Introduction to Effective Field Theories

Matthias Neubert (matthias.neubert@uni-mainz.de)
Cluster of Excellence PRISMA+ and
Mainz Institute for Theoretical Physics (MITP)
Johannes Gutenberg University Mainz

Cluster of Excellence
PRISMA+
Precision Physics,
Fundamental Interactions
and Structure of Matter

mitp Mainz Institute for Theoretical Physics

(Virtual) Bad Honnef Physics School on
Method of Effective Field Theory and Lattice Field Theory
12-30 July 2021

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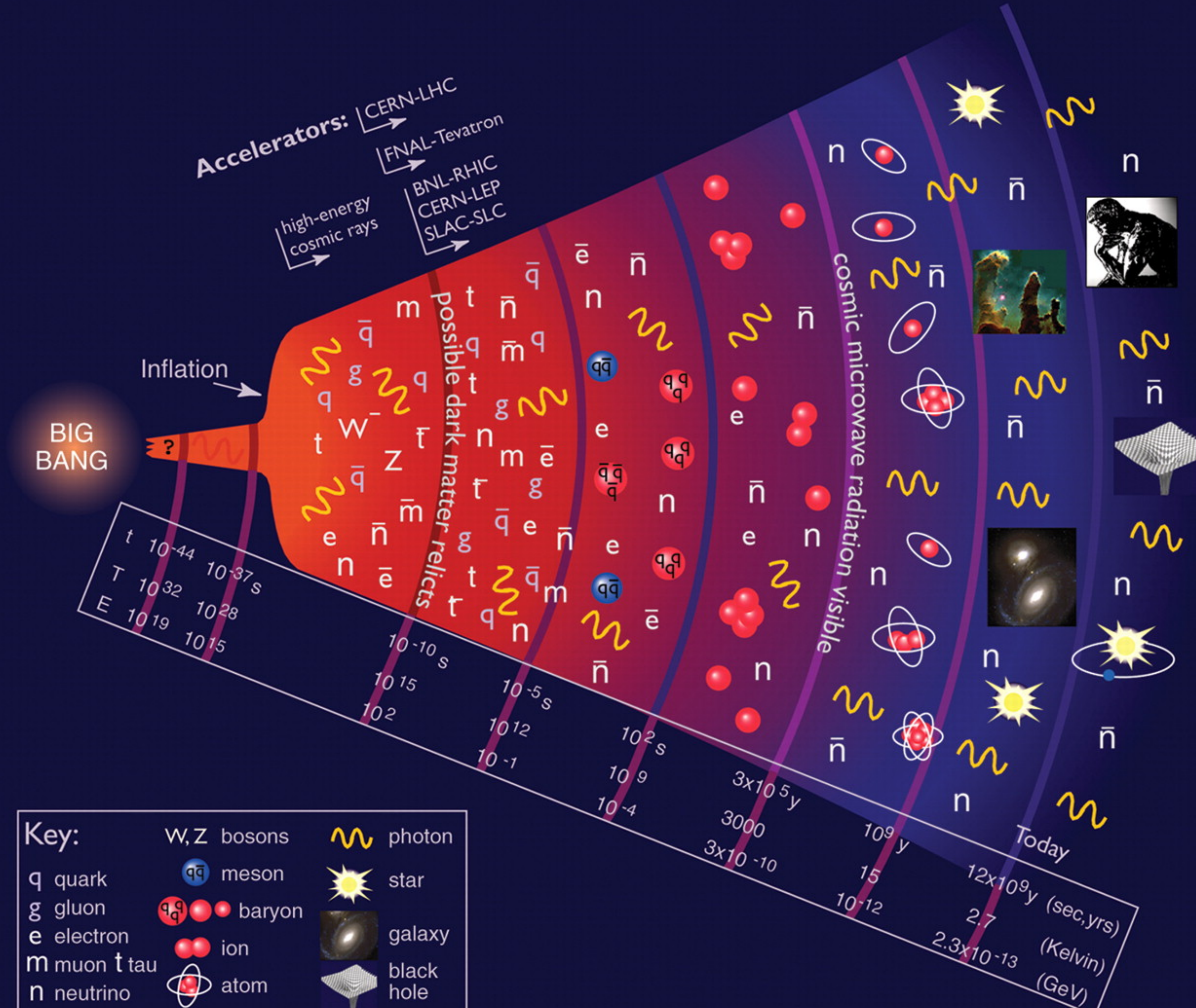

