On asymptotic symmetries in spatially flat FLRW

Martín Enríquez Rojo & Till Heckelbacher

arXiv:2011.01960

25/01/2021

Overview

1. Introduction
   - Historical review
   - Objectives and applications
2. Asymptotically decelerating spatially flat FLRW at $\mathscr{I}^+$
   - Short description of FLRW
   - Asymptotic metrics
3. Results
4. Current and future research

Introduction - Historical review

Early stage:

- Asymptotic structure of asymptotically flat spacetimes at $\mathscr{I}^+$ [Bondi, van der Burg & Metzner (1962) and Sachs (1962)].
- AdS_3/CFT_2 correspondence [Brown & Henneaux (1986)] $\rightarrow$ AdS/CFT correspondence [Maldacena (1999)].

Modern era:

- Inclusion of superrotations [de Boer & Solodukhin (2003) and Barnich & Trossaert (2010)].
- Infrared triangle: asymptotic symmetries $\leftrightarrow$ memory effects $\leftrightarrow$ soft theorems [Strominger & Zhiboedov (2014) and Strominger (2017)].

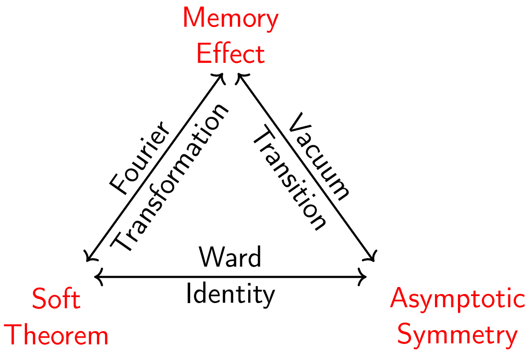

Figure: The infrared triangle according to [Strominger (2017)]

Introduction - Objectives and applications

Diversification into a variety of topics, including flat holography [Strominger (2014), Stieberger & Taylor (2019), Donnay et al. (2020)], black hole entropy [Hawking, Perry & Strominger (2016), Averin, Gómez, Dvali & Lüst (2016) , Donnay et al. (2016), Grumiller et al. (2020)], extension to spatial and timelike flat infinity [Prabhu et al. (2019), Khairnar et al. (2000)], extension to (A)dS_4 [Compère et al. (2000)], string theory [Avery et al. (2016), Afshar et al. (2019)], swampland [Lüst et al. (2020)] , membrane paradigm/fluid-gravity duality [Penna (2016), Penna (2017)] ...

... **but what about cosmological spacetimes?** Very little has been done.

- Cosmological gravitational memory effect [Kehagias et al. (2016), Wald et al. (2016), Shiu et al. (2017)].
- Asymptotic diffeomorphisms in cosmological spacetimes [Shiromizu et al. (1999), Ferreira et al. (2017), Bonga & Prabhu (2020)].

Spatially flat FRW spacetimes are conformal to flat spacetimes. In Bondi coordinates

$$u = \eta - \sqrt{x^i x_i}\ , \quad r = \sqrt{x^i x_i}\ , \quad z = \frac{x^1 + i x^2}{x^3 + \sqrt{x^i x_i}}\ , \quad \bar{z} = \frac{x^1 - i x^2}{x^3 + \sqrt{x^i x_i}}\ ,$$

their metrics can be written as

$$\mathrm{d}s^2 = \left(\frac{r+u}{L}\right)^{2k} \left(-\mathrm{d}u^2 - 2\mathrm{d}u\mathrm{d}r + \frac{4r^2}{(1+z\bar{z})^2}\mathrm{d}z\mathrm{d}\bar{z}\right)\ ,$$

where η is the conformal time and $k = 2/(3\omega+1)$, $\omega = p/\rho$.
These spacetimes can be divided into accelerated ($-1 < \omega < -1/3$, $k < 0$) and decelerated ($-1/3 < \omega < 1$, $k > 0$) expansion.

It turns out that only the decelerated have future null infinity $\mathscr{I}^+ \to$ analysis restricts to them.

