Methods of Effective Field Theory and Lattice Field Theory
Bad Honnef Physics Schoo
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Nuclear physics with applications to neutrino physics (EFT): I

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Plan for the lectures

- Big picture:
  - Nuclear probes of physics beyond the Standard Model
  - Connecting scales using EFTs: from BSM to nuclear physics

- EFTs for low-energy QCD and nuclear physics
  - Chiral perturbation theory for mesons and baryons (π, N)
  - Chiral EFT for multi-nucleon systems

- Application to neutrino physics (neutrinoless double beta decay)
  - Lepton Number Violation (LNV) in the Standard Model EFT
  - Nuclear scale realizations
  - Where are we?

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1st lecture

2nd lecture

A note on references

- References to the original papers will be sloppy (e.g. “Weinberg ’79”) and incomplete: apologies!

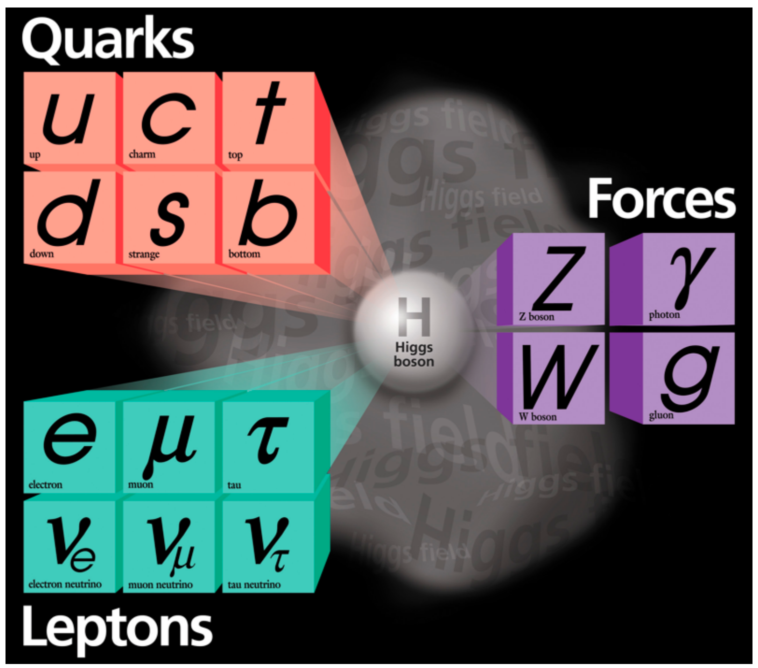
- An incomplete list of excellent lectures and reviews with references to the original literature:

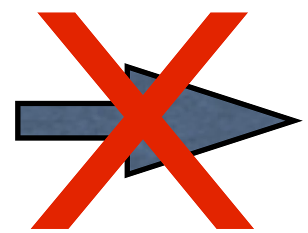
    - A. Manohar, hep-ph/9606222
    - G. Ecker, hep-ph/9805500
    - P. Bedaque and U. Van Kolck, nucl-th/0203055
    - E. Epelbaum, H.-W. Hammer, U.-G. Meiβner, 0811.1338 [nucl-th]
    - H.-W. Hammer, S. Konig, U.Van Kolck 1906.12122 [nucl-th]

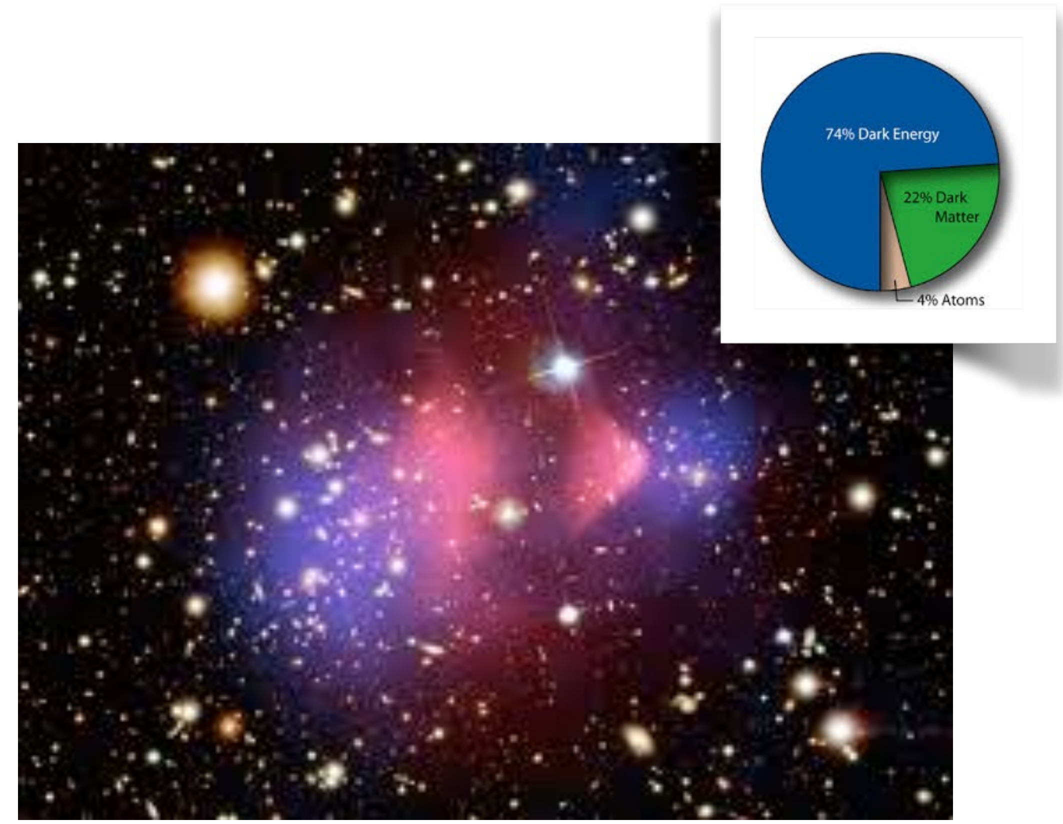
Big picture:
nuclear probes of physics beyond the standard model

New physics: why?

- The SM is remarkably successful, but it's probably not the whole story







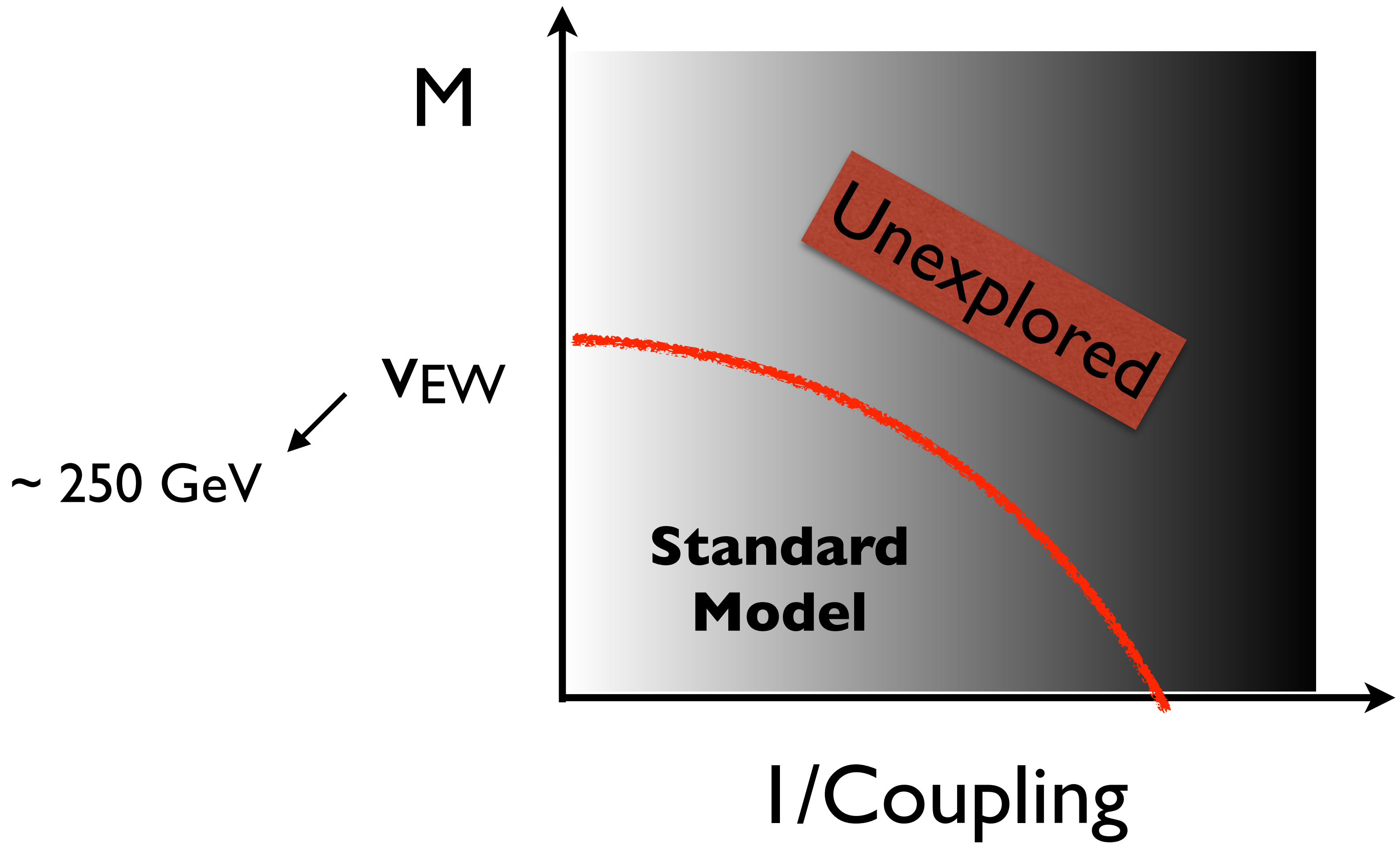
No Baryonic Matter, no Dark Matter, no Dark Energy, no Neutrino Mass

Do forces unify at high E? What is the origin of families? …

Addressing these puzzles likely requires new degrees of freedom

New physics: where?

- Where is the new physics? Is it Heavy? Is it Light & weakly coupled?



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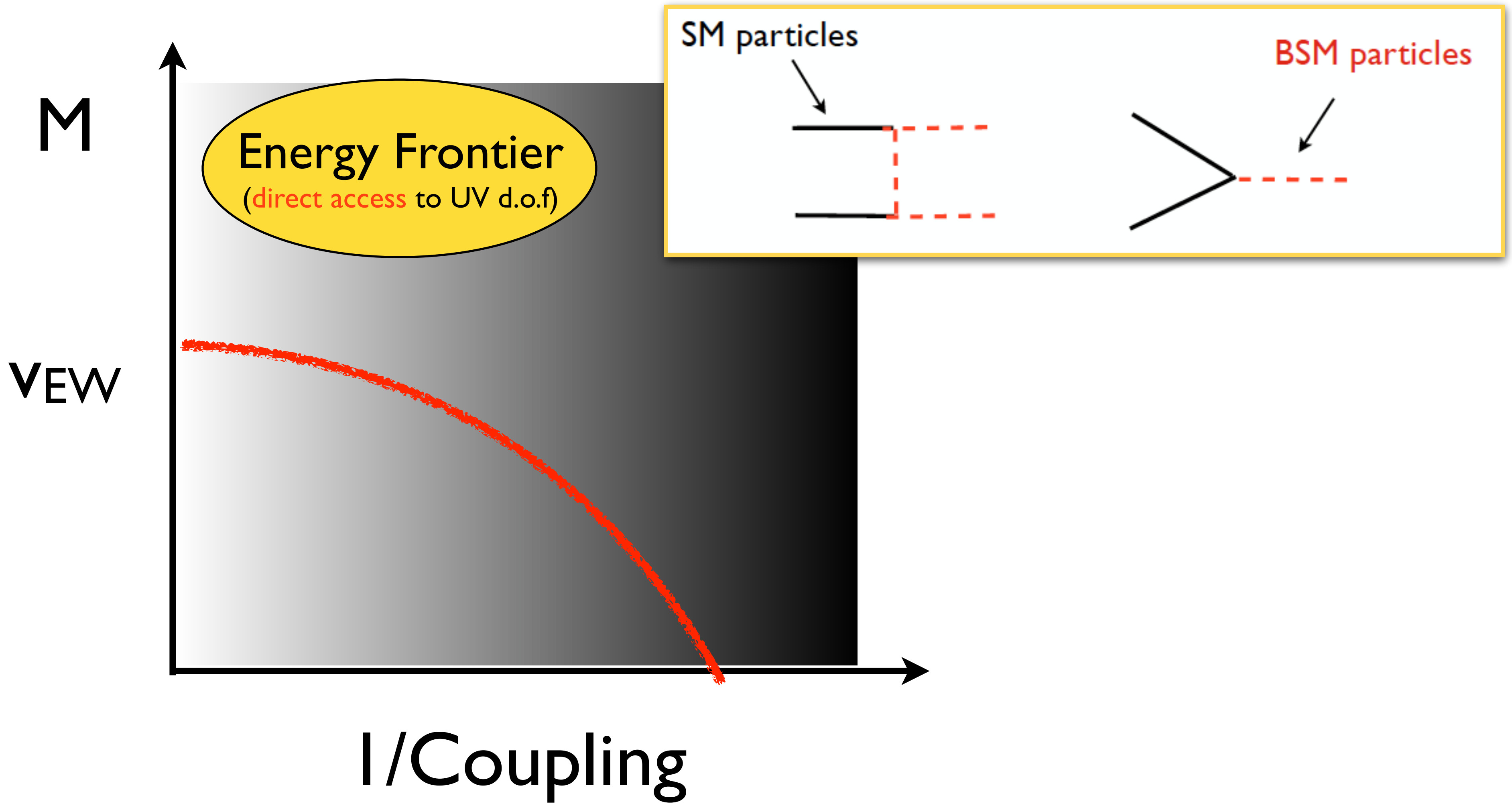

