CP Violation in Kaons

precise again

Emmanuel Stamou
emmanuel.stamou@epfl.ch

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

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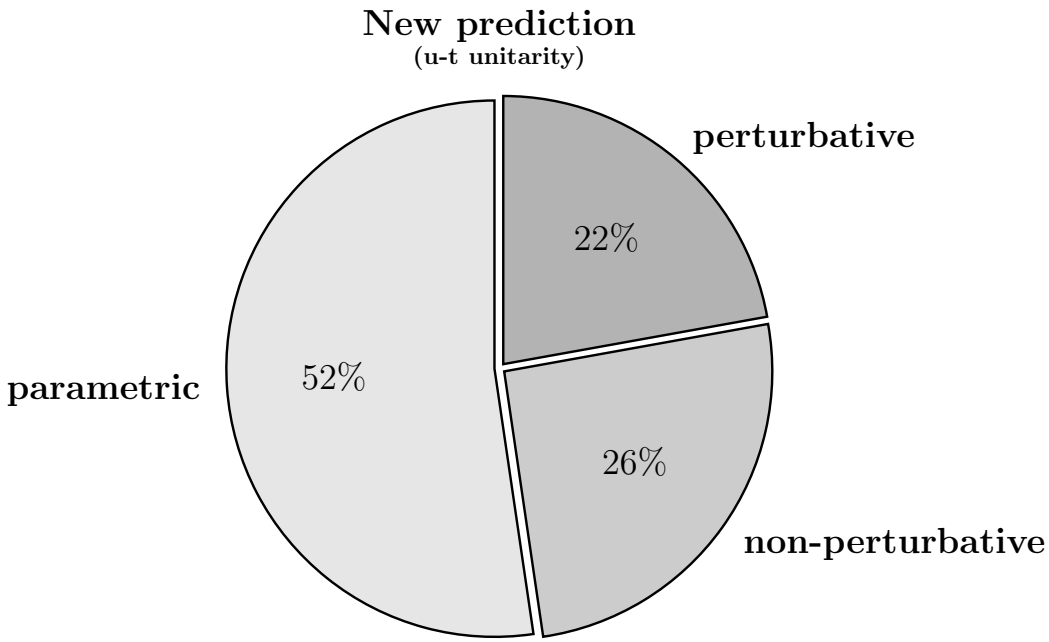
arXiv:1911.06822: in collaboration with J. Brod and M. Gorbahn

Summary and outline

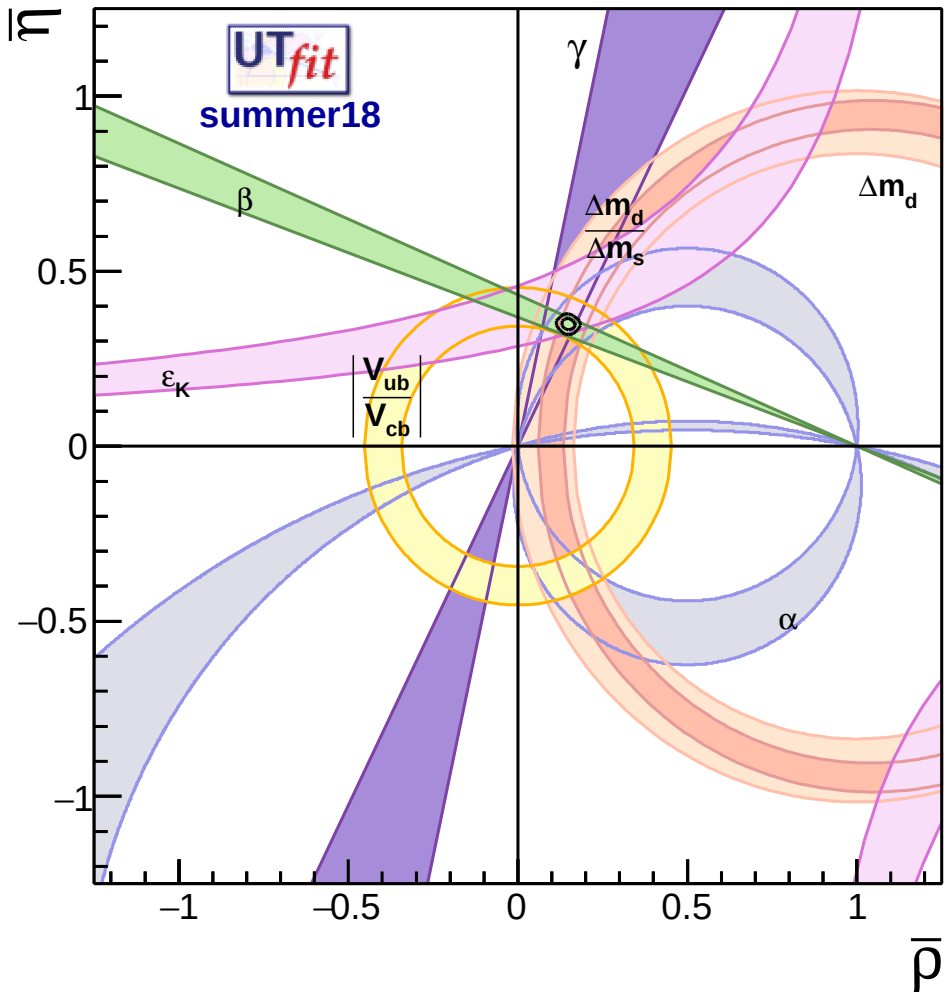
- The parameter ϵ_K measures CP violation in the Kaon sector
- ϵ_K is a powerful constraint for BSM models
- Rearrangement of perturbative series leads to a significant improvement of the perturbative uncertainty in ϵ_K