BIG BANG
Inflation
Accelerators:
CERN-LHC
FNAL-Tevatron
BNL-RHIC
CERN-LEP
SLAC-SLC
high-energy cosmic rays
possible dark matter relicts
cosmic microwave radiation visible
t 10-44 10-37 s
T 10 32 10 28
E 10 19 10 15
10 -10 s
10 15
10 2
10 -5 s
10 12
10 -1
10 2 s
10 9
10 -4
3x10 5 y
3000
3x10 -10
10 9 y
15
10 -12
Today
12x10 9 y (sec,yrs)
2.7 (Kelvin)
2.3x10 -13 (GeV)
Key:
q quark
g gluon
e electron
m muon t tau
n neutrino
W, Z bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole

Effective Field Theory

Effective field theories are a very powerful tool in quantum field theories:

- systematic formalism for the analysis of **multi-scale problems** ("Taylor expansion of Feynman graphs")
- simplifies **practical calculations**, often makes them feasible
- particularly important in QCD, where short-distance effects are calculable perturbatively but long-distance effects are not
- provides new perspective on **renormalization**
- basis of **factorization** (i.e. scale separation) and **resummation** of large logarithmic terms

Effective Field Theory

Useful reviews (there are many more):

- J. Polchinsky, hep-th/9210046
- A. Buras, hep-ph/9806471
- M. Neubert, hep-ph/0512222
- …
- Proceedings of the Les Houches Summer School “EFT in Particle Physics and Cosmology” (July 2017), available online; see in particular the lectures by A. Manohar (arXiv:1804.05863) and M. Neubert (arXiv:1901.06573)

Effective Field Theory

Lecture I:

- General concepts of EFT, scale separation, integrating out high-energy modes, Wilsonian effective action, low-energy effective Lagrangian, modern view on QFT

Lecture II:

- Matching and running, concrete examples

Lecture III:

- Applications of EFT, the Standard Model as an EFT, interesting insights about particle physics, concrete examples

Lecture I: Concepts of Effective Field Theory

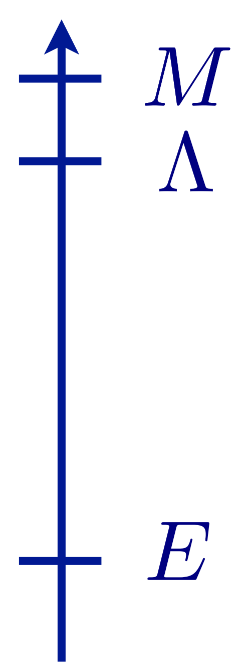
Derivation of the effective Lagrangian

Consider a QFT with a characteristic (fundamental) high-energy scale *M*

We are interested in performing experiments at energies $E \ll M$

<u>Step 1:</u> Choose a cutoff $\Lambda < M$ and divide all quantum fields into high- and low-frequency components ($\omega > \Lambda$ and $\omega < \Lambda$):

$$\phi = \phi_L + \phi_H$$



Recall:

$$\phi(x) = \int \frac{d^3k}{(2\pi)^3\, 2E_k} \left(a_k\, e^{-ik\cdot x} + a_k^\dagger\, e^{ik\cdot x} \right)$$

$$\omega = E_k = \sqrt{\vec{k}^2 + m^2}$$

Derivation of the effective Lagrangian

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Physics (i.e. Green functions) at low energies $E \ll \Lambda$ is entirely described in terms of the fields ϕ_L; Green functions of these fields can be derived from the generating functional:

$$Z[J_L] = \int \mathcal{D}\phi_L \, \mathcal{D}\phi_H \, e^{iS(\phi_L,\phi_H) + i\int d^Dx \, J_L(x) \, \phi_L(x)}$$

$$\langle 0 | \, T\{\phi_L(x_1) \ldots \phi_L(x_n)\} \, 0 \rangle = \frac{1}{Z[0]} \left(-i \, \frac{\delta}{\delta J_L(x_1)} \right) \ldots \left(-i \, \frac{\delta}{\delta J_L(x_n)} \right) Z[J_L] \Big|_{J_L=0}$$