- Two approaches, both needed to uncover new physics

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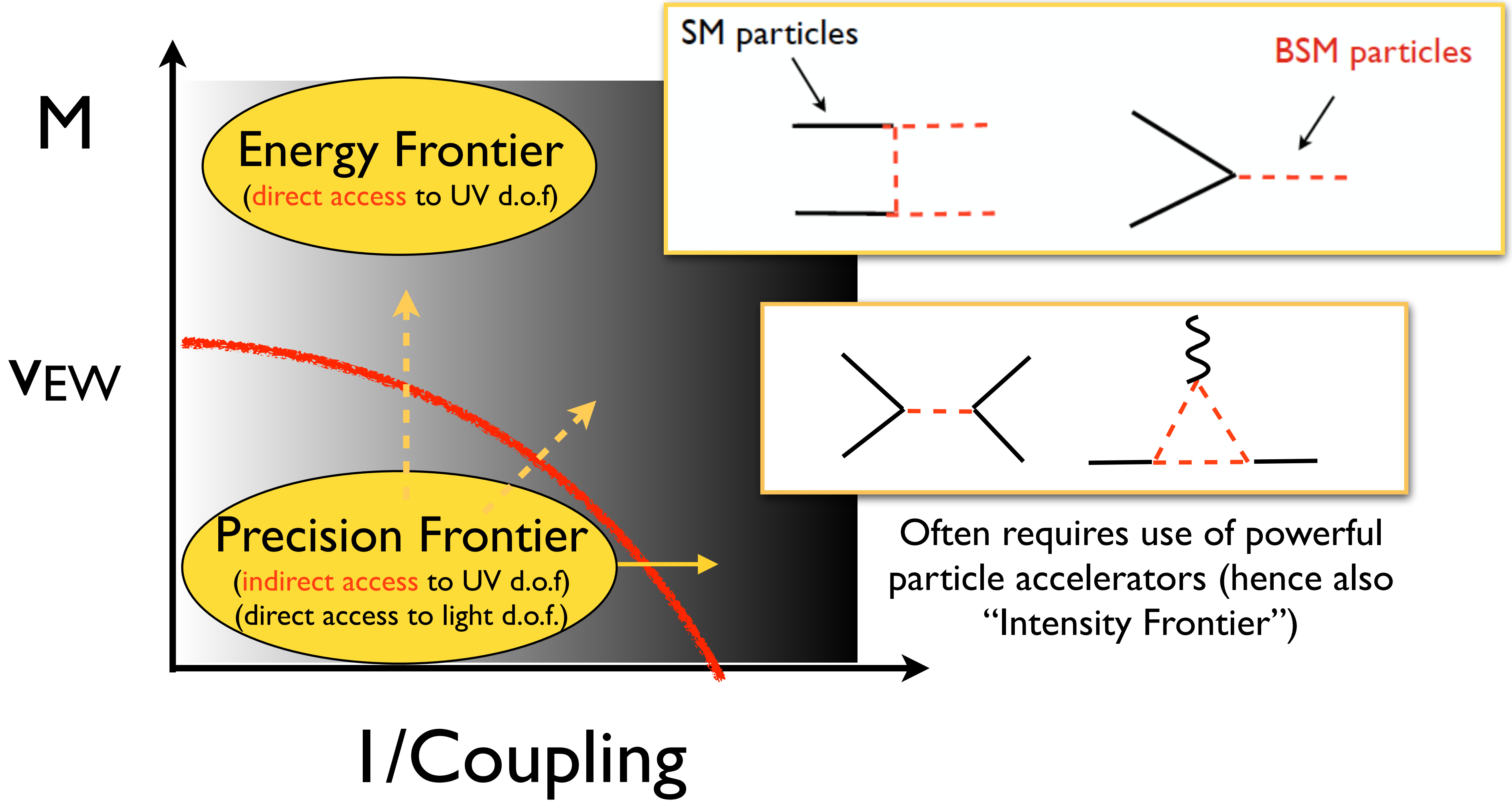

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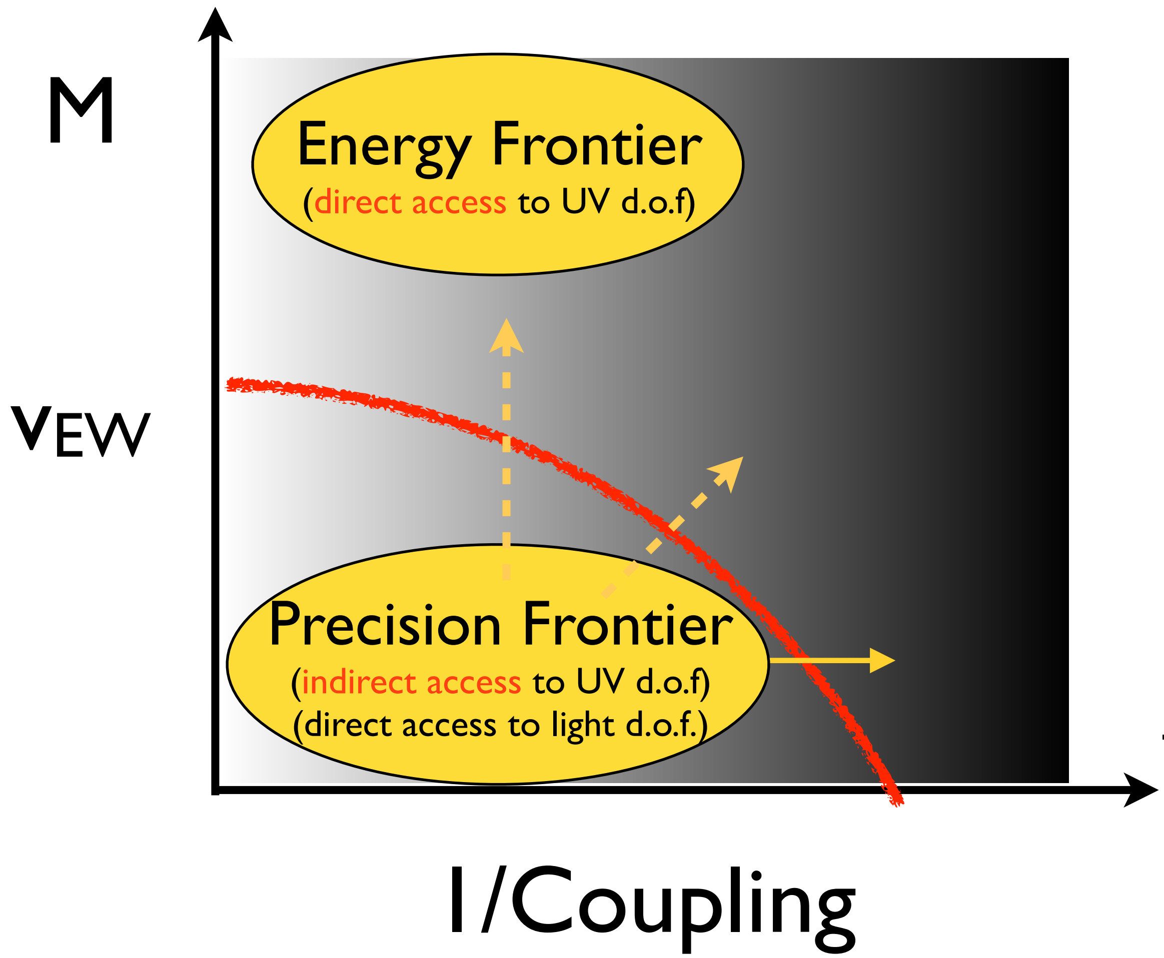

- EWSB mechanism
- Higgs properties
- Direct access to heavy particles
- ...

- L and B non conservation
- CP violation (w/o flavor)
- Flavor violation: quarks, leptons
- Multi-TeV scale interactions
- Neutrino properties
- Dark sectors
- ...

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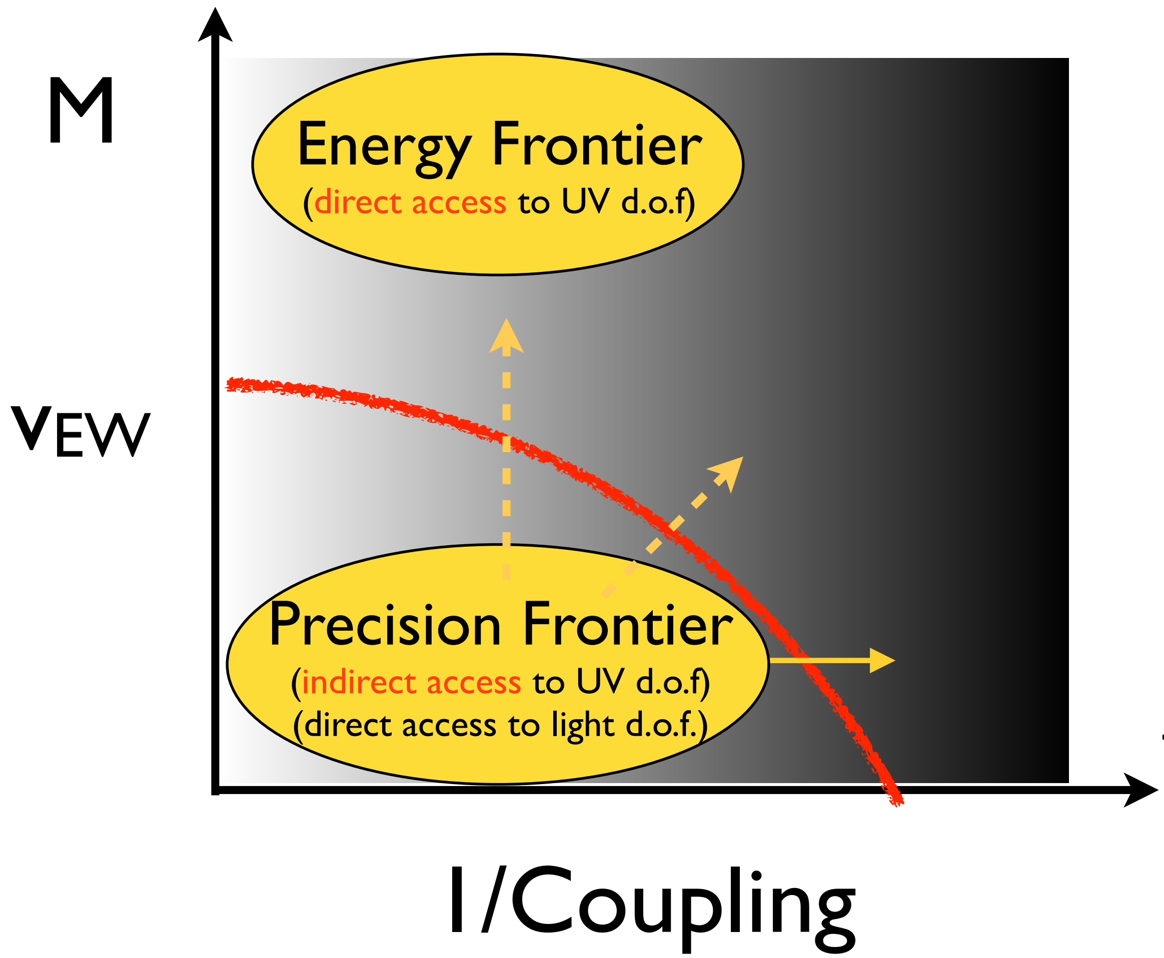

