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QED CORRECTIONS TO N_{eff} IN THE SM

13. 04. 2021

Latest Advances in the Physics of BBN and Neutrino Decoupling

In collaboration with **Jack J. Bennett**, **Gilles Buldgen**, **Pablo F. de Salas**, **Stefano Gariazzo**, **Sergio Pastor**, and **Yvonne Y. Y. Wong**

arXiv:2012.02726 [hep-ph],
JCAP 2003 (2020) 003, arXiv:1911.04504 [hep-ph],
JCAP 07 (2019) 014, arXiv:1905.11290 [astro-ph.CO].

N_{eff} in the SM

$N_{eff} = 3$ under the assumptions

- primordial plasma is an ideal gas
- neutrino decoupling happens instantaneous
- neutrino decoupling happens at $T >> m_e$

All three assumptions are violated in reality!

A new Benchmark Value for N_{eff} in the SM

$$N_{\rm eff}^{\rm SM} = 3.0440 \pm 0.0002,$$

arXiv:2012.02726 [hep-ph]

$$\left.\begin{aligned} N_{\rm eff} &= 2.96^{+0.34}_{-0.33}, \\ \sum m_\nu &< 0.12\ \text{eV}, \end{aligned}\right\} \quad \begin{aligned} &95\,\%,\ \textit{Planck}\ \text{TT,TE,EE+lowE} \\ &\text{+lensing+BAO.} \end{aligned}$$

arXiv:1807.06209 [astro-ph.CO]

A new Benchmark Value for N_{eff} in the SM

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arXiv:2012.02726 [hep-ph]

Disclaimer:
This talk is strongly focussed on arXiv:1911.04504, arXiv:2012.02726.
Other notable recent contributions include:

- Escudero 2001.04466
- Akita / Yamaguchi 2005.07047
- Froustey / Pitrou / Volpe 2008.01074
- [I apologise if I missed your work]

Motivations for a Precision Computation

Upcoming experiment:

- CMB S4 will decrease observational error by a order of magnitude
- Theory error should be much smaller, so that the theory prediction can be treated as an "exact number"

Controversy in the literature:

$$N_{\text{eff}} = 3.044$$ arXiv:1905.11290 [hep-ph]

$$N_{\text{eff}} = 3.052$$ arXiv:1512.02205 [astro-ph.CO]

So what's new?

New ingredients in arXiv:2012.02726 [hep-ph]:

- Use new `FortEPiaNO` code
- Include thermal correction to the QED equation of state
- complete numerical evaluation of the neutrino–neutrino collision integral in the presence of neutrino flavour oscillations
- assessment of the uncertainties:

(i) numerical convergence.
(ii) the approximate modelling of the weak collision integrals in the presence of flavour oscillations,
(iii) measurement errors in the physical parameters of the weak sector.

So what's new?

New ingredients in arXiv:2012.02726 [hep-ph]:

- Use new `FortEPiaNO` code

- Include thermal correction to the QED equation of state
  [see Pablo Salas' talk for all other aspects]

- complete numerical evaluation of the neutrino–neutrino collision integral in the presence of neutrino flavour oscillations

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QED Equation of State

QED equation of state can be computed from partition function Z

In practice lnZ is expanded in powers of e

$$\ln Z = \ln Z^{(0)} + \ln Z^{(2)} + \ln Z^{(3)} + \cdots$$

From this, contributions to energy, pressure and entropy care computed

$$P^{(n)} = \frac{T}{V} \ln Z^{(n)},$$

$$\rho^{(n)} = \frac{T^2}{V} \frac{\partial \ln Z^{(n)}}{\partial T} = -P^{(n)} + T \frac{\partial P^{(n)}}{\partial T},$$

$$s^{(n)} = \frac{1}{V} \frac{\partial \left[T \ln Z^{(n)} \right]}{\partial T} = \frac{\rho^{(n)} + P^{(n)}}{T},$$

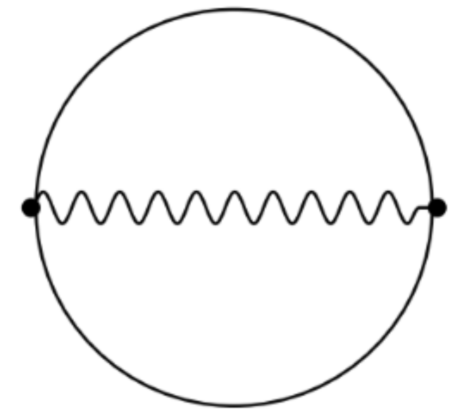
QED Equation of State

At zeroth order one finds ideal gas

$$P^{(0)} = \frac{T}{\pi^2} \int_0^\infty \mathrm{d}p\, p^2 \;\ln\left[\frac{(1+\mathrm{e}^{-E_e/T})^2}{(1-\mathrm{e}^{-E_\gamma/T})}\right],$$

$$\rho^{(0)} = \frac{1}{\pi^2} \int_0^\infty \mathrm{d}p\, p^2 \left[\frac{2E_e}{e^{E_e/T}+1} + \frac{E_\gamma}{e^{E_\gamma/T}-1}\right],$$

