Department of Physics
Tohoku University
Sendai, Japan.

《Exercises》

Sec. 1

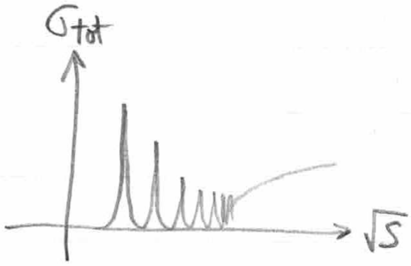
1-1) Show that the total cross section for $e^+e^- \to \gamma^* \to q\bar{q}$ is given by

$$\sigma_{tot}(e^+e^- \to \gamma^* \to q\bar{q}) = \frac{96\pi^2\alpha^2}{s^2} \cdot N_c Q_q^2 \cdot \mathrm{Im}\, G(\vec{x}=\vec{0}; E)$$

via optical theorem.

1-2) Show that the above cross section exhibits a peak structure corresponding to the bound states (resonance states).

Hint: second reference in the front page of Sec. 1.



Sec. 2

2-1) Show that

$$Z(\infty; 2,1; 1,1) \equiv \sum_{n=2}^{\infty} \sum_{k=1}^{n-1} \frac{1}{n^2 k}$$

is equal to $\zeta(3) = \sum_{n=1}^{\infty} \frac{1}{n^3}$.

This is one example to demonstrate that nested sums can often be expressed by simpler basis sums/integrals.

Hint: Show that $Z(\infty; 2,1; 1,1) = \int_0^1 dx \frac{1}{x} \int_0^x dy \frac{1}{1-y} \int_0^y dz \frac{1}{1-z}$ by expanding the integrand first. You can also find many other ways to evaluate this integral.

Sec. 3

3-1) Show that, in the leading-log approx., the perturbative series of $V_{QCD}(r)$ exhibits asymptotically factorial ($n!$) growth at high orders. Show that the minimal term is order Λ_{QCD}.

Hint: reference paper in the front page of Sec. 3.