Introduction HTL, NRQCD and pNRQCD at finite temperature. Part 1.

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FONDO EUROPEO DE DESENVOLVEMENTO REXIONAL
"Unha maneira de facer Europa"
EXCELENCIA MARÍA DE MAEZTU

Week 2

	Mon Jul 19	Tue Jul 20	Wed Jul 21	Thu Jul 22	Fri Jul 23
15:00	**Pineda**	**Pineda**	**Escobedo**	**Fodor**	**Moult**
16:30	Break	Break	Break	Break	Break
16:50	**Escobedo**	**Fodor**	**Zhao**	**Cirigliano**	**Zhao**
18:20	End	End	End	End	End

QCD

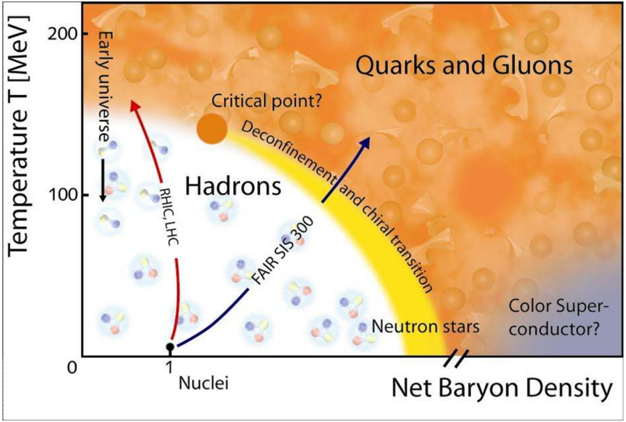
QCD is the theory of the strong interactions and it explains the origin of most of the mass observed in the universe. $m_{proton} \gg m_u, m_d$

The most relevant features of QCD are

- Confinement
- Asymptotic freedom

...or at least this is what happens in the vacuum.

QCD phase diagram

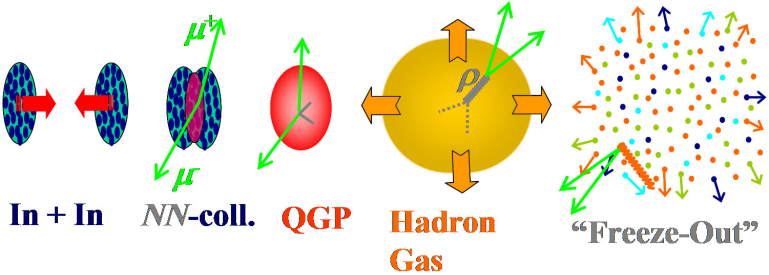


Quark gluon plasma

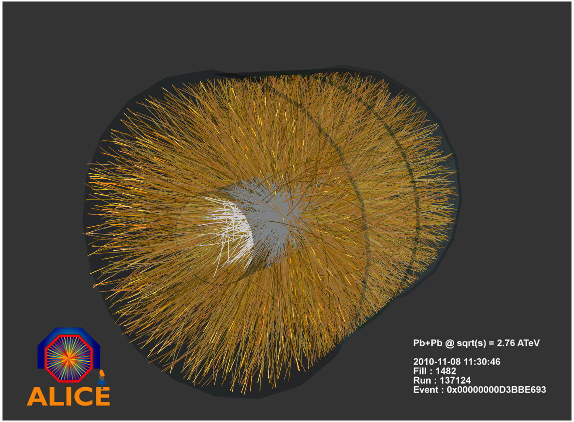
- If instead of the vacuum we consider a system with a given temperature, the mean energy of the particles is related with the temperature.
- If the temperature is very high asymptotic freedom implies that particles interact weekly, almost a free gas.
- In this way we can scape confinement. Deconfinement.
- Understanding deconfinement is a way to understand confinement. It was also a phase that was present in the early universe.

Heavy ion collisions

They are performed nowadays at LHC and RHIC



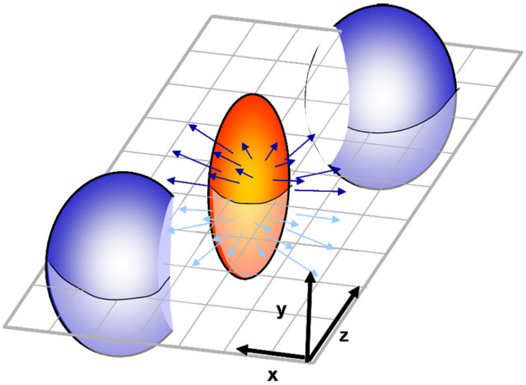
Difficulties in heavy ion collisions



Difficulties in heavy ion collisions

- We want to understand QGP.
- This is just one of the many stages that takes place in heavy ion collisions.
- At the end what we see in the detectors is the same type of particles as in proton-proton collisions, but a lot of them.

Elliptic flow in AA collisions



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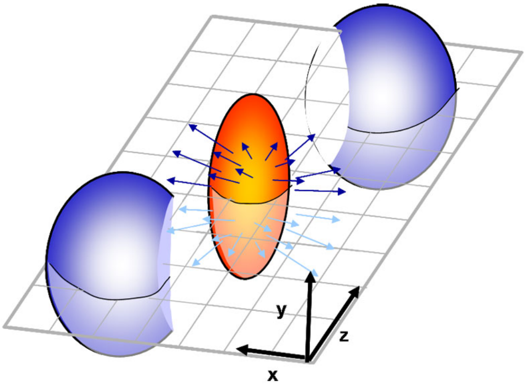

- The overlapping region has an elliptic form.

