$h \to b\bar{b}$ AT NLO IN SMEFT

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SMEFT

$$\mathcal{L}^{\rm SMEFT} = \mathcal{L}^{\rm SM} + \sum_{i=1}^{59} C_i(\mu) Q_i(\mu) + (\text{dim-8 and higher})$$

- Q_i are 59 dimension-6 operators (2499 parameters with flavour indices)
- $C_i \sim 1/\Lambda_{\rm NP}^2$ are Wilson coefficients. $C_i \neq 0 =$BSM (COST?)
- well defined QFT: renormalizable order by order in $1/\Lambda_{\rm NP}^2$

Motivation for $h \to b\bar{b}$ at NLO in SMEFT

1) **Phenomenology**:

- $h \to b\bar{b}$ largest Higgs branching fraction
- can measure Higgs SM Yukawa at (sub)percent level at Higgs factory
- $\Rightarrow$ NLO SMEFT calculation sets long-term baseline for analysis in EFT

2) **SMEFT development**:

- reveals many non-trivial features of SMEFT at NLO in (relatively) simple setting
- analytic results useful for benchmarking automated codes for NLO SMEFT

$h \to b\bar{b}$ AS SMEFT LABORATORY

Some things we dealt with in full NLO calculation:
[Jonathan Cullen, B.P., Darren Scott: arXiv:1904.06358]

- renormalize e, M_W, M_Z, m_b, C_i, plus external b-quark, h-boson fields (45 C_i appear at NLO, checks 100s of entries in 1-loop anom. dims. [Alonso, Jenkins, Manohar, Trott])
- gauge invariance: tadpoles and gauge fixing in SMEFT
- hybrid renormalization schemes and decoupling relations

Outline of NLO calculation

Basic outline:

- specify input parameters and renormalization scheme
- write down LO and UV counterterm amplitudes for $h \to bb$
- calculate one-loop $h \to bb$ and 2-point functions for counterterms
- calculate real emissions of photons, gluons, add together with UV-renormalized virtual corrections to get IR finite answer

In general, every piece of calculation gets dim-4 (SM) and dim-6 (SMEFT) contributions, dim-8 terms are dropped

Input parameters

We use mass basis Lagrangian parameters:

$$\alpha_s, \alpha, m_f, m_H, M_W, M_Z, V_{ij},\ C_i$$

- C_i are renormalized in the $\overline{\text{MS}}$ scheme
- M_W, M_Z renormalized in on-shell scheme
- renormalization scheme for e and $m_b \neq 0$ kept flexible
- M_H, m_t, m_τ kept non-zero but not renormalized at NLO. all other $m_f = 0$.
- we approximate $V_{ij} = \delta_{ij}$

The LO amplitude

- LO decay amplitude

$$i\mathcal{M}^{(0)}(h \to b\bar{b}) = -i\bar{u}(p_b)\left(\mathcal{M}_L^{(0)} P_L + \mathcal{M}_L^{(0)*} P_R\right) v(p_{\bar{b}})$$

- split into dim-4 and dim-6 contributions

$$\mathcal{M}_L^{(0)} = \mathcal{M}_L^{(4,0)} + \mathcal{M}_L^{(6,0)}$$

- Explicit results ($\hat{v}_T \equiv 2M_w \hat{s}_w/e$, $\hat{c}_w \equiv \cos\theta_W = M_W/M_Z$)

$$\mathcal{M}_L^{(4,0)} = \frac{m_b}{\hat{v}_T}\,,$$

$$\mathcal{M}_L^{(6,0)} = m_b \hat{v}_T \left[C_{H\Box} - \frac{C_{HD}}{4}\left(1 - \frac{\hat{c}_w^2}{\hat{s}_w^2}\right) + \frac{\hat{c}_w}{\hat{s}_w} C_{HWB} - \frac{\hat{v}_T}{m_b}\frac{C_{bH}^*}{\sqrt{2}} \right]$$