- perturbative charm-quark uncertainties at the permil level (total relative perturbative $9\% \to 3\%$)
- good convergence ➜opens path for further improvements

Flavour and CP Violation

- CKM Unitarity (Probing the self-consistency of the SM)



[UTfit collaboration 2018]

- ϵ_K an important ingredient of the SM fit

The NP flavour Problem

- Generic extensions of the SM come with multiple **new sources** of flavour and CP violation (supersymmetry, extra-dimensions, composite-Higgs,...)
- If NP-scale close to the EW scale ($\Lambda_{\text{NP}} \sim$ TeV) tension with flavour data (**NP Flavour Problem**)

➜ Need non-trivial flavour structure for TeV-scale NP (MFV, flavour-symmetries, partial-compositeness,...)

ϵ_K has been providing among the most stringent constraints of BSM models
(e.g. it "excludes" composite-Higgs models with anarchic Yukawas)

$$\mathcal{L}_{\text{NP}} = \frac{C}{\Lambda^2_{\text{NP}}}(\bar{s}\Gamma d)(\bar{s}\Gamma d) + \text{h.c.}$$

Sensitivity to Λ_{NP} ➜ precise SM prediction

K-$\overline{K}$ mixing

- Neutral Kaon mixing described by

$$i\frac{d}{dt}\begin{pmatrix}|K^0\rangle\\|\overline{K}^0\rangle\end{pmatrix} = \hat{H}\begin{pmatrix}|K^0\rangle\\|\overline{K}^0\rangle\end{pmatrix} = \left(\hat{M} - i\frac{\hat{\Gamma}}{2}\right)\begin{pmatrix}|K^0\rangle\\|\overline{K}^0\rangle\end{pmatrix}$$

- The Hamiltonian can be split into a hermitian $\hat{M}$ and and anti-hermitian $i\hat{\Gamma}$ contribution
- The $\Delta S = 2$ (FCNC) transition is mediated by the off-diagonal entries
- The Hamiltonian is diagonalised by

$$|K_S\rangle = p|K^0\rangle + q|\overline{K}^0\rangle$$
$$|K_L\rangle = p|K^0\rangle - q|\overline{K}^0\rangle$$

ϵ_K: indirect CP violation

- If CP is conserved $K_L \not\to \pi\pi$, but the mixing allows it:

$$\epsilon_K \equiv \frac{\langle (\pi\pi)_{I=0} | K_L \rangle}{\langle (\pi\pi)_{I=0} | K_S \rangle} = \frac{1-\lambda_0}{1+\lambda_0}$$

- With $\lambda_0 = \frac{q}{p} \frac{\langle (\pi\pi)_{I=0} | \overline{K}^0 \rangle}{\langle (\pi\pi)_{I=0} | K^0 \rangle}$
- It can be shown that $\lambda_0 = 1 - i\phi \frac{\Delta M_K}{\Delta M_K + i\Delta\Gamma_K/2}$
- with the weak phase $\phi = arg(-M_{12}/\Gamma_{12})$

[Nierste in Anikeev et al; hep-ph/0201071]

ϵ_K: indirect CP violation

$$\epsilon_K = e^{i\phi_\epsilon} \sin\phi_\epsilon \frac{1}{2} arg\left(-\frac{M_{12}}{\Gamma_{12}}\right)$$

$$\phi_\epsilon \equiv arctan \frac{\Delta M_K}{\Delta\Gamma_K/2}$$

Almost something we can compute with

- M_{12} : dispersive part of the $\Delta S = 2$ amplitude
- Γ_{12} : absorptive part of the $\Delta S = 2$ amplitude