Derivation of the effective Lagrangian

Step 2: Since the high-frequency fields ϕ_H do not appear in the generating functional, we can **"integrate them out"** in the path integral:

$$Z[J_L] \equiv \int \mathcal{D}\phi_L \, e^{iS_\Lambda(\phi_L)+i\int d^Dx\, J_L(x)\,\phi_L(x)}$$

where

$$e^{iS_\Lambda(\phi_L)} = \int \mathcal{D}\phi_H \, e^{iS(\phi_L,\phi_H)}$$

and $S_\Lambda(\phi_L)$ is called the **Wilsonian effective action**

Dependence on the cutoff Λ enters via the condition on the frequencies of the fields

Derivation of the effective Lagrangian

Step 3: Effective action is **non-local** on the scale $\Delta t \sim 1/\omega$, corresponding to the propagation of high-energy modes that have been removed from the Lagrangian

Since the remaining fields have energies $\omega < \Lambda$, the non-local effective action can be expanded in an **infinite series of local operators:**

$$S_\Lambda(\phi_L) = \int d^D x \, \mathcal{L}_\Lambda^{\text{eff}}(x)$$

where:

$$\mathcal{L}_\Lambda^{\text{eff}}(x) = \sum_i g_i \, Q_i(\phi_L(x))$$

coupling constants (Wilson coefficients) → g_i

local operators built out of fields ϕ_L and their derivatives → Q_i

Dimensional analysis

Does a Lagrangian consisting of an infinite number of interactions and hence an infinite number of (renormalized) coupling constants give any predictive power?

- not if one adopt an old-fashioned view about renormalization and renormalizable QFTs
- but not all is lost...

Use **naive dimensional analysis** to estimate the size of individual terms in the infinite sum to a given matrix element

Dimensional analysis

As is common practice in particle physics, we adopt units where $\hbar = c = 1$, such that $[m] = [E] = [p] = [x^{-1}] = [t^{-1}]$ are all measured in the same units (mass units)

Denote by $[g_i] = -\gamma_i$ the mass dimension of the coupling constants in the effective Lagrangian

Since by assumption the theory has only a single fundamental scale *M*, it follows that:

$$g_i = C_i \, M^{-\gamma_i}$$

where by **naturalness** we expect that $C_i = \mathcal{O}(1)$

Dimensional analysis

At low energy, it follows that the contribution of a given term $g_i\, Q_i$ to an observable (which for simplicity we assume to be dimensionless) scales like:

$$C_i \left(\frac{E}{M}\right)^{\gamma_i} = \begin{cases} O(1)\,; & \text{if } \gamma_i = 0 \\ \ll 1\,; & \text{if } \gamma_i > 0 \\ \gg 1\,; & \text{if } \gamma_i < 0 \end{cases}$$

Therefore, only operators with $\gamma_i \leq 0$ are important for $E \ll M$

This is what makes the effective Lagrangian useful!

Depending on the precision goal, one can truncate the infinite sum over terms by only retaining operators whose γ_i value is smaller than a certain value

Dimensional analysis

Since the Lagrangian has mass dimension *D* = dimensionality of spacetime (the action is dimensionless), it follows that

$$\delta_i = [Q_i] = D + \gamma_i$$

Hence we can summarize:

Dimension	Importance for $E \to 0$	Terminology
$\delta_i < D$, $\gamma_i < 0$	grows	relevant operators (super-renormalizable)
$\delta_i = D$, $\gamma_i = 0$	constant	marginal operators (renormalizable)
$\delta_i > D$, $\gamma_i > 0$	falls	irrelevant operators (non-renormalizable)

Only a **finite number** of relevant and marginal operators exist!