- $Q_{bH} \sim (H^\dagger H)(\bar{b}_L b_R H)$+h.c. contributes to LO Feynman diagram
- other Q_i, where $Q_i \stackrel{\text{SSB}}{\longrightarrow} v_T^2 Q_i^{(4)}$ appear from rotation to mass basis

$$Q_{H\Box} = (H^\dagger H)\Box(H^\dagger H), \quad Q_{HWB} = H^\dagger \sigma^I H W^I_{\mu\nu} B^{\mu\nu}, \quad Q_{HD} = \left(H^\dagger D_\mu H\right)^* \left(H^\dagger D_\mu H\right)$$

The counterterm amplitude

dimension-4 counterterm is

$$\delta\mathcal{M}_L^{(4)} = \frac{m_b}{\hat{v}_T}\left(\frac{\delta m_b^{(4)}}{m_b} - \frac{\delta\hat{v}_T^{(4)}}{\hat{v}_T} + \frac{1}{2}\delta Z_h^{(4)} + \frac{1}{2}\delta Z_b^{(4),L} + \frac{1}{2}\delta Z_b^{(4),R*}\right)$$

dimension-6 counterterm is

$$\begin{aligned}\delta\mathcal{M}_L^{(6)} =& \frac{m_b}{\hat{v}_T}\left(\frac{\delta m_b^{(6)}}{m_b} - \frac{\delta\hat{v}_T^{(6)}}{\hat{v}_T} + \frac{1}{2}\delta Z_h^{(6)} + \frac{1}{2}\delta Z_b^{(6),L} + \frac{1}{2}\delta Z_b^{(6),R*}\right)\\ &+ \mathcal{M}_L^{(6,0)}\left(\frac{\delta m_b^{(4)}}{m_b} + \frac{\delta\hat{v}_T^{(4)}}{\hat{v}_T} + \frac{1}{2}\delta Z_h^{(4)} + \frac{1}{2}\delta Z_b^{(4),L} + \frac{1}{2}\delta Z_b^{(4),R*}\right)\\ &- \frac{\hat{v}_T^2}{\sqrt{2}}C_{bH}^*\left(\frac{\delta\hat{v}_T^{(4)}}{\hat{v}_T} - \frac{\delta m_b^{(4)}}{m_b}\right) + m_b\hat{v}_T\left[C_{HWB} + \frac{\hat{c}_w}{2\hat{s}_w}C_{HD}\right]\delta\left(\frac{\hat{c}_w}{\hat{s}_w}\right)^{(4)}\\ &+ m_b\hat{v}_T\left(\delta C_{H\Box} - \frac{\delta C_{HD}}{4}\left(1 - \frac{\hat{c}_w^2}{\hat{s}_w^2}\right) + \frac{\hat{c}_w}{\hat{s}_w}\delta C_{HWB} - \frac{\hat{v}_T}{m_b}\frac{\delta C_{bH}^*}{\sqrt{2}}\right)\end{aligned}$$

where

$$\frac{\delta\hat{v}_T}{\hat{v}_T} \equiv \frac{\delta M_W}{M_W} + \frac{\delta\hat{s}_w}{\hat{s}_w} - \frac{\delta e}{e}$$

and

$$\frac{\delta\hat{s}_w}{\hat{s}_w} = -\frac{\hat{c}_w^2}{\hat{s}_w^2}\left(\frac{\delta M_W}{M_W} - \frac{\delta M_Z}{M_Z}\right), \quad \delta\left(\frac{\hat{c}_w}{\hat{s}_w}\right)^{(4)} = -\frac{1}{\hat{c}_w\hat{s}_w}\left(\frac{\delta\hat{s}_w^{(4)}}{\hat{s}_w}\right)$$

Calculational procedure

One-loop $h \to bb$ matrix elements and two-point functions for counterterms involve many Feynman diagrams and dim-6 operators

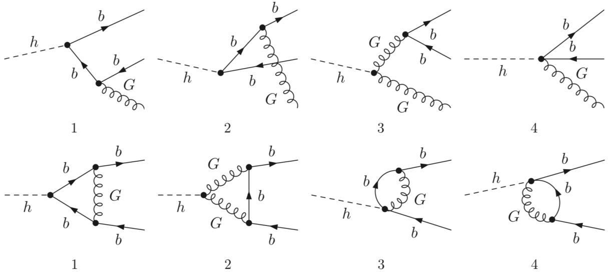
- **automation**: Feynrules (in-house model file, including gauge fixing and ghosts), Feynarts, FormCalc, Package X
- loop integrals obtained analytically in terms of Passarino-Veltmann integrals and standard functions