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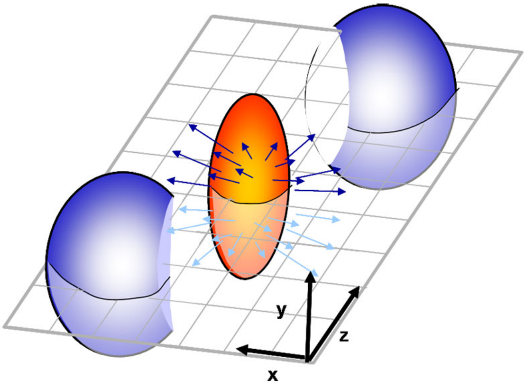

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- The geometrical asymmetry creates an asymmetry in pressure.

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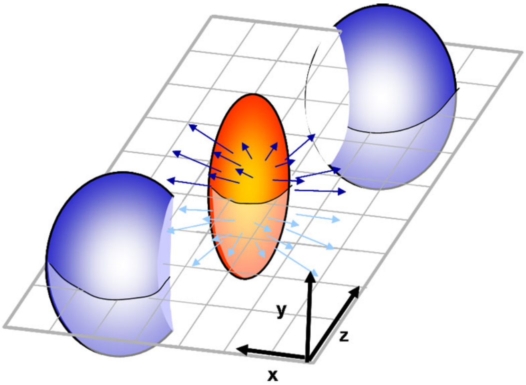

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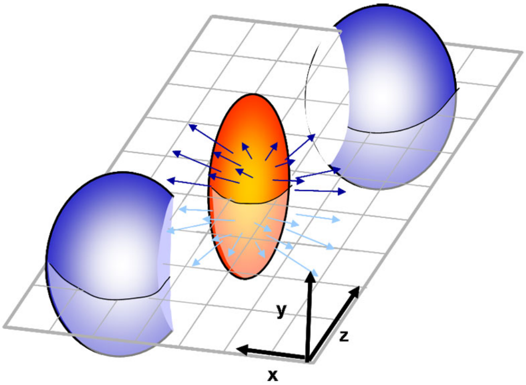
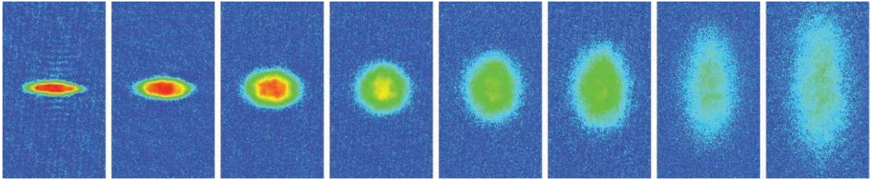


- The overlapping region has an elliptic form.
- The geometrical asymmetry creates an asymmetry in pressure.
- Pressure gradients create an asymmetry in the momenta of the detected particles.
- Can be computed using hydrodynamics. Needs as input the equation of state of QCD matter.

Elliptic flow in AA collisions

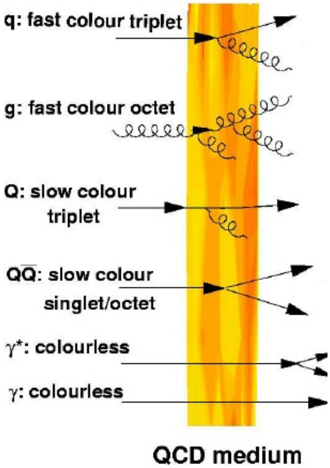
An example from unitary Fermi-gas



Hara, Hemmer, Gehm, Granade and Thomas, *Science* 13, V. 298, 2179-2182

Completely different system but the physical principle behind the expansion is the same.

Hard probes

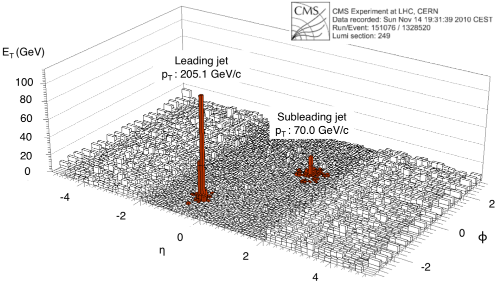


Probes that are created at the beginning of the collision (typically because its creation needs a high energy) that get modified in a substantial way and that are relatively easy to detect.

- Jet quenching.
- Heavy quark diffusion.
- Quarkonium suppression.
- Photon production.

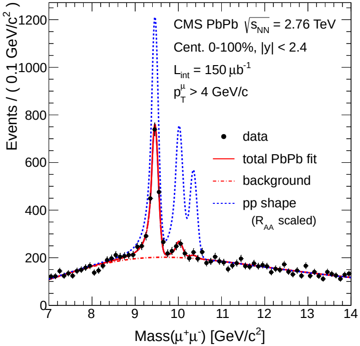
Picture taken from d'Enterria (2007)

Jet quenching



- It is a measure of the opacity of the medium to boosted particles.
- Its strength depends on a transport coefficient called $\hat{q}$.

Quarkonium suppression



- It was predicted by Matsui and Satz that quarkonium melts in heavy-ion collisions and that this is a signal of the formation of a quark-gluon plasma.
- The EFT study of this problem is the main topic of these lectures.
- One of the most interesting quantities is the nuclear modification factor, R_{AA}.

$$R_{AA} = \frac{\text{Number of quarkonia in AA collisions}}{\text{Number of nucleon-nucleon collisions} \cdot \text{Number of quarkonia in pp collisions}}$$

Motivation

- To characterize and understand the quark-gluon plasma that is created in heavy-ion collisions.
- Use the phenomenon of quarkonium suppression as a tool to extract information about the quark-gluon plasma.

Content

- Introduce Hard Thermal Loop perturbation theory as an useful tool to study the properties of the quark-gluon plasma.
- Discuss the study of quarkonium suppression using Non-relativistic EFTs.