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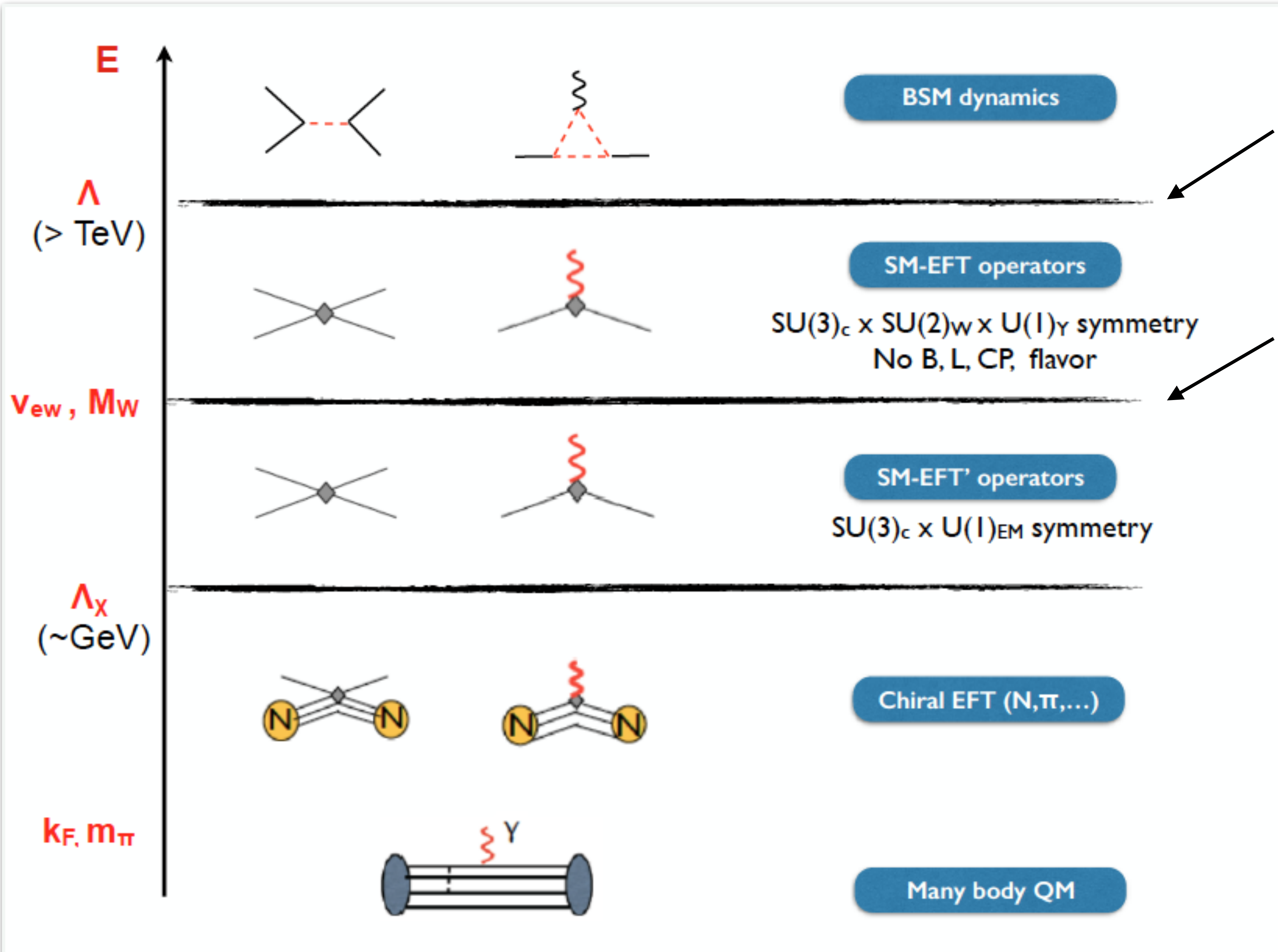
Nuclear probes play a prominent role at the Precision Frontier

Connecting scales

To connect UV physics to nuclei, use multiple EFTs



Matching to BSM scenarios

Perturbative matching within SM

Connecting scales

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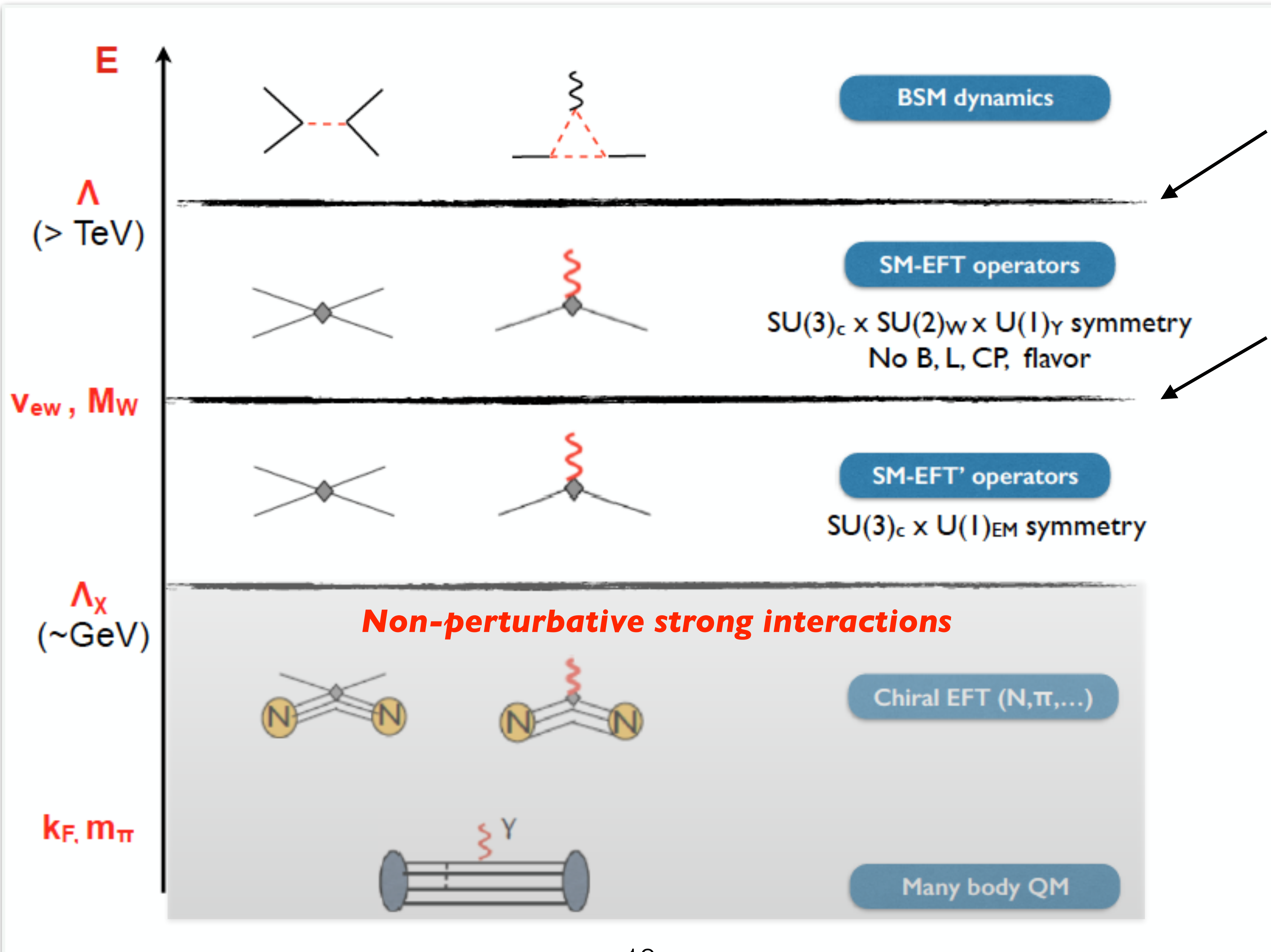

Matching to BSM scenarios

Perturbative matching within SM

Hadronic matrix elements

Nuclear matrix elements

EFTs for low-energy QCD and nuclear physics

Outline

- Chiral symmetry and its breaking
- Chiral Perturbation Theory (ChPT) for Goldstone modes (π)
- Heavy Baryon ChPT (N=n,p)
- EFT for multi-nucleon systems (NN, NNN, …)

Chiral symmetry

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu,a} + i\bar{q}_L\gamma^\mu D_\mu q_L + i\bar{q}_R\gamma^\mu D_\mu q_R - \bar{q}_L m_q q_R - \bar{q}_R m_q^\dagger q_L$$

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \qquad q_{L,R} = \left(\frac{1 \mp \gamma_5}{2}\right) q \qquad m_q = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$$

Chiral symmetry

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a} + i\bar{q}_L \gamma^\mu D_\mu q_L + i\bar{q}_R \gamma^\mu D_\mu q_R - \bar{q}_L m_q q_R - \bar{q}_R m_q^\dagger q_L$$

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- For $m_q = 0$, invariant under independent U(3) transformations of left- and right-handed quarks:

$L,R \in SU(3)$

$$\underbrace{SU(3)_L \times SU(3)_R}_{\text{Chiral group G}} \times \left[U(1)_V \times U(1)_A\right]$$

$$q_L \rightarrow L\, q_L$$
$$q_R \rightarrow R\, q_R$$

Chiral group G

Chiral symmetry

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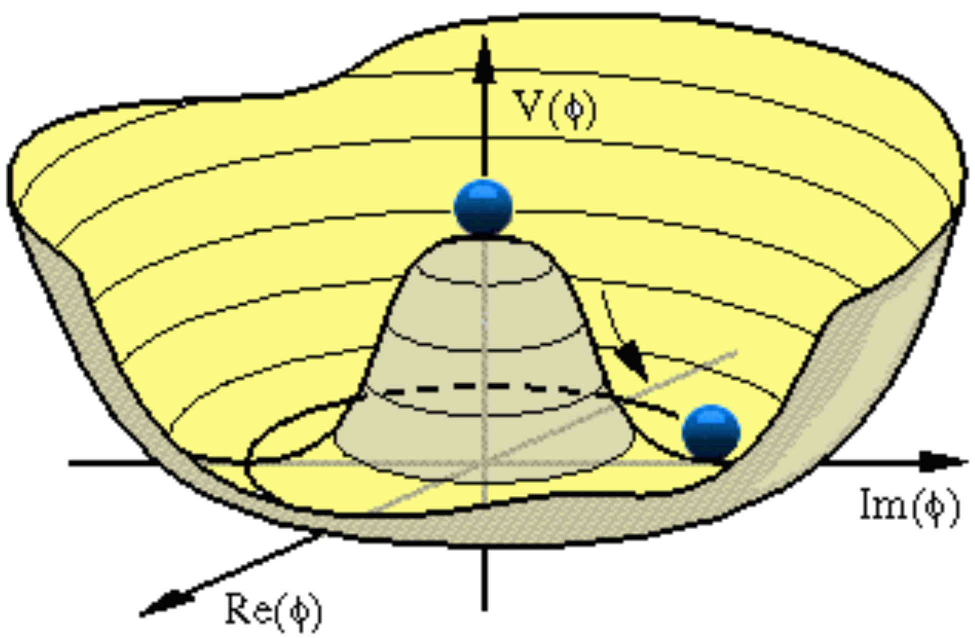
- Symmetry is broken explicitly by $m_q \neq 0$ and "spontaneously"

$$\partial_\mu \left(\bar{q}\gamma^\mu T^a q\right) = \bar{q}\,[T^a, m_q]\, q \qquad \partial_\mu \left(\bar{q}\gamma^\mu \gamma^5 T^a q\right) = \bar{q}\,\{T^a, m_q\}\, i\gamma_5\, q$$

Spontaneous Symmetry Breaking

- Action is invariant under symmetry group, but ground state is not
- Continuous symmetry: degenerate physically equivalent minima
- Excitations along the valley of minima → massless states in the spectrum (Goldstone Bosons)