Decay rate also requires real emission corrections $h \to bb(g, \gamma)$

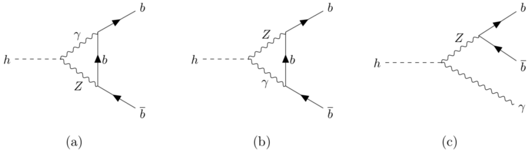
- squared matrix elements generated with automated tools
- 3-body phase space integrals done by hand

EXAMPLE: QCD-QED CORRECTIONS

- QCD corrections by far simplest to calculate [Gauld, B.P., Scott '16]
- UV-renormalized one-loop amplitudes have IR divergences canceled by real emissions



- most corrections involving photons can obtained analogously, exception is graphs involving $h\gamma Z$ vertex



Analytic structure of $h\gamma Z$ corrections

$$\Gamma_{h\gamma Z} \propto v_b \left[2(C_{HB} - C_{HW})\hat{c}_w \hat{s}_w + C_{HWB}(\hat{c}_w^2 - \hat{s}_w^2)\right] F_{h\gamma Z}\left(\frac{M_Z^2}{m_H^2}, \frac{\mu^2}{m_H^2}, \frac{m_b^2}{m_H^2}\right)$$

$$\begin{aligned}
F_{h\gamma Z}\left(z, \hat{\mu}^2, b\right) &= \frac{3}{4}\beta(8z-5) - \beta^3\left(\frac{39}{4} + \frac{z}{b}\right) - \frac{4}{3}\beta^2\pi^2\bar{z} + \frac{4}{3}\pi^2 z\bar{z} + 6\beta\Bigg(\beta^2 - \frac{2}{3}z \\
&+ \frac{(2b-\beta^2)z^2}{12b^2}\Bigg)\ln(b) + 2(\beta^2 - z)\bar{z}\ln(x_z)^2 - 4\beta_z z\bar{z}\ln(x_{\beta z}) \\
&+ \ln(x)\Bigg(-\frac{1}{8}\left(15 + 7\beta^4 + 8z(4z-7) + \beta^2(2+8z)\right) + 2(z-\beta^2)\bar{z}\ln(x_z) \\
&+ 4(\beta^2 - z)\bar{z}\ln(1 - xx_z) + 2(\beta^2 - z)\bar{z}\ln(x_{\beta z})\Bigg) \\
&+ \ln(x_z)\left(\frac{\beta\beta_z z\left(\beta^2(2b+z) - 2bz\right)}{2b^2} + 2(z-\beta^2)\bar{z}\ln(x_{\beta z})\right) \\
&+ 4\beta z\bar{z}\ln(\bar{z}) + \frac{\beta^3(\beta^2+2b)z^2\ln(z)}{2b^2} - 6\beta^3\ln\left(\hat{\mu}^2\right) \\
&+ 4(\beta^2 - z)\bar{z}\left(\mathrm{Li}_2\left(\frac{x}{x_z}\right) + \mathrm{Li}_2\left(xx_z\right)\right)
\end{aligned}$$

where

$$\beta = \sqrt{1-4b}, \quad \beta_z = \sqrt{1 - \frac{4b}{z}}, \quad x = \frac{1-\beta}{1+\beta}, \quad x_z = \frac{1-\beta_z}{1+\beta_z}, \quad x_{\beta z} = \frac{\beta - \beta_z}{\beta + \beta_z}, \quad \bar{z} = 1 - z$$

Cross-checks and features

Results involve 45 different Wilson coefficients (generally complex). Cross-checks:

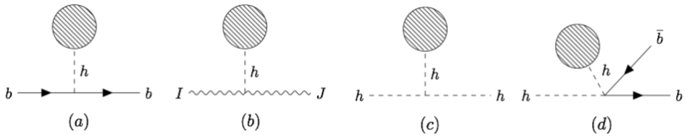
- all UV and IR poles cancel (and μ-dependence consistent with RG eqns)
- SM results reproduced from dim. 4 terms
- all results calculated in unitary and Feynman gauge with full agreement

Interesting features:

- structure of wave-function renormalization of b-quark field
- Higgs-Z mixing
- Ward identities and electric charge renormalization
- structure of tadpole contributions
- decoupling relations and hybrid renormalization schemes (mix of $\overline{\text{MS}}$ and on-shell)

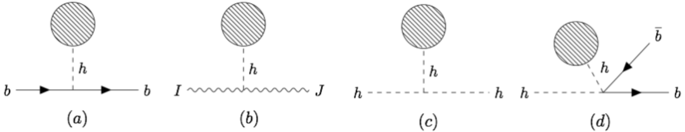
Tadpoles I

- we used FJ tadpole scheme [Fleischer, Jegerlehner '80]
- discussion in [Denner, Jenniches, Lang, Sturm '16] shows FJ scheme implemented by simply calculating all tadpole contributions to n-point functions
- tadpoles needed for $h \to bb$ in SMEFT



- we calculated tadpoles in Feynman and unitary gauge and verified:
  (1) tadpoles cancel in on-shell scheme
  (2) mass and electric charge counterterms, and matrix elements +wavefunction renormalization separately gauge invariant after adding tadpoles
- structure of tadpoles contributions in SMEFT richer than in SM

Tadpoles II



$IJ \in \{\gamma\gamma, \gamma Z, WW, ZZ\}$

- (a) contributes to δm_b in SM and SMEFT, but also to δZ_b^L in SMEFT

$$\delta Z^L_{b,\mathrm{tad.}} = -\frac{i\sqrt{2}\hat{v}_T^2}{m_H^2 m_b}\mathrm{Im}(C_{bH})T^{(4)}$$

- (b) contributes to $\delta M_W, \delta M_Z$ in SM and SMEFT, also to δe in SMEFT ($IJ = \gamma\gamma$)

$$\frac{\delta e^{\mathrm{cl.4},(6)}}{e} = \frac{1}{16\pi^2}\left[c_{h\gamma\gamma}A_0(m_H^2) + 4\hat{c}_w\hat{s}_w C_{HWB}\left(4M_W^2 - 3A_0(M_W^2)\right)\right] - 2c_{h\gamma\gamma}\frac{\hat{v}_T}{m_H^2}T^{(4)}_{\mathrm{un.}}$$

- (c) contributes to δZ_h in SMEFT (through C_{HD} and $C_{H\Box}$), but not in SM
- (d) contributes to $h \to bb$ matrix element in SMEFT, but not in SM

Enhanced NLO corrections I: QCD corrections

- QCD/QED corrections generate $\ln m_b/m_H$ terms when $\mu = m_H$:

$$\frac{\Gamma^{(1)}_{g,\gamma}}{\Gamma^{(4,0)}} \approx \ln^2\left(\frac{m_b^2}{m_H^2}\right)\frac{\hat{v}_T^2}{\pi}\left(C_F\alpha_s C_{HG} + Q_b^2\alpha c_{h\gamma\gamma}\right) + c_{m_b}\ln\left(\frac{m_b^2}{m_H^2}\right)\frac{3}{2}\left(\frac{C_F\alpha_s + Q_b^2\alpha}{\pi}\right)\left[1 + 2\hat{v}_T^2\left(C_{H\Box} - \frac{C_{HD}}{4}\left(1 - \frac{\hat{c}_w^2}{\hat{s}_w^2}\right) + \frac{\hat{c}_w}{\hat{s}_w}C_{HWB} - \frac{\hat{v}_T}{m_b}\frac{C_{bH}}{2\sqrt{2}}\right)\right]$$