QED Equation of State O(e²)

The first correction comes from $\ln Z^{(2)} = -\frac{1}{2}$ 

$$P^{(2)} = \frac{T}{V}\ln Z^{(2)} = -\frac{e^2T^2}{12\pi^2}\int_0^\infty \mathrm{d}p\,\frac{p^2}{E_p}n_D - \frac{e^2}{8\pi^4}\left(\int_0^\infty \mathrm{d}p\,\frac{p^2}{E_p}n_D\right)^2 + \frac{e^2m_e^2}{16\pi^4}\iint_0^\infty \mathrm{d}p\,\mathrm{d}\tilde{p}\,\frac{p\tilde{p}}{E_pE_{\tilde{p}}}\ln\left|\frac{p+\tilde{p}}{p-\tilde{p}}\right| n_D\tilde{n}_D,$$

Kapusta / Gale, Finite-temperature field theory: Principles and applications.

Usually the log-dependent term is neglected

QED Equation of State O(e²)[no log]

Neglecting the log term yields

$$P^{(2)} = \frac{T}{V} \ln Z^{(2)} = -\frac{e^2 T^2}{12\pi^2} \int_0^\infty \mathrm{d}p \, \frac{p^2}{E_p} n_D - \frac{e^2}{8\pi^4} \left(\int_0^\infty \mathrm{d}p \, \frac{p^2}{E_p} n_D \right)^2$$

$$\rho^{(2)\not{\ln}} = -\frac{e^2 T^2}{12\pi^2} \int_0^\infty \mathrm{d}p \frac{p^2}{E_p} \left(n_D + T \partial_T n_D \right) + \frac{e^2}{8\pi^4} \left(\int_0^\infty \mathrm{d}p \frac{p^2}{E_p} n_D \right)^2 - \frac{e^2}{4\pi^4} \left(\int_0^\infty \mathrm{d}p \frac{p^2}{E_p} \, n_D \right) \left(\int_0^\infty \mathrm{d}p \frac{p^2}{E_p} \, T \partial_T n_D \right),$$

QED Equation of State O(e^2)

Adding the log term yields

$$P^{(2)} = \frac{T}{V}\ln Z^{(2)} = -\frac{e^2T^2}{12\pi^2}\int_0^\infty \mathrm{d}p\,\frac{p^2}{E_p}n_D - \frac{e^2}{8\pi^4}\left(\int_0^\infty \mathrm{d}p\,\frac{p^2}{E_p}n_D\right)^2 + \frac{e^2m_e^2}{16\pi^4}\iint_0^\infty \mathrm{d}p\,\mathrm{d}\tilde{p}\,\frac{p\tilde{p}}{E_pE_{\tilde{p}}}\,\ln\left|\frac{p+\tilde{p}}{p-\tilde{p}}\right|\,n_D\tilde{n}_D,$$

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$$\rho^{(2)\,\ln} = \frac{e^2m_e^2}{16\pi^4}\iint_0^\infty \mathrm{d}p\,\mathrm{d}\tilde{p}\,\frac{p\tilde{p}}{E_pE_{\tilde{p}}}\ln\left|\frac{p+\tilde{p}}{p-\tilde{p}}\right|\,n_D\left(2T\partial_T\tilde{n}_D - \tilde{n}_D\right)$$

Avoiding Mistakes

- The previous formulae look a bit complicated, and one may be tempted to simplify them…

- It is well known that some properties of a system of interaction particles can be described in the **quasiparticle picture**, i.e., by absorbing the effects of the interactions into modified dispersion relations and couplings of effective quasiparticles [Landau, Weldon, Klimov…]

- So maybe one can simply insert modified dispersion relations into the ideal gas energy and pressure?

$$E_\gamma^2(p) \to E_\gamma^2(p,T) = p^2 + \delta m_\gamma^2(T),$$
$$E_e^2(p) \to E_e^2(p,T) = p^2 + m_e^2 + \delta m_e^2(p,T)$$

Avoiding Mistakes

However: While the quasiparticle picture works reasonably well in transport equations [e.g. Arnold/Yaffe/Moore hep-ph/0209353], it in general does not describe bulk properties well.

