Effective Actions for Cosmic String Simulation

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- Motivation: Axion abundance
- Reminder: String structure, multiscale problem
- Effective description with separation scale r_0
- Implementation in 2+1 Dimensions
- Alternative implementation, works in 3+1 D
- Results so far

Axion

Complex scalar φ, QCD-anomalous global U(1) symmetry:

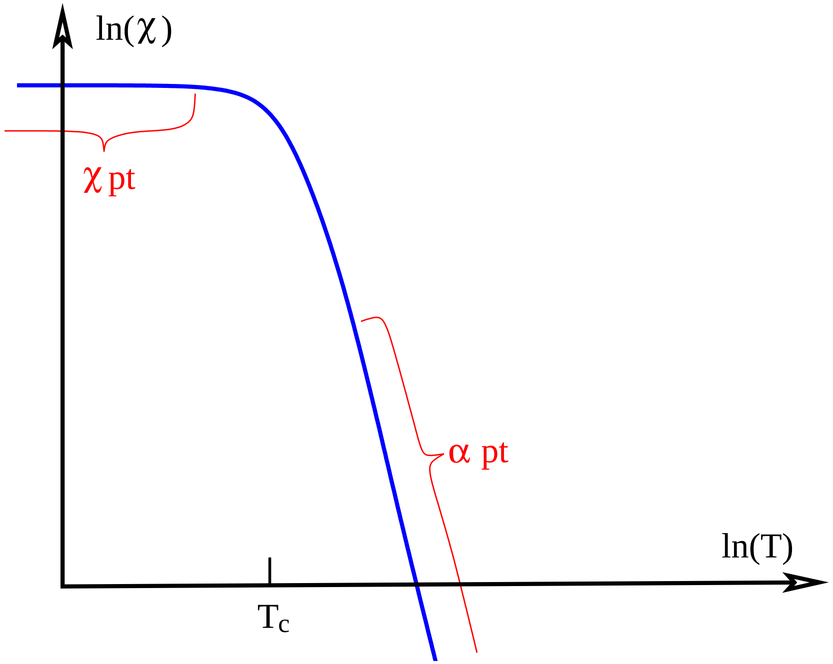
$$-\mathcal{L} = \partial_\mu \varphi \partial^\mu \varphi^* + \frac{m_r^2}{8 f_a^2} \left(2\varphi^* \varphi - f_a^2 \right)^2 - \mathcal{L}_{\varphi,QCD}$$

with $\mathcal{L}_{\varphi,QCD}$ an interaction Lagrangian with QCD degrees of freedom which makes the U(1) $\mathrm{SU}_c(3)$ anomalous.

Symmetry spontaneously breaks, $\varphi = \frac{f_a}{\sqrt{2}} e^{i\theta_A}$.
One value, $\theta_A = 0$, is P, CP conserving
QCD gives small U(1)-violating "tilt" to potential:

$$V(\theta_A) = \chi(T)(1 - \cos\theta_A)\,.$$

$\chi(T)$: what we expect



Low T: χ-pt works.

$\chi \simeq \frac{m_u m_d}{(m_u+m_d)^2} m_\pi^2 f_\pi^2$

Hi T: standard pert-thy works(??)

Low T: $\chi(T \ll T_c) = (76 \pm 1\ \mathrm{MeV})^4$ Cortona *et al*, arXiv:1511.02867

High T: $\chi(T \gg T_c) \propto T^{-8}$ Gross Pisarski Yaffe Rev.Mod.Phys.53,43(1981)

but with much larger errors.

Initial state of φ field?

most likely: **randomly different in different places!**

- Inflation stretches quantum fluctuations to classical ones: $\Delta\varphi \sim H_{\text{infl.}}$. If $N_{\text{efolds}} H^2 > f_a^2$, scambles field.

  If not: need $H < 10^{-5} f_a$ to avoid excess "isocurvature" fluctuations in axion field

- Gets scrambled *after* inflation if Universe was ever really hot $T > f_a \sim 10^{11}$ GeV.

Random starting conditions for field breaking a global U(1) symmetry: expect a network of cosmic strings!