- double logs of IR origin remain and are largest NLO correction
  - $Q_{HG} = H^\dagger H G^A_{\mu\nu}G^{A\mu\nu}$ gives hgg vertex strongly constrained from Higgs production
- $c_{m_b} = 1$ in on-shell scheme, $c_{m_b} = 0$ ($\overline{\text{MS}}$ scheme) for m_b.
- $\Rightarrow$ **QCD/QED prefers $\overline{\text{MS}}$ scheme for m_b** (running mass resums single UV logs)

Enhanced NLO corrections II: tadpoles

- QCD and QED corrections prefer $\overline{\text{MS}}$ scheme for m_b and e
- however, in $\overline{\text{MS}}$ scheme tadpoles contribute m_t^4 enhanced corrections. Example: SM in $m_t \to \infty$ limit

$$\overline{\text{MS}}\text{ scheme:} \qquad \frac{\overline{\Gamma}_t^{(4,1)}}{\Gamma^{(4,0)}} \approx -\frac{N_c}{2\pi^2}\frac{m_t^4}{\hat{v}_T^2 m_H^2} \approx -15\%$$

$$\text{on-shell scheme:} \qquad \frac{[\Gamma_t]^{\mathrm{O.S.}(4,1)}}{\Gamma^{(4,0)}} = \frac{m_t^2}{16\pi^2\hat{v}_T^2}\left(-6 + N_c\frac{7-10\hat{c}_w^2}{3\hat{s}_w^2}\right) \approx -3\%$$

- similar behaviour in SMEFT contributions to decay rate
- $\Rightarrow$ **EW corrections prefer on-shell scheme for m_b and e large corrections from top loops (and heavy EW bosons)**

Combining EW and QCD corrections is a non-trivial problem. Would like to calculate QCD in $\overline{\text{MS}}$ scheme, but EW in on-shell scheme...

Decoupling relations I

- decoupling relations connect $\overline{\text{MS}}$ parameters in SM, with those in low-energy theory where top and heavy bosons integrated out:

$$\overline{m}_b(\mu) = \zeta_b(\mu, m_t, m_H, M_W, M_Z)\overline{m}_b^{(\ell)}(\mu)$$

- decoupling constants ζ_i contain contributions from heavy particles.
- ζ_b calculated by relating on-shell with $\overline{\text{MS}}$ mass in SM and low-energy theory:

$$m_b = z_b^{-1}(\mu, m_b, m_t, m_H, M_W, M_Z)\overline{m}_b(\mu) = \left[z_b^{(\ell)}(\mu, m_b)\right]^{-1}\overline{m}_b^{(\ell)}(\mu)$$

$$\Rightarrow \zeta_b(\mu, m_t, m_H, M_W, M_Z) = \left.\frac{z_b(\mu, m_b, m_t, m_H, M_W, M_Z)}{z_b^{(\ell)}(\mu, m_b)}\right|_{m_b\to 0}$$

- works analogously for electric charge. The connection between low energy parameters and experiment are:

from B-physics: $\overline{m}_b^{(\ell)}(\overline{m}_b^{(\ell)}) \approx 4.2\ \text{GeV}$

from LEP: $\overline{\alpha}^{(\ell)}(M_Z) = \alpha(M_Z)\left(1 + \frac{100\alpha}{27\pi}\right), \quad \alpha(M_Z) \approx 1/129,\ \alpha \approx 1/137$

Decoupling relations II

- dim.4 contributions to ζ_i well known, we calculated dim.6 corrs. Example:

$$\zeta_e^{(4,1)} = \frac{\alpha}{\pi}\left[-\frac{1}{12} - \frac{7}{8}\ln\left(\frac{\mu^2}{M_W^2}\right) + \frac{N_c}{6}Q_t^2\ln\left(\frac{\mu^2}{m_t^2}\right)\right]$$

$$\zeta_e^{(6,1)} = \frac{\alpha}{\pi}\left[\sqrt{2}\hat{v}_T m_t N_c Q_t\left(\hat{c}_w\frac{\mathrm{Re}(C_{tB})}{e} + \hat{s}_w\frac{\mathrm{Re}(C_{tW})}{e}\right)\ln\left(\frac{\mu^2}{m_t^2}\right) + 9\frac{C_W}{e}\hat{s}_w M_W^2\ln\left(\frac{\mu^2}{M_W^2}\right)\right] + \frac{\delta e^{\mathrm{cl.4(6)}}}{e}\Bigg|_{\mathrm{fin.},\, m_b\to 0}$$