Plan

Summary

There are two different formalisms

Outline

In this section we are going to introduce how to make QFT computations at finite temperature. There are two different formalisms:

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Imaginary time formalism

- Computation of static quantities in Euclidean space. (Partition function)
- Used in lattice QCD.
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Imaginary time formalism

- Computation of static quantities in Euclidean space. (Partition function)
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Real time formalism

- Requires considering both chronologically and antichronologically ordered fields.
- Straight-forward to generalize to out of equilibrium.

Partition function and expectation values

$$Z = Tr\left(e^{-\frac{H}{T}}\right)$$

Expectation value of the time evolution operator from time t to the complex time $t - \frac{i}{T}$.

$$\langle A \rangle = \frac{1}{Z} Tr\left(A e^{-\frac{H}{T}}\right)$$

Note that

$$\langle A\left(t - \frac{i}{T}\right)\rangle = \frac{1}{Z} Tr\left(A\left(t - \frac{i}{T}\right) e^{-\frac{H}{T}}\right) = \frac{1}{Z} Tr\left(e^{\frac{H}{T}} A(t) e^{-\frac{H}{T}} e^{-\frac{H}{T}}\right)$$
$$= \frac{1}{Z} Tr\left(A(t) e^{-\frac{H}{T}}\right) = \langle A(t) \rangle$$

Feynman rules

We use the notation $\hat{\phi}(\tau, \mathbf{x}) = \phi(-i\tau, \mathbf{x})$. We wish to compute correlators of the type

$$\langle \mathcal{T}_\tau \hat{\phi}(\tau_1)\hat{\phi}(\tau_2)\cdots\hat{\phi}(\tau_n)\rangle \,,$$

where τ_i have values between 0 and $\frac{1}{T}$ and $\mathcal{T}_\tau$ means ordering in increasing values of τ. The following Feynman rules can be used:

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- Use the same rules as in a standard QFT computation in Euclidean time.
- The energies have discrete values instead of continuous:
  - Bosons, $2\pi n T$, with n and integer.
  - Fermions, $\pi T(2n+1)$.

Dimensional reduction

What is the the physics of small energy and momentum modes?

Fermions

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Bosons

- Physics dominated by the $n = 0$ mode.
- The physics of the $n = 0$ mode can be described by an EFT in an space with one dimension less.

Electrostatic QCD

EFT for zero frequency bosonic modes with momentum smaller than T. Braaten and Nieto (1995).

$$\mathcal{L}_{EQCD} = \frac{1}{4}F^a_{ij}F^a_{ij} + \frac{1}{2}(D_iA_0)^a(D_iA_0)^a + \frac{1}{2}m_E^2 A_0^a A_0^a + \frac{1}{8}\lambda_E(A_0^a A_0^a)^2 + \cdots$$

- QCD in 3 dimensions plus a scalar field.
- $m_E \sim gT$ (electrostatic screening), $g_E = g\sqrt{T}$ (coupling constant) and $\lambda_E \sim g^4 T$.

In this EFT we have two different types of modes

- Modes with momentum of order gT. If g is small we can use perturbation theory, but the expansion parameter is $\frac{g_E^2}{m_E} \sim g$ (instead of α_s).
- Modes with momentum of order $g^2 T$. Magnetic modes.

Magnetostatic QCD

EFT valid for zero frequency bosonic modes with momentum smaller than gT. Braaten and Nieto (1995).

$$\mathcal{L}_{MQCD} = \frac{1}{4} F^a_{ij} F^a_{ij} + \cdots$$

Up to corrections, it is QCD in three dimensions.

- Super-renormalizable.
- It is a non-perturbative theory even if g is arbitrary small.
- The physics of low energy magnetic modes is non-perturbative.

The doubling of degrees of freedom

Consider the following expectation value

$$Tr(\phi(t_1)\phi(t_2)\rho)$$

Using the interaction picture (such that the interacting field coincides with the Heisenberg field at $t = -\infty$)

$$Tr(U^\dagger(t_1, -\infty)\phi_I(t_1)U(t_1, t_2)\phi_I(t_2)U(t_2, -\infty)\rho)$$

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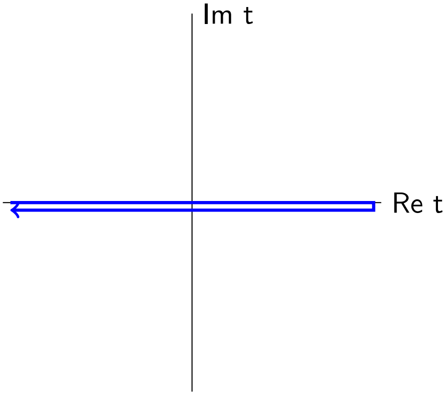
- There are both chronologically ordered and anti-chronologically ordered fields.
- In $T = 0$ collinear physics, the LSZ reduction formula ensures that all the physics is encoded in time-ordered correlators. At finite T, this is not the case.

The Schwinger-Keldysh contour



Instead of considering chronologically and anti-chronologically ordered operators, we introduce the concept of ordering along the Schwinger-Keldish contour

- The upper(lower) branch is chronologically(anti-chronologically) ordered.
- At $t = t_i$ (with $t_i \to -\infty$ if we consider adiabatic switching) we have the density matrix. In the case of thermal equilibrium, we can represent it with the vertical line from t_i to $t_i - i/T$.