Reminder: Cosmic String

Spont. breaking of U(1) allows for cosmic string solution:

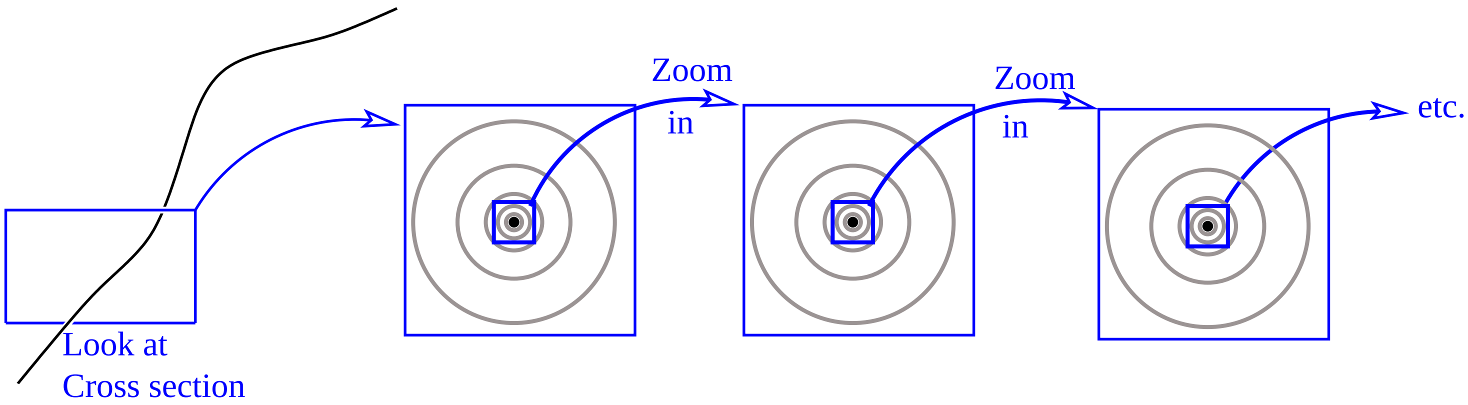
$$\varphi(z, r, \phi) = \frac{f_a}{\sqrt{2}} f(r) e^{i(\phi - \phi_0)}, \qquad f(r) \begin{cases} \propto m_r r & m_r r \ll 1 \\ \to 1 & m_r r \gg 1 \end{cases}$$

Solution is a *soliton* which is protected *topologically* from dissipating under local dynamics.
Get rid of string only by annihilating with anti-string.

Kibble mechanism: random initial conditions contain dense network of strings, which evolve towards *scaling solution*
Inter-string spacing $R \sim H^{-1}$ the Hubble scale

m_r to H ratio does **not** scale out

$$E_{\rm str} = \int dz \int d\phi \int r\,dr \left(\nabla\phi^* \nabla\phi \simeq f_a^2/2r^2 \right) \simeq \pi \ell f_a^2 \int_{m_r^{-1}}^{\sim H^{-1}} \frac{r\,dr}{r^2}$$



Series of "sheaths" around string:

equal energy in each $\times 2$ scale.

Assume $m_r \sim f_a$ and $H \sim T^2/m_{pl}$: $m_r/H \sim 10^{30}$.

Log-large string tension $T_{\rm str} = \pi f_a^2 \ln(10^{30}) \equiv \pi f_a^2 \kappa$ with $\kappa \sim 70$

What controls string network dynamics?

1. String tension $\propto \pi f_a^2 \kappa$ makes string "crack," cut off loops, form cusps and kinks etc
2. String radiation to Goldstones $\propto \pi f_a^2$
   Damps fine structure on string.
3. Inter-string forces $\propto \pi f_a^2$
   Helps strings collide, loops form, loops dissipate

Note, 1. scales with κ, the others do not.

Large κ reduces importance of string-Goldstone couplings.

A three-scale problem!

Radiation removes "short" scale structure – down to scale $L_{\rm ss} \sim H^{-1}/\kappa \sim H^{-1}/70$. 3 scales:

- UV: Higgs-mass or core size scale (m_r or $L_{\rm core}$)
- IR: Inter-string spacing, axion mass (H or H^{-1})
- In between H^{-1}/κ: scale of string structure ($L_{\rm ss}$)

Do I need correct κ and $L_{\rm ss}H$ hierarchy?

Yes.

- Gets the right density and structure of strings
- Tension (weight) of string decides how much potential must "tilt" before string network gets annihilated. Network collapses when $dm_a/dt \sim \pi f_a^2 \kappa$
- Production and longevity of loops

We need to generate and resolve this hierarchy.