- relation between NLO decay rate using low-energy parameters vs. SM params:

$$\overline{\Gamma}_\ell^{(4,1)} = \overline{\Gamma}^{(4,1)} + 2\overline{\Gamma}^{(4,0)}\left(\zeta_b^{(4,1)} + \zeta_e^{(4,1)}\right),$$

$$\overline{\Gamma}_\ell^{(6,1)} = \overline{\Gamma}^{(6,1)} + 2\overline{\Gamma}^{(4,0)}\left(\zeta_b^{(6,1)} + \zeta_e^{(6,1)}\right) + 2\overline{\Gamma}^{(6,0)}\zeta_b^{(4,1)} + \sqrt{2}C_{bH}\frac{(\overline{v}^{(\ell)})^3}{\overline{m}_b^{(\ell)}}\overline{\Gamma}^{(4,0)}\left(\zeta_b^{(4,1)} + \zeta_e^{(4,1)}\right)$$

- illustrative results: QCD-QED corrections and EW corrections in $m_t \to \infty$ limit:

$$\overline{\Gamma}_{\ell,g,\gamma} = \overline{\Gamma}_{g,\gamma}\,, \qquad \overline{\Gamma}_{\ell,t} = [\Gamma_t]^{\mathrm{O.S.}}$$

  - interpretation: QCD-QED corrections in $\overline{\mathrm{MS}}$ scheme (UV logs resummed), heavy-particle EW corrections in on-shell (tadpoles cancel)

Numerical results

- independent renormalization scales for Wilson coefficents (μ_C) and $\overline{\text{MS}}$ parameters (μ_R)
- varying $\mu_C, \mu_R = m_H$ by factors of 2 and adding in quadrature:

$$\Delta^{\rm LO}(m_H, m_H) = (1 \pm 0.08) + \frac{(\bar{v}^{(\ell)})^2}{\Lambda_{\rm NP}^2} \Big\{ (3.74 \pm 0.36)\tilde{C}_{HWB} + (2.00 \pm 0.21)\tilde{C}_{H\Box} - (1.41 \pm 0.07)\frac{\bar{v}^{(\ell)}}{\overline{m}_b^{(\ell)}}\tilde{C}_{bH} + (1.24 \pm 0.14)\tilde{C}_{HD} \pm 0.35\tilde{C}_{HG} \pm 0.19\tilde{C}_{Hq}^{(1)} \pm 0.18\tilde{C}_{Ht} \pm 0.11\tilde{C}_{Hq}^{(3)} + \dots \Big\}$$

$$\Delta^{\rm NLO}(m_H, m_H) = 1.13^{+0.01}_{-0.04} + \frac{(\bar{v}^{(\ell)})^2}{\Lambda_{\rm NP}^2} \Big\{ \left(4.16^{+0.05}_{-0.14}\right) \tilde{C}_{HWB} + \left(2.40^{+0.04}_{-0.09}\right) \tilde{C}_{H\Box} + \left(-1.73^{+0.04}_{-0.03}\right) \frac{\bar{v}^{(\ell)}}{\overline{m}_b^{(\ell)}}\tilde{C}_{bH} + \left(1.33^{+0.01}_{-0.04}\right) \tilde{C}_{HD} + \left(2.75^{+0.49}_{-0.48}\right) \tilde{C}_{HG} + \left(-0.12^{+0.04}_{-0.01}\right) \tilde{C}_{Hq}^{(3)} + \left(-0.08^{+0.05}_{-0.01}\right) \tilde{C}_{Ht} + \left(0.06^{+0.00}_{-0.05}\right) \tilde{C}_{Hq}^{(1)} + \left(0.00^{+0.07}_{-0.04}\right) \frac{\tilde{C}_{tG}}{g_s} + \dots \Big\}$$