The two-point correlators

The consider the partition function and generating functional

$$Z = Tr\left(\bar{\mathcal{T}}\left(\mathrm{e}^{i\int_{-\infty}^{\infty} dt(H(t)+j_2(t)\phi(t))}\right)\mathcal{T}\left(\mathrm{e}^{-i\int_{-\infty}^{\infty} dt(H(t)+j_1(t)\phi(t))}\right)\rho(-\infty)\right)$$

Then

$$Tr(\mathcal{T}\phi(t_1)\phi(t_2)) = -\left.\frac{\partial^2 Z}{\partial j_1(t_1)\partial j_1(t_2)}\right|_{j_1=j_2=0}$$

$$Tr(\bar{\mathcal{T}}\phi(t_1)\phi(t_2)) = -\left.\frac{\partial^2 Z}{\partial j_2(t_1)\partial j_2(t_2)}\right|_{j_1=j_2=0}$$

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- Even if we want to compute the time-ordered correlator, at one loop we need to consider fields in the lower branch of the contour.
- The previous rules are fulfilled in thermal equilibrium and Gaussian states.

Propagator of a scalar field in thermal equilibrium

$$\Delta(p) = \begin{pmatrix} \Delta_{11}(p) & \Delta_{12}(p) \\ \Delta_{21}(p) & \Delta_{22}(p) \end{pmatrix}$$

where

$$\Delta_{11}(p) = \frac{i}{p^2 - m^2 + i\epsilon} + 2\pi\delta(p^2 - m^2)n_B(|p_0|)$$

$$\Delta_{22}(p) = (\Delta_{11}(p))^*$$

$$\Delta_{12}(p) = 2\pi(\theta(-p_0) + n_B(|p_0|))\delta(p^2 - m^2)$$

$$\Delta_{21}(p) = \Delta_{12}(-p)$$

Keldysh representation

The following relation is fulfilled by definition

$$\Delta_{11} + \Delta_{22} = \Delta_{12} + \Delta_{21}$$

It is convenient to define a new set of fields which exploits this

$$\Delta_R = \Delta_{11} - \Delta_{12} = \frac{i}{p^2 - m^2 + isgn(p_0)\epsilon}$$
$$\Delta_A = \Delta_{11} - \Delta_{21} = -(\Delta_R)^*$$
$$\Delta_S = \Delta_{11} + \Delta_{22} = 2\pi\delta(p^2 - m^2)(1 + 2n_B(|p_0|))$$

which are called respectively retarded, advanced and symmetric propagator. Note that, at tree level, only the symmetric propagator depends on the temperature.

Keldysh representation II

$$\Delta_R(t) = \theta(t)Tr([\phi(t), \phi(0)]\rho)$$
$$\Delta_A(t) = -\theta(-t)Tr([\phi(t), \phi(0)]\rho)$$
$$\Delta_S(t) = Tr(\{\phi(t), \phi(0)\}\rho)$$

In the limit in which ϕ is a classical field the symmetric propagator dominates over the others.

Keldysh representation III

$$\Delta_{11} = \frac{1}{2}(\Delta_S + \Delta_A + \Delta_R)$$
$$\Delta_{12} = \frac{1}{2}(\Delta_S + \Delta_A - \Delta_R)$$
$$\Delta_{21} = \frac{1}{2}(\Delta_S - \Delta_A + \Delta_R)$$
$$\Delta_{22} = \frac{1}{2}(\Delta_S - \Delta_A - \Delta_R)$$

In the classical limit, the symmetric propagator dominates. Therefore, all time orderings give the same result.

Keldysh representation for self-energies

$$\Pi_R = \Pi_{11} + \Pi_{12}$$
$$\Pi_A = \Pi_{11} + \Pi_{21}$$
$$\Pi_S = \Pi_{11} + \Pi_{22}$$

Fluctuation-dissipation theorem

In thermal equilibrium

$$\Delta_{12}(p) = \Delta_{21}(-p) = -\mathrm{e}^{-\frac{p_0}{T}}\Delta_{21}(p)$$

this implies that

$$\Delta_S(p) = (1 + 2n_B(|p_0|))sgn(p_0)(\Delta_R(p) - \Delta_A(p))$$

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Consequence

In the limit $p_0 \ll T$ Bose-Einstein condensation takes place. The symmetric propagator is enhanced by a factor $\frac{T}{|p_0|}$ and the field is approximately classical.

A practical one loop computation

1. Compute the retarded propagator

$$\Delta_R = \Delta_{0,R} + \Delta_{0,R}\Pi_R\Delta_R$$

2. Use $\Delta_A = -(\Delta_R)^*$
3. Use the fluctuation-dissipation theorem to obtain Δ_S.
4. Having the retarded, advanced and symmetric propagator we can compute any time-ordering.

The case of fermions

$$S(p) = (p_\mu \gamma^\mu + m)\tilde{\Delta}(p)$$

where $\tilde{\Delta}$ is analogue to Δ but changing n_B to $-n_F$.

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Pauli exclusion principle instead of Bose enhancement. The symmetric propagator of low energy fermions is suppressed.

Summary and key ideas

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- The retarded propagator encodes the information about the spectral properties, how the energy depends on the momentum and the decay width. This is true also beyond tree level.
- The perturbative expansion takes a simple form for the retarded propagator.
- In thermal equilibrium, having the retarded propagator allows to obtain all the time-orderings.
- Low energy bosonic fields are approximately classical because the symmetric propagator dominates.

Plan

Degrees of freedom in a perturbative quark-gluon plasma

- Particles with energy of order πT.
  - Since $\alpha_s(\pi T) \ll 1$, perturbation theory works well for these particles.
  - Tree level is a reasonable approximation. Free streaming particles with rare collisions.
- Particles with energy of order gT.
  - From EQCD, we know that thermal effects are leading order for these particles.
  - Bosonic degrees of freedom dominate and are classical up to corrections of order g.
- Particles with energy of order $g^2 T$.
  - From MQCD, non-perturbative degrees of freedom even if g is small.
  - Bosonic degrees of freedom dominate and are classical up to corrections of order g^2.

Physical picture

- Hard particles, with energy of order T. Approximately point like particles that could be modelled with a Boltzmann equation.

