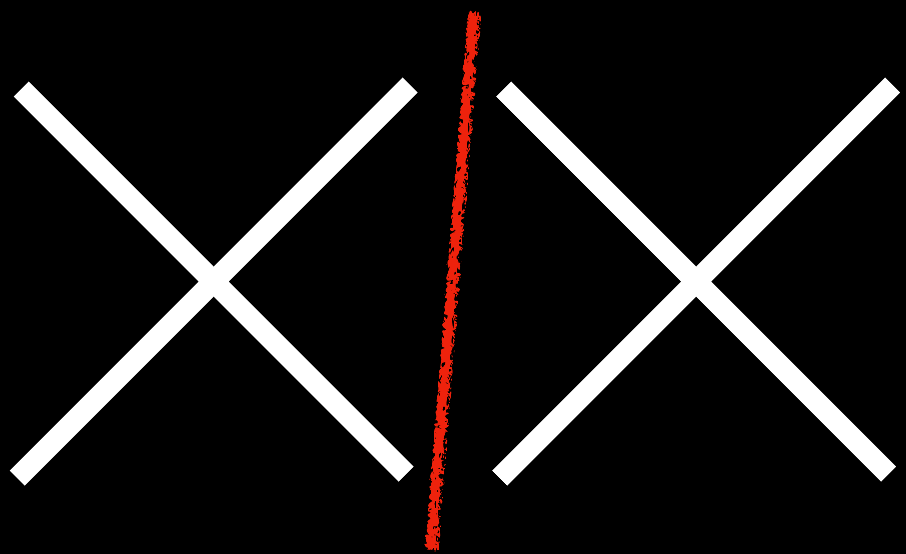
$$A_{tree} = 3(tf_s - sf_t) = 3(uf_t - tf_u)$$



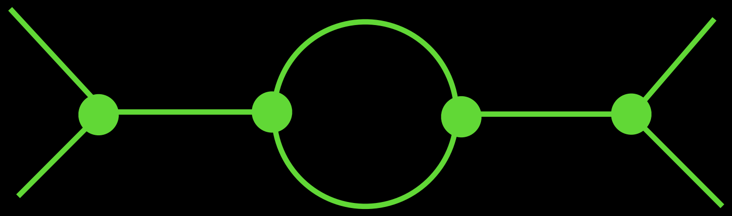


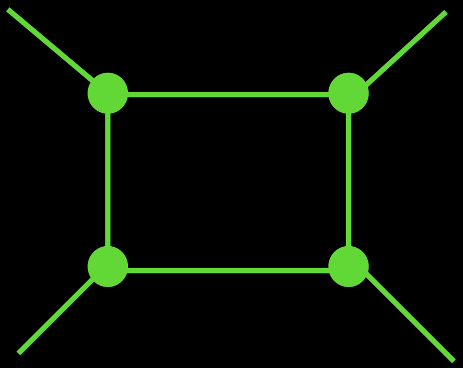
“gauge invariance” we will use to organise the computation

Result



flavour bubble



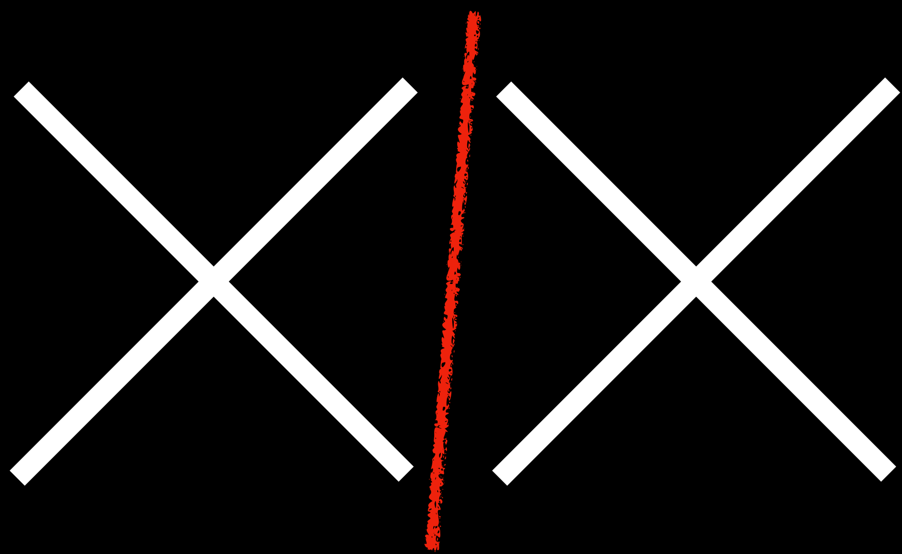


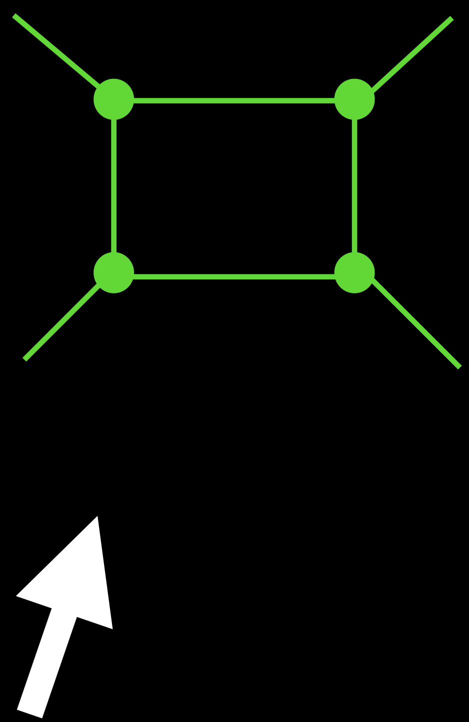
flavour box

& flavour triangle…

Possible flavour structures emerging from the cut procedure

Result





flavour box

Can choose a "gauge" with only boxes!

Result

- Same procedure for the other channels (related by crossing)

- Sum over channel guarantees crossing invariance

- ***Boxes*** are flavour-space tensors: *how many are they?*

- ***They are 3, and furnish a 1 + 2 reducible representation***

Result

- Out of the 4 dimensional space available at $\mathcal{O}(p^4)$, we exclude a 2 dimensional space (almost!)
- Perform a simple integral, to find that *only the singlet combination of boxes survives*

$$A_{div} = \# \; (s^2 + t^2 + u^2) \; (F_s + F_t + F_u)$$

$$F_s \equiv tr(F_I F_K F_J F_L)$$

$$(F_I)_{JK} = i f_{IJK}$$

Conclusions

Balance of the results

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B. What are those coefficients? $F_s + F_t + F_u$ ✓

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