- in general, scale uncertainties in LO result overlap with NLO one, and scale uncertainties decrease between LO and NLO
- exception is C_{HG}, which gives large corrections unrelated to RG eqns.
- scale variation of C_{HG} gives rise to C_{tG}, which is a significant contribution although it is two-loop order

Corrections to LO results

	SM	$\tilde{C}_{HWB}$	$\tilde{C}_{H\Box}$	$\tilde{C}_{bH}$	$\tilde{C}_{HD}$
NLO QCD-QED	18.2%	17.9%	18.2%	18.2%	18.2%
NLO large-m_t	-3.1%	-4.6%	3.2%	3.5%	-9.0%
NLO remainder	-2.2%	-1.9%	-1.2 %	0.6%	-2.0%
NLO correction	12.9%	11.3%	20.2%	22.3%	7.1%

Table: Size of NLO corrections to different terms in LO decay rate, split into QCD-QED, large m_t, and remaining components.

- applying SM K-factor to dim.6 operators bad approximation for EW corrections

Summary

- calculated full NLO corrections to $h \to bb$ in SMEFT
- forms basis for serious phenomenological analysis of $h \to bb$ in SMEFT
- calculation involved many non-trivial conceptual issues in SMEFT, and can be used as a template for future calculations combining EW and QCD corrections

Backup Slides

INPUT PARAMETERS II

- many non-trivial SMEFT effects are involved in rotation to mass basis
- example 1: Higgs vev $\langle H^\dagger H \rangle \equiv v_T^2/2$

$$\frac{1}{v_T} = \frac{1}{\hat{v}_T}\left(1 + \hat{v}_T^2 \frac{\hat{c}_w}{\hat{s}_w}\left[C_{HWB} + \frac{\hat{c}_w}{4\hat{s}_w} C_{HD}\right]\right); \quad \hat{v}_T = 2M_W \hat{s}_W/e; \quad \hat{c}_w = \frac{M_W}{M_Z}$$

- example 2: covariant derivative in mass basis

$$\begin{aligned}
D_\mu = \partial_\mu - i\frac{e}{\hat{s}_w}&\left[1 + \frac{\hat{c}_w^2 \hat{v}_T^2}{4\hat{s}_w^2} C_{HD} + \frac{\hat{c}_w \hat{v}_T^2}{\hat{s}_w} C_{HWB}\right]\left(\mathcal{W}_\mu^+ \tau^+ + \mathcal{W}_\mu^- \tau^-\right) \\
&- i\left[\frac{e}{\hat{c}_w \hat{s}_w}\left(1 + \frac{(2\hat{c}_w^2 - 1)\hat{v}_T^2}{4\hat{s}_w^2} C_{HD} + \frac{\hat{c}_w \hat{v}_T^2}{\hat{s}_w} C_{HWB}\right)\left(\tau^3 - \hat{s}_w^2 Q\right)\right. \\
&\left. + \, e\left(\frac{\hat{c}_w \hat{v}_T^2}{2\hat{s}_w} C_{HD} + \hat{v}_T^2 C_{HWB}\right) Q\right] \mathcal{Z}_\mu - ieQ\mathcal{A}_\mu \,,
\end{aligned}$$

- gauge fixing is also much more involved than SM

Fermion w.f. renormalization in SMEFT

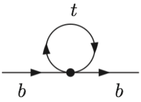
- Decompose two-point function of fermion f as

$$\Gamma^f(p) = i(\not{p} - m_f) + i\left[\not{p}\left(P_L\Sigma_f^L(p^2) + P_R\Sigma_f^R(p^2)\right) + m_f\left(\Sigma_f^S(p^2)P_L + \Sigma_f^{S*}(p^2)P_R\right)\right]$$