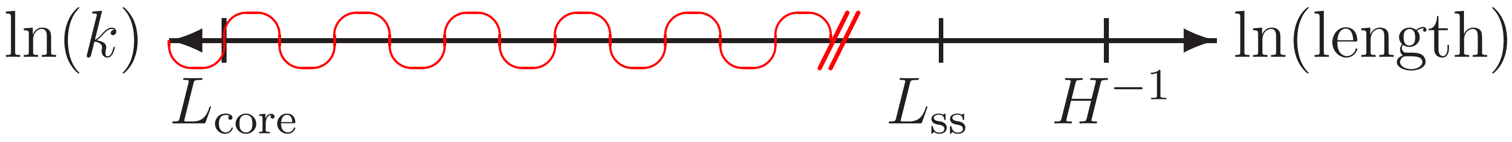
Can't I just use mesh refinement?

To reach spacing a along string, I need:

- Of order L/a boxes along each string
- Time step of order a

Replaces $(L/a)^4$ effort with $(L/a)^2$ effort.
Might get from (10^3) on a side to (10^6) on a side
But that's far from (10^{30}) on a side. Not enough!

An effective description



Scales from m_r to just-under $L_{\rm ss}$ are boring:

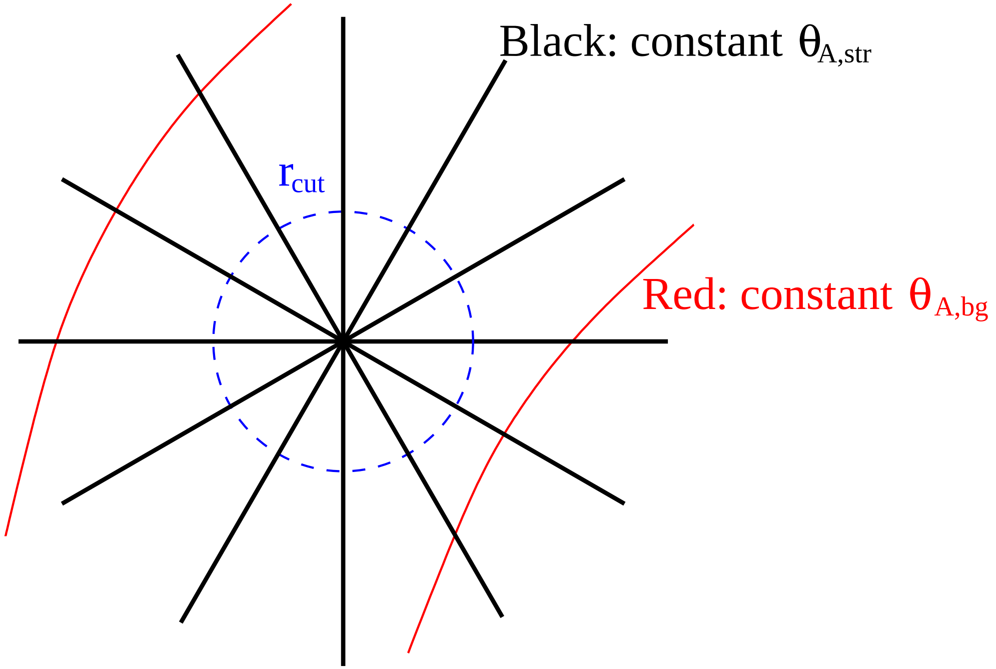
Just straight global string with log-scale tension.

Integrate them out *a la* **Abholkar and Quashnock**:

- Strings, tension $\mathbf{T} = \kappa' \pi f_a^2$, $\kappa' = \ln(m_r/m_{\rm reg})$
- Goldstone θ_A fields throughout space
- Kalb-Ramond coupling between string,Goldstone:

$$\mathcal{L} = \mathcal{L}_{NG} + f_a^2 \partial_\mu \theta_A \partial^\mu \theta_A / 2 + \mathcal{L}_{KR}$$

Effective Theory in More Detail



Put tube, size $r_{\rm cut}$ around string.
Field gradient
$\nabla\theta_A = \nabla\theta_{A,\rm str} + \nabla\theta_{A,\rm bg}$

Outside: θ_A winds by 2π in going around circle.

