Introduction HTL, NRQCD and pNRQCD at finite temperature. Part 2.

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UNIÓN EUROPEA
FONDO EUROPEO DE DESENVOLVEMENTO REXIONAL
"Unha maneira de facer Europa"
EXCELENCIA MARÍA DE MAEZTU

1. Introduction

2. Thermal equilibrium properties
   - The case $M \gg \frac{1}{r} \gg T \sim E$
   - The case $M \gg \frac{1}{r} \gg T \gg E \gg m_D$
   - The case $M \gg \frac{1}{r} \gg T \gg m_D \gg E$
   - The case $M \gg \frac{1}{r} \sim T \gg m_D \gg E$
   - The case $M \gg T \gg \frac{1}{r} \sim m_D \gg E$

3. Quarkonium evolution in a medium
   - The $\frac{1}{r} \gg T \sim m_D \gg E$ regime
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Today

We are going to apply all this to the study of quarkonium suppression.

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- Heavy quarks can only be created at the beginning of the collision. It is a hard process.
- However, the existence of a medium changes the probability that a bound state is formed and its lifetime.
- Measuring R_{AA}, the ratio of quarkonium states measured in heavy-ion collisions divided by the naive extrapolation of pp data, we can extract information about the medium.

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Screening

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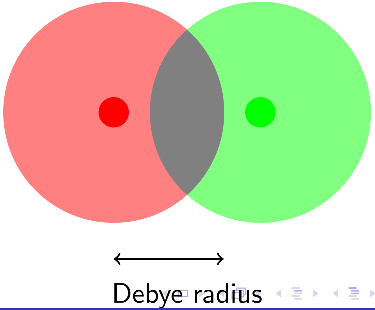
$$V(r) = -\alpha_s \frac{e^{-m_D r}}{r}$$

At finite temperature



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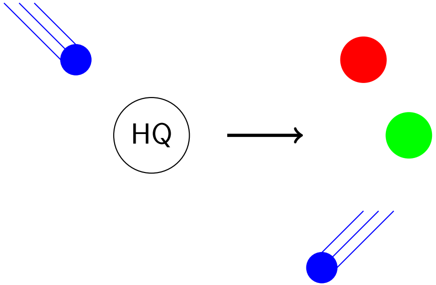
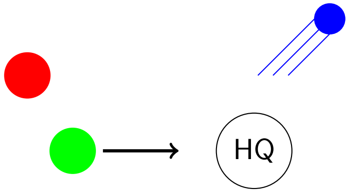


Recombination



Two heavy quarks coming from different origin may recombine to form a new quarkonium state.

Energy scales

Heavy quarkonium

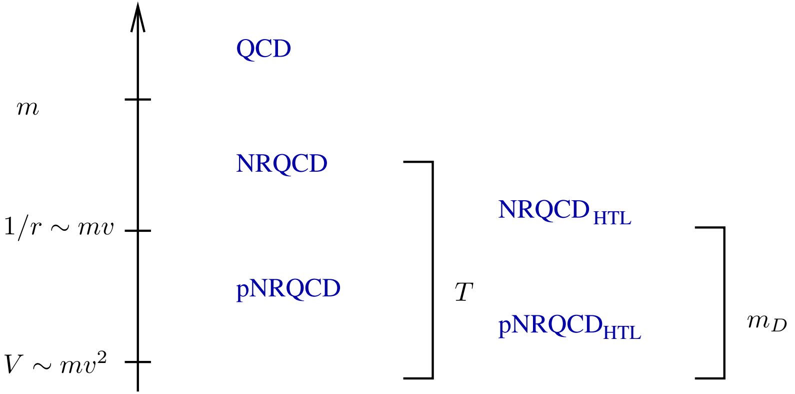
Quarkonium is a non-relativistic system. We can identify three energy scales

- The scale of the heavy quark mass m.
- The scale of the inverse of the typical radius $\frac{1}{r} \sim mv$.
- The scale of the binding energy $E \sim mv^2$.

Weakly coupled quark-gluon plasma

- Particles with energy of order T. Naive perturbation theory works well at this energy scale.
- Particles with energy of order gT. Screening of Chromoelectric fields. HTL resummation is needed.
- Particles with energy of order $g^2 T$. Non-perturbative even if g is small. We expect Chromomagnetic screening to happen at this scale.

EFTs to study quarkonium in a medium



Brambilla, Ghiglieri, Vairo and Petreczky (PRD78 (2008) 014017)
M.A.E and Soto (PRA78 (2008) 032520)

Heavy particles at finite temperature

$$\frac{1}{\mathrm{e}^{\frac{M}{T}}\mathrm{e}^{\frac{p^2}{2MT}}\pm 1} \sim \mathrm{e}^{-\frac{M}{T}}\mathrm{e}^{-\frac{p^2}{2MT}}$$

- Thermal equilibrium for heavy particles implies the dilute limit.
- In many out of equilibrium situations, we are also interested in the dilute limit. Very few bottom quarks are created in heavy-ion collisions at LHC.