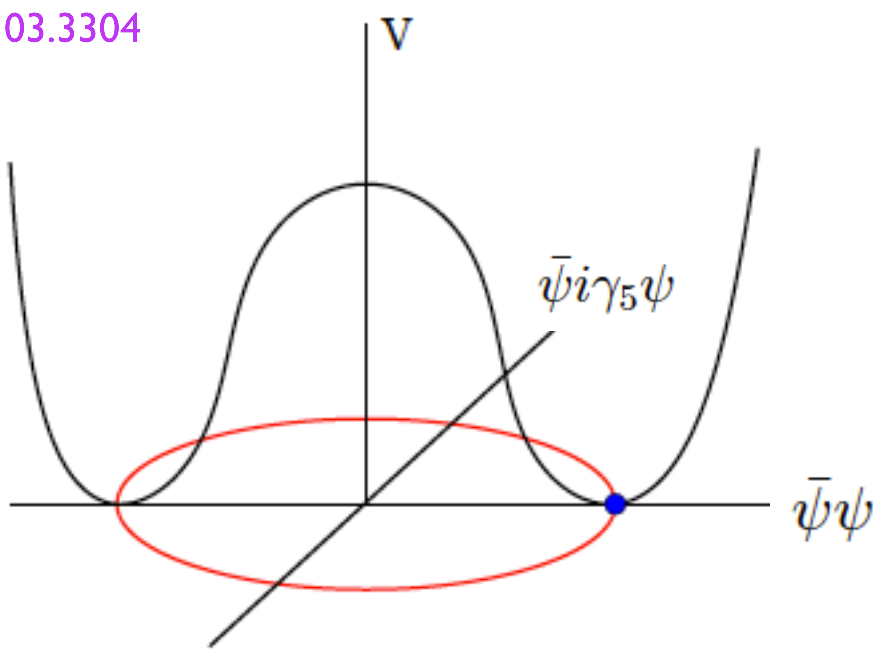
SSB of chiral SU(3)

- Empirical & theoretical evidence of breaking pattern

$$G = SU(3)_L \times SU(3)_R \;\rightarrow\; H = SU(3)_{V=L+R}$$

$$q_L \rightarrow L\,q_L \qquad q_R \rightarrow R\,q_R$$

Figure from M. Creutz
1103.3304



$$\langle 0|\bar{q}q|0\rangle = \langle 0|\bar{q}_L q_R|0\rangle + \langle 0|\bar{q}_R q_L|0\rangle \neq 0$$

- Vector subgroup SU(3)$_V$ (L=R) unbroken and symmetry is approximately manifest in the QCD spectrum
- Axial generators broken (no parity doublets, pseudoscalar mesons are the lightest hadrons)

- Goldstone's theorem: massless states appear in the spectrum, in one-to-one correspondence with the broken generators. Identified π,K,η

Low-energy EFT for GBs

- At low-E, the only d.o.f. are fluctuations along the vacuum manifold (Goldstone modes)
- Even though $M_{\pi,K,\eta} \neq 0$ (due to $m_q \neq 0$), π, K, η are still the lightest hadrons
- Use EFT methods to analyze the low-energy dynamics:
  - Identify relevant d.o.f: GBs plus possibly matter fields
  - Write down all interactions consistent with chiral symmetry
  - Order interactions according to power counting

Relevant ratio of scales (EFT expansion parameter): E/Λ, $M_{\pi,K}/\Lambda$

Λ: scale of lowest QCD resonances ~ O(1 GeV)

Fields and their transformations

Callan, Coleman, Wess, Zumino '69

- SSB pattern G→H: GB fields ~ coordinates of the vacuum manifold G/H

$\boldsymbol{\phi} = (\phi_1, \ldots, \phi_N)$

- Example: O(N) linear sigma model

$$\mathcal{L} = \frac{1}{2}\partial_\mu \boldsymbol{\phi} \cdot \partial^\mu \boldsymbol{\phi} - \lambda(\boldsymbol{\phi} \cdot \boldsymbol{\phi} - v^2)^2$$

Vacuum manifold $\phi_1^2 + \phi_2^2 + \ldots + \phi_N^2 = v^2$

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N=3: G=O(3), H=O(2), G/H = S^2

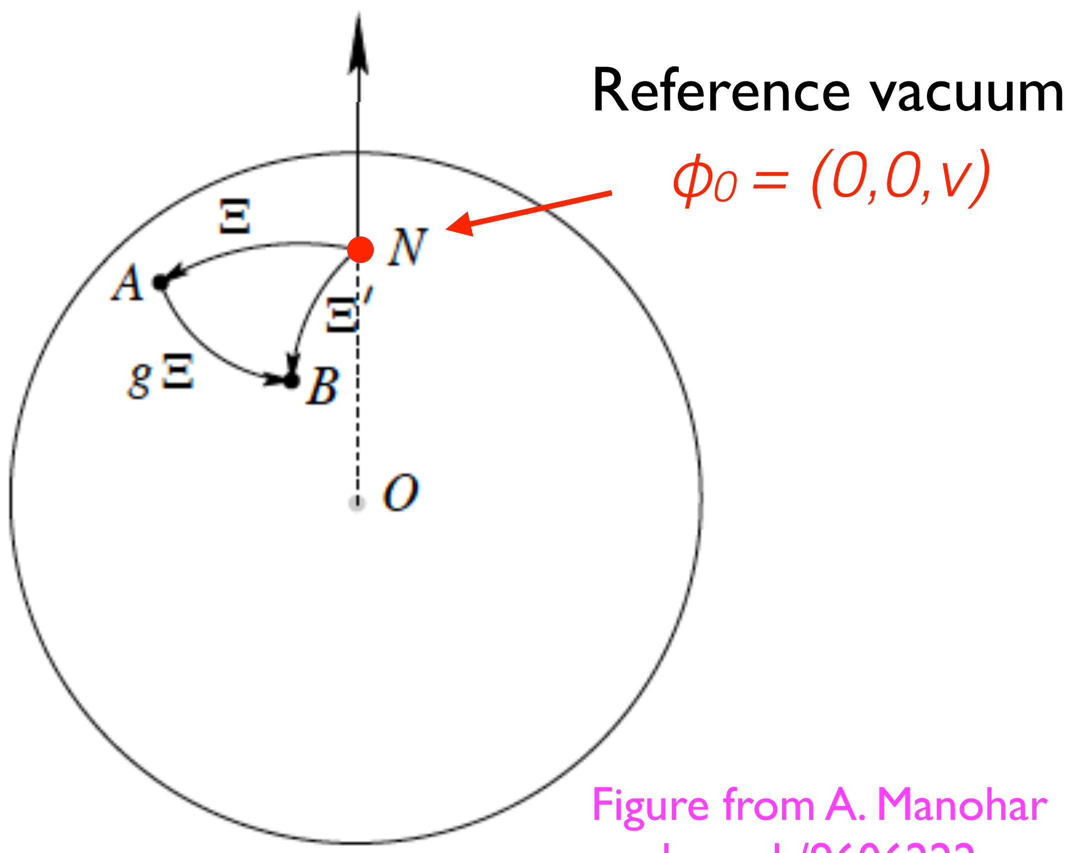

Figure from A. Manohar hep-ph/9606222

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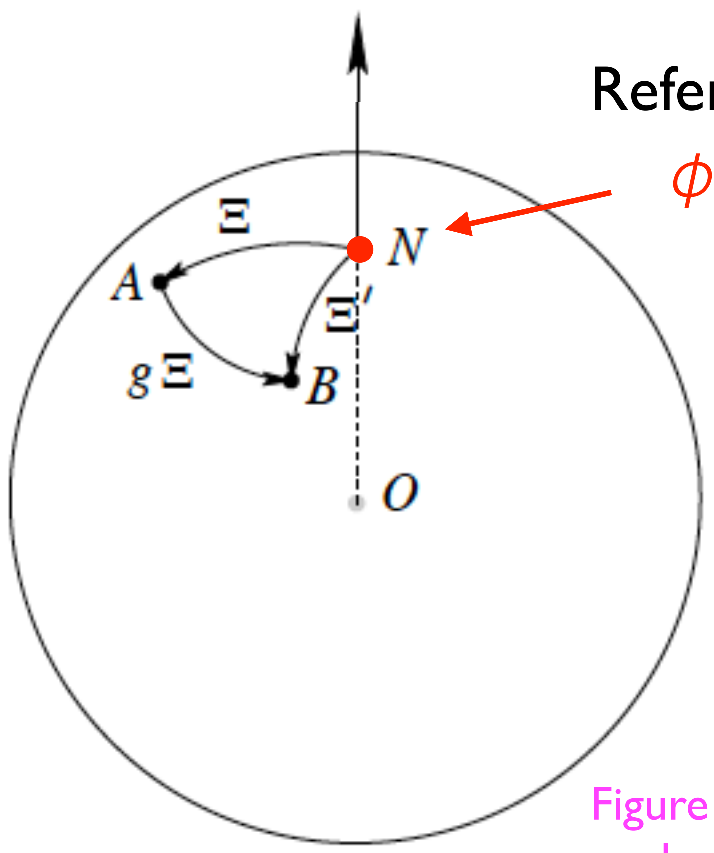
N=3: G=O(3), H=O(2), G/H = S^2

$$\phi_{\text{vac}}(x) = \Xi(x)\,\phi_0$$

$$\Xi(x) = e^{i\pi^a(x)A^a} \xrightarrow{N=3} e^{i(\pi^1(x)J^1 + \pi^2(x)J^2)}$$

Goldstone fields ($\pi^a(x)$) broken generators (A^a)



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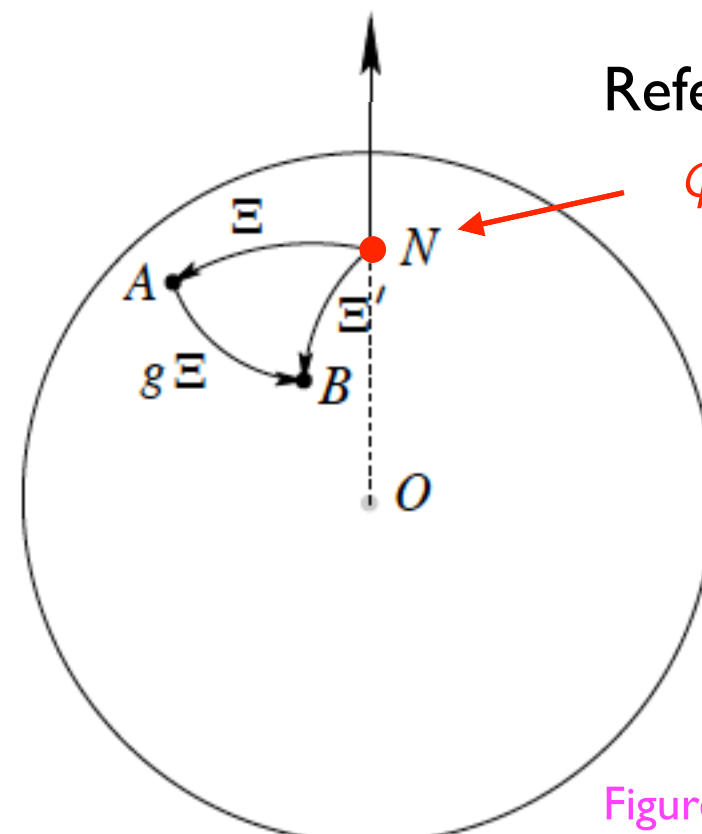
Goldstone fields ($\pi^a(x)$)   broken generators (A^a)

$$g\, \Xi(x) = \Xi'(x)\, h(g, \Xi(x))$$

$h \in H$

$g \in G$



Reference vacuum

$\phi_0 = (0,0,v)$

Figure from A. Manohar hep-ph/9606222

Fields and their transformations

- SSB pattern G→H: GB fields ~ coordinates of the vacuum manifold G/H

- GBs & massive fields (ψ) transformation

$$\Xi(\pi) = e^{i\pi^a A^a}$$

$$\Xi(\pi') = g\ \Xi(\pi)\ h^{-1}(g,\pi)$$

$h \in H$
$g \in G$

Ψ $h_\Psi(h)$

$$\Psi \xrightarrow{g} \Psi' = h_\Psi(h(g,\pi))\ \Psi$$

Representation of unbroken subgroup H under which ψ transforms

Non-linear representation of the group G, linear when restricted to H

Back to SU(n)$_L$ x SU(n)$_R$ n = 2,3

Transformation

$$g = \begin{pmatrix} L & 0 \\ 0 & R \end{pmatrix}$$

Generators

$$V^a = T^a_L + T^a_R$$ Unbroken

$$A^a = T^a_L - T^a_R$$ Broken

$$V^a = \begin{pmatrix} T^a & 0 \\ 0 & T^a \end{pmatrix}$$

$$A^a = \begin{pmatrix} T^a & 0 \\ 0 & -T^a \end{pmatrix}$$

SU(n) generators

Back to $SU(n)_L \times SU(n)_R$ n = 2,3

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SU(n) generators

$$\Xi(\pi) = e^{i\pi^a A^a} = \begin{pmatrix} u(\pi) & 0 \\ 0 & u^\dagger(\pi) \end{pmatrix}$$

$$u(\pi) = e^{i\pi^a T^a}$$

$$\begin{pmatrix} u(\pi) & 0 \\ 0 & u^\dagger(\pi) \end{pmatrix} \longrightarrow \begin{pmatrix} L & 0 \\ 0 & R \end{pmatrix} \begin{pmatrix} u(\pi) & 0 \\ 0 & u^\dagger(\pi) \end{pmatrix} \begin{pmatrix} h^{-1} & 0 \\ 0 & h^{-1} \end{pmatrix}$$

h^{-1}, element of $SU(n)_V$

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h^{-1}, element of $SU(n)_V$

$$u \rightarrow L\, u\, h^{-1} = h\, u\, R^\dagger$$

$$u^2 \equiv U \rightarrow L\, U\, R^\dagger$$

Explicit parameterization (SU(2))