Inside: string has tension $\pi f_a^2 \kappa_0$, with $\kappa_0 = \ln(m_r r_{\rm cut})$

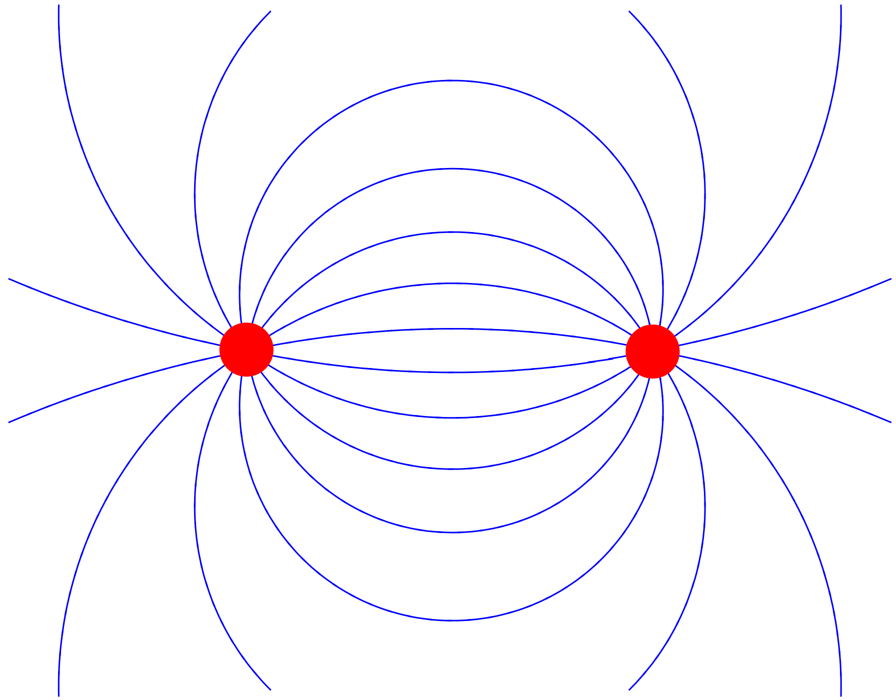
Boundary: Find $\oint T_{ij}$ and apply as force-per-length

$$\frac{dF_i}{dz} = \oint T_{ij}\, d\hat{n}_j = 2\pi f_a^2 \epsilon_{ijk} \nabla_j \theta_{A,\rm bg} \hat{s}_k$$

Clean example: 2+1 Dimensions

Suppose φ doesn't vary in z-direction: 2+1D.

String-antistring pair: lines of constant phase



Looks just like dipole. Because it is!

2+1 Dim: Dual EM description

Outside $r_{\rm cut}$: θ_A is all the physics.

Strict analogy to electromagnetism:

$$
\begin{aligned}
F^{\mu\nu} &\equiv \epsilon^{\mu\nu\alpha}\partial_\alpha\theta_A \\
\partial_\mu F^{\mu\nu} &= \epsilon^{\mu\nu\alpha}\partial_\mu\partial_\alpha\theta_A = 0 \\
\epsilon^{\alpha\mu\nu}\partial_\alpha F_{\mu\nu} &= -2\partial_\alpha\partial^\alpha\theta_A = 0 \\
\oint \vec{E}\cdot d\hat{n} &= \oint \frac{d\theta_A}{dl}dl = 2\pi N_{\rm wind}
\end{aligned}
$$

replace strings with point-charges of charge $\pm 2\pi$.

Mass $=$ tension $\pi f_a^2\kappa_0$: choose any value you want!

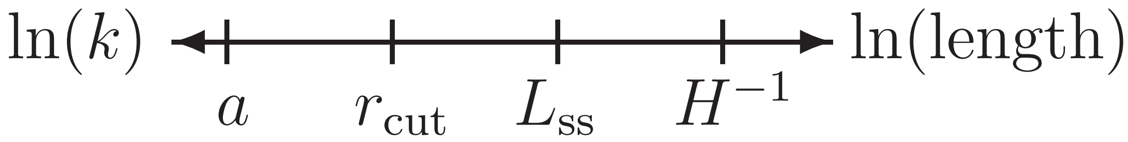
Put it on the lattice?

Actually not super-easy.

Large-k modes on lattice propagate slower than light.

Allows Cherenkov radiation of high-k modes!

FIX by smearing charge in a ball (form factor at large k)



Need r_{cut}/a and $L_{\text{ss}}/r_{\text{cut}}$ each a factor of "several"

Two extrapolations: large $L_{\text{ss}}/r_{\text{cut}}$ and large r_{cut}/a.

But factor $L_{\text{ss}}/a \sim 10^{28}$ now only $L_{\text{ss}}/a \sim 5 \times 5$.

Simulations: what did we learn?

- Strings in 2+1D are really different than in 3+1D.
  - Nonrelativistic
  - Tight, slowly-decaying binary pairs
- Walls "pull together" strings when $E_{\rm wall} \sim E_{\rm str}$

  Wasting energy in walls
- Axion production very weakly κ dependent, close to misalignment value *and can be lower*

How do I generalize this method to 3+1 D?