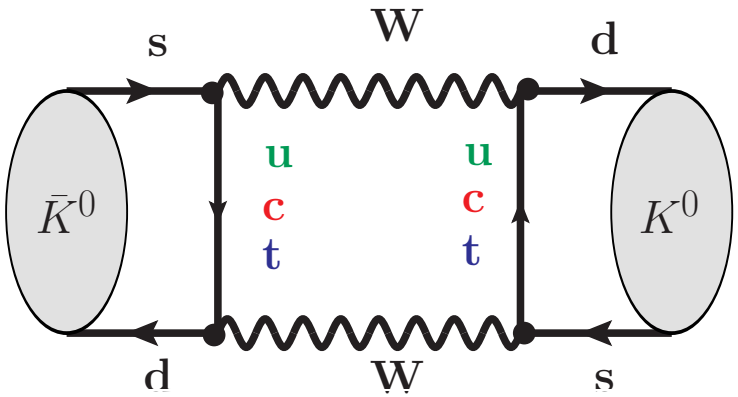
In **PDG-phase convention** M_{12} and Γ_{12} nearly real, from which it can be shown that

$$\epsilon_K = e^{i\phi_\epsilon} \sin\phi_\epsilon \left(\frac{Im(M_{12})}{\Delta M_K} + \xi\right)$$

$$\xi = -\frac{Im(\Gamma_{12})}{2Re(\Gamma_{12})}$$ (non-perturbative / long-distance)

(Estimates of ξ from: ϵ'/ϵ [Nierste 02, Buras et al 08], ChPT [Buras et al 10], lattice [Blum et al 15, Bai et al 15])

$Im(M_{12})$



- Use separation of scale (EFT) to **factorize** computation of $Im(M_{12})$

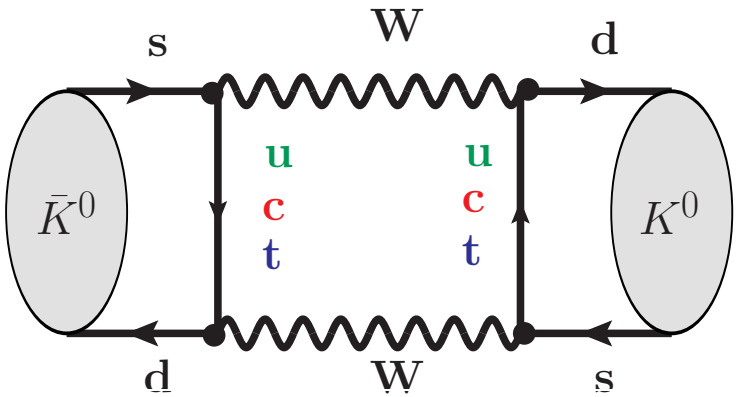
$$\mu_{\text{had}} < \mu_c \ll \mu_W \sim m_t, M_W, M_Z$$

- The ME of $Q^{\Delta S=2} = (\bar{s}_L \gamma^\mu d_L)(\bar{s}_L \gamma_\mu d_L)$ factorises

$$\langle \mathcal{H}_{\text{eff}} \rangle = \underbrace{\langle Q^{\Delta S=2} \rangle(\mu_{\text{had}}) U(\mu_{\text{had}}}_{\text{hadronic matrix element}}, \underbrace{\mu_c) U(\mu_c, \mu_W) C(\mu_W)}_{\text{pertubative evolution of } C}$$

- $\hat{B}_K = \frac{3}{2} b(\mu_{\text{had}}) \frac{\langle \overline{K}^0 | Q^{\Delta S=2} | K^0 \rangle}{f_K^2 M_K^2}$ compute on the lattice [Aoki et al 07]

The CKM power-counting for K-$\overline{K}$ mixing



$$\lambda_i \equiv V_{id}V_{is}^* \text{ and } \lambda \sim 0.23$$

	Im[...]	**Re[...]**	**order**
$\boldsymbol{\lambda_t^2}$	λ^{10}	λ^{10}	m_t^2/M_W^2
$\boldsymbol{\lambda_c\lambda_t}$	λ^6	λ^6	$m_c^2/M_W^2 \log(m_c/m_t)$
$\boldsymbol{\lambda_c^2}$	λ^6	λ^2	m_c^2/M_W^2
$\boldsymbol{\lambda_u\lambda_t}$	λ^6	λ^6	$m_c^2/M_W^2 \log(m_c/m_t)$
$\boldsymbol{\lambda_u^2}$	$\mathbf{0}$	λ^2	m_c^2/M_W^2

Eliminate either λ_u or λ_c via two different unitarity conditions

$\lambda_u = -\lambda_t - \lambda_c$ "ct"-unitarity (traditional computation, 3 imaginary parts)
$\lambda_c = -\lambda_t - \lambda_u$ "ut"-unitarity (this computation, 2 imaginary parts)

CKM unitarity conditions in the $\Delta S = 2$ Hamiltonian

- One unitarity condition **must** be used to cancel divergences (GIM)
- At LO the results are trivially related
- **Beyond LO** different conditions correspond to a rearrangement/reshuffling of the perturbation series
- A priori it is **not clear** that results can be trivially translated to one another
- The $\Delta S = 2$ Hamiltonian depends on the **CKM-phase convention**

Questions

➜ Is there a phase-independent notion of splitting the amplitude into "independent" pieces?