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- Hard particles, with energy of order T. Approximately point like particles that could be modelled with a Boltzmann equation.
- Soft particles, with energy of order gT. At leading order we can ignore fermions and focus on the classical part of the bosonic degrees of freedom.
- Taking this into account. The Vlasov equation should provide a good qualitative picture of the physics of soft particles.

The Vlasov equation

Consider a system of electrons interacting with a classical electromagnetic field. The qualitative behaviour of the electromagnetic field is similar to that of the soft gluonic field in a quark-gluon plasma.

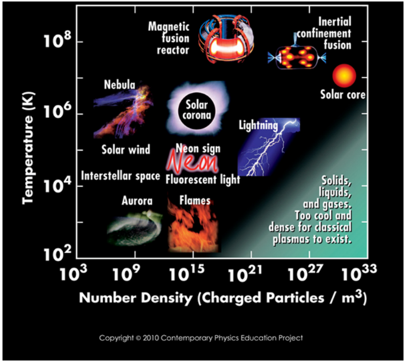
$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{x}} + e(\mathbf{E} + \mathbf{v} \times \mathbf{B})\frac{\partial f}{\partial \mathbf{k}} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{B} = \mathbf{j}_{cl} + \frac{\partial \mathbf{D}}{\partial t} \qquad \nabla \cdot \mathbf{D} = \rho_{cl}$$

with $\mathbf{D} = \mathbf{E} + \mathbf{P}$.

Plasma physics



- The Vlasov equation is used to study plasma physics.
- Many of the phenomena that are seen in electromagnetic plasmas also appear in the quark-gluon plasma.
- We can see Hard Thermal Loops as an EFT of QED or QCD that gives a quantum description of plasma phenomena.

Vlasov equation:solution strategy

1. Take a given value of $\mathbf{E}$ and $\mathbf{B}$.

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4. Obtain the induced current and charge from δf.
5. Obtain the polarization, **P**, and the displacement, **D**, using $\mathbf{D} = \mathbf{E} + \mathbf{P}$.

Obtaining f

Start expanding around thermal equilibrium

$$f = f^0 + f^1 + \cdots$$

then the Vlasov equation takes the form

$$\frac{\partial f^1}{\partial t} + \mathbf{v}\frac{\partial f^1}{\partial \mathbf{x}} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B})\frac{\partial f^0}{\partial \mathbf{k}}$$

Due to isotropy, the magnetic field does not contribute. The solution, with the condition that the fields vanish in the distant past, is

$$f^1(\mathbf{x}, \mathbf{k}, t) = -e\frac{df^0}{dk}\int_{-\infty}^{t} dt' \frac{\mathbf{k}}{k}\mathbf{E}\left(t', \mathbf{x} - \frac{\mathbf{k}}{k}(t - t')\right) \mathrm{e}^{\epsilon t'}$$

Obtaining the induced current

$$\mathbf{j}_{ind}(t,\mathbf{x}) = e\int \frac{d^3k}{(2\pi)^3}\frac{\mathbf{k}}{k}f^1(\mathbf{x},\mathbf{k},t)$$

therefore,

$$\mathbf{j}_{ind}(t,\mathbf{x}) = -e^2\int \frac{d^3k}{(2\pi)^3}\frac{\mathbf{k}}{k}\frac{df^0}{dk}\int_0^\infty dt'\frac{\mathbf{k}}{k}\mathbf{E}\left(t-\tau,\mathbf{x}-\frac{\mathbf{k}}{k}\tau\right)\mathrm{e}^{-\epsilon\tau}$$

Obtaining the induced current II

In momentum space

$$\mathbf{j}(q_0,\mathbf{q}) = ie^2 m_D^2 \int \frac{d\Omega}{4\pi} \frac{\mathbf{k} k_i E_i(q_0,\mathbf{q})}{k^2\left(\left(q_0 - \frac{\mathbf{q}\mathbf{k}}{k} + i\epsilon\right)\right)}$$

where

$$m_D^2 = -\int \frac{k^2 dk}{2\pi^2} \frac{df^0}{dk}$$

Obtaining the polarization and the displacement

From

$$\frac{\partial \mathbf{P}}{\partial t} = \mathbf{j}_{ind}$$

we get in momentum space $\mathbf{P} = \frac{i}{q_0}\mathbf{j}_{ind}$. Then $\mathbf{D} = \mathbf{E} + \frac{i}{q_0}\mathbf{j}_{ind}$.
The dielectric tensor is defined as $D_i = \epsilon_{ij} E_j$, then

$$\epsilon_{ij} = \delta_{ij} - \frac{m_D^2}{q_0} \int \frac{d\Omega}{4\pi} \frac{k_i k_j}{k^2 \left(q_0 - \frac{\mathbf{qk}}{k} + i\epsilon \right)}$$

The dielectric tensor

Due to symmetry considerations, the dielectric tensor can be split into a longitudinal and a transverse component

$$\epsilon_{ij} = \epsilon_T \left(\delta_{ij} - \frac{q_i q_j}{q^2}\right) + \epsilon_L \frac{q_i q_j}{q^2}$$

we obtain

$$\epsilon_T = 1 - \frac{m_D^2}{2q_0^2}\left(x^2 + \frac{x(1-x^2)}{2}\log\left(\frac{x+1+i\epsilon}{x-1+i\epsilon}\right)\right)$$

$$\epsilon_L = 1 - \frac{m_D^2}{q^2}\left(\frac{x}{2}\log\left(\frac{x+1+i\epsilon}{x-1+i\epsilon}\right) - 1\right)$$

where $x = \frac{q_0}{q}$.