The dilute limit

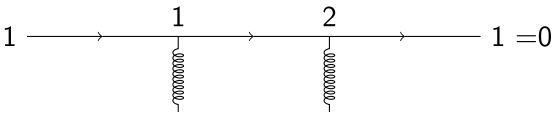
If ψ is a dilute heavy fermion, $Tr(\psi\psi^\dagger\rho) \gg Tr(\psi^\dagger\psi\rho)$ This implies for the propagator in the Schwinger-Keldysh contour that

$$S_{21}(t) \gg S_{12}(t)$$
$$S_{11}(t) = \theta(t)S_{21}(t) - \theta(-t)S_{12}(t) \sim \theta(t)S_{21}(t)$$
$$S_R(t) \sim S_{11}(t) \sim \theta(t)S_{21}(t)$$

The NRQCD heavy quark propagator in the Schwinger-Keldysh contour

$$S(p_0,\mathbf{p})=\frac{1+\gamma_0}{2}\begin{pmatrix}\frac{i}{p_0-\frac{p^2}{2M}+i\epsilon} & 0\\ 2\pi\delta\left(p_0-\frac{p^2}{2M}\right) & \frac{-i}{p_0-\frac{p^2}{2M}-i\epsilon}\end{pmatrix}$$

In the computation of any chronologically ordered propagator of heavy quarks we can consider that all the vertexes involving heavy quarks are of type 1



Heavy particles at finite temperature

Thermal equilibrium properties

- Binding energies, decay widths.
- Contained in the time-ordered propagator.
- Diagrams in which all vertices involving heavy particles are of type 1.
- We can define medium modified non-relativistic EFTs useful to compute the time-ordered correlator.

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Evolution of the heavy quark distribution

- The information needed to compute the nuclear modification factor is encoded in the 12 propagator.
- Since the 12 propagator is zero in the dilute limit, we need to go to NLO in this expansion.

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- We are going to study different cases, starting from $T \sim E$ and increasing the temperature until $T \gg \frac{1}{r} \sim m_D$.

The case $T \sim E$

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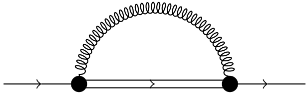
- Since $\frac{1}{r} \gg T$, the matching between NRQCD and pNRQCD can be done at $T = 0$.
- Thermal modifications enter thought the thermal modification of the propagator of ultrasoft gluons.
- Therefore, the power counting of pNRQCD indicates that thermal corrections are of order $\alpha_s r^2 T^3$. Of the size of the Bethe-logs.

Thermal corrections

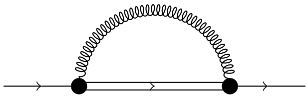


$$\delta h_s(p_0) = -i\frac{2g^2 C_F}{3} r^i \int \frac{d^4k}{(2\pi)^4} \frac{ik_0^2}{p_0 - k_0 - h_o + i\epsilon} \frac{2\pi\delta(k^2)}{e^{\frac{|k_0|}{T}} - 1} r^i$$

- At tree level and in the Coulomb gauge, only transverse gluons are affected by the medium.
- Note that δh_s is an operator

$$\frac{i}{p_0 - h_o + i\epsilon} = \int \frac{d^3q}{(2\pi)^3} |\mathbf{q}, o\rangle \frac{i}{p_0 - \frac{q^2}{M} + i\epsilon} \langle \mathbf{q}, o|$$

Binding energy



$$\delta E_n = Re\langle n|\delta h(E_n)|n\rangle$$

No simple analytical formula in the general case. Note that

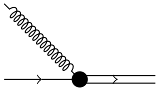
$$\delta h(E_n) = r^i f(E_n - h_o, T) r^i = \int \frac{d^3q}{(2\pi)^3} f(E_n - q^2/M, T) r^i |\mathbf{q}, o\rangle\langle\mathbf{q}, o| r^i$$

Only in the limit $T \gg \Delta E$, f is a polynomial in $E_n - h_o$ and simple expressions can be obtained.

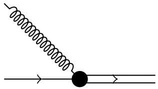
The decay width

$$\Gamma_n = -2Im\langle n|\delta h(E_n)|n\rangle = \frac{4}{3}C_F\alpha_s\langle n|r^i\frac{(h_o - E_n)^3}{e^{\frac{h_o-E_n}{T}}-1}r^i|n\rangle$$

Its related to a process called gluo-dissociation ($g + HQ \to Q + \bar{Q}$).



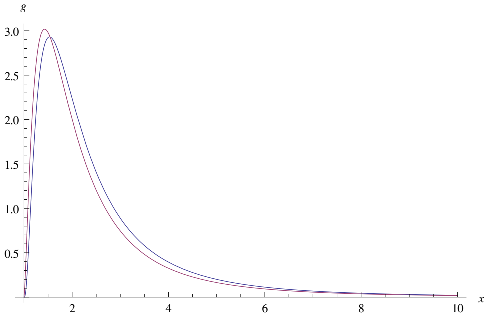
Gluo-dissociation



- First computed by Bhanov and Peskin (1979) using OPE and the large-N_c limit.
- The pNRQCD result is a generalization to finite N_c.
- Since both the singlet and the octet have binding energy of a similar size, the energy of the gluon must be of order E.
- Since the gluon is on-shell, the momentum is also of order E.

Gluo-dissociation cross-section

$$\Gamma_{GD} = \int \frac{d^3k}{(2\pi)^3} n_B(k)\sigma_{GD}(k) \qquad \sigma_{GD}(k) = \sigma_R g(k/E_n)$$



Purple is the large-N_c limit and blue is the full result.