➜ Is there a notion of a "better" unitarity condition?

The CKM phase-(in)dependent $\Delta S = 2$ Hamiltonian

- The $\Delta S = 2$ Hamiltonian depends on the **CKM-phase convention**
- However the convention-independence of $\epsilon_K \propto arg(M_{12}/\Gamma_{12})$ can be made explicit
- **Trick:** factor out $1/(\lambda_u^*)^2$ in $\Delta S = 2$ and $1/\lambda_u^*$ in $\Delta S = 1$ Hamiltonian

$$\mathcal{H}^{\Delta S=2} = \frac{G_F^2 M_W^2}{4\pi^2} \frac{1}{(\lambda_u^*)^2} Q_{S2} \Big\{ f_1 \, C_1 + i J \, [f_2 \, C_2 + f_3 \, C_3] \Big\} + \text{h.c.} + \dots$$

[arXiv:1911.06822]

- C_1, C_2, C_3 are real Wilson coefficients
- J, f_1, f_2, f_3 are real CKM structures
- $J = \text{Im}(V_{us} V_{cb} V_{ub}^* V_{cs}^*), f_1, f_2, f_3$ are phase-convention invariant
- Real part of $f_1 = |\lambda_u|^4$ is unique, splitting of f_2 and f_3 not

The traditional Hamiltonian (ct-unitarity)

$$f_2 = Re(\lambda_t \lambda_u^*) \text{ and } f_3 = Re(\lambda_c \lambda_u^*)$$

$$\mathcal{H}^{\Delta S=2} = \frac{G_F^2 M_W^2}{4\pi^2} Q_{S2} \left\{ \lambda_t^2 C^{tt} + \lambda_c^2 C^{cc} + \lambda_c \lambda_t C^{ct} \right\} + \text{h.c.} + \dots$$

Match to phase-convention "independent" Hamiltonian

- $C^{cc} = \mathcal{C}_1 \quad C^{ct} = 2\mathcal{C}_1 + \mathcal{C}_3 \quad C^{tt} = 2\mathcal{C}_1 + \mathcal{C}_2 + \mathcal{C}_3$
- We find that $\mathcal{C}_1$, which is relevant for ΔM_K, enters all pieces

➜ **This is <u>bad</u> news**

- ΔM_K is long-distance dominated, bad convergence behaviour of $\mathcal{C}_1$

[Brod, Gorbahn 11]

➜ We artificially included $\mathcal{C}_1$ into the prediction of ϵ_K!

We can do better.

The new Hamiltonian (ut-unitarity)

$$f_2 = 2Re(\lambda_t \lambda_u^*) \text{ and } f_3 = |\lambda_u|^2$$

This ut-unitarity was proposed for a lattice computation [Christ et al 12]

$$\mathcal{H}^{\Delta S=2} = \frac{G_F^2 M_W^2}{4\pi^2} Q_{S2} \left\{ \lambda_t^2 C^{tt} + \lambda_u^2 C^{uu} + \lambda_u \lambda_t C^{ut} \right\} + \text{h.c.} + \dots$$