Landau damping

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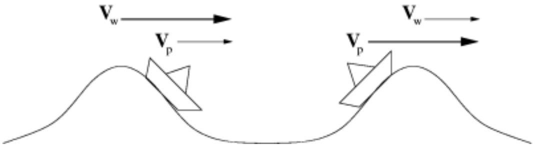
- This does not imply that the electric field in coordinate space is complex.
- The complex nature of the dielectric tensor in momentum space is a signal of dissipation. There is a exchange of energy between the particles and the electromagnetic field in the plasma.
- In next lecture, we are going to discuss the effect of Landau damping in quarkonium suppression.

Landau damping, physical interpretation



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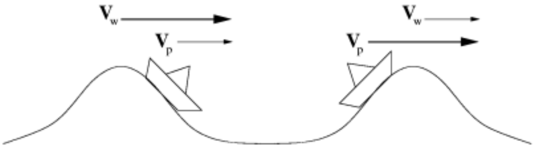

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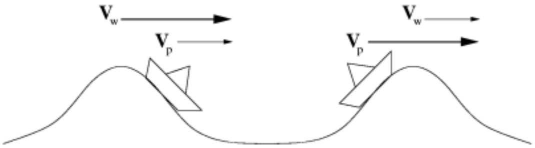

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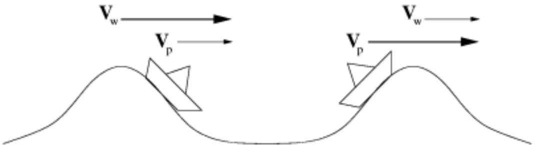

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- The particles lose or gain energy depending on their momentum.
- Picture taken from Laundau damping's page on wikipedia.

Debye screening

Exercise

What is the electric field created by a static point-like charge?

$$\nabla \mathbf{D} = \epsilon_L \nabla \mathbf{E} = \rho_{cl}$$

we get

$$\mathbf{E} = -\alpha \nabla \left(\frac{e^{-m_D r}}{r} \right)$$

The field created by a static charge is screened at distances larger than the Debye radius $r_D \sim \frac{1}{m_D}$. This phenomenon can lead to the dissociation of quarkonium.

The plasmon

Consider the case in which there are no external sources. It must be fulfilled that

$$\nabla \mathbf{D} = \nabla(\epsilon_L \mathbf{E}) = 0$$

either $\mathbf{E} = 0$ or $\epsilon_L = 0$.

The plasmon

Consider the case in which there are no external sources. It must be fulfilled that

$$\nabla \mathbf{D} = \nabla(\epsilon_L \mathbf{E}) = 0$$

either $\mathbf{E} = 0$ or $\epsilon_L = 0$.

- The $\epsilon_L = 0$ condition defines a relation (dispersion relation) between q_0 and q.
- There exists a new propagating mode that is not present in the vacuum, the plasmon.

Transverse waves

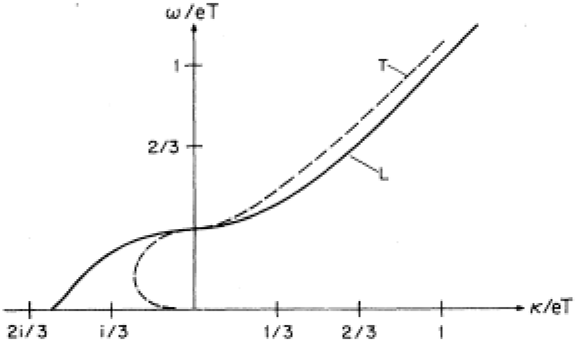
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{B} = \frac{\partial \mathbf{D}}{\partial t} \quad \nabla \cdot \mathbf{D} = 0$$

Assuming that **E** is transverse, we get a wave equation

$$q^2 \mathbf{E} = \epsilon_T q_0^2 \mathbf{E} \quad q^2 \mathbf{B} = \epsilon_T q_0^2 \mathbf{B}$$

Plasmon and transverse wave dispersion relation

Picture taken from Weldon (1982)



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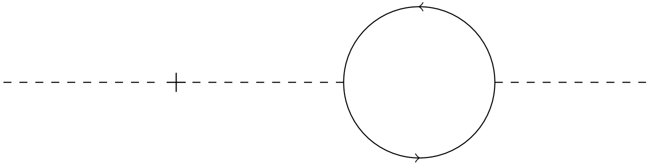
- The computation of the gluon propagator is quite similar. However, there we would need to consider also gluon and ghost loops.
- The procedure was already outlined before:
    - Compute the retarded self-energy, Π_R.
    - Compute the retarded propagator, Δ_R, from the self-energy.
    - Obtain the advanced and symmetric propagator using the fluctuation-dissipation theorem.

The temporal photon propagator, Coulomb gauge



$$\frac{i}{q^2+i\epsilon}+\frac{i}{q^2+i\epsilon}i\Pi_{00}(q_0,\mathbf{q})\frac{i}{q^2+i\epsilon}\sim\frac{i}{q^2+\Pi_{00}(q_0,\mathbf{q})+i\epsilon}$$

Π_{00}, power counting

- Π_{00} can be divided into a vacuum part and a thermal part.

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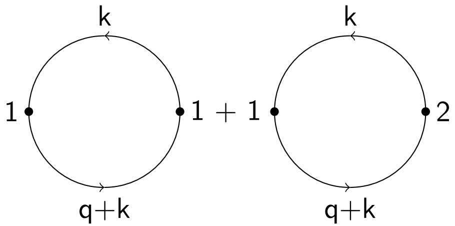
Π_{00}, power counting

- Π_{00} can be divided into a vacuum part and a thermal part.
- If $q \gg T$ the thermal part is suppressed by Boltzmann factors, $\mathrm{e}^{-\frac{p}{T}}$.
- If $q \sim T$, Π_{00} is of size $e^2 T^2$.
- If $q \ll T$, Π_{00} is still of size $e^2 T^2$. Therefore, if the momentum q is of order eT, the self-energy is not a perturbation any more and must be resummed.