We haven't found a way yet.

It looks possible but challenging

MPI-parallelizing would be harder still.

Other short-distance physics?

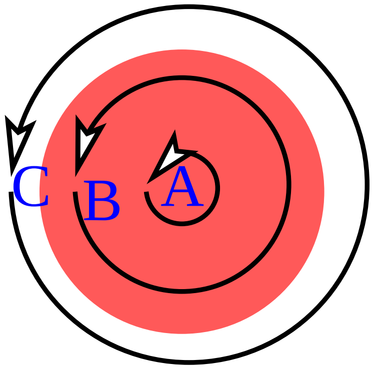
We need something which:

- Gives strings with large tension
- Strings obey expected Nambu-Goto behavior
- Strings force $\oint \partial_\phi \theta_A = 2\pi$
- No, or at least only heavy, DOF away from string

Abelian Higgs Model: Tension-Only Strings

$$\mathcal{L}(\varphi, A_\mu) = \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + (D_\mu \varphi)^*(D^\mu \varphi) + \frac{\lambda}{8}\left(2\varphi^*\varphi - f_a^2\right)^2$$

with $D_\mu = \partial_\mu - ieA_\mu$ covariant derivative



$$\oint \partial_\phi \varphi \, d\phi = 2\pi f_a \quad \text{but}$$

$$\oint D_\phi \varphi \, d\phi = (2\pi - B_{\text{encl}}) f_a$$

A: full $\nabla\varphi$ energy.

B: partial. C: cancels.

Outside string, B compensates $\nabla\varphi$.

Finite tension $T \simeq \pi f_a^2$. No long-range interactions.

Trick: global strings, local cores

Hybrid theory with A_μ and two scalars

$$\begin{aligned}\mathcal{L}(\varphi_1, \varphi_2, A_\mu) \;=\; & \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \\ & + \frac{\lambda}{8}\left[(2\varphi_1^*\varphi_1 - f^2)^2 + (2\varphi_2^*\varphi_2 - f^2)^2\right] \\ & + |(\partial_\mu - iq_1 e A_\mu)\varphi_1|^2 + |(\partial_\mu - iq_2 e A_\mu)\varphi_2|^2\end{aligned}$$

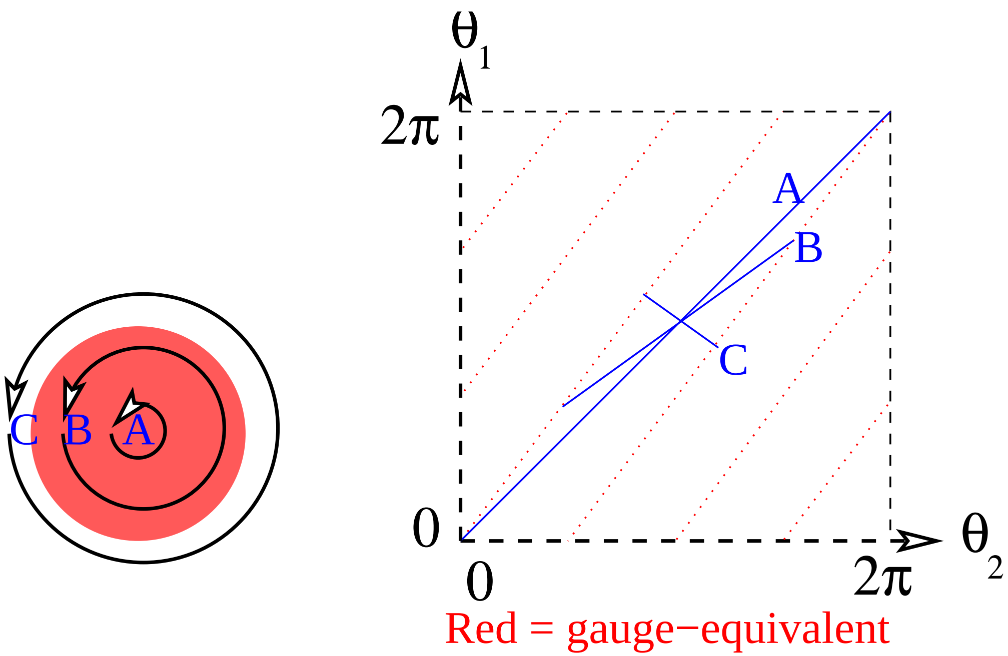
Pick $q_1 \neq q_2$, say, $q_1 = 4$, $q_2 = 3$.