Match to phase-convention "independent" Hamiltonian

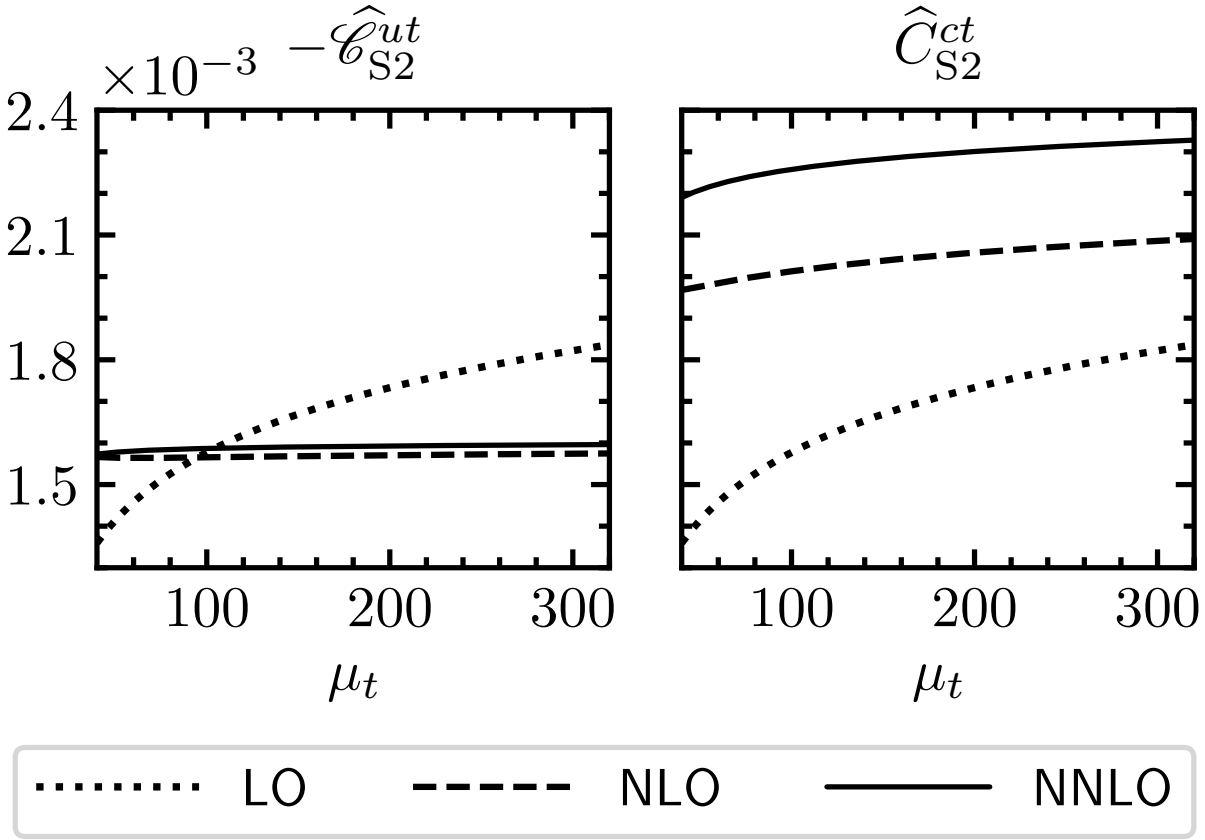
- $C^{uu} = \mathcal{C}_1 \quad C^{tt} = \mathcal{C}_2 \quad C^{ut} = \mathcal{C}_3$
- We find that $Re(M_{12})$ and $Im(M_{12})$ are now disentangled

➜ $Im(M_{12})$ without ΔM_K pollution, **this is good news!**

- It can be shown that all Wilson coefficients and anomalous dimensions in this setup can be extracted from the "traditional" results [arXiv:1911.06822]

Comparison of ct- and ut-unitarity

- Estimate the residual theories uncertainties from scale variation (μ_t, μ_c)



- Miserable convergence behaviour of ct-unitarity (right panel)
- Excellent convergence behaviour of ut-unitarity (left panel)

$$\eta_{ut} = 0.402 \pm 0.005 \quad \textbf{@ NNLO} \quad \text{[arXiv:1911.06822]}$$

$$\eta_{tt} = 0.55 \pm 0.02 \quad \textbf{@ NLO} \quad \text{[Buras et al 90 with our variation]}$$

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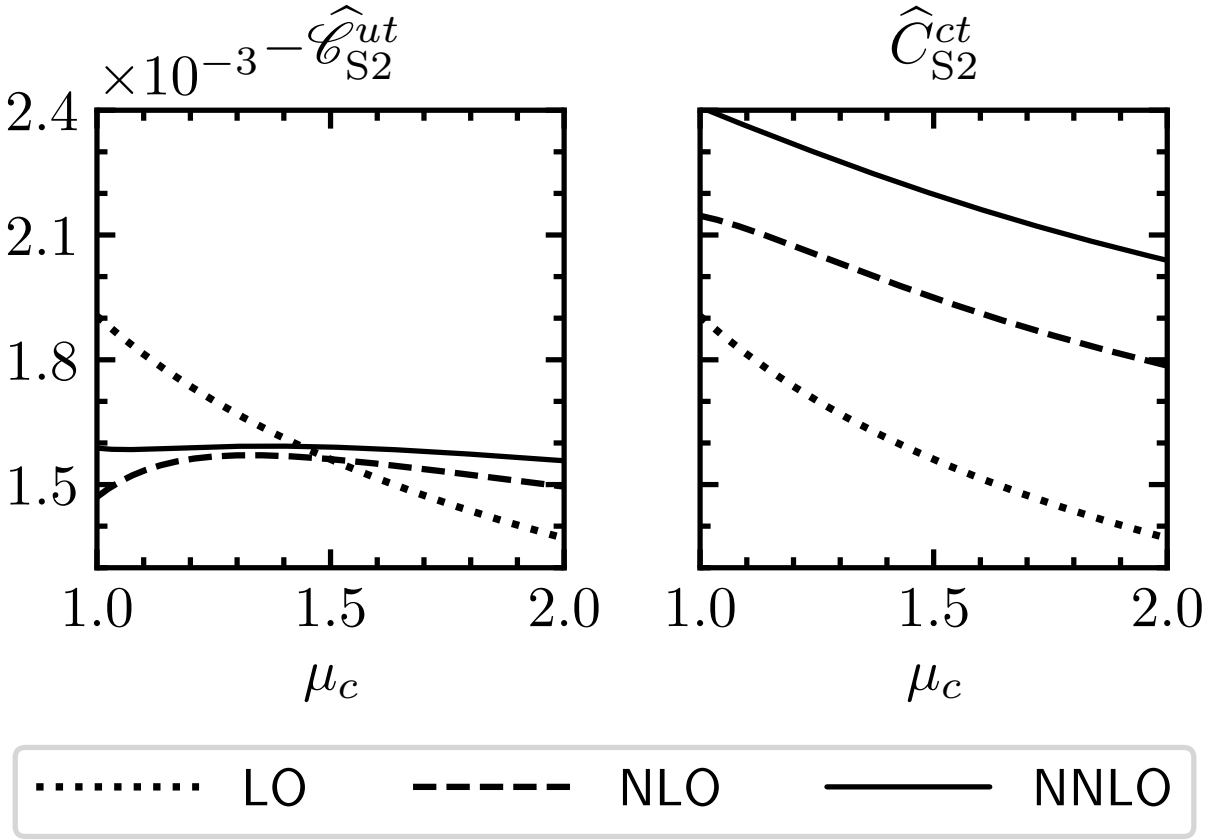