- In the present context it only works well to order e^2

- The quasiparticle approximation describes interaction rates well, but "double counts" the effect of the interactions when being used to compute bulk properties

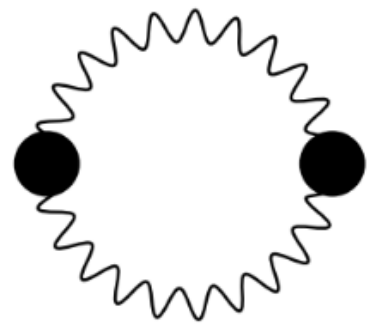
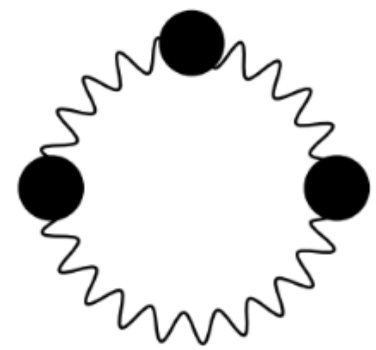
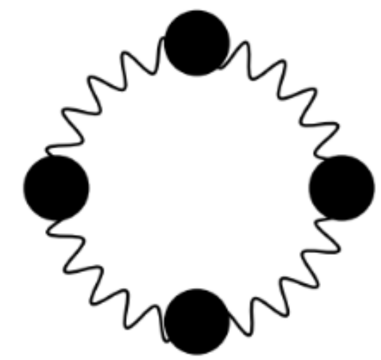
  ⇒ need to add factor 1/2 in front of corrections!

  This factor 1/2 was missed in [1512.02205], which explains the huge value $N_{\rm eff} = 3.052$

  Discrepancy in te literature solved!

QED Equation of State O(e³)

The next correction comes from

$\ln Z^{(3)} = \frac{1}{2}\Big[\frac{1}{2}$  $- \frac{1}{3}$  $+ \frac{1}{4}$  $+ \cdots \Big]$

Kapusta / Gale, Finite-temperature field theory: Principles and applications.

It reads

$$P^{(3)} = \frac{T}{V} \ln Z^{(3)} = \frac{e^3 T}{12\pi^4} I^{3/2}(T) \qquad \rho^{(3)} = \frac{e^3 T^2}{8\pi^4} I^{1/2} \partial_T I$$