Dimensional analysis

Comments:

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- **“relevant” operators** are usually unimportant, since they are forbidden by some symmetry (otherwise they give rise to a hierarchy problem)
- **“marginal” operators** are all there is in “renormalizable” QFTs
- **“irrelevant” operators** are the most interesting ones, since they tell us something about the fundamental scale *M*

Example 1: ϕ^4 - theory at weak coupling

Use the free Lagrangian to derive the mass dimension of all fields and couplings, assuming the theory is weakly coupled:

$$S = \int d^D x \left(\frac{1}{2}\, \partial_\mu \phi \, \partial^\mu \phi - \frac{m^2}{2}\, \phi^2 - \frac{\lambda}{4!}\, \phi^4 \right)$$

In *D* dimensions, it follows that:

$$[\phi] = \frac{D}{2} - 1 \,, \quad [m] = 1 \,, \quad [\lambda] = 4 - D$$

Hence:

- mass term is a relevant operator
- interaction term is marginal in D=4 (relevant in D<4)

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Hence:

- an operator containing n_1 fields ϕ and n_2 derivatives has dimension:

$$\delta_i = n_1 \left(\frac{D}{2} - 1 \right) + n_2\,, \qquad \gamma_i = (n_1 - 2) \left(\frac{D}{2} - 1 \right) + (n_2 - 2)$$

- for $D>2$, adding fields or derivatives increases the dimension!

Example 2: QED (and QCD)

Use the free Lagrangian to derive the mass dimension of all fields and couplings, assuming the theory is weakly coupled:

$$S = \int d^D x \left[\bar{\psi} \, (i \not{D} - m) \, \psi - \frac{1}{4} \, F_{\mu\nu} F^{\mu\nu} \right]$$

where $D^\mu = \partial^\mu + ieA^\mu \, , \; F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$

In *D* dimensions, it follows that:

$$[\psi] = \frac{D-1}{2} \, , \quad [A^\mu] = \frac{D-2}{2} \, , \quad [m] = 1 \, , \quad [e] = \frac{4-D}{2}$$

Hence:

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Hence:

- for *D*>2, adding fields or derivatives increases the dimension of an operator!

Comments

Interactions modify the naive scaling behavior of composite operators and couplings as derived from the free action ("anomalous dimensions", see lecture II)

At strong coupling, this may switch a finite number of terms in the action between the categories "relevant", "marginal" and "irrelevant"

- e.g. "walking technicolor" (an irrelevant coupling is dynamically enhanced to become nearly marginal)
- IR fixed point in *D*=3 scalar field theory (ϕ^4 interaction is naively relevant in *D*=3, but at strong coupling this is offset by quantum effects and there is a zero in the β-function, which is responsible for critical behavior in many systems)

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- corrections to the naive scaling laws are most important for marginal couplings, because arbitrarily small corrections can make them relevant or irrelevant
- these effects are typically logarithmic and lead to effects such as asymptotic freedom and confinement in QCD

Examples of EFTs

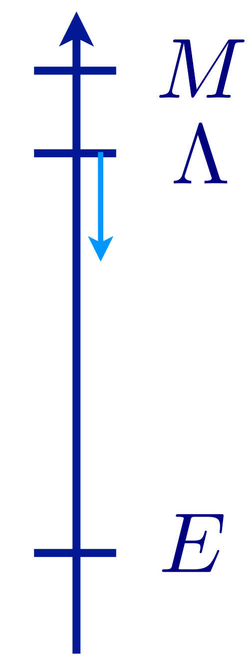
High-energy theory	Fundamental scale	Low-energy theory
11-d M theory (?)	?	String theory (?)
String theory (?)	$M_S \sim 10^{18}$ GeV	QFT
GUT (?)	$M_{GUT} \sim 10^{16}$ GeV	SUSY (?)
SUSY (?)	$M_{\not{SUSY}} \sim 10^{4+x}$ GeV	SMEFT
SMEFT	$M_W \sim 10^2$ GeV	Fermi theory
QCD	$m_b \sim 5$ GeV	HQET, NRQCD
	$M_{\chi SB} \sim 1$ GeV	ChPT

- String/M theory: fundamental theory is non-local and even spacetime breaks down at short distances
- QCD at low energy: example with strong coupling, where the relevant d.o.f. (hadrons) are different from the d.o.f. of QCD

Running couplings / Wilson coefficients

Often the fields ϕ_H correspond to heavy particles, whose effects become unimportant at low energies

But the frequency decomposition implies that **high-energy excitations of massless particles** (such as gauge bosons) are also integrated out from the low-energy effective theory



Consider now the situation where we lower the cutoff Λ without crossing the threshold for a heavy particle that could be integrated out:

- the structure of the operators Q_i in the effective Lagrangian remains the same
- hence, the effect of lowering the cutoff must be entirely absorbed into the values of the coupling constants g_i

Follows that $g_i = g_i(\Lambda)$ are **running,** Λ-dependent parameters!

