Introduction to Effective Field Theories

Matthias Neubert (matthias.neubert@uni-mainz.de)
Cluster of Excellence PRISMA+ and
Mainz Institute for Theoretical Physics (MITP)
Johannes Gutenberg University Mainz

Cluster of Excellence PRISMA+
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mitp Mainz Institute for Theoretical Physics

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Lecture III: Applications of Effective Field Theory

Low-energy properties of the Standard Model

Many aspects of particle physics at energies below the electroweak scale can be understood by studying the EFT obtained by integrating out some heavy SM particles (e.g. the top quark, the electroweak gauge bosons W and Z, and the Higgs boson)

We have already discussed the example of the effective weak interactions at low energies (Fermi theory) in some detail

As a second interesting example, we now briefly consider Higgs-boson production at hadron colliders

Higgs production at the LHC

The protons that collide in the LHC contain only light quarks (*up*, *down*, and a little bit of *strange*), which in the SM have negligible couplings to the Higgs boson, and gluons, which do not couple to the Higgs boson at all

How, then, is the Higgs boson produced in pp collisions at the LHC?

Higgs production at the LHC

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How, then, is the Higgs boson produced in pp collisions at the LHC?

We can gain insight by exploiting the fact that the Higgs boson is lighter than the top quark

We can then construct an effective low-energy theory for Higgs physics, in which the top quark is integrated out

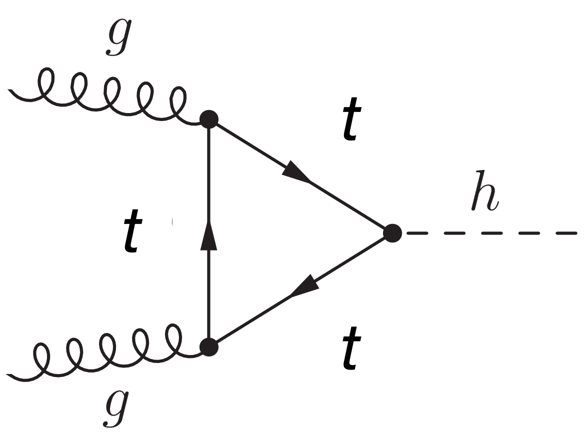
This EFT must be built in the broken phase of the electroweak symmetry

Higgs production at the LHC

In this effective low-energy theory, direct couplings of the Higgs boson to pairs of gluons and photons arise at the level of **irrelevant dimension-5 operators,** with coefficients that scale like $1/m_t$, e.g.:

$$\mathcal{L}_{hgg} = \frac{y_t}{\sqrt{2}m_t}\,\frac{\alpha_s}{12\pi}\,h\,G^a_{\mu\nu}G^{\mu\nu,a} = \frac{\alpha_s}{12\pi}\,\frac{h}{v}\,G^a_{\mu\nu}G^{\mu\nu,a}$$

and similarly for the Higgs-photon coupling. These operators appear first at one-loop order, via the exchange of a virtual top-quark



The **effective *hgg* interaction** provides the dominant production mechanism for the Higgs boson in the "gluon-gluon fusion process" at the LHC

Standard Model as an effective field theory

Many interesting insights can be gained by considering the Standard Model (SM) as a low-energy effective theory of some more fundamental theory (supersymmetry?, extra dimensions?, new strongly coupled physics?, GUT?, …)

We will denote the **scale of New Physics** by Λ; this could be as large as 10^{16} GeV for some applications, but as small as 10^{3} GeV for others

The SM Lagrangian should then be extended to an effective Lagrangian, which besides the SM terms contains additional, **irrelevant operators**

These operators respect the symmetries of the SM (gauge invariance, Lorentz symmetry) but are otherwise unrestricted

Standard Model as an effective field theory

We will discuss a couple of interesting aspects of SM physics from the perspective of this construction:

- SMEFT up to dimension-6 order
- neutrino masses and the see-saw mechanism
- anomalous magnetic moment of the muon
- conservation of baryon and lepton numbers (accidental symmetries)
- proton decay

Standard Model Effective Field Theory (SMEFT)

As early as in 1985, Buchmüller and Wyler constructed the EFT extension of the SM up to dimension-6 order

The building blocks are covariant derivatives, (dual) field-strength tensors for the three gauge groups $SU(3)_c \times SU(2)_L \times U(1)_Y$, fermion bilinears, and the Higgs doublet

The most non-trivial task is finding a minimal basis of linearly independent operators, since operators can be related in non-trivial ways via the equations of motion, integration by parts, Fierz identities, and the Bianchi identity

The minimal basis ("Warsaw basis": Grzadkowski, Iskrzyński, Misiak and Rosiek, 2010) contains **one dimension-5 operator** and **63 dimension-6 operators** (barring flavor indices)