- Choice of GB fields

$$u^2 = U \;=\; e^{i\mathbf{\Phi}/F} \qquad \mathbf{\Phi} = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}$$

$$u \;\rightarrow\; L\,u\,h^{-1} \;=\; h\,u\,R^\dagger$$

$$U \rightarrow L\,U\,R^\dagger$$

$$L, R \in SU(2)_{L,R}$$

$$h \in SU(2)_V$$

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$$L, R \in SU(2)_{L,R}$$
$$h \in SU(2)_V$$

- Matter fields ~ representations of unbroken subgroup $SU(2)_V$

$$N = \begin{pmatrix} p \\ n \end{pmatrix} \qquad N \;\to\; h(g,\pi)N$$

Chiral covariant derivative

$$\nabla_\mu N = (\partial_\mu + \Gamma_\mu)N \to \; h(g,\pi)\,\nabla_\mu N$$

Convenient building block to construct invariants

$$u_\mu \equiv i\left(u\partial_\mu u^\dagger - u^\dagger\partial_\mu u\right) \to h(g,\pi)u_\mu h^{-1}(g,\pi)$$

$$\Gamma_\mu \equiv \frac{1}{2}\left(u\partial_\mu u^\dagger + u^\dagger\partial_\mu u\right)$$

Effective Lagrangian (π)

- Require invariance under the non-linear realization of G=SU(n)xSU(n)
- Organize it as an expansion in powers of derivatives (low E expansion) and explicit symmetry breaking (quark mass)

$$\mathcal{L}_\pi = \mathcal{L}_\pi^{(2)} + \mathcal{L}_\pi^{(4)} + \mathcal{L}_\pi^{(6)} + \ldots$$

$$\mathcal{L}_\pi^{(2)} = \frac{F^2}{4} \mathrm{Tr}\left[\partial_\mu U \, \partial^\mu U^\dagger + 2B \, m_q \left(U + U^\dagger\right)\right]$$

- Counting rules: $\partial \sim p$, $m_q \sim p^2$ (to be explained in a moment)

- m_q term easiest to derive if assume $m_q \to L \, m_q \, R^\dagger$: L_{QCD} is invariant

- Two 'low energy constants' (LECs) not determined by symmetry: F, B

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- Noether's currents: identify F with pion decay constant F= F_π

$$j_R^{\mu a} = \frac{iF^2}{2} \mathrm{Tr} \left(T^a \, U \partial_\mu U^\dagger \right)$$

$$j_L^{\mu a} = \frac{iF^2}{2} \mathrm{Tr} \left(T^a \, U^\dagger \partial^\mu U \right)$$

$$j_A^{\mu a} = j_R^{\mu a} - j_L^{\mu a} = -F \, \partial^\mu \pi^a \; + \ldots$$

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- m_q term gives GB mass terms $M_{PS}^2 \sim B\ m_q \sim p^2$

$$\mathcal{L}_2 \supset \partial_\mu \pi^- \partial^\mu \pi^+ \boxed{- B\,(m_u+m_d)\,\pi^+\pi^- - B\,(m_u+m_s)\,K^+K^-} + \ldots$$

Effective Lagrangian (π)

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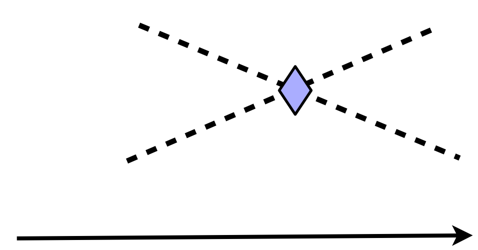
- 2n-GB interaction vertices in terms of F, B: ππ scattering, etc.

$$\mathcal{L}_2 \supset \frac{1}{12f^2} \langle (\Phi \overleftrightarrow{\partial}_\mu \Phi)(\Phi \overleftrightarrow{\partial^\mu} \Phi) \rangle$$



$$A_{\pi\pi}^{(2)} \sim \frac{p_{\rm ext}^2}{F^2}$$

Power counting (1)

- Higher derivatives and/or mass insertions in effective Lagrangian:

$$\mathcal{L} = \frac{F^2}{4}\left[\mathrm{Tr}\,\partial_\mu U\,\partial^\mu U + \frac{1}{\Lambda^2}\mathcal{L}_\pi^{(4)} + \frac{1}{\Lambda^4}\mathcal{L}_\pi^{(6)} + \ldots\right]$$

$$A_{\pi\pi}^{(2)} \sim \frac{p_{\rm ext}^2}{F^2} \qquad\qquad A_{\pi\pi}^{(4)} \sim \frac{p_{\rm ext}^2}{F^2}\,\frac{p_{\rm ext}^2}{\Lambda^2}$$

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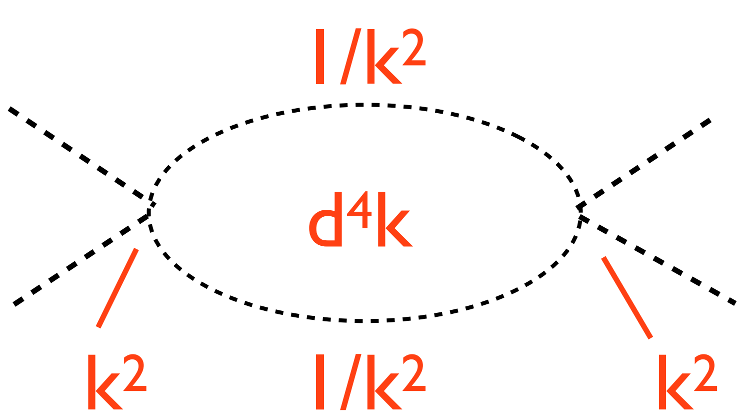
$$A_{\pi\pi}^{(2)} \sim \frac{p_{\mathrm{ext}}^2}{F^2} \qquad\qquad A_{\pi\pi}^{(4)} \sim \frac{p_{\mathrm{ext}}^2}{F^2}\,\frac{p_{\mathrm{ext}}^2}{\Lambda^2}$$

- What about loops with $\mathcal{L}_2$ vertices?

$$\Lambda_{\mathrm{loop}} \sim 4\pi\,F$$



$$A_{\pi\pi}^{(\mathrm{loop})} \sim \frac{p_{\mathrm{ext}}^4}{16\pi^2\,F^4}\,\log\frac{p_{\mathrm{ext}}^2}{\mu^2}$$

Estimate straightforward in mass-independent regulators and subtraction schemes (such as dim reg): amplitude can only contain powers of p, while μ appears only in logs

Power counting (I)

- Higher derivatives and/or mass insertions in effective Lagrangian:

$$\mathcal{L} = \frac{F^2}{4}\left[\mathrm{Tr}\,\partial_\mu U\,\partial^\mu U + \frac{1}{\Lambda^2}\mathcal{L}_\pi^{(4)} + \frac{1}{\Lambda^4}\mathcal{L}_\pi^{(6)} + \ldots\right]$$

$$A_{\pi\pi}^{(2)} \sim \frac{p_{\rm ext}^2}{F^2} \qquad\qquad A_{\pi\pi}^{(4)} \sim \frac{p_{\rm ext}^2}{F^2}\,\frac{p_{\rm ext}^2}{\Lambda^2}$$

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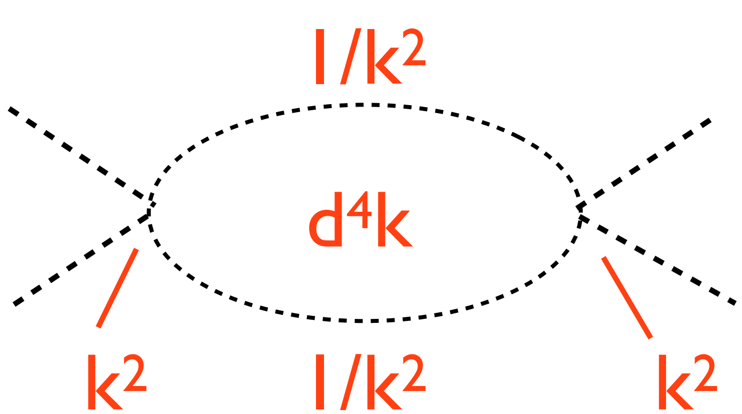


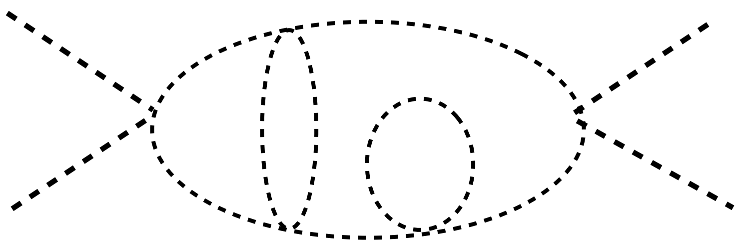
$$A_{\pi\pi}^{(\rm loop)} \sim \frac{p_{\rm ext}^4}{16\pi^2\ F^4}\ \log\frac{p_{\rm ext}^2}{\mu^2}$$

Dependence on regularization / renormalization scale in loops (μ) canceled by the p^4 low-energy constants: S-matrix elements are μ-independent

Power counting (2)

Weinberg '79

- Weinberg's general argument



#derivatives + 2 #m_q

#vertices of type i

$$\mathcal{A} \sim \int (d^4p)^L \frac{1}{(p^2)^I} \prod_i \left[p^{d_i}\right]^{V_i} \sim p^\nu$$

$$\nu = \sum_i V_i d_i - 2I + 4L = \sum_i V_i(d_i - 2) + 2L + 2$$

$$L = I - \sum_i V_i + 1$$

d_i -2 > 0 due to chiral symmetry

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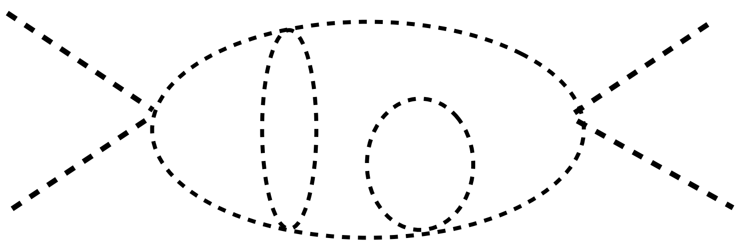

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d_i -2 > 0 due to chiral symmetry

- Low-energy expansion with even powers of ν
- Loop divergences can be reabsorbed by higher order L_{eff}
- EFT is renormalizable (and predictive) to a given order in p/Λ