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- We can integrate out the scale T to go from pNRQCD to a new EFT that we call $pNRQCD_{HTL}$.
- We can use $pNRQCD_{HTL}$ to compute the effects of the scales E and m_D.

$pNRQCD_{HTL}$

$$\mathcal{L}_{pNRQCD_{HTL}} = \mathcal{L}_{HTL} + \int d^3r\, Tr\left\{S^\dagger[i\delta_0 - h_s - \delta V_s]S + O^\dagger[iD_0 - h_o - \delta V_o]O \right.$$
$$\left. + O^\dagger \mathbf{r} g\mathbf{E} S + S^\dagger \mathbf{r} g\mathbf{E} O + \frac{1}{2} O^\dagger \mathbf{r} g\mathbf{E} O + \frac{1}{2} O^\dagger O \mathbf{r} g\mathbf{E}\right\}$$

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- pNRQCD with a modified potential plus HTL for gluons and light quarks.

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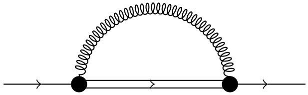
$$+O^\dagger \mathbf{r} g \mathbf{E} S + S^\dagger \mathbf{r} g \mathbf{E} O + \frac{1}{2} O^\dagger \mathbf{r} g \mathbf{E} O + \frac{1}{2} O^\dagger O \mathbf{r} g \mathbf{E}\Big\}$$

- pNRQCD with a modified potential plus HTL for gluons and light quarks.
- δV_x is a complex quantity of size $\alpha_s r^2 T^3$ or smaller.

δV_s at one loop



But now the octet propagator must be expanded

$$\frac{i}{E - h_o - k_0 + i\epsilon} \sim \frac{i}{-k_0 + i\epsilon} - \frac{i(E - h_o)}{(-k_0 + i\epsilon)^2} + \cdots$$

The first term would give a contribution of size $\alpha_s r^2 T^3$, but it vanishes. The second term gives a contribution of order $\alpha_s r^2 T^2 E$.

Expansion in E/T

Consider the linear term in E

$$\frac{i}{p_0 - h_s + i\epsilon} r^i (p_0 - h_o) r^i \frac{i}{p_0 - h_s + i\epsilon}$$

We need to isolate the double pole since the single pole can be reabsorbed by a field redefinition.

Note that $p_0 - h_o = p_0 - h_s - (h_o - h_s) = p_0 - h_s - (V_o - V_s)$.

We can also use that

$$r^i (p_0 - h_s) r^i = \frac{1}{2}\left([r^i, p_0 - h_s] r^i + (p_0 - h_s) r^2 + r^i [p_0 - h_s, r^i] + r^2 (p_0 - h_s)\right)$$

$$= \frac{1}{2}([[h_s, r^i], r^i] + \{r^2, (p_0 - h_s)\})$$

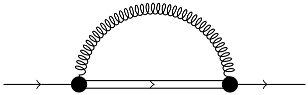
Expansion in E/T II

Using that $[[h_s, r^i], r^i] = -\frac{6}{M}$

$$\frac{i}{p_0 - h_s + i\epsilon} r^i (p_0 - h_o) r^i \frac{i}{p_0 - h_s + i\epsilon} = \frac{i}{p_0 - h_s + i\epsilon} \left(-\frac{3}{M} - \frac{N_c \alpha_s r}{2} \right) \frac{i}{p_0 - h_s + i\epsilon}$$

where we have neglected terms that do not contribute to the double pole.

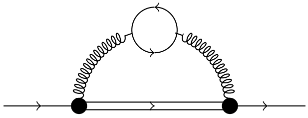
δV_s at one loop



$$\delta V_s = \frac{\pi}{9} N_c C_F \alpha_s^2 T^2 r + \frac{2\pi}{3M} C_F \alpha_s T^2 + \mathcal{O}(\alpha_s r^2 E^3)$$

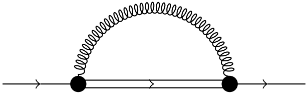
Corrections of order $\alpha_s r^2 E^3$ have been computed, but we do not discuss them here. At this order δV_s is real.

δV_s at two loop



- Contributes at order $\alpha_s^2 r^2 T^3 \sim \alpha_s r^2 T m_D^2$. Sub-leading at this temperature regime.
- It has an imaginary part and an infrared divergence. More on this later.

Contribution scale E



- Contribution of size $\alpha_s r^2 TE^2$ or smaller.
- We can use the HTL propagator but in the limit $k \gg m_D$.

$$\delta E_n = \mathcal{O}(\alpha_s r^2 T m_D^2)$$

$$\delta\Gamma_n = \frac{1}{3}N_c^2 C_F \alpha_s^3 T - \frac{16}{3M} C_F \alpha_s T E_n + \frac{8}{3} N_c C_F \alpha_s^2 T \frac{1}{Mn^2 a_0} + \mathcal{O}(\alpha_s r^2 E^3)$$

Summary

$$\delta E_n = \frac{\pi}{9} N_c C_F \alpha_s^2 T^2 \frac{a_0}{2}[3n^2 - l(l+1)] + \frac{\pi}{3} C_F^2 \alpha_s^2 T^2 a_0 + \mathcal{O}(\alpha_s r^2 E^3)$$

$$\Gamma_n = \frac{1}{3} N_c^2 C_F \alpha_s^3 T - \frac{16}{3M} C_F \alpha_s T E_n + \frac{8}{3} N_c C_F \alpha_s^2 T \frac{1}{M n^2 a_0} + \mathcal{O}(\alpha_s r^2 E^3)$$

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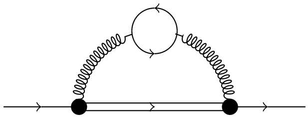
- We can use $pNRQCD_{HTL}$ to compute the effects of the scales m_D and E.