Modern quantum field theory

"Theorem of modesty":

- no QFT ever is complete on all length and energy scales
- all QFTs are low-energy effective theories valid in some energy range, up to some cutoff Λ

Give up renormalizability as a construction criterion for "decent" QFTs:

- at low energy, any effective theory will automatically reduce to a "renormalizable" QFT, meaning that "non-renormalizable" interactions give rise to small contributions ~$(E/M)^n$
- this does not make renormalization irrelevant, but it provides a different point of view (Wilsonian approach)

Modern quantum field theory

Forget the folklore about “cancellations of infinities”

Take a more physical viewpoint:

- low-energy physics depends on the **short-distance dynamics** of the fundamental theory only through a small number of **relevant and marginal couplings,** and possibly through some irrelevant couplings if our measurements are sufficiently precise
- this finite number of couplings can be renormalized (i.e., infinities can be removed consistently) using a finite number of experimental data

Modern quantum field theory

Forget the folklore about “cancellations of infinities”

Take a more physical viewpoint:

- contrary to the old paradigm of strictly forbidding non-renormalizable interactions, we always expect them to be present and give rise to small effects, which may or may not be observable at a given level of accuracy
- this provides an **“indirect way”** to search for hints of physics beyond the (current) Standard Model:

low-energy, high-precision measurements

Modern quantum field theory

Instead, relevant ("super-renormalizable") interactions cause problems!

Consider, e.g., the mass term $m^2\phi^2$ in scalar field theory

Dimensional analysis suggests that $m^2 \sim M^2 \sim \Lambda_{\mathrm{UV}}^2$

But then a light scalar particle should not be present in the low-energy effective theory!

Hierarchy problem!

The same argument applies for all mass terms in any QFT!

… and likewise for the cosmological constant!

Modern quantum field theory

New paradigm: EFTs must be **natural** in the sense that **all mass terms should be forbidden** by (exact or broken) symmetries!

Indeed:

- **gauge invariance:** forbids mass terms for gauge fields (photons and gluons in the Standard Model)
- **chiral symmetry:** forbids mass terms for fermions (all matter fields in the Standard Model)

Explains why the SM is a chiral gauge theory!

- **Supersymmetry:** links the masses of scalars and fermions and hence, in combination with chiral symmetry, forbids mass terms for scalar fields (solves the hierarchy problem)

Exercises for lecture I

1. Derive the scaling relations for all fields and coupling parameters in the Standard Model Lagrangian in a D-dimensional spacetime. Show that in $D = 4$ dimensions only a single relevant coupling exists (which one?).

2. Consider the quantum field theory for a light Dirac fermion of mass m and a heavy real scalar boson of mass $M \gg m$, which are coupled by a Yukawa interaction. The corresponding Lagrangian is

$$\mathcal{L} = \bar{\psi}\,(i\partial\!\!\!/ - m)\,\psi + \frac{1}{2}\,(\partial_\mu\phi\,\partial^\mu\phi - M^2\phi^2) - g\bar{\psi}\psi\phi\,.$$

   If we are interested in processes with energy $E \ll M$, the heavy scalar field can be integrated out from the theory. Derive the explicit form of the resulting effective action $S_{\text{eff}}(\psi,\bar{\psi})$ using the functional integral approach, and perform its expansion in local operators. Derive the scaling relations for the operators and coupling constants in this expansion.

3. Since the Lagrangian in the previous problem is at most quadratic in the field ϕ, the effective Lagrangian after integrating out the heavy scalar field can also be derived by solving the Euler-Lagrange equation (i.e. the classical equation of motion) for ϕ. Show that the formal solution to this equation is

$$\phi = -\frac{g}{M^2 + \Box}\,\bar{\psi}\psi\,.$$

   Use this result to eliminate ϕ from the original Lagrangian, and show that this leads to the same result as the functional method in the previous problem.