Intermediate summary

- ChPT quite successful in the meson sector

- Example: ππ scattering

Phase shifts from K →ππeν

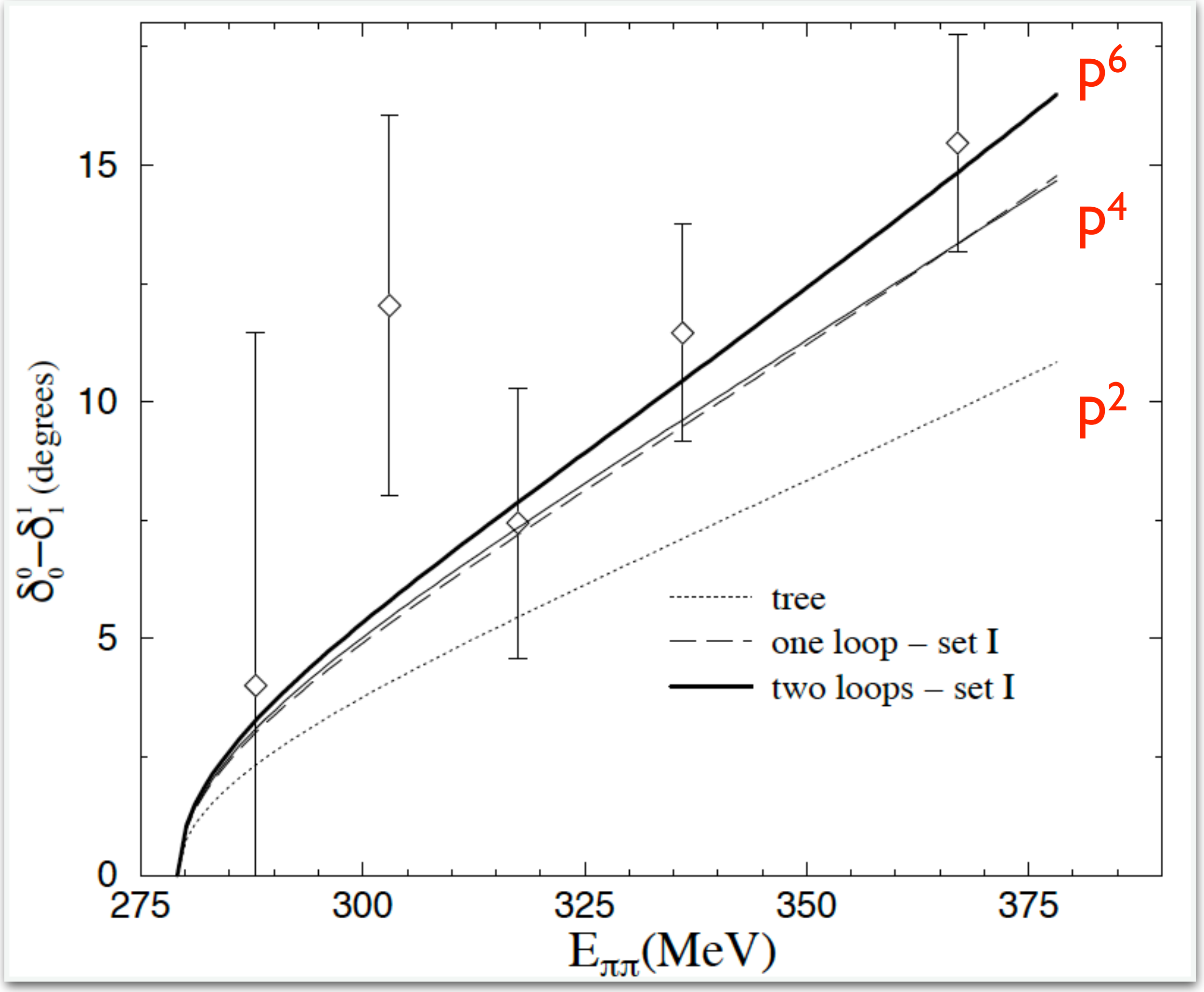

Figure from G. Ecker hep-ph/9805500

- *Many* applications, including semileptonic and non-leptonic weak decays of pseudo scalar mesons, radiative corrections, etc.

ChPT with baryons (1)

- Presence of $m_N \sim \Lambda_\chi$ spoils manifest power counting as $i\partial_0 N \sim m_N N$
- But nucleons interacting with `soft' pions are nearly on shell

$$p^\mu = m_N v^\mu + k^\mu \qquad \frac{k}{m_N} \ll 1$$

- Propagator takes the form (up to relative corrections ~ k/m_N):

$$i\frac{\not{p} + m_N}{p^2 - m_N^2 + i\epsilon} = i\frac{m_N(1+\not{v}) + \not{k}}{2m_N v \cdot k + k^2 + i\epsilon} \rightarrow \left(\frac{1+\not{v}}{2}\right)\frac{i}{v \cdot k + i\epsilon}$$

Projects on to 'large' particle components of Dirac spinor

Scales as $1/p_{soft}$

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Scales as $1/p_{soft}$

- What Lagrangian generates this?

ChPT with baryons (2)

Georgi '90, Jenkins-Manohar '91

- To get manifest power counting, write Lagrangian in terms of v-dependent fields N_v so that $i\partial_0 N_v \sim k_0 N_v << \Lambda_\chi N_v$

$$N(x) = e^{-im_N v\cdot x}\Big(N_v(x) + H_v(x)\Big)$$

$$N_v(x) = e^{im_N v\cdot x}\frac{1+\not{v}}{2}N(x) \qquad H_v(x) = e^{im_N v\cdot x}\frac{1-\not{v}}{2}N(x)$$

$$\downarrow$$

$$\bar{N}(i\not{\partial} - m_N)N \rightarrow \bar{N}_v iv\cdot\partial N_v + \ldots$$

- Baryon bilinears expressed in terms of $v^\mu, \quad S^\mu = \frac{i}{2}\gamma_5\sigma^{\mu\nu}v_\nu$

$$\bar{N}_v\gamma_5 N_v = 0 \qquad \bar{N}_v\gamma_\mu\gamma_5 N_v = 2\bar{N}_v S_\mu N_v$$

$$\bar{N}_v\gamma_\mu N_v = \bar{N}_v v_\mu N_v \qquad \bar{N}_v\sigma_{\mu\nu}N_v = 2\epsilon_{\mu\nu\alpha\beta}v^\alpha\bar{N}_v S^\beta N_v$$

Effective Lagrangian (π,N)

- Use building blocks N, ∇N, u, … and organize according to standard counting rules $\partial \sim p$, $m_q \sim p^2$

$$\mathcal{L}_{\pi N} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \ldots$$

$$\boxed{\mathcal{L}_{\pi N}^{(1)} = \bar{N}_v\, iv \cdot \nabla\, N_v \; + \; g_A \bar{N}_v\, S \cdot u\, N_v}$$

$$v^\mu = (1, \vec{0})$$
$$S^\mu = (0, \vec{\sigma}/2)$$

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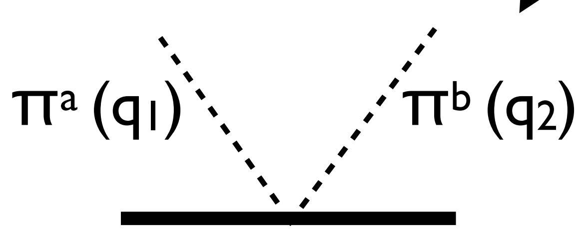
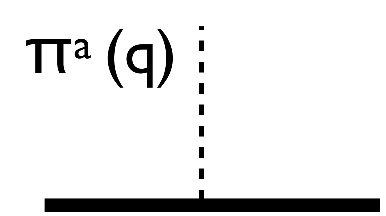


$$\frac{1}{4F^2} v \cdot (q_1 + q_2) \epsilon^{abc} \tau^c$$

+ 2N4π, …



$$\frac{g_A}{F} S \cdot q \tau^a$$

+ 2N3π, …

Effective Lagrangian (π,N)

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$$v^\mu = (1,\vec{0}) \qquad S^\mu = (0,\vec{\sigma}/2)$$

- In higher orders both p/Λ_χ and p/m_N terms appear

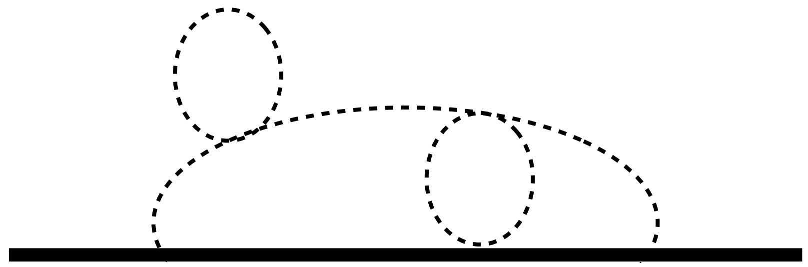
$$\mathcal{L}_{\pi N}^{(2)} : \frac{1}{2m_N}\bar{N}_v\left((v\cdot\nabla)^2 - \nabla\cdot\nabla\right)N_v,$$

$$\bar{N}_v\,(v\cdot u)^2\, N_v, \quad \bar{N}_v\, u\cdot u\, N_v, \quad \ldots$$

← Non-relativistic expansion of kinetic energy

Power counting with nucleons

- Weinberg's general argument for connected amplitudes



$$\mathcal{A} \sim \int (d^4p)^L \frac{1}{(p^2)^{I_\pi}} \frac{1}{(p)^{I_N}} \prod_i \left[p^{d_i}\right]^{V_i} \sim p^\nu$$

$$\nu = \sum_i V_i(d_i + n_i/2 - 2) + 2L - E_N/2 + 2$$

$$L = I_\pi + I_N - \sum_i V_i + 1$$

$$2I_N + E_N = \sum_i V_i n_i$$

n_i = # of nucleon fields in the vertex

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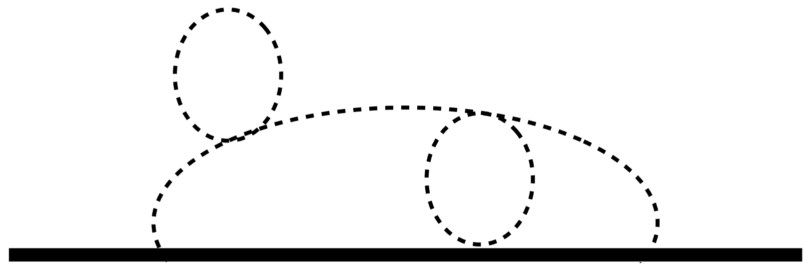

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n_i = # of nucleon fields in the vertex

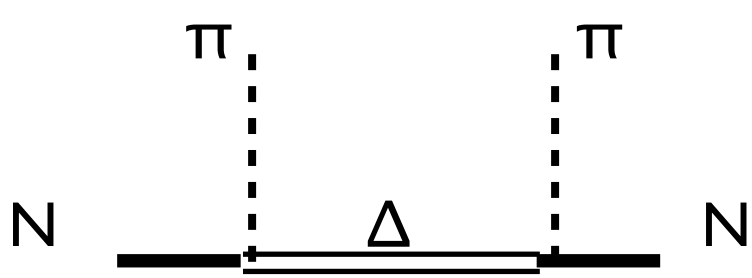
- Low-energy expansion contains all powers of ν
- Convergence pattern not too natural in certain cases: impact of the Δ?

See lectures by Martin Hoferichter

The role of the Δ(1232)

Jenkins-Manohar '91, Hemmert-Holstein-Kambor '97

- Unnaturally large values of some LECs can be understood in terms of large contributions from the Δ-decuplet:



$$\delta \equiv m_\Delta - m_N = 293 \text{ MeV}$$

LEC~1/δ instead of ~1/Λ

- Can include Δ in the EFT with power counting:

$$Q \sim m_\pi \sim \delta \ll \Lambda$$

Δ-ful

instead of

$$Q \sim m_\pi \ll \delta \sim \Lambda$$

Δ-less

- Improved convergence, LECs have more natural size

Electroweak interactions

- Start from QCD with external electroweak sources $s(x), p(x), l_\mu(x), r_\mu(x)$

$$\mathcal{L} = \bar{q}_L i \not{D} q_L + \bar{q}_R i \not{D} q_R - \bar{q}_R (s + ip) q_L - \bar{q}_R (s - ip) q_L + \bar{q}_L \gamma^\mu l_\mu q_L + \bar{q}_R \gamma^\mu r_\mu q_R$$

$$q = \begin{pmatrix} u \\ d \end{pmatrix}$$

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$$q = \begin{pmatrix} u \\ d \end{pmatrix}$$

- In the Standard Model (at low E):

$$s + ip = m_q + \text{coupling to Higgs}$$

$$l_\mu = -e Q_L^{\rm em} A_\mu \; + \; Q_L^{\rm w} J_\mu^{\rm lept} \; + \; Q_L^{\rm w\dagger} J_\mu^{\rm lept\dagger}$$

$$r_\mu = -e Q_R^{\rm em} A_\mu$$

$$Q_L^{\rm em} = Q_R^{\rm em} = \begin{pmatrix} 2/3 & 0 \\ 0 & -1/3 \end{pmatrix} \qquad Q_L^{\rm w} = -2\sqrt{2} G_F \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad J_\mu^{\rm lept} = \bar{e}_L \gamma_\mu \nu_{eL}$$

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$$q = \begin{pmatrix} u \\ d \end{pmatrix}$$