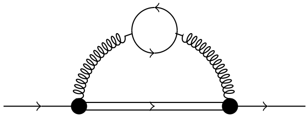
δV_s at two loop



$$\delta V_s = -\frac{3}{2}\zeta(3) C_F \frac{\alpha_s}{\pi} r^2 T m_D^2 + \frac{2}{3}\zeta(3) N_c C_F \alpha_s^2 r^2 T^3$$

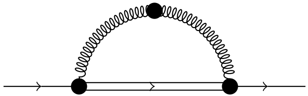
$$+ i \left[\frac{C_F}{6} \alpha_s r^2 T m_D^2 \left(\frac{1}{\epsilon} + \gamma_E + \log\pi - \log\frac{T^2}{\mu^2} + \frac{2}{3} - 4\log 2 - 2\frac{\zeta'(2)}{\zeta(2)} \right) + \frac{4\pi}{9} \log 2 N_c C_F \alpha_s^2 r^2 T^3 \right]$$

δV_s at two loop



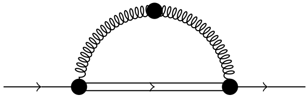
- It has an imaginary part. Its origin is Landau damping, since it comes from space-like gluons.
- It has an infrared divergence. This is not a problem since δV_s is a Wilson coefficient.

Contribution from the scale m_D



- Contribution of size $\alpha_s r^2 T m_D^2$ or smaller.
- Since $m_D \gg E$, the octet propagator has to be expanded as in the matching to $pNRQCD_{HTL}$.

Contribution from the scale m_D

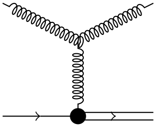


$$\delta V_s = \frac{C_F}{6}\alpha_s r^2 m_D^3 - i\frac{C_F}{6}\alpha_s r^2 T m_D^2\left(\frac{1}{\epsilon} - \gamma_E + \log\pi + \log\frac{\mu^2}{m_D^2} + \frac{5}{3}\right)$$

Binding energy

$$\delta E_n = \frac{\pi}{9} N_c C_F \alpha_s^2 T^2 \frac{a_0}{2}[3n^2 - l(l+1)] + \frac{\pi}{3} C_F^2 \alpha_s^2 T^2 a_0$$
$$+\left(\frac{C_F}{6}\alpha_s m_D^3 - \frac{3}{2}\zeta(3) C_F \frac{\alpha_s}{\pi} T m_D^2 + \frac{2}{3}\zeta(3) N_c C_F \alpha_s^2 T^3\right) \frac{a_0^2 n^2 (5n^2 - 3l(l+1) + 1}{2}$$
$$+\mathcal{O}(r^2 \alpha_s T E^2, r^2 \alpha_s r^2 \frac{m_D^4}{T})$$

Decay width, inelastic parton scattering

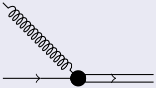


$$\Gamma_n = \frac{C_F}{3}\alpha_s T m_D^2 \left(\log\frac{T^2}{m_D^2} + 1 + 4\log 2 + 2\frac{\zeta'(2)}{\zeta(2)} - 2\gamma_E\right)\frac{a_0^2 n^2(5n^2 - 3l(l+1)+1}{2}$$

$$+\mathcal{O}(r^2\alpha_s TE^2, \alpha_s r^2\frac{m_D^4}{T})$$

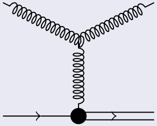
Inelastic parton scattering vs. gluo-dissociation

Gluo-dissociation



- $g + HQ \to Q + \bar{Q}$
- Incoming gluon has always energy and momentum of order E.
- If $T \gg E$, contribution of size $\alpha_s r^2 T E^2$.

Inelastic parton scattering



- $p + HQ \to p + Q + \bar{Q}$
- Incoming gluon can have any energy. Additional power of α_s.
- If $T \gg E$, contribution of size $\alpha_s r^2 T m_D^2$.

Inelastic scattering cross-section

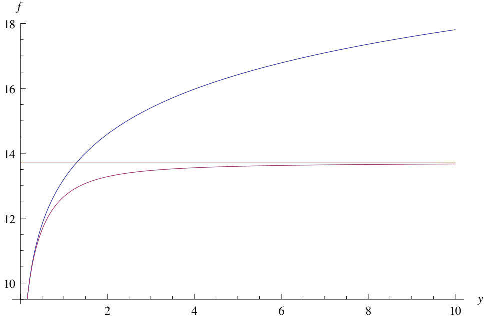
$$\Gamma_{IS} = \int \frac{d^3k}{(2\pi)^3} (n_B(k)(1+n_B(k))\sigma_{IS,g}(k) + n_F(k)(1-n_F(k))\sigma_{IS,q}(k))$$

$$\sigma_{IS,q} = \sigma_R f(m_D a_0, k a_0)$$

In the following slide we are going to show the result for quarks.