SMEFT at dimension-5 order

Remarkably, only a single operator built out of SM fields can be constructed at dimension-5 order:

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm SM}^{(4)} + \frac{1}{\Lambda}\sum_k C_k^{(5)} Q_k^{(5)} + \frac{1}{\Lambda^2}\sum_k C_k^{(6)} Q_k^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right)$$

$$Q_{\nu\nu} = \varepsilon_{jk}\varepsilon_{mn}\varphi^j\varphi^m (l_p^k)^T C l_r^n \equiv (\widetilde{\varphi}^\dagger l_p)^T C (\widetilde{\varphi}^\dagger l_r)$$

This operator:

- **violates lepton number** by two units
- gives **non-zero neutrino masses** and mixings after EWSB
- is crucially important to exist, given the experimental evidence for neutrino masses and mixings!

Neutrino masses

The discovery of non-zero neutrino masses is often described as a departure from the SM

But this is no longer true if we consider the SM as an effective low-energy theory

Without a right-handed neutrino (which indeed is not part of the SM), it is impossible to write a neutrino mass term at the level of relevant or marginal operators

However, **it is possible** to write a gauge-invariant neutrino mass term at the level of **irrelevant operators** of dimension ≥ 5:

$$\mathcal{L}_{\text{neutrino mass}} = \frac{g}{M}\left(\tilde{l}_L^T \Phi^*\right) C \left(\tilde{\Phi} l_L\right)$$

Neutrino masses

After electroweak symmetry breaking, this gives rise to a Majorana mass term of the form:

$$\mathcal{L}_{\text{neutrino mass}} = -\frac{v^2 g}{2M}\,\tilde{\nu}_L^T C\,\nu_L$$

The SM as an effective field theory **predicted** that neutrinos should be massive, with $m_\nu \sim v^2/M$ suppressed by the fundamental scale of some new physics

Experiments hint at the fact that the fundamental scale relevant for the generation of neutrino masses may be very heavy,

$$M \sim 10^{14}\ \text{GeV}$$

which is not far from the scale of grand unification

Neutrino masses and the see-saw mechanism

Extensions of the SM containing **heavy, right-handed neutrinos** (with masses that are naturally of order *M*) provide explicit examples of fundamental theories which yield such a Majorana mass term when the heavy, right-handed neutrinos are integrated out **(see-saw mechanism)**

SMEFT at dimension-6 order

At dimension-6 order, operators can contain zero, two or four fermion fields:

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{SM}}^{(4)} + \frac{1}{\Lambda}\sum_k C_k^{(5)} Q_k^{(5)} + \frac{1}{\Lambda^2}\sum_k C_k^{(6)} Q_k^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right)$$

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				

$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I \varphi)(\bar l_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde\varphi\, G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r)\tau^I \widetilde\varphi\, W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I \varphi)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r)\varphi\, G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar u_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r)\tau^I \varphi\, W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar d_p \gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu\varphi)(\bar u_p \gamma^\mu d_r)$

Operators containing no or two fermion fields

$(\bar LL)(\bar LL)$		$(\bar RR)(\bar RR)$		$(\bar LL)(\bar RR)$	
Q_{ll}	$(\bar l_p \gamma_\mu l_r)(\bar l_s \gamma^\mu l_t)$	Q_{ee}	$(\bar e_p \gamma_\mu e_r)(\bar e_s \gamma^\mu e_t)$	Q_{le}	$(\bar l_p \gamma_\mu l_r)(\bar e_s \gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar q_s \gamma^\mu q_t)$	Q_{uu}	$(\bar u_p \gamma_\mu u_r)(\bar u_s \gamma^\mu u_t)$	Q_{lu}	$(\bar l_p \gamma_\mu l_r)(\bar u_s \gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar q_p \gamma_\mu \tau^I q_r)(\bar q_s \gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar d_p \gamma_\mu d_r)(\bar d_s \gamma^\mu d_t)$	Q_{ld}	$(\bar l_p \gamma_\mu l_r)(\bar d_s \gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar l_p \gamma_\mu l_r)(\bar q_s \gamma^\mu q_t)$	Q_{eu}	$(\bar e_p \gamma_\mu e_r)(\bar u_s \gamma^\mu u_t)$	Q_{qe}	$(\bar q_p \gamma_\mu q_r)(\bar e_s \gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar l_p \gamma_\mu \tau^I l_r)(\bar q_s \gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar e_p \gamma_\mu e_r)(\bar d_s \gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar u_s \gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar u_p \gamma_\mu u_r)(\bar d_s \gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar q_p \gamma_\mu T^A q_r)(\bar u_s \gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar u_p \gamma_\mu T^A u_r)(\bar d_s \gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar d_s \gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar q_p \gamma_\mu T^A q_r)(\bar d_s \gamma^\mu T^A d_t)$