Schwinger-Keldysh structure of Π_R



$$\propto \tilde{\Delta}_{11}(k)\tilde{\Delta}_{11}(q+k) - \tilde{\Delta}_{21}(k)\tilde{\Delta}_{12}(q+k)$$

Computation of Π_R

- We are interested in the thermal part.
- At tree level, only the symmetric propagator depends on the medium

$$\frac{1}{4}\Big[\tilde{\Delta}_S(k)(\tilde{\Delta}_A(q+k)+\tilde{\Delta}_R(q+k))+(\tilde{\Delta}_R(k)+\tilde{\Delta}_A(k))\tilde{\Delta}_S(q+k)$$

$$-\tilde{\Delta}_S(k)(\tilde{\Delta}_A(q+k)-\tilde{\Delta}_R(q+k))-(-\tilde{\Delta}_A(k)+\tilde{\Delta}_R(k))\tilde{\Delta}_S(q+k)\Big]$$

$$=\frac{1}{2}\Big[\tilde{\Delta}_S(k)\tilde{\Delta}_R(q+k)+\tilde{\Delta}_A(k)\tilde{\Delta}_S(q+k)\Big]$$

Using the change of variable $k'=-q-k$

$$\tilde{\Delta}_S(k)\tilde{\Delta}_R(q+k)$$

Computation of Π_R II

$$\int \frac{d^4k}{(2\pi)^4} f(k^\mu, q^\mu)\tilde{\Delta}_S(k)\tilde{\Delta}_R(k+q) \sim$$

$$\int \frac{d^4k}{(2\pi)^4} f(k^\mu, q^\mu)\frac{2\pi\delta(k_0^2-k^2)}{e^{\frac{|k_0|}{T}}+1}\frac{i}{(q_0+k_0)^2-(\mathbf{q}+\mathbf{k})^2+isgn(q_0+k_0)\epsilon}$$

In the second line we have neglected the terms that do not depend on the temperature. We can use

$$\delta(k_0^2-k^2)=\frac{1}{2|k_0|}(\delta(k_0+k)+\delta(k_0-k))$$

Computation of Π_R III

$$\frac{1}{2}\int\frac{d^3k}{(2\pi)^3}\frac{1}{k(e^{\frac{k}{T}}+1}\left[f(k^\mu,q^\mu)|_{k_0=k}\frac{i}{q_0^2-q^2+2kq_0-2\mathbf{kq}+isgn(q_0+k)\epsilon}\right.$$
$$\left.+f(k^\mu,q^\mu)|_{k_0=-k}\frac{i}{q_0^2-q^2-2kq_0-2\mathbf{kq}+isgn(q_0-k)\epsilon}\right]$$

HTL limit

$$\frac{i}{q_0^2-q^2+2kq_0-2\mathbf{kq}+isgn(q_0+k)\epsilon}\sim\frac{1}{2k}\frac{i}{q_0-q\cos\theta+i\epsilon}-\frac{q_0^2-q^2}{4k^2}\frac{i}{(q_0-q\cos\theta+i\epsilon)^2}$$

where θ is the angle between $\mathbf{q}$ and $\mathbf{k}$

Computation of Π_R IV

We need to compute integrals of the type

$$\int_0^{\pi} \frac{d\theta \cos^n \theta}{(q_0 - q\cos\theta + i\epsilon)^m}$$

and

$$\int_0^{\infty} \frac{dkk}{e^{\frac{k}{T}} + 1} = \frac{\pi^2}{12}$$

Final result

$$\Pi_{00,R}(q_0, q) = m_D^2 \left(1 - \frac{q_0}{2q} \log\left(\frac{q_0 + q + i\epsilon}{q_0 - q + i\epsilon}\right)\right)$$

Δ_R

$$\Delta_{00,R}(q_0,q) = \frac{i}{q^2\epsilon_L(q_0,q) + isgn(q_0)\epsilon}$$

As we discussed before, $\epsilon_L = 0$ corresponds to poles in the temporal photon propagator, a plasmon.
Note that $\Delta_A(q) = -(\Delta_R(q))^*$.

Δ_S, fluctuation-dissipation theorem

$$\Delta_{00,S}(q_0,q) = (1+2n_B(|q_0|))sgn(q_0)Re\Delta_{00,R}(q_0,q) \sim \frac{2T}{q_0}Re\Delta_{00,R}(q_0,q)$$

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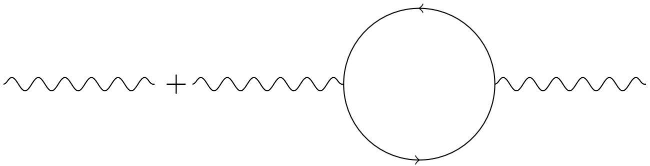
Observations

- Since $\frac{T}{q_0}$ is of size $\frac{1}{e}$. In the HTL limit, the symmetric propagator is larger than the retarded and advanced.
- Δ_R has a branch cut for space-like momentum, $q_0 < q$. ϵ_L is complex due to the phenomenon of Landau damping.
- Δ_R has plasmon poles for time-like momentum for q_0 such that $\epsilon_L(q_0,q) = 0$. This leads to a Dirac delta in Δ_S.
- As a function of q_0, Δ_S is a continuum function from 0 to q that goes to zero when $q_0 = q$. It has a Dirac delta for a value of q_0 bigger than q.

The transverse photon propagator



$$\frac{i\left(\delta_{ij}-\frac{q_iq_j}{q^2}\right)}{q_0^2-q^2+i\epsilon}+\frac{i\left(\delta_{ik}-\frac{q_iq_k}{q^2}\right)}{q_0^2-q^2+i\epsilon}i\Pi_{kl}\frac{i\left(\delta_{lj}-\frac{q_lq_j}{q^2}\right)}{q_0^2-q^2+i\epsilon}$$

The transverse photon propagator is sensitive to Π_T, the projection of Π_{ij} to the subspace transverse to $\mathbf{q}$.