Inelastic scattering cross-section for $1S$

$m_D a_0 = 0.001$



$\frac{1}{r} \gg T \gg m_D$, $T \sim \frac{1}{r} \gg m_D$ and $T \gg \frac{1}{r} \sim m_D$. Discrepancy between blue and red lines signals a failure of color dipole approximation.

Non-perturbative scenario

In the limit in which all the thermal scales are smaller than $\frac{1}{r}$ but bigger than E, we can really encode all the information we need from the medium in non-perturbative and gauge invariant parameters.

$$\Gamma = \kappa\langle r^2\rangle \qquad \delta E_n = \frac{1}{2}\gamma\langle r^2\rangle$$

$$\kappa = \frac{g^2}{6\,N_c}\,\mathrm{Re}\int_{-\infty}^{+\infty} ds\,\langle\,\mathrm{T}\,E^{a,i}(s,\mathbf{0})E^{a,i}(0,\mathbf{0})\rangle$$

$$\gamma = \frac{g^2}{6\,N_c}\,\mathrm{Im}\int_{-\infty}^{+\infty} ds\,\langle\,\mathrm{T}\,E^{a,i}(s,\mathbf{0})E^{a,i}(0,\mathbf{0})\rangle$$

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κ coincides with the heavy quark diffusion coefficient.

Non-perturbative scenario II

- The heavy quark diffussion coefficient also appears when we study the diffusion of D and B mesons in a plasma. It can be determined by
  - Fitting a model to experimental data.
  - Direct computation in Lattice QCD or AdS/CFT.
  - From Lattice computation of the decay width of quarkonium.
  - Perturbative computations of κ exists to NLO. Convergence is not particularly good.
- γ can be determined by computing the mass shift of quarkonium in Lattice QCD.

Outline

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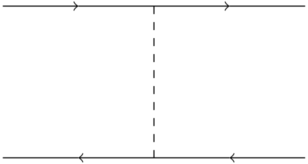
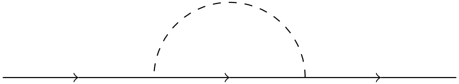
- The contribution from the scale m_D is the same as in the previous case.

Matching NRQCD to $pNRQCD_{HTL}$

.





The longitudinal gluon does not have thermal corrections at three level in the Coulomb gauge. Therefore, V_s does not receive thermal corrections at LO.

Matching NRQCD to $pNRQCD_{HTL}$ II

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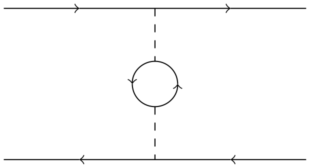

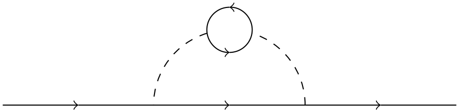

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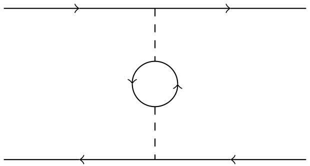

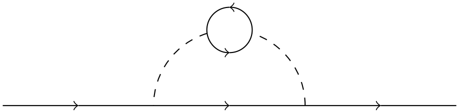

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- Has been computed analytically. Results occupies a full slide.

Matching NRQCD to $pNRQCD_{HTL}$ II

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- Has been computed analytically. Results occupies a full slide.
- It has an imaginary part, related with inelastic parton scattering. This imaginary part has an infrared divergence that cancels the ultraviolet divergence from the m_D region.

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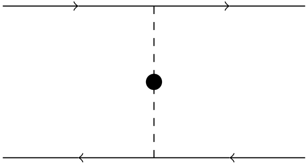
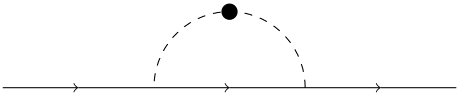
- We can integrate out the scales $\frac{1}{r}$ and m_D at the same time. We arrive to a modified version of pNRQCD.

Matching $NRQCD_{HTL}$ to pNRQCD



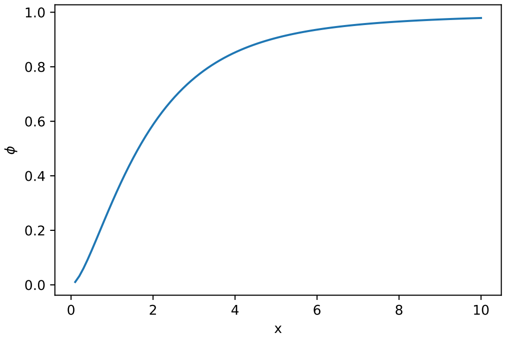


$$V_s = -\alpha_s C_F \frac{e^{-m_D r}}{r} - i\alpha_s C_F T \phi(r m_D)$$

$$\phi(x) = 2\int \frac{dzz}{(z^2+1)^2}\left(1 - \frac{\sin(xz)}{xz}\right)$$

This is the Laine potential (Laine et al, 2007).