$(\bar LR)(\bar RL)$ and $(\bar LR)(\bar LR)$		B-violating	
Q_{ledq}	$(\bar l_p^j e_r)(\bar d_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar q_p^j u_r)\varepsilon_{jk}(\bar q_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar q_p^j T^A u_r)\varepsilon_{jk}(\bar q_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar l_p^j e_r)\varepsilon_{jk}(\bar q_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar l_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar q_s^k \sigma^{\mu\nu} u_t)$		

Operators containing four fermion fields

SMEFT at dimension-6 order: no or two fermions

Dimension-6 operators containing no or two fermions give rise to:

- anomalous gauge couplings (3- and 4-boson vertices)
- modifications of the Higgs potential
- modified Higgs couplings to fermions
- modified Higgs couplings to gauge bosons
- oblique corrections (EWPTs)
- anomalous dipole moments
- new sources of CP violation

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
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Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
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$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I \varphi)(\bar l_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde\varphi\, G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r)\tau^I \widetilde\varphi\, W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I \varphi)(\bar q_p \tau^I \gamma^\mu q_r)$
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SMEFT at dimension-6 order: four fermions

Dimension-6 operators containing four fermions give rise to new contact interactions:

- flavor-conserving 4-fermion interactions
- flavor-changing 4-fermion interactions involving quarks or leptons (or both)
- new sources of CP violation
- processes violating baryon and/or lepton number
- …

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
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		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
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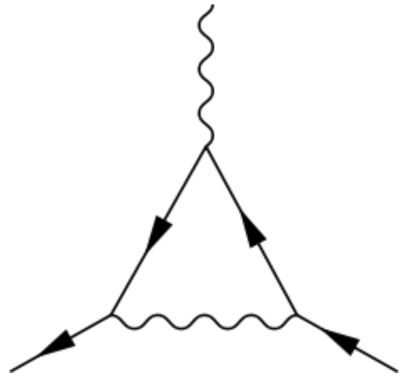
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j\sigma_{\mu\nu}e_r)\varepsilon_{jk}(\bar{q}_s^k\sigma^{\mu\nu}u_t)$		

We will now explore two examples in more detail

Anomalous magnetic moment of the muon

In a celebrated calculation, which was the birth of modern QFT, Schwinger computed the anomalous magnetic moment of the electron in 1948 and found:

$$\mu_e = \frac{g_e}{2m_e} \text{ , with } \quad a_e = \frac{g_e - 2}{2} = \frac{\alpha}{2\pi} + \ldots$$



In the meantime, radiative corrections in the SM have been calculated up to 5-loop order in QED corrections, including also electroweak loop effects involving heavy SM fields and hadronic corrections (vacuum polarization and light-by-light scattering)

Anomalous magnetic moment of the muon

The current state is as follows:

$$a_\mu^{\text{exp}} = 116\,592\,061\,(41) \times 10^{-11}$$ Muon (g-2) collaboration (April 2021)

$$a_\mu^{\text{SM}} = 116\,591\,810\,(43) \times 10^{-11}$$ Muon (g-2) theory initiative (2021)

Hint for a departure from the SM by 4.2σ:

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (251 \pm 59) \times 10^{-11}$$

How may this result be affected if the SM is considered as an effective field theory?

Anomalous magnetic moment of the muon

At lowest order in perturbation theory, the relevant operators are the dimension-6 electroweak dipole operators:

Q_{eW}	$(\bar{l}_p \sigma^{\mu\nu} e_r)\tau^I \varphi W^I_{\mu\nu}$
Q_{eB}	$(\bar{l}_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$

After EWSB, their contribution to a_μ scales like:

$$a_\mu^{\mathrm{SMEFT}} = a_\mu^{\mathrm{SM}} + C_{\mathrm{NP}} \frac{m_\mu v}{\Lambda^2}$$

Explaining the anomaly via the extra contribution yields:

$$\Lambda = C_{\mathrm{NP}}^{1/2} \times (102 \pm 12)\,\mathrm{TeV}$$

For an O(1) coefficient C_{NP} this would correspond to a large new-physics scale, but for $C_{\mathrm{NP}} \approx y_\mu$ the scale is lowered to 2.5 TeV, which is within the reach of the LHC

Baryon and lepton number conservation

In the construction of the SM, the conservation of baryon and lepton number is *not* imposed as a condition

There are no corresponding $U(1)$ symmetries of the Lagrangian

How can we understand that in nature we have not seen any hints of baryon- or lepton-number violating processes?

Baryon and lepton number conservation

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The answer is that **it is impossible to construct a relevant or marginal operator** that would respect the gauge symmetries of the SM and violate baryon or lepton number!