with

$$I(T) = \int_0^\infty \mathrm{d}p \left(\frac{p^2 + E_p^2}{E_p} \right) n_D$$

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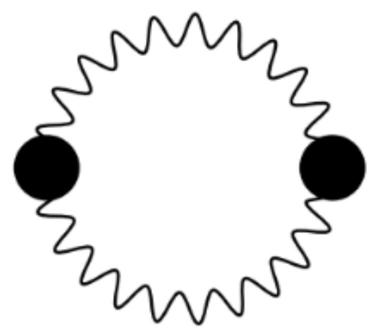
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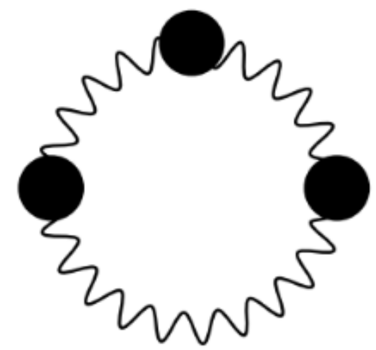
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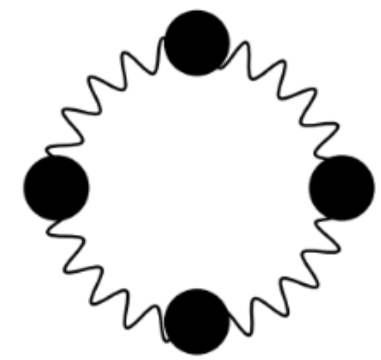
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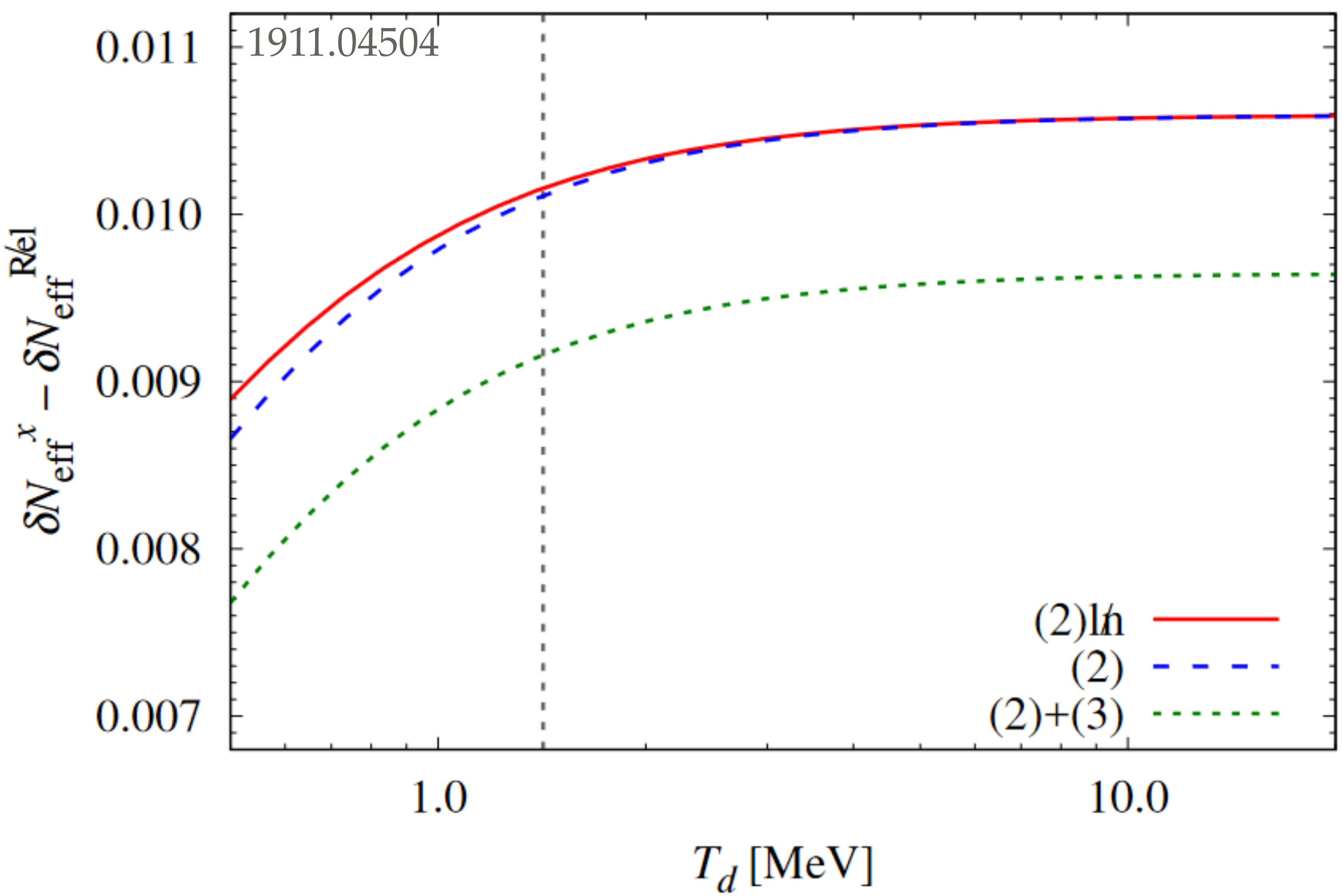
$$I(T) = \int_0^\infty \mathrm{d}p \left(\frac{p^2 + E_p^2}{E_p}\right) n_D$$