Two rotation symmetries, $\varphi_1 \to e^{i\theta_1}\varphi_1$, $\varphi_2 \to e^{i\theta_2}\varphi_2$

$q_1\theta_1 + q_2\theta_2$ gauged, $q_2\theta_1 - q_1\theta_2$ global (Axion)

Two scalars, one gauge field

String where *each* scalar winds by 2π:



B-field *almost* compensates gradients outside string. $f_a^2 = f^2/(q_1^2+q_2^2)$.

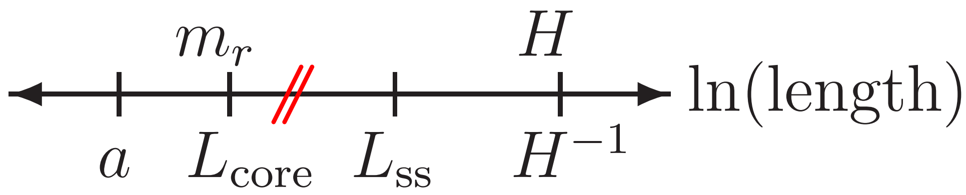
$$T \simeq 2\pi f^2 \,, \quad \frac{dF}{dl} = \frac{f^2}{(q_1^2+q_2^2)r} \,, \quad \kappa_{\rm eff} = 2(q_1^2+q_2^2) \,.$$

Two scalars, one gauge field

- Strings have Abelian-Higgs core $\rightarrow$ **Tension**
- Outside core: $q_1\theta_2 - q_2\theta_1 =$ **Axions**
- Ratio of tension to f_a tunable:
  can get string tension right!

Heavy degrees of freedom, also off-string.
But in *right limit*, their role should be small.

Scales in this model



Lattice spacing $a \ll m_r^{-1}$ micro-scale $\ll L_{\rm ss} \ll H^{-1}$
and H^{-1} must fit in the box.
But this *should* work for 2048^3 boxes...??

Must systematically explore $m_r a \to 0$ and $m_r L_{\rm ss} \to \infty$

Comments on $m_r a$

String must be several lattice spacings thick, eg, $m_r a \ll 1$

- Improved actions: $(m_r a)^4$ corrections (usually)
- BUT when strings become very relativistic
  Lorentz contraction: $(\gamma m_r a)$ is relevant
- $\gamma m_r a \sim 1$ mistreated (radiates heavy modes?)
- Prevalence of large γ should be $\propto \gamma^{-2}$

Corrections $\propto (m_r a)^2$ even with improved action.
We extrapolated this limit in establishing axion production.
But I am still nervous.