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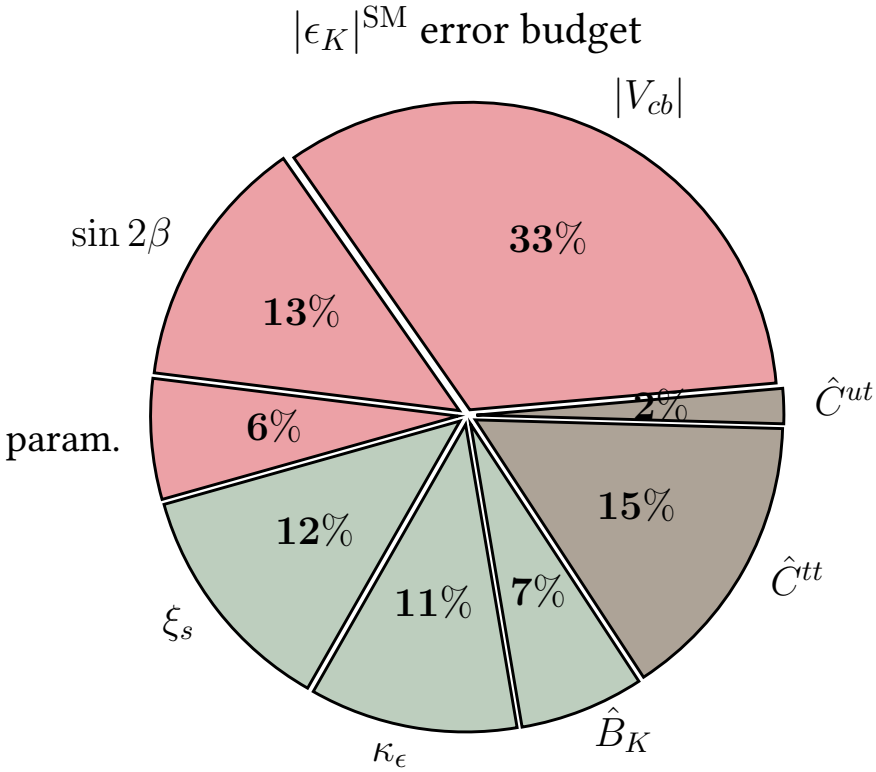
SM prediction of ϵ_K with PDG input

$$|\epsilon_K| = \kappa_\epsilon C_\epsilon \widehat{B}_K |V_{cb}|^2 \lambda^2 \bar{\eta} \Big(|V_{cb}|^2 (1-\bar{\rho}) \eta_{tt} S(x_t) - \eta_{ut} S(x_c, x_t) \Big)$$

[Buras et al 08, 10]



- $\hat{B}_K = 0.7625(97)$

  [FLAG19, 1902.08191]

- $|\epsilon_K|^{\rm SM} = \mathbf{2.16(18) \times 10^{-3}}$
- $|\epsilon_K|^{\rm exp} = \mathbf{2.228(11) \times 10^{-3}}$