Properties of ϕ



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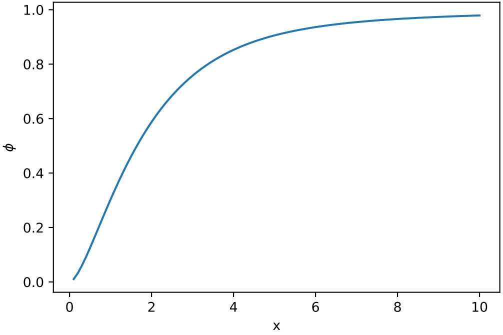

- ϕ goes to zero as x^2 for small values of x. Reflects the fact that if $rm_D \ll 1$, gluons with momentum of order m_D see quarkonium as a small color dipole.

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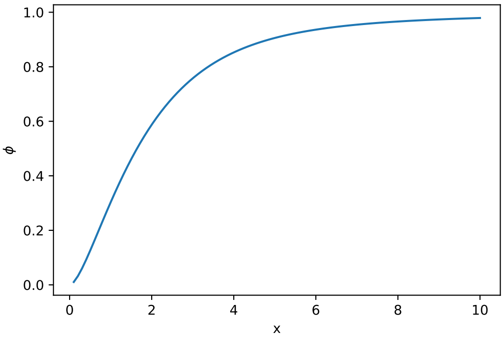

- ϕ goes to zero as x^2 for small values of x. Reflects the fact that if $rm_D \ll 1$, gluons with momentum of order m_D see quarkonium as a small color dipole.
- ϕ goes to a constant as $x \to \infty$. If $rm_D \gg 1$ the medium sees quarkonium as two uncorrelated heavy quarks.$\Gamma_{HQ} = \Gamma_Q + \Gamma_{\bar{Q}}$.

The melting of quarkonium

- Historically, screening was considered the main source of quarkonium dissociation. In this picture quarkonium melts when $rm_D \sim 1$. The Yukawa potential no longer supports bound state solutions.

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- If $\Gamma \sim E$, the lifetime of quarkonium is of the order of its formation time. Alternative criteria for the melting of quarkonium.
- In the Laine potential $ImV_s \gg ReV_s$. Dissociation happens at $T \gg \frac{1}{r} \gg m_D$.

1. Introduction
2. Thermal equilibrium properties
   - The case $M \gg \frac{1}{r} \gg T \sim E$
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3. Quarkonium evolution in a medium
   - The $\frac{1}{r} \gg T \sim m_D \gg E$ regime
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- This is what we need to compute R_{AA}.
- As we mentioned in the introduction, for this we need to compute the evolution of the 12 propagator.
- Note that the 12 propagator, $Tr(S^\dagger(t, r_2)S(t, r_1)\rho)$, is the density matrix of quarkonium.

Quarkonium as an Open quantum system

- We consider a *universe* consisting in heavy quarks (system) in a medium of light quarks and gluons (environment). The density matrix $\rho(S, E)$ describes the state of the universe. Its evolution is unitary.

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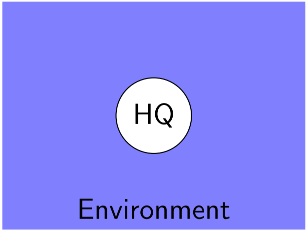

The master equation

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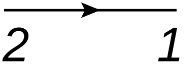
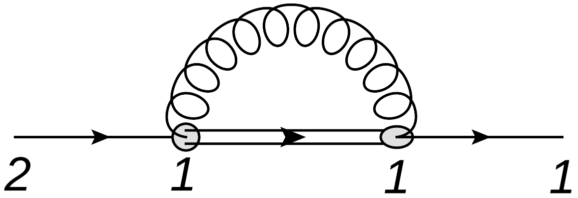
- We can recover the Schrödinger equation and the Boltzmann equation as limits of the master equation in specific regimes.
- We need to derive the master equation from QCD. This has been done in:
  - Perturbation theory. Akamatsu (2015,2020), Blaizot and Escobedo (2017,2018).
  - Potential non-relativistic QCD (pNRQCD) in the $\frac{1}{r} \gg T$ regime. Brambilla et al. (2016,2017).

$$\partial_t \rho_s = -i[h_s, \rho_s] - i\Sigma\rho_s + i\rho_s\Sigma^\dagger + \mathcal{F}(\rho_o)$$



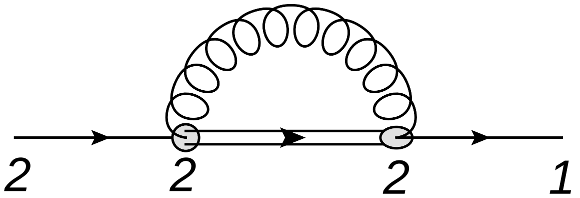
We use the non-equilibrium quantum field theory formalism to perform this computation. Tree level is equivalent to the $T = 0$ LO evolution

$$\partial_t \rho_S = -i[h_s, \rho_s] - i\Sigma\rho_s + i\rho_s\Sigma^\dagger + \mathcal{F}(\rho_o)$$



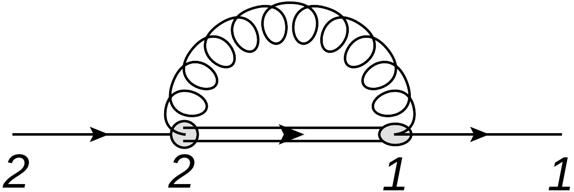
Self-energy diagram contributes to screening and to the decay width.

$$\partial_t \rho_S = -i[h_s, \rho_s] - i\Sigma\rho_s + i\rho_s\Sigma^\dagger + \mathcal{F}(\rho_o)$$



Hermitian conjugate of the previous diagram. We can reorganize this to diagrams in a redefinition of h_s and a term that represents the decay width.

$$\partial_t \rho_S = -i[h_s, \rho_s] - i\Sigma\rho_s + i\rho_s\Sigma^\dagger + \mathcal{F}(\rho_o)$$



The number of singlets is increased due to the octets in the medium that absorb a gluon.