Transverse photon retarded propagator

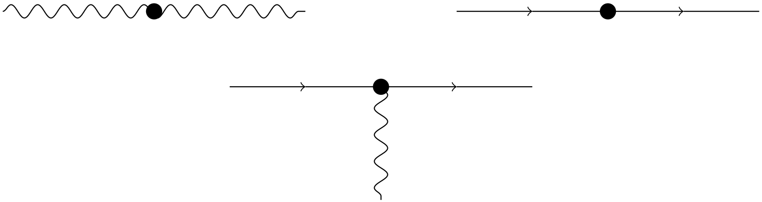
The computation can be done similarly to the case of the temporal photon

$$\Delta_{ij,R} = \frac{i\left(\delta_{ij} - \frac{q_i q_j}{q^2}\right)}{q_0^2 \epsilon_T(q_0, q) - q^2 + i sgn(q_0)}$$

As expected from our previous analysis of the Vlasov equation, $\epsilon_T = \frac{q^2}{q_0^2}$ corresponds to poles in the propagator. ϵ_T has an imaginary part for space-like momentum.

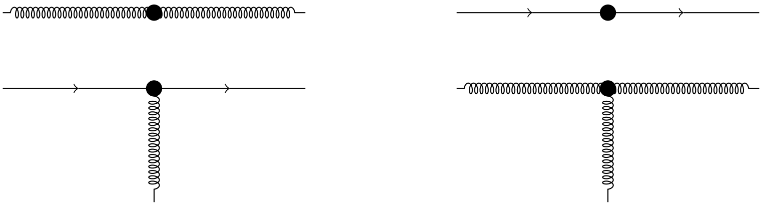
HTL in QED

Apart from the photon propagator, there are other processes in QED for which the HTL resummation needs to be performed when all external particles are soft.



Resummation is needed for any process involving 2 electrons and an arbitrary number of photons.

HTL in QCD



In QCD, resummation is needed for processes involving two quarks and any number of gluons and for processes involving any number of gluons.

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- However, we can write an effective action in the imaginary time formalism. From the Euclidean correlator we can obtain the retarded correlator and, using the fluctuation-dissipation theorem, any time-ordering

$$\Delta_R(q_0) = -i\Delta_{\mathcal{E}}(q_0 + i\epsilon)$$

with $\Delta_{\mathcal{E}}(iw_n)$ the euclidean correlator.

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  with $\Delta_{\mathcal{E}}(iw_n)$ the euclidean correlator.
- The effective action is Gauge invariant, invariant under spatial translations and rotations. Lorentz invariance is broken by the presence of the medium.

HTL effective action II

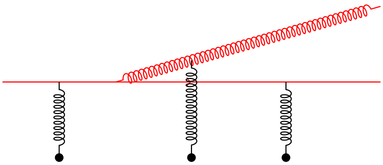
Gluons

$$\delta\mathcal{L}_g = -m^2\, Tr \int \frac{d\Omega}{4\pi} F_{\mu\alpha} \frac{k^\alpha k^\beta}{(kD)^2} F^\mu_\beta$$

Quarks

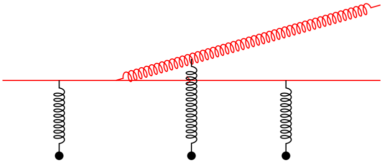
$$\delta\mathcal{L}_q = i m_f^2 \bar{\psi} \int \frac{d\Omega}{4\pi} \frac{\not{k}}{kD} \psi$$

Jet broadening



- High energy partons lose energy when traversing a medium.
- Radiative energy loss. The parton gains virtuality due to medium interaction and this induces the emission of high energy gluons.
- The interaction of the medium with a parton with a large p^+ momentum (light-cone coordinates) is local in the x^- coordinate and can be encoded in a so-called scattering potential.

Jet broadening II



$$V_{HTL}(x_\perp) = 2\alpha_s C_A T \left[K_0(m_D x_\perp) + \log\left(\frac{m_D x_\perp}{2}\right) + \gamma_E\right]$$

For a recent discussion, check Barata, Mehtar-Tani, Soto-Ontoso and Tywoniuk (2021).

Equation of state for neutron stars

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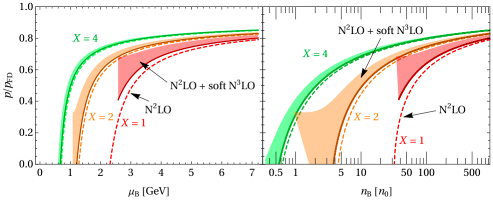
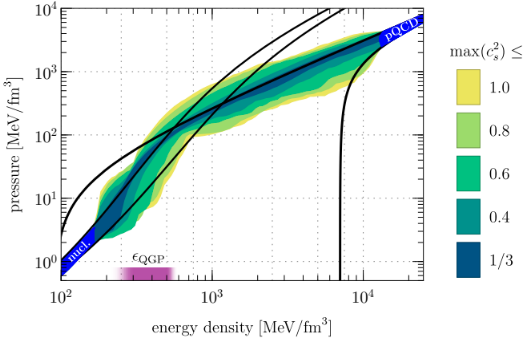
- This is the case of the nuclear matter in neutron stars.
- Gravitational waves are sensitive to the equation of state.
- HDL expansion gives reliable results for high energy densities.

Equation of state for neutron stars II



Picture taken from Gorda, Kurkela, Paatelainen, Säppi and Vuorinen (2021).



Picture taken from Annala, Gorda, Kurkela, Nättilä and Vuorinen (2020)

Thanks!