- This correction was previously neglected
- It turns out to be important

Impact on N_{eff} - Instantaneous Decoupling Estimate

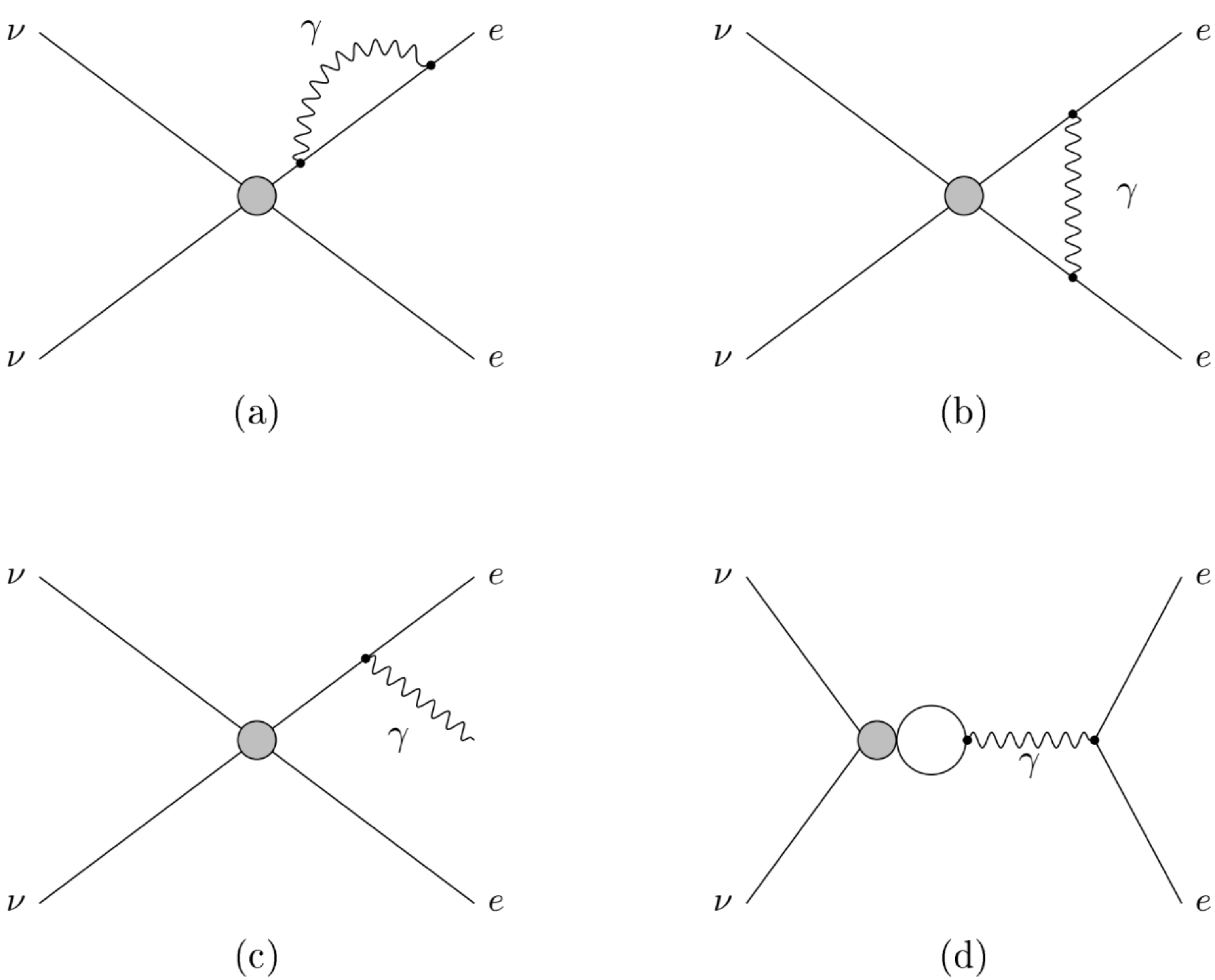
In the instantaneous decoupling approximation one can estimate:

$$\delta N_{\rm eff} = 3\left[\left(1+\frac{\delta s}{s^{(0)}\big|_{T_d/m_e\to\infty}}\right)^{-4/3}-1\right]$$



What was not accounted for?

- We did not incorporate thermal QED corrections to collision integrals



- Estimate in [2001.04466] suggests $\delta N_{eff} \sim -0.0007$

- To be investigated in more detail…

Impact on N_{eff}

Estimate in 1911.04504

x	$\delta N_{\rm eff}^{x}$ ($T_d = 1.3453$ MeV)	$\delta N_{\rm eff}^{x}$ ($T_d^{m_{\rm th}} = 1.46$ MeV)
	QED equation of state corrections	
$\not{R}$el	0.039895	0.033903
(2)$\not{l}$n	0.010121	0.010173
(2) ln	−0.000050	−0.000043
(3)	−0.000952	−0.000951
(4)	$\simeq 3.5 \times 10^{-6}$	$\simeq 3.5 \times 10^{-6}$
	Weak rate corrections	
$m_{\rm th}$	−0.000080	−0.000067
Total	**0.048937**	**0.043019**

- Instantaneous decoupling estimate works well

Full numerical reels from 2012.02726

Standard-model corrections to $N_{\rm eff}^{\rm SM}$	Leading-digit contribution
m_e/T_d correction	+0.04
$\mathcal{O}(e^2)$ FTQED correction to the QED EoS	+0.01
Non-instantaneous decoupling+spectral distortion	−0.005
$\mathcal{O}(e^3)$ FTQED correction to the QED EoS	−0.001
Flavour oscillations	+0.0005
Type (a) FTQED corrections to the weak rates	$\lesssim 10^{-4}$

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- Instantaneous decoupling estimate works well
- Adding the O(e³) term is equally important as adding neutrino oscillations!

Full numerical reels from 2012.02726

[see Pablo Salas' talk for all other aspects]

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