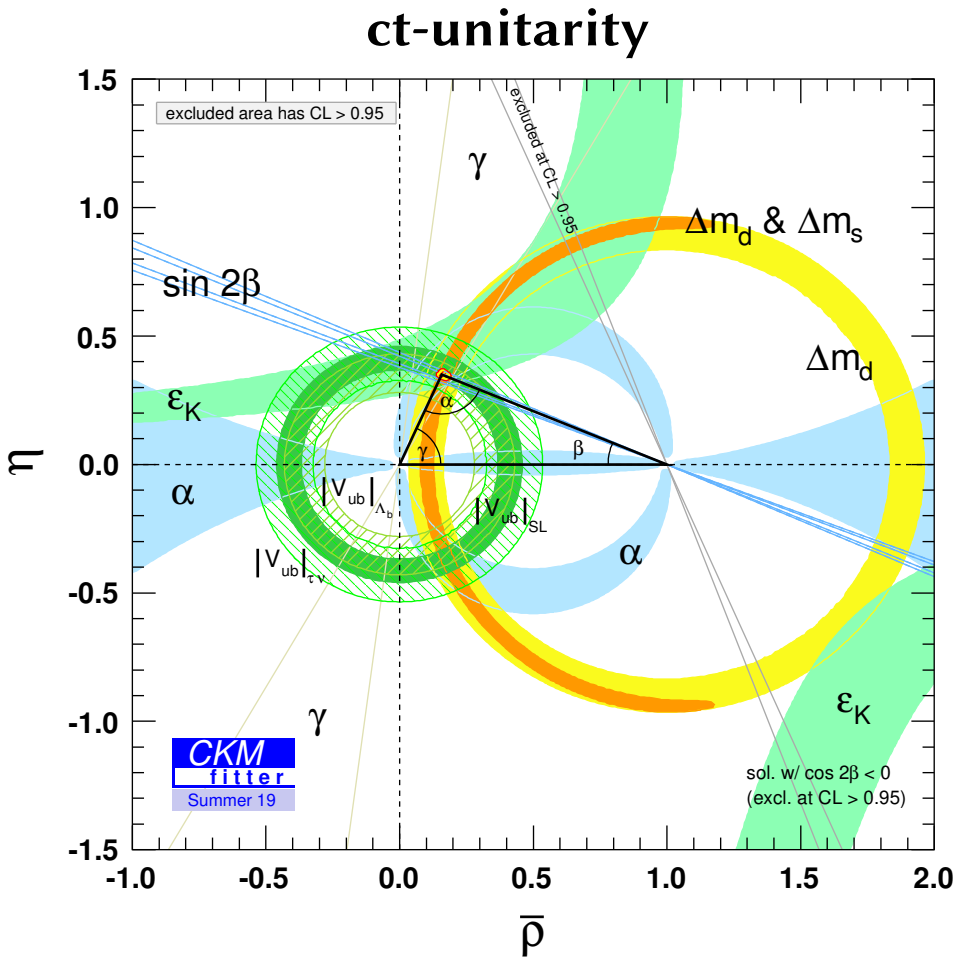
ϵ_K with exclusive V_{cb}

- ϵ_K highly sensitive to $|V_{cb}|$
- Use exclusive determination $|V_{cb}|^{\text{excl}} = 0.403(8)$
- And lattice value $\kappa_\epsilon = 0.923(6)$

➜ $\boldsymbol{|\epsilon_K|^{\text{SM}} = 1.81(14) \times 10^{-3}}$

- 3σ away from $\boldsymbol{|\epsilon_K|^{\text{exp}} = 2.228(11) \times 10^{-3}}$