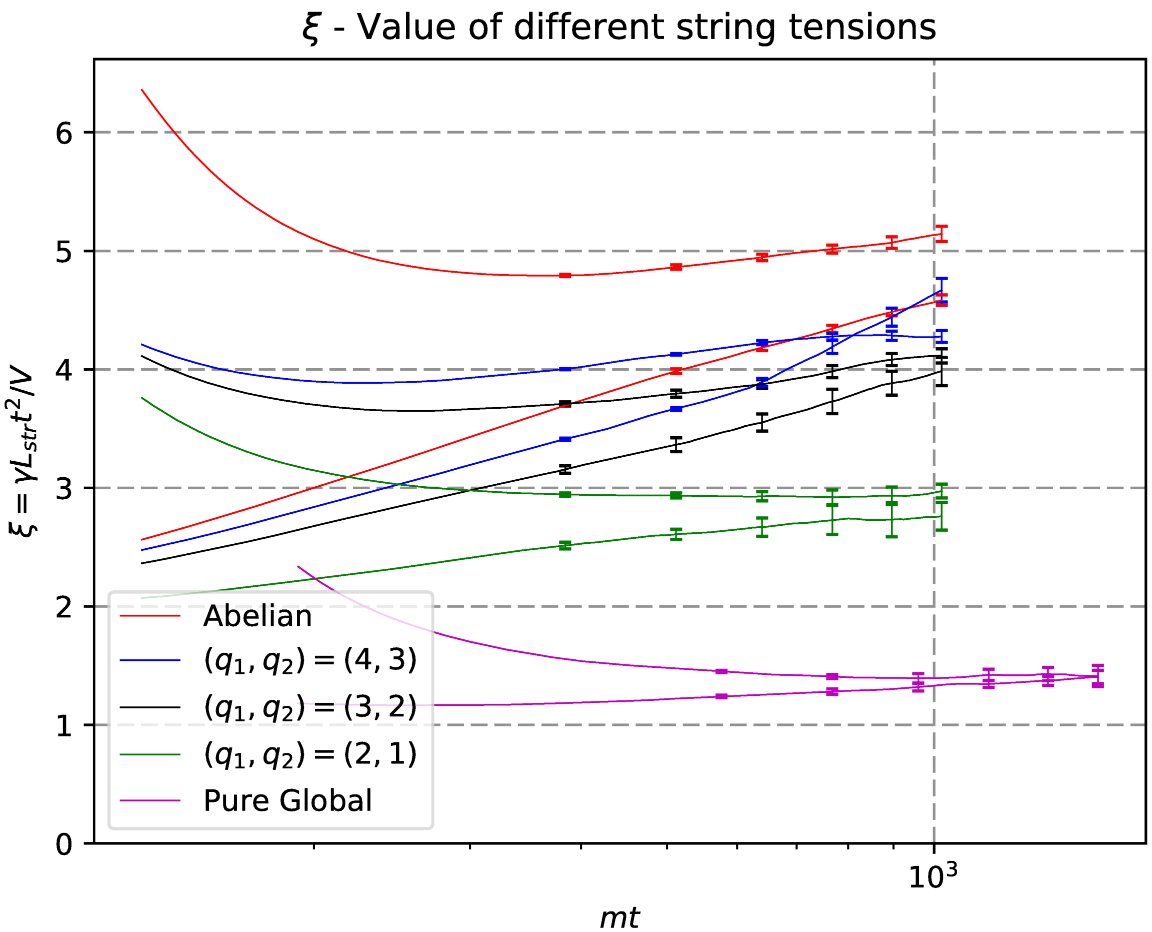
Comments on $L_{\rm ss} m_r$

Thickness should be small compared to structure,
$L_{\rm ss} m_r \gg 1$.

- Technically never strictly true – right after string intercommutations (kinks)
- **when** string has curvature radius $R \gg 1/m_r$ then its behavior should be well captured with **exponentally** small corrections
- Late stages of loop collapse also mistreated

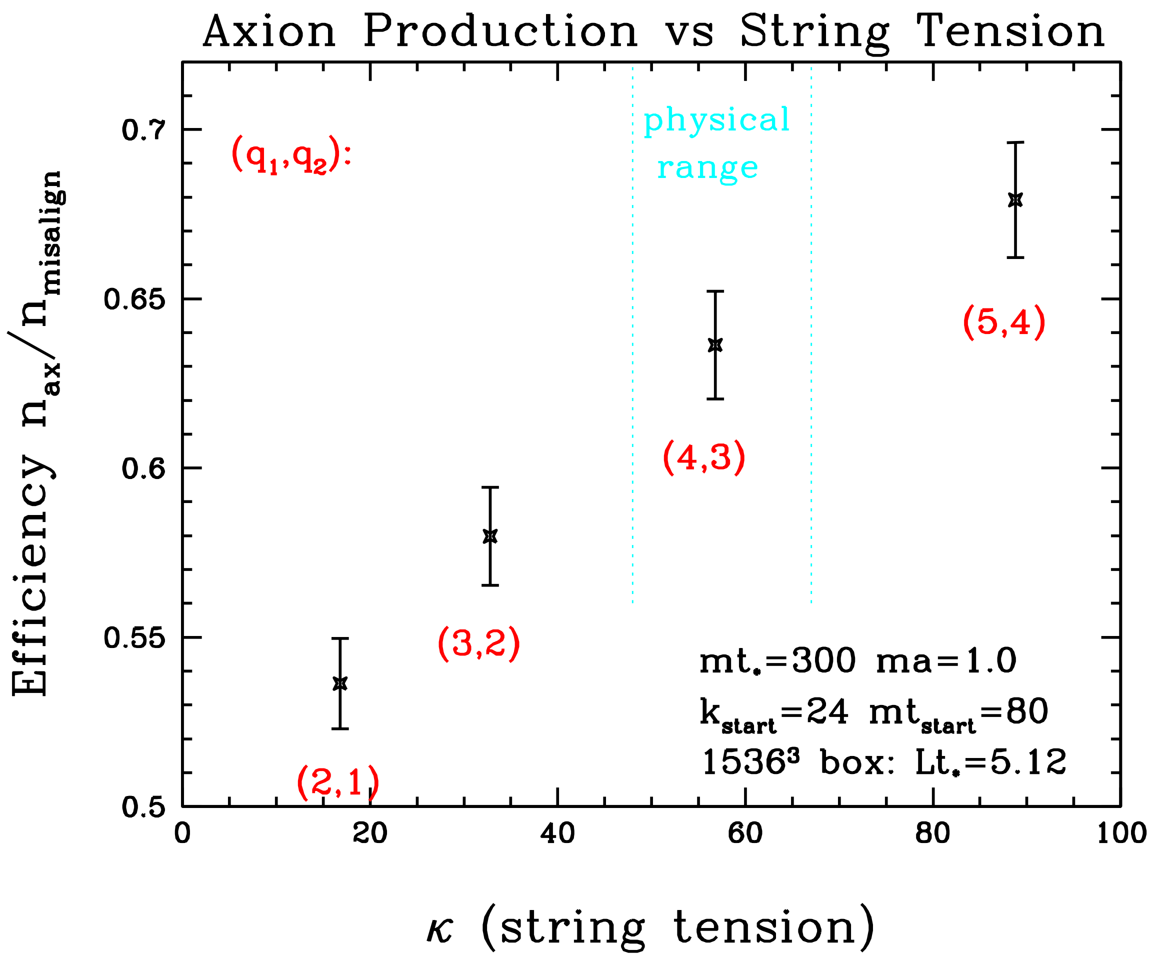
For counting axions, expect $\propto 1/(L_{\rm ss} m_r)$ corrections