Hence, at the level of renormalizable interactions, baryon and lepton number are **accidental symmetries** of the SM

Proton decay in SMEFT

We have seen that in SMEFT lepton number can be violated starting at dimension-5 order, allowing for processes such as neutrinoless double *β* decay

Baryon number can be violated starting at dimension-6 order. What does this imply for proton decay?

The relevant operators contain three quark fields (ΔB=1) and one lepton field (ΔL=1); at dimension-6 order they are:

$$
\begin{array}{c|c}
Q_{duq} & \varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right] \\
Q_{qqu} & \varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right] \\
Q_{qqq} & \varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right] \\
Q_{duu} & \varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]
\end{array}
$$

The proton can be made sufficiently long-lived by raising the fundamental scale Λ into the 10^{16} GeV range

Proton decay in SMEFT

Now imagine that you would not know about the existence of quarks (no one has seen any), but you do know about the existence of protons and pions

Then an effective Lagrangian for proton decay could be:

$$\mathcal{L}_{\text{proton decay}} = g\,\pi\,\bar{\psi}_e\,\psi_p$$

This is a marginal operator, and hence proton decay would not be suppressed by any large mass scale!

In some sense, we see that the **longevity of the proton** provides a hint for its substructure: replacing a fundamental field by a **composite of several fields** raises the dimension of the operators and hence gives rise to additional suppression

Proton decay in SMEFT

The same trick can be applied to other fine-tuning problems

For example, the hierarchy problem can be solved by supposing that the Higgs boson is **not an elementary scalar** particle, but instead a bound state of a pair of elementary fermions

If this is were case, then the Higgs mass term would correspond to a 4-fermion operator, which is irrelevant

This kind of “Higgs compositeness” is the main idea behind **technicolor theories**

Summary

Effective field theories are a very powerful tool in quantum field theory

The can be of great practical use, but also provide the conceptual tools to understand scale separation (factorization) and renormalization in an intuitive and systematic way

Effective field theories are abundant, since any QFT can be considered as an effective low-energy theory of some more fundamental theory, which is often not yet known

Because of this fact, effective field theories provide the tools to perform indirect searches for new physics beyond the Standard Model

Exercises for lecture III

1. Draw all one-loop Feynman diagrams (in unitary gauge) mediating the Higgs-boson decay $h \to \gamma\gamma$ in the Standard Model. Since m_h^2 is numerically quite a bit smaller than $4m_t^2$ and $4m_W^2$, it turns out to be a reasonable approximation to integrate out the top quark and the W boson, thereby shrinking the loop diagrams to a point and producing an effective $h\gamma\gamma$ interaction. Write down the explicit form of the operator describing this interaction in the low-energy EFT. (Do *not* evaluate any loop diagrams!)

2. Starting from the effective Lagrangian of the SMEFT at dimension-6 order, work out (at tree level) the modifications to the Higgs-boson couplings to the heavy electroweak gauge bosons $W^\pm$ and Z^0 arising from the operators $Q_{\varphi D}$, $Q_{\varphi W}$, $Q_{\varphi B}$ and $Q_{\varphi WB}$. To this end, set $\varphi = \frac{v+h}{\sqrt{2}}$ in the relevant operators, and express the gauge fields W_μ^3 and B_μ (and their field strengths) in terms of the mass eigenstates Z_μ^0 and A_μ (the photon).

3. Work out (at tree level) the modifications to the Higgs–fermion couplings arising from the operators $Q_{e\varphi}$, $Q_{u\varphi}$ and $Q_{d\varphi}$ in the SMEFT Lagrangian. Allow for a generic flavor structure by writing the contributions of these operators in the form

$$\mathcal{L}_{\text{SMEFT}} \ni -\sum_{p,r=1}^{3} \left[Y_e^{pr} \left(\bar{l}_p \, \varphi \, e_r \right) + \text{h.c} \right] + \frac{1}{\Lambda^2} \sum_{p,r=1}^{3} \left[C_{e\varphi}^{pr} \left(\varphi^\dagger \varphi \right) \left(\bar{l}_p \, \varphi \, e_r \right) + \text{h.c} \right],$$

and similarly for the other two operators, where p, r are generation indices and $C_{e\varphi}^{pr}$ are arbitrary complex coefficients. The first term in the above result is the standard Yukawa interaction. Set $\varphi = \frac{v+h}{\sqrt{2}}$ and identify the fermion mass terms (no Higgs fields) and the fermion–Higgs couplings (terms linear in h). Use field redefinitions to bring the mass matrix (including the dimension-6 contributions) to diagonal form. Are the fermion–Higgs couplings in the mass basis flavor diagonal, as in the SM?