Very similar reasoning.

$$\partial_t \rho_o = -i[H_o, \rho_o] - \frac{1}{2}\{\Gamma, \rho_o\} + \mathcal{F}_1(\rho_s) + \mathcal{F}_2(\rho_o)$$

- Computationally costly system of coupled equations. The density matrix contains many information, especially in the non-Abelian case (QCD).
- Total number of heavy particles is conserved. $Tr(\rho_s) + Tr(\rho_o)$ is a constant of the evolution.
- Only if $T \gg E$ the interaction with the medium can be considered local in time and the evolution is Markovian.

The Lindblad equation

Any master equation that is:

- Markovian
- Preserves the properties that a density matrix must fulfil (Hermitian, positive semi-definite, trace is conserve).

Can be written as a Lindblad (or GKSL) equation (Lindblad (1976), Gorini, Kossakowski and Sudarshan (1976)).

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$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_n \left(C_n \rho C_n^\dagger - \frac{1}{2}\{C_n^\dagger C_n, \rho\} \right)$$

At strong coupling the screening length $\frac{1}{m_D}$ is of the order of the temperature. When all the thermal scales are smaller than $\frac{1}{r}$ but bigger than E the evolution equation is Markovian and can be written in the Lindblad form.

$$\partial_t \rho = -i[H(\gamma), \rho] + \sum_k (C_k \rho C_k^\dagger - \frac{1}{2}\{C_k^\dagger C_k, \rho\})$$

there is a transition singlet-octet

$$C_i^{so} = \sqrt{\frac{\kappa}{N_c^2 - 1}} r_i \begin{pmatrix} 0 & 1 \\ \sqrt{N_c^2 - 1} & 0 \end{pmatrix}$$

and octet to octet

$$C_i^{oo} = \sqrt{\frac{(N_c^2 - 4)\kappa}{2(N_c^2 - 1)}} r_i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

The Monte Carlo Wave function method

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- The Monte Carlo wave function method is useful to reduce the computational cost. It also clarifies the physical picture of the Lindblad equation.
- Note that to compare with experimental data, one needs to compute the evolution of quarkonium many times. The temperature profile seen by quarkonium depends on the impact parameter of the collision, the point in which quarkonium is created...

The Monte-Carlo Wave Function method

Take the Lindblad equation

$$\partial_t \rho = -i[H(\gamma), \rho] + \sum_k (C_k(\kappa)\rho C_k^\dagger(\kappa) - \frac{1}{2}\{C_k^\dagger(\kappa)C_k(\kappa), \rho\})$$

Let us define

$$\Gamma_n = C_n^\dagger C_n \qquad \Gamma = \sum_n \Gamma_n$$

and

$$H_{eff} = H - i\Gamma/2$$

$\rho(t) = \sum_n p_n |\Psi_n(t)\rangle\langle\Psi_n(t)|$. If we know how to evolve the case $\rho(t) = |\Psi(t)\rangle\langle\Psi(t)|$, it is straightforward to generalize.

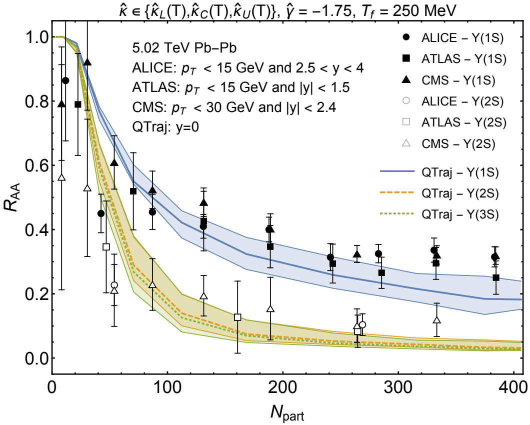
The Monte-Carlo Wave Function method

The algorithm to evolve from t to $t + dt$

- With probability $1 - \langle\Psi(t)|\Gamma|\Psi(t)\rangle dt$.
  - Evolve the wave-function with $(1 - iH_{eff}\,dt)|\Psi(t)\rangle$. In our case, this implies solving a 1D Schrödinger equation because H_{eff} does not mix states with different color or angular momentum.
- With probability $\langle\Psi(t)|\Gamma_n|\Psi(t)\rangle dt$.
  - Take a quantum jump, $|\Psi(t)\rangle \to C_n|\Psi(t)\rangle$.
  - Only here transitions between different color and angular momentum are allowed.
- Normalize the resulting wave-function.

The average of this stochastic evolution of the wave-function is equivalent to the Lindblad equation for the density matrix.