- "Spurion" transformation of the sources (so that $\mathcal{L}$ is invariant)

$$(s + ip) \to R(s + ip)L^\dagger$$
$$l_\mu \to L l_\mu L^\dagger + i L \partial_\mu L^\dagger$$
$$r_\mu \to R r_\mu R^\dagger + i R \partial_\mu R^\dagger$$

$$Q_L^{\text{em,w}} \to L Q_L^{\text{em,w}} L^\dagger \qquad Q_R^{\text{em}} \to R Q_R^{\text{em}} R^\dagger$$

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$$q = \begin{pmatrix} u \\ d \end{pmatrix}$$

- Modified building blocks and their transformations:

$$D_\mu U = \partial_\mu U - i l_\mu U + i U r_\mu$$

$$D_\mu U \to L\, D_\mu U\, R^\dagger$$

$$\nabla_\mu N = (\partial_\mu + \Gamma_\mu)N$$

$$\Gamma_\mu \equiv \frac{1}{2}\left(u(\partial_\mu - i r_\mu)u^\dagger + u^\dagger(\partial_\mu - i l_\mu)u\right)$$

$$\nabla_\mu N \to h\, \nabla_\mu N$$

$$u_\mu \equiv i\left(u(\partial_\mu - i r_\mu)u^\dagger - u^\dagger(\partial_\mu - i l_\mu)u\right)$$

$$\chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u \qquad \chi = 2B(s+ip)$$

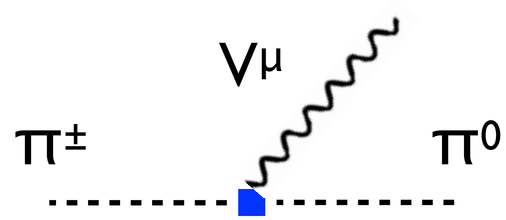
$$\mathcal{Q}_L^{\mathrm{em,w}} = u^\dagger Q_L^{\mathrm{em,w}} u \qquad \mathcal{Q}_R^{\mathrm{em}} = u Q_R^{\mathrm{em}} u^\dagger$$

$$O \to h O h^{-1}$$

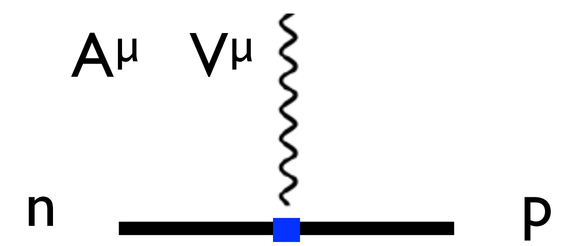
Examples

- Weak charged-current interaction vertices (mesons, baryons)

$$\sim F_\pi q^\mu$$



$$\sim (p_1^\mu + p_2^\mu)$$



$$\sim (v^\mu - 2g_A S^\mu)$$

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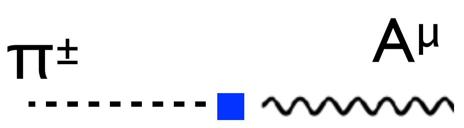

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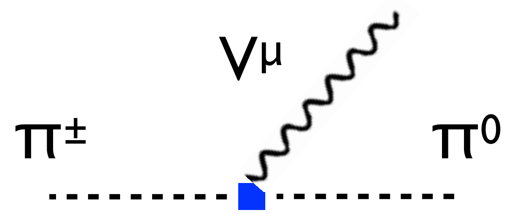

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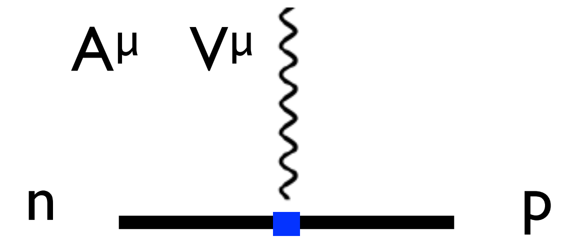
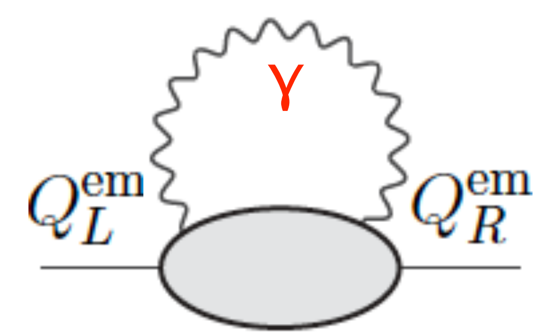


$$\sim (v^\mu - 2 g_A S^\mu)$$

- Effects of virtual photons: pion mass splitting



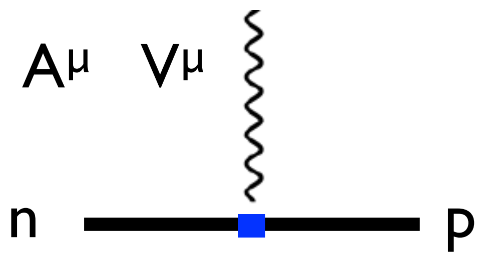
$$\mathcal{L}_\pi^{(e^2)} = e^2 Z F^4 \mathrm{Tr}\left[Q_L^{em} U Q_R^{em} U^\dagger\right] \supset -2e^2 Z F^2 \, \pi^+ \pi^- \; + \; \ldots$$

LEC determined in terms of the pion electromagnetic mass splitting

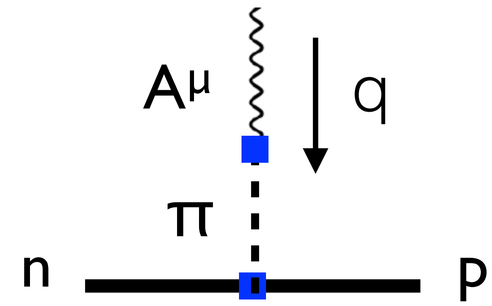
Nucleon weak currents

- "V-A" current relevant for single and double beta decay

- Leading order (O(p))

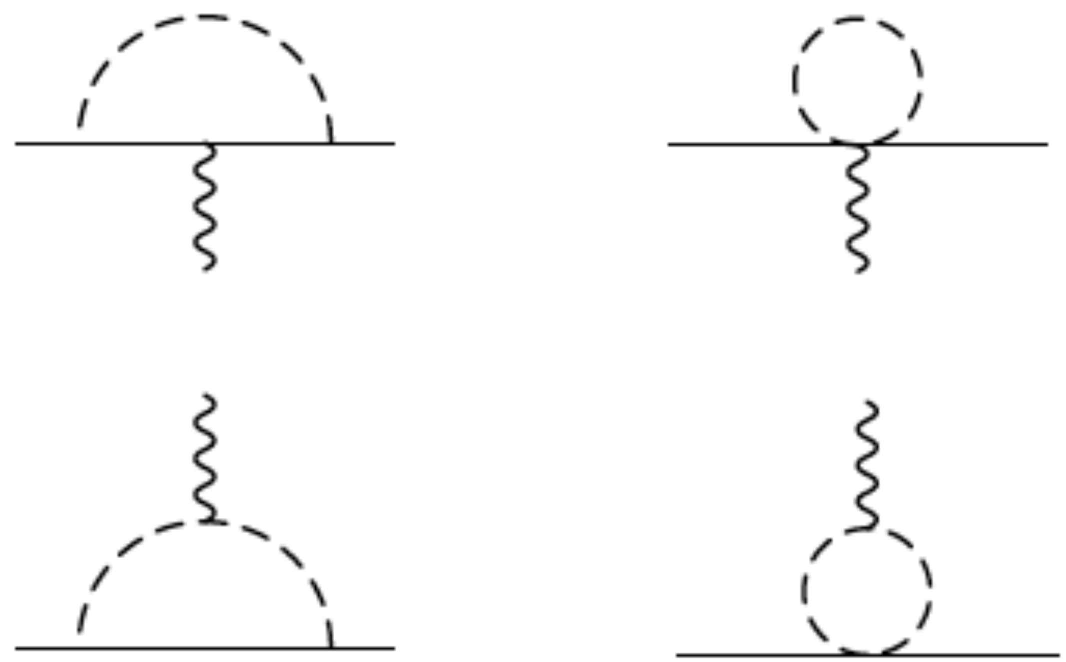


$$\sim (v^\mu - 2g_A S^\mu)$$



$$\sim -2g_A \frac{S \cdot q}{\vec{q}^2 + m_\pi^2} q^\mu$$

- N2LO (O(p³))



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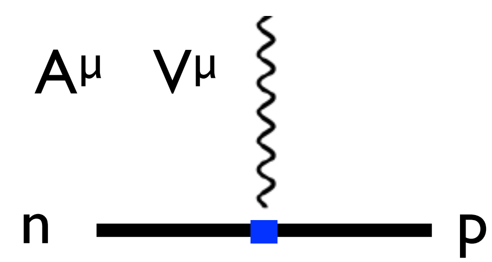

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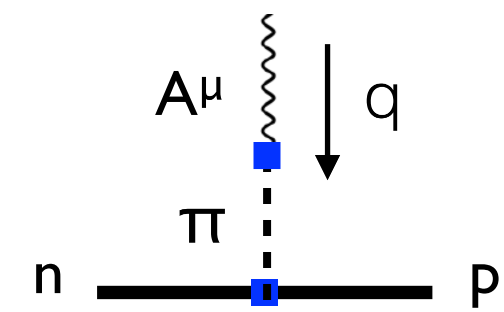

$$\sim -2 g_A \frac{S \cdot q}{\vec{q}^2 + m_\pi^2} q^\mu$$

- Including recoil ($1/m_N$) and $O(p^3)$ effects (form factors):

$$J_V^\mu = g_V(\mathbf{q}^2)\left(v^\mu + \frac{p^\mu + p'^\mu}{2m_N}\right) + i g_M(\mathbf{q}^2)\,\epsilon^{\mu\nu\alpha\beta}\,\frac{v_\alpha S_\beta q_\nu}{m_N},$$

$$J_A^\mu = -2 g_A(\mathbf{q}^2)\left(S^\mu - \frac{S \cdot (p + p')}{2m_N} v^\mu + \frac{S \cdot q}{\mathbf{q}^2 + m_\pi^2} q^\mu\right).$$

$$g_\alpha(q^2) = g_\alpha(0)\left[1 + \frac{\langle r_\alpha \rangle^2}{6} q^2 + \ldots\right] \qquad g_V(0) = 1 \qquad g_A(0) = g_A \qquad g_M(0) = 1 + \kappa_v = 4.71$$

Multi-nucleon systems

- Many scales in the problem: tall order for EFT

E

a^{-1} ($^1S_0$) = 8.3 MeV

a^{-1} ($^3S_1$) = 36 MeV

p_D ~ 45 MeV

[B_D = 2.224 MeV]

m_π ~140 MeV

Λ~GeV

- EFT needs to account for shallow nuclear bindings and cope with large scattering lengths (for k>1/a, re-sum all "ak" terms and expand in k/Λ), e.g.

$$T = \frac{4\pi}{m_N}\frac{1}{\left(-\frac{1}{a}+\frac{r_{0s}}{2}k^2+....-ik\right)} \rightarrow -\frac{4\pi}{m_N}\frac{1}{\left(-\frac{1}{a}+ik\right)}\left[1+O(k^2)\right]$$

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$p_D \sim 45$ MeV $\quad$ [$B_D = 2.224$ MeV]

$m_\pi \sim 140$ MeV

$\Lambda \sim$ GeV

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$$T = \frac{4\pi}{m_N} \frac{1}{\left(-\frac{1}{a} + \frac{r_{0s}}{2}k^2 + \ldots . - ik\right)} \to -\frac{4\pi}{m_N} \frac{1}{\left(-\frac{1}{a} + ik\right)} \left[1 + O(k^2)\right]$$

$$\mathcal{A} = \sum_\nu \left(\frac{Q}{\Lambda}\right)^\nu F_\nu\left(\frac{Q}{\mu}, g_i(\mu)\right)$$