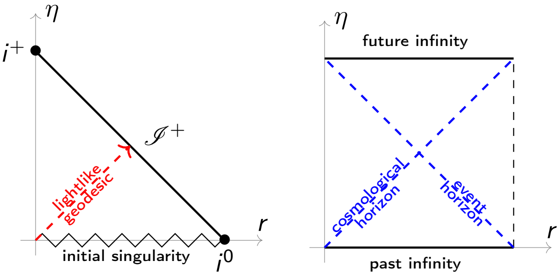

Figure: Conformal diagram of spatially flat decelerating FLRW (left) and accelerating FLRW (right)

Our project: analysis of asymptotically decelerating spatially flat FLRW metrics at $\mathscr{I}^+$. Concretely, we delve into the *asymptotic symmetry corner of the cosmological infrared triangle*, aiming to extend the flat space analysis to a more phenomenologically relevant framework.

This is an unavoidable step towards applicability and testability of the theory.

Requirements:

- Vacuum space is unperturbed FLRW.
- Compatible with Bondi gauge and frame.
- Allow for general cosmological perturbations but preserve (to leading order) homogeneous, isotropic and spatially flat profile, as well as k-fluid background.
- Closed in the r-expansion upon application of asymptotic diffeomorphism.
- Trace and components of the energy momentum tensor remains at most finite when appropriately scaled.

Ansatz:

$$\mathrm{d}s^2 = \left(\frac{r+u}{L}\right)^{2k} \left\{ -\left(1 - \Phi - \frac{2m}{r}\right) \mathrm{d}u^2 - 2\left(1 - \Psi - \frac{K}{r}\right) \mathrm{d}u\mathrm{d}r - 2(r\Theta_A + U_A + \frac{1}{r}N_A)\mathrm{d}u\mathrm{d}x^A + \left((1+\Omega)r^2\gamma_{AB} + rC_{AB} + h_{AB}\right)\mathrm{d}x^A\mathrm{d}x^B \right\} .$$

(Some) Results

Asymptotic diffeomorphisms:

$$\xi = \xi^u(u,z,\bar{z})\partial_u + \left[r\xi^{r(V)}(z,\bar{z}) + \xi^{r(0)} + \frac{1}{r}\xi^{r(1)} \right] \partial_r + \left[V^B(z,\bar{z}) + \frac{1}{r}\xi^{B(1)} + \frac{1}{r^2}\xi^{B(2)} \right] \partial_B \ ,$$

with

$$\xi^{B(1)} = -\frac{(1-\Psi)}{1+\Omega} D^B \xi^u \ ,$$

$$\xi^{B(2)} = \frac{1}{2}\left(\frac{1-\Psi}{(1+\Omega)^2} C^{AB} D_A \xi^u + \frac{K}{1+\Omega} D^B \xi^u \right)$$

$$\xi^{r(0)} = \frac{1}{1+k}\left[-\frac{1}{2(1+\Omega)} D_A((1+\Omega)\xi^{A(1)}) - \frac{1}{2(1+\Omega)} \Theta^A D_A \xi^u + ku\xi^{r(V)} - k\xi^u \right]$$

$$\xi^{r(1)} = \frac{1}{2(1+k)(1+\Omega)} \left[\frac{C^A_B \Theta_A D^B \xi^u}{1+\Omega} - 2k(1+\Omega)\left(u^2\xi^{r(V)} - u\xi^{r(0)} - u\xi^u \right) - D_A\left((1+\Omega)\xi^{A(2)} \right) - U^A D_A \xi^u \right] \ .$$

Effect on asymptotic data:

$$\begin{aligned}
\delta\Phi =& V^A D_A\Phi + \xi^u\partial_u\Phi - 2(1-\Psi)\partial_u\xi^{r(0)} - 2k(1-\Phi)\xi^{r(V)} \\
& - 2(1-\Phi)\partial_u\xi^u + 2\Theta_A\partial_u\xi^{A(1)} \\
\delta m =& \xi^u\partial_u m - k(1-\Phi)\xi^u - \left((1-2k)m - ku(1-\Phi)\right)\xi^{r(V)} \\
& - k(1-\Phi)\xi^{r(0)} + V^A D_A m + \frac{1}{2}\xi^{A(1)}D_A\Phi + K\partial_u\xi^{r(0)} \\
& - (1-\Psi)\partial_u\xi^{r(1)} + m\partial_u\xi^u + U_A\partial_u\xi^{A(1)} + \Theta_A\partial_u\xi^{A(2)} \\
\delta\Psi =& V^A\partial_A\Psi + \xi^u\partial_u\Psi - (1+2k)(1-\Psi)\xi^{r(V)} - (1-\Psi)\partial_u\xi^u \\
\delta K =& \xi^u\partial_u K + V^A D_A K + K\partial_u\xi^u + \xi^{A(1)}D_A\Psi - \Theta_A\xi^{A(1)} \\
& + 2k(1-\Psi)\left(u\xi^{r(V)} - \xi^u - \xi^{r(0)}\right) + 2kK\xi^{r(V)} \\
\delta\Omega =& V^C D_C\Omega + \xi^u\partial_u\Omega + 2(1+k)\xi^{r(V)} + (1+\Omega)D_A V^A
\end{aligned}$$

$$
\begin{aligned}
\delta C_{AB} =& \xi^u \partial_u C_{AB} + V^C D_C C_{AB} + C_{AC} D_B V^C + C_{BC} D_A V^C \\
&+ 2(1+\Omega)\gamma_{AB}((1+k)\xi^{r(0)} - ku\xi^{r(V)} + k\xi^u) + \gamma_{AB}\xi^{C(1)} D_C \Omega \\
&+ (1+\Omega)(D_A \xi_B^{(1)} + D_B \xi_A^{(1)}) + \Theta_A D_B \xi^u + \Theta_B D_A \xi^u \\
&+ (1+2k) C_{AB} \xi^{r(V)} \\
\delta \Theta_A =& V^B D_B \Theta_A + \xi^u \partial_u \Theta_A + (1+2k)\Theta_A \xi^{r(V)} + \Theta_B D_A V^B \\
&- (1-\Psi)\partial_A \xi^{r(V)} + \Theta_A \partial_u \xi^u + (1+\Omega)\partial_u \xi_A^{(1)} \\
\delta U_A =& (2k\Theta_A + \partial_u U_A)\xi^u + (1+2k)\Theta_A \xi^{r(0)} + 2k\xi^{r(V)}(U_A - u\Theta_A) \\
&+ V^B D_B U_A + \xi^{B(1)} D_B \Theta_A + \Theta_B D_A \xi^{B(1)} + U_B D_A V^B \\
&- (1-\Psi) D_A \xi^{r(0)} + K D_A \xi^{r(V)} - (1-\Phi) D_A \xi^u + U_A \partial_u \xi^u \\
&+ C_{AB} \partial_u \xi^{B(1)} + (1+\Omega)\partial_u \xi_A^{(2)}
\end{aligned}
$$

- Introduced the class of metrics to be considered asymptotically decelerating spatially flat FLRW at future null infinity $\mathscr{I}^+$.
- Obtained supertranslation and superrotation-like diffeomorphisms together with their effect on the asymptotic data.
- Investigated how these tranformations act on a pure FRW background and noticed that, not only C_{zz} and $C_{\bar{z}\bar{z}}$, but all the asymptotic data can be generated out of the vacuum.
- Furthermore, we have realized that central inhomogeneities are more involved to describe than Schwarzschild in the flat case. The Sultana-Dyer black hole background, has been explored and we have realized that it is not covered by our ansatz.
- Considered simple cosmologically perturbed backgrounds and analyzed how the perturbations affect the remaining asymptotic data under the action of the asymptotic diffeomorphisms.

Current and future research

Working at present:

- Extend our ansatz to include anisotropic spacetimes (e.g. Bianchi Type I).
- Extend our ansatz to include cosmological black holes (e.g. Sultana-Dyer and primordial black holes).
- Understanding the asymptotic symmetry algebra.
- Cosmological holographic flow.

Long-term goals:

- On-shell implementation for GR and alternative gravity theories.
- Extension to accelerating spatially flat FLRW spacetimes.*
- Entropy of cosmological black holes (and horizons).
- Cosmological (non-linear) memory effects (scalar, vector and tensor).
- Scattering problem in cosmology.

Thank you for your attention!