String network Density



Physical tension: 2048^3 lattice may not be quite enough!

Results: Axion production



Axions produced vary mildly with increasing string tension

But *both* am_r and $1/(m_r L_{\rm ss})$ are marginal!

Better numerics: Mesh tricks?

Effective theories + mesh refinement? Might work.

Alternative: Use 2-scalar Higgs along string core,
θ_A-only at large distances (adaptive mesh DOF)

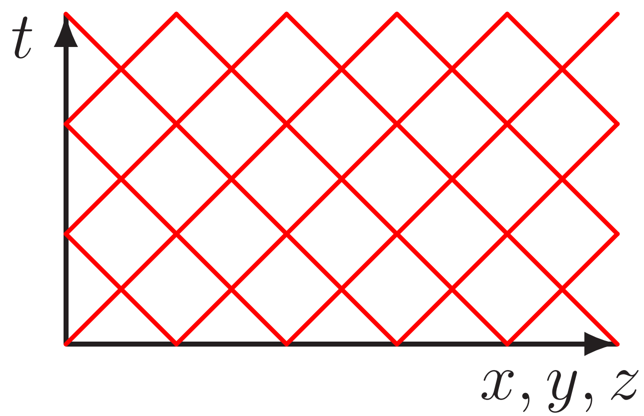
Both to be explored, in their infancy

Better numerics? Algorithm

Many-core, GPU both limited by RAM bandwidth

Bandwidth to L_2 cache is several times higher.

Complex fix: causal diamonds



One core does each diamond (fits in L_2)

Only boundary read/write to RAM

Simple fix: site ordering

```
for(kbig=0; kbig < N; kbig += 16)
for(jbig=0; jbig < N; jbig += 16)
for(i=0; i<N; i++)
for(ksmall=0;ksmall<16;ksmall++)
for(jsmall=0;jsmall<16;jsmall++) {
k=ksmall+kbig; j=jsmall+jbig;

(UPDATE) }
```

Back to axions for a moment

Axion production: $n_{\rm ax}(T=T_*) = (13\pm 2) H(T_*) f_a^2$

Hubble law: $H^2 = \dfrac{8\pi\varepsilon}{3m_{\rm pl}^2}\,,$

Equation of state: $\varepsilon = \dfrac{\pi^2 T^4 g_*}{30}\,, \quad s = \dfrac{4\varepsilon}{3T}\,, \quad g_*(1{\rm GeV}) \simeq 73$

Susceptibility: $\chi(T) \simeq \left(\dfrac{1\,{\rm GeV}}{T}\right)^{7.6} (1.02(35)\times 10^{-11}{\rm GeV}^4)$

Dark matter: $\dfrac{\rho}{s} = 0.39\,{\rm eV}$

One finds $T_* = 1.54\,{\rm GeV}$ and $m_a = 26.2 \pm 3.4\,\mu{\rm eV}$

Conclusions

- *Realistic* global strings have small cores $\Rightarrow$ high tension
- Brute-force will not work
- Right effective description known. Multiple realizations
- Most elegant – θ_A plus explicit string DOF –
  Works in 2+1D but not implemented in 3+1D
- Two scalar / frustrated network is alternative
- Getting all hierarchies is a challenge.