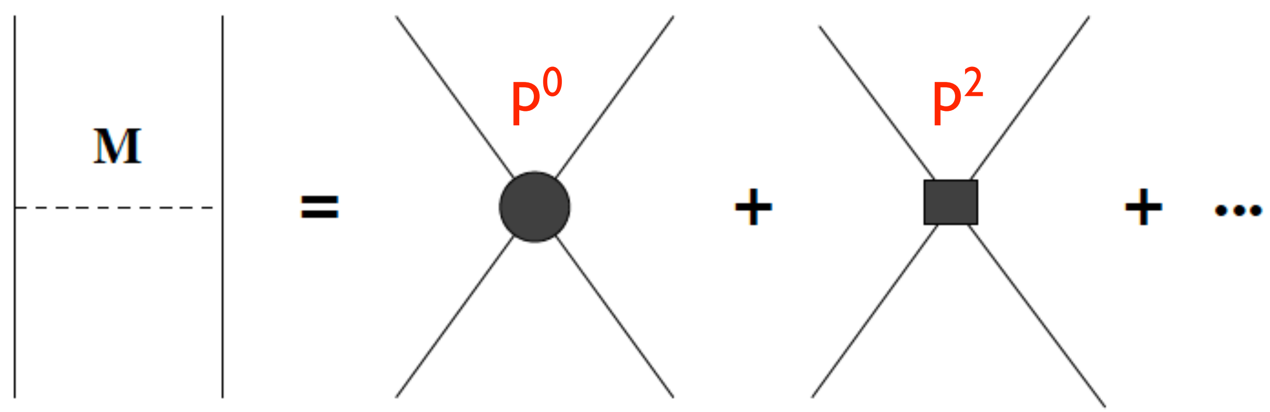
More generally, F_ν may require a non-perturbative calculation

Multi-nucleon EFT

Weinberg '91, …

- In addition to π & πN, new NN interactions should be considered, ordered according to # of derivatives and m_q insertions
- Two chiral-invariant terms with no-derivatives

$$\mathcal{L}_{NN}^{(0)} = -\frac{C_S}{2}\,\bar{N}N\,\bar{N}N - \frac{C_T}{2}\,(\bar{N}\vec{\sigma}N)\cdot(\bar{N}\vec{\sigma}N)\ +\ \ldots$$



Can think of these as arising from exchange of heavier mesons M=σ,ρ,ω,…

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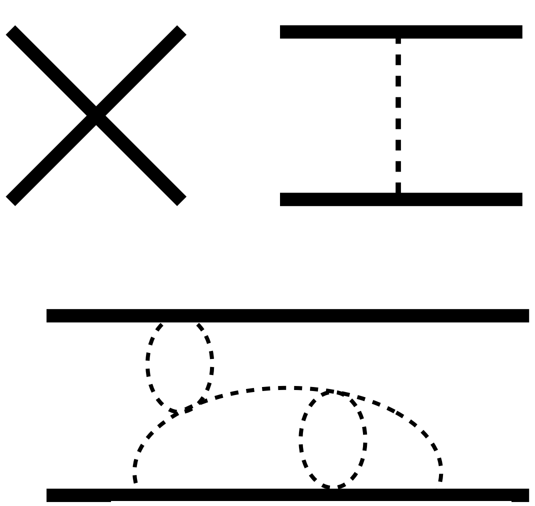
- Scaling of connected A→A nucleon amplitudes ($A = E_N/2$)



$$\mathcal{A} \sim Q^{\nu} \qquad Q \sim m_\pi$$

$$\nu = \sum_i V_i(d_i + n_i/2 - 2) + 2L - E_N/2 + 2$$

Multi-nucleon EFT

Weinberg '91, …

- In addition to π & πN, new NN interactions should be considered, ordered according to # of derivatives and m_q insertions
- Two chiral-invariant terms with no-derivatives

$$\mathcal{L}_{NN}^{(0)} = -\frac{C_S}{2}\,\bar{N}N\,\bar{N}N - \frac{C_T}{2}\,(\bar{N}\vec{\sigma}N)\cdot(\bar{N}\vec{\sigma}N)\ +\ \ldots$$

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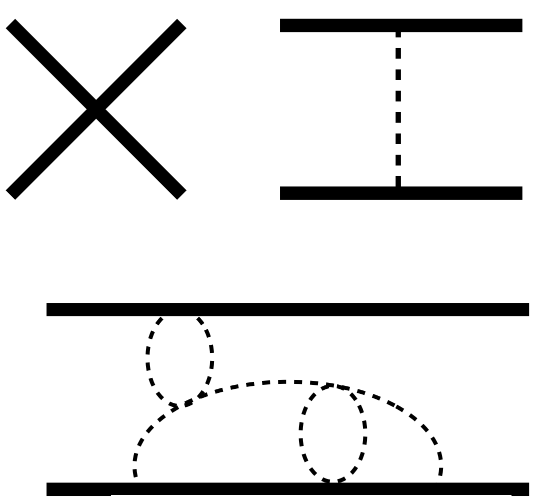

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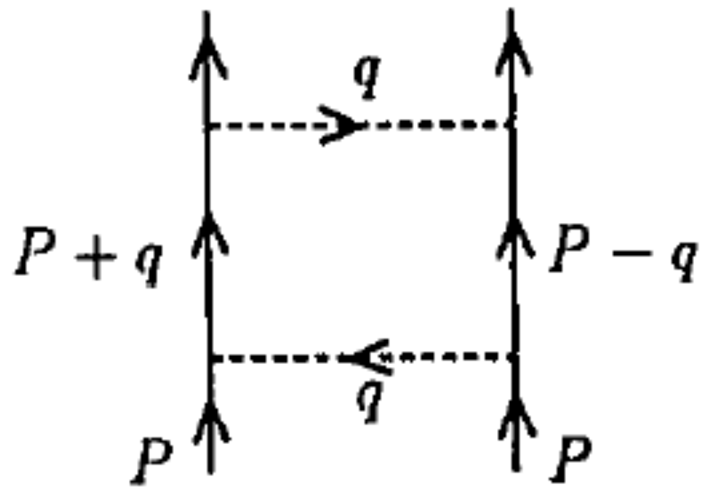
Simple ordering of diagrams?

Infrared enhancements

- The above result is troubling: everything is perturbative, no nuclei! Bound states should arise from failure of perturbation theory
- IR divergence in “reducible” diagrams (NN intermediate states)!



$$\int dq^0 \,(q^0 + i\epsilon)^{-1}(q^0 - i\epsilon)^{-1}$$

Contour of integration ‘pinched’ between nucleon poles

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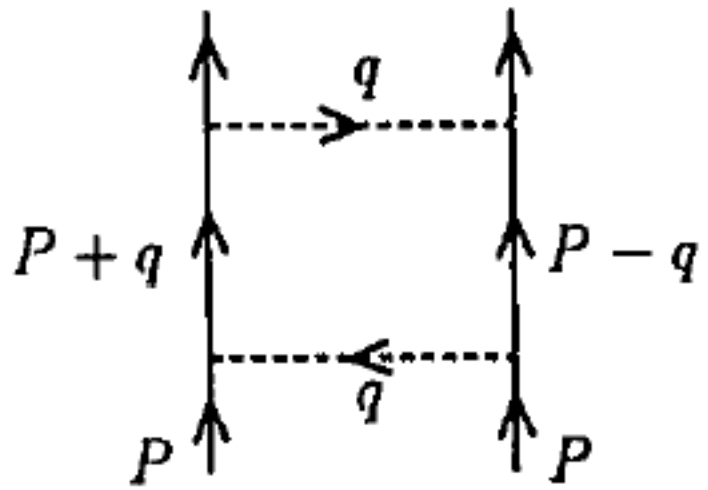

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Contour of integration 'pinched' between nucleon poles

- Physically, a new scale has to be taken into account: $Q^2/(2m_N)$

$$q^0 = \mp i\epsilon, \quad \longrightarrow \quad q^0 \simeq \pm[\vec{q}^{\,2}/2m_N - i\epsilon]$$

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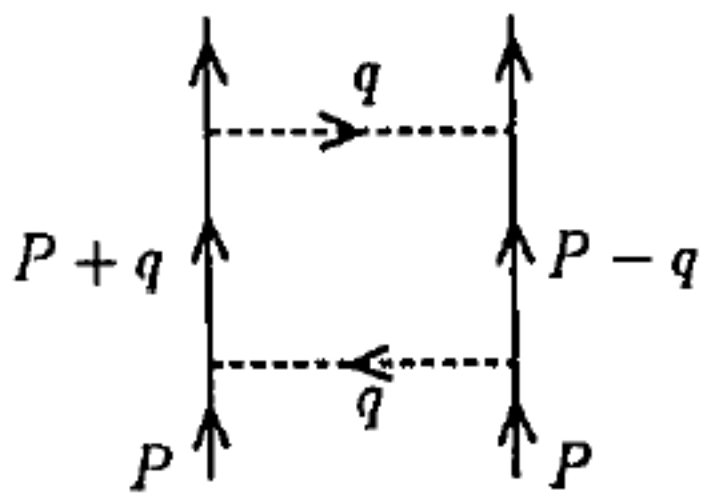
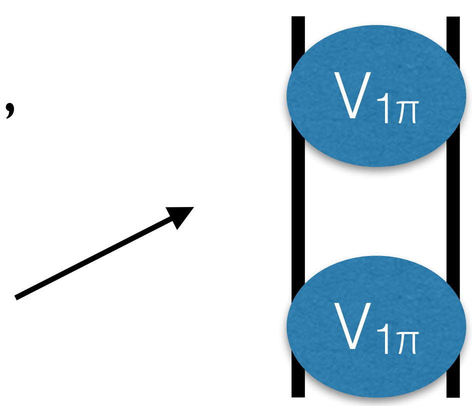


Recover 'old fashioned' perturbation theory
$G = 1/(E - q^2/m_N)$



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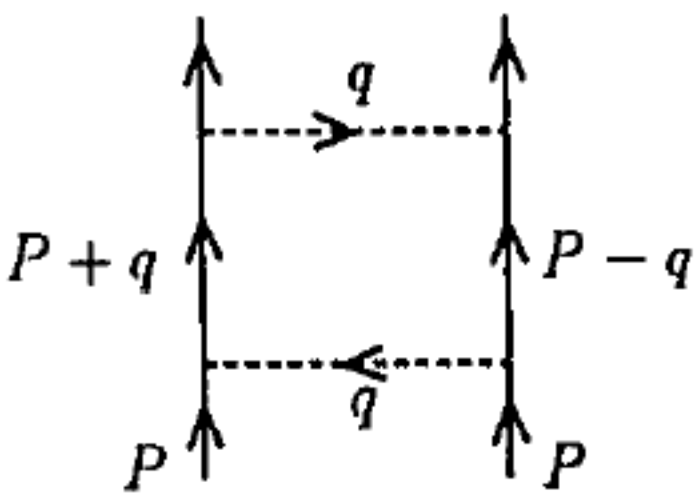

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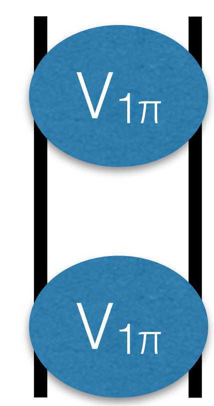

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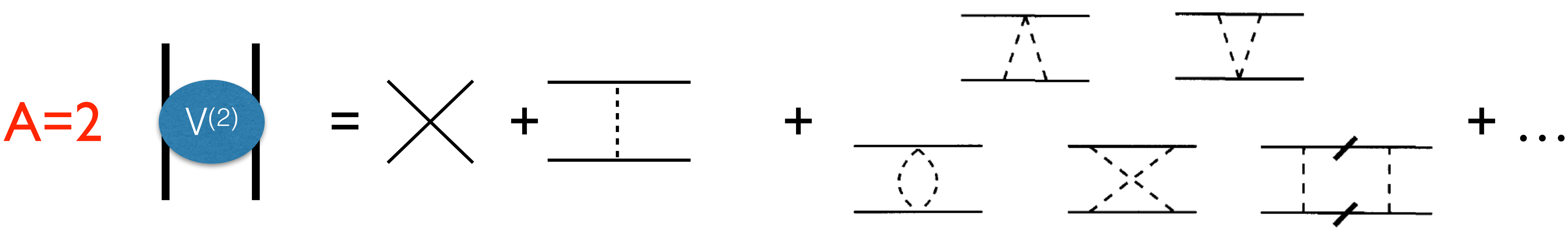
- Nucleon propagator $S(q) = 1/(q_0 - q^2/2m_N)$ scales as $1/Q$ or m_N/Q^2. (enhanced by $m_N/Q >> 1$), depending on which pole is picked up. Similar story for $\int dq_0$

Potentials and amplitudes

- Weinberg's prescription:
    - Define n-nucleon potentials $V^{(n)}$: connected n→n nucleon diagrams free of 'pinch' poles (no m_N dependence), for which chiral power counting works: $V^{(n)} = V_0^{(n)} + V_1^{(n)} + \ldots \quad V_k^{(n)} \sim Q^k$



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