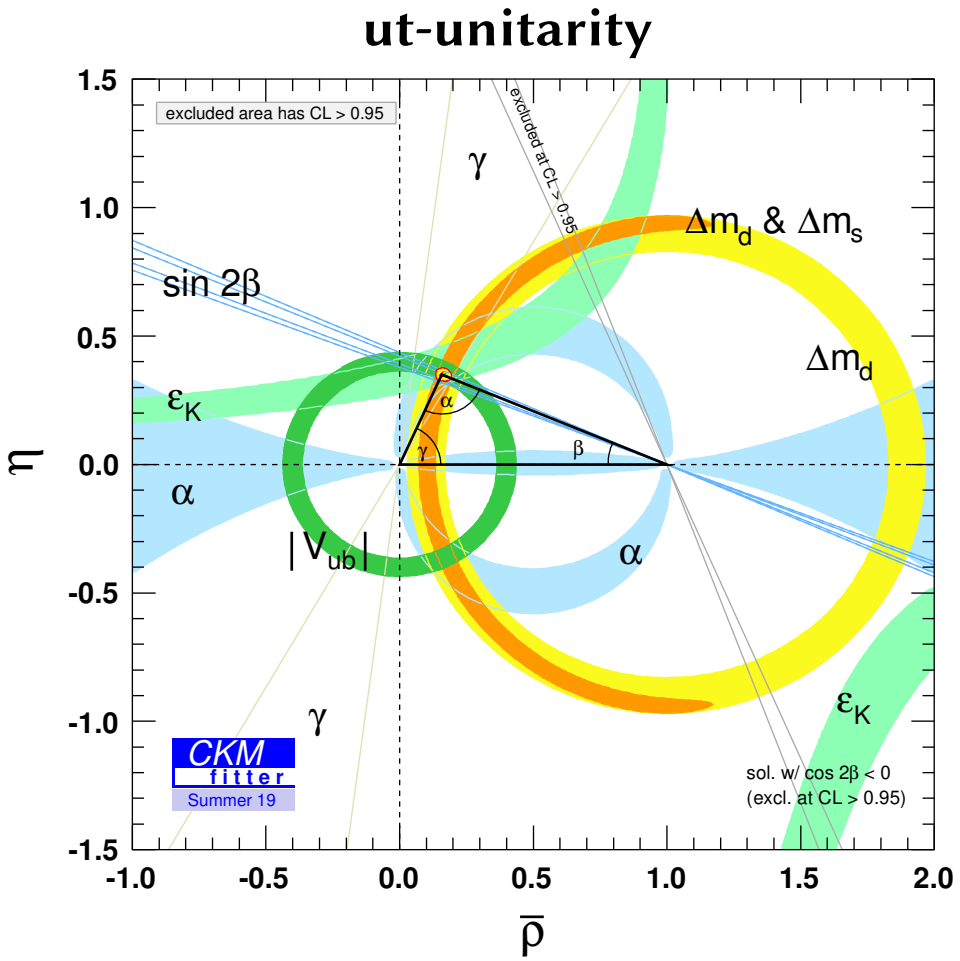
ϵ_K with ut-unitarity at CKMfitter



- CKMfitter collaboration implemented "ut"-unitarity
- Visible improvement
- **Future directions**
  - NNLO corrections to η_{tt}

    [in progress Brod, Gorbahn, Stamou, Yu]
  - QED/EW correction of the order 2%
  - lattice computation with dynamical charm still worthwhile

ϵ_K with ut-unitarity at CKMfitter



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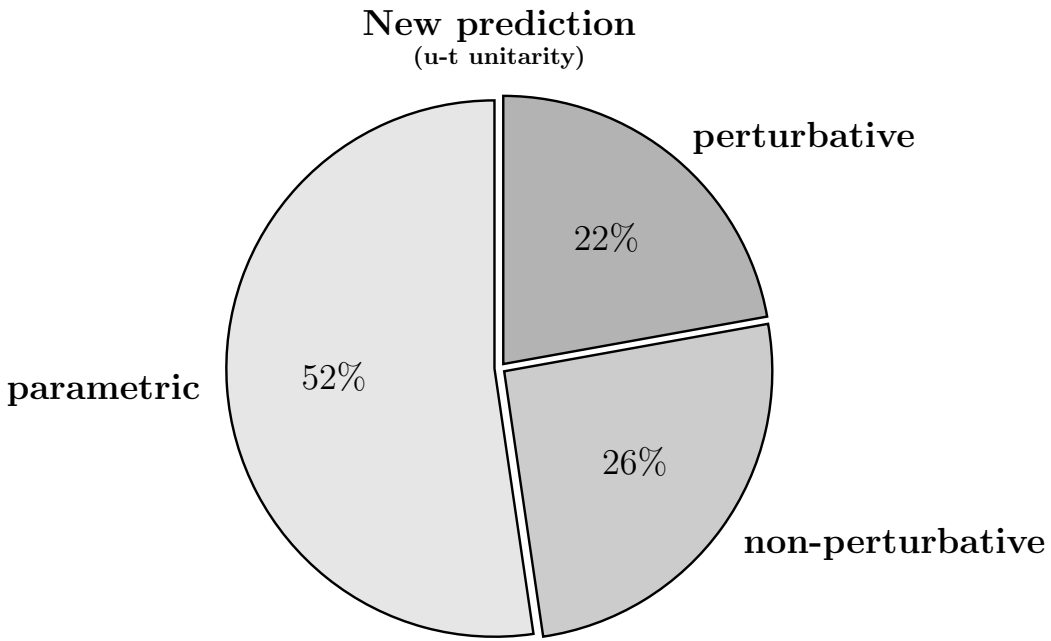
Summary and Conclusions

- ϵ_K is a powerful constraint for BSM models
- Rearrangement of perturbative series reduces impact of charm uncertainty to permil level
- $\boldsymbol{\eta_{ut} = 0.402(5)}$



- good convergence ➜opens path for further improvements
- future NNLO η_{tt} can reduce the total SD uncertainty to the percent level