Results


$\hat{\kappa} \in \{\hat{\kappa}_L(T), \hat{\kappa}_C(T), \hat{\kappa}_U(T)\}$, $\hat{\gamma} = -1.75$, $T_f = 250$ MeV
5.02 TeV Pb–Pb
ALICE: p_T < 15 GeV and 2.5 < y < 4
ATLAS: p_T < 15 GeV and |y| < 1.5
CMS: p_T < 30 GeV and |y| < 2.4
QTraj: y=0
ALICE – Υ(1S)
ATLAS – Υ(1S)
CMS – Υ(1S)
ALICE – Υ(2S)
ATLAS – Υ(2S)
CMS – Υ(2S)
QTraj – Υ(1S)
QTraj – Υ(2S)
QTraj – Υ(3S)
R_{AA}
N_{part}
0.0
0.2
0.4
0.6
0.8
1.0
0
100
200
300
400

Outline

- The evolution of this case is not Markovian. $\frac{d\rho}{dt}$ depends on the values of ρ at previous times. In other words, there are memory effects.

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- We can mathematically evolve $\rho(t')$ to $\rho(t)$ using $T = 0$ evolution. In principle, this introduces sub-leading corrections.
- However, these corrections can lead to negative probabilities. Therefore, we do not obtain a Lindblad equation any more.

The evolution equation can be written as

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_{nm} h_{nm} \left(L_i^n \rho L_i^{m\dagger} - \frac{1}{2}\{L_i^{m\dagger} L_i^n, \rho\} \right)$$

where h_{nm} are the elements of the matrix

$$h = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

If h were a positive definite matrix, then it would always be possible to redefine the operators L_i^n in such a way that the evolution equation would be of the Lindblad form.

The evolution equation can be written as

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_{nm} h_{nm}\left(L_i^n \rho L_i^{m\dagger} - \frac{1}{2}\{L_i^{m\dagger} L_i^n, \rho\}\right)$$

$$L_i^0 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} r^i\,, \qquad L_i^1 = \begin{pmatrix} 0 & 0 \\ 0 & \frac{N_c^2-4}{2(N_c^2-1)} A_i^{oo\dagger} \end{pmatrix}$$

$$L_i^2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} r^i\,, \qquad L_i^3 = \begin{pmatrix} 0 & \frac{1}{N_c^2-1} A_i^{os\dagger} \\ A_i^{so\dagger} & 0 \end{pmatrix}$$

$$A_i^{uv}(t) = \frac{g^2}{2N_c}\int_{t_0}^{t} dt_2\, e^{ih_u(t_2-t)}\, r^j\, e^{ih_v(t-t_2)}\, \langle E^{a,j}(t_2,\mathbf{0}) E^{a,i}(t,\mathbf{0})\rangle$$

Naive perturbative solution

$$\frac{d\rho}{dt} + i[H, \rho] = \epsilon \mathcal{F}[\rho(t)]$$

Let us consider the rhs as a perturbation ($E \gg \Gamma$).

$$\rho(t) = \rho_0(t) + \epsilon\rho_1(t) + \cdots$$

$$\rho_0(t) = \mathrm{e}^{-iHt}\rho_0(0)\mathrm{e}^{iHt}$$

$$\rho_1(t) = \int_0^t dt' \mathrm{e}^{-iH(t-t')}\mathcal{F}[\rho(t')]\mathrm{e}^{iH(t-t')}$$

Problem: $\rho_1(t)$ can be larger than $\rho_0(t)$ at large times.

Multiple-scale analysis

We need to apply a mathematical method to solve differential equations with secular terms. We define a small time $\tau = \epsilon t$ such that

$$\rho(t,\tau) = \rho_0(t,\tau) + \epsilon\rho_1(t,\tau) + \cdots$$

$$\frac{d\rho_0}{dt} + i[H,\rho_0] = 0$$

$$\frac{d\rho_1}{dt} + \frac{d\rho_0}{d\tau} + i[H,\rho_1] = \epsilon\mathcal{F}[\rho_0(t)]$$

We need to absorb in $\frac{d\rho_0}{d\tau}$ the secular terms

$$\frac{d\rho_0}{d\tau} = \lim_{T\to\infty}\frac{1}{T}\int_0^T dt' \mathrm{e}^{-iH(T-t')}\mathcal{F}[\rho(t')]\mathrm{e}^{iH(T-t')}$$

Multiple-scale analysis II

- Multiple-scale analysis includes at LO corrections that do not oscillate fast compare with the intrinsic time scale of quarkonium $1/E$. Rotating-wave approximation.
- Using these arguments a Boltzmann equation can be justified from pNRQCD if $E \gg \Gamma$.

An open question

An EFT approach valid to compute the 12 propagator when $T \sim \frac{1}{r}$ or higher.

1. Introduction
2. Thermal equilibrium properties
   - The case $M \gg \frac{1}{r} \gg T \sim E$
   - The case $M \gg \frac{1}{r} \gg T \gg E \gg m_D$
   - The case $M \gg \frac{1}{r} \gg T \gg m_D \gg E$
   - The case $M \gg \frac{1}{r} \sim T \gg m_D \gg E$
   - The case $M \gg T \gg \frac{1}{r} \sim m_D \gg E$
3. Quarkonium evolution in a medium
   - The $\frac{1}{r} \gg T \sim m_D \gg E$ regime
   - The $\frac{1}{r} \gg T \sim E$ regime

Thanks!