- In on-shell renormalization scheme

$$\begin{aligned}\delta Z_f^L &= -\widetilde{\mathrm{Re}}\,\Sigma_f^L(m_f^2) + \Sigma_f^S(m_f^2) - \Sigma_f^{S*}(m_f^2)\\ &\quad - m_f^2\frac{\partial}{\partial p^2}\widetilde{\mathrm{Re}}\left[\Sigma_f^L(p^2) + \Sigma_f^R(p^2) + \Sigma_f^S(p^2) + \Sigma_f^{S*}(p^2)\right]\Bigg|_{p^2=m_f^2},\\ \delta Z_f^R &= -\widetilde{\mathrm{Re}}\,\Sigma^{f,R}(m_f^2) - m_f^2\frac{\partial}{\partial p^2}\widetilde{\mathrm{Re}}\left[\Sigma_f^L(p^2) + \Sigma_f^R(p^2) + \Sigma_f^S(p^2) + \Sigma_f^{S*}(p^2)\right]\Bigg|_{p^2=m_f^2}\end{aligned}$$

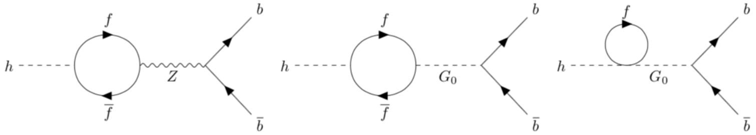
- $\Sigma_f^S(m_f^2) - \Sigma_f^{S*}(m_f^2)$ vanishes in SM, but is proportional $\mathrm{Im}(C_i)$ in SMEFT.
- appears in many places in renormalization of amplitude – example:



$$Z_b^L = \frac{1}{\epsilon}\left[-\frac{m_t^3}{m_b}\left((2N_c+1)\left(C_{qtqb}^{(1)} - C_{qtqb}^{(1)*}\right) + c_{F,3}\left(C_{qtqb}^{(8)} - C_{qtqb}^{(8)*}\right)\right)\right] + \text{finite}$$

HIGGS-Z (GOLDSTONE) MIXING

- unlike SM, in SMEFT Higgs can mix into Z and neutral Golstone boson G_0



- h-G_0 mixing is that between real and imaginary parts of doublet:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} -\sqrt{2} i \phi^+(x) \\ \left[1 + C_{H,\text{kin.}}\right] h(x) + i \left[1 - \frac{\hat{v}_T^2}{4} C_{HD}\right] \phi^0(x) + v_T \end{pmatrix}$$

- mixing is therefore proportional to imaginary parts of Wilson coefficients and reads

$$\eta_5 = \frac{\sqrt{2}}{\hat{v}_T} \mathrm{Im}\left[N_c m_b C_{bH} - N_c m_t C_{tH} + m_\tau C_{\tau H} + \ldots\right]$$

- this term exactly cancels one appearing in renormalization of Q_{bH} (i.e. that in $\dot{C}_{bH}$ calculated in[Jenkins, Manohar, Trott '13])

Electric charge renormalization

SM: $ff\gamma$ vertex related to two-point fcns through Ward identities:

- result

$$\frac{\delta e^{(4)}}{e} = \frac{1}{2}\frac{\partial \Sigma_T^{AA(4)}(k^2)}{\partial k^2}\bigg|_{k^2=0} - \frac{\left(v_f^{(4)} - a_f^{(4)}\right)}{Q_f}\frac{\Sigma_T^{AZ(4)}(0)}{M_Z^2}$$

- $v_f^{(4)} - a_f^{(4)} = -Q_f \hat{s}_w/\hat{c}_w \Rightarrow \delta e^{(4)}$ independent of fermion charge

SMEFT: we find by renormalizing $ff\gamma$ vertex directly (not using Ward identities)

- result

$$\frac{\delta e^{(6)}}{e} = \frac{1}{2}\frac{\partial \Sigma_T^{AA(6)}(k^2)}{\partial k^2}\bigg|_{k^2=0} + \frac{1}{M_Z^2}\left(\frac{\hat{c}_w}{\hat{s}_w}\Sigma_T^{AZ(6)}(0) - \frac{\hat{v}_T^2}{4\hat{c}_w\hat{s}_w}C_{HD}\Sigma_T^{AZ(4)}(0)\right)$$

- $v_f^{(6)} - a_f^{(6)} = 2C_{Hf}\hat{v}_T^2/4\hat{c}_w\hat{s}_w \Rightarrow$ Naive application of SM Ward identity is wrong, due to operators of form $Q_{Hf} = (H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{f}\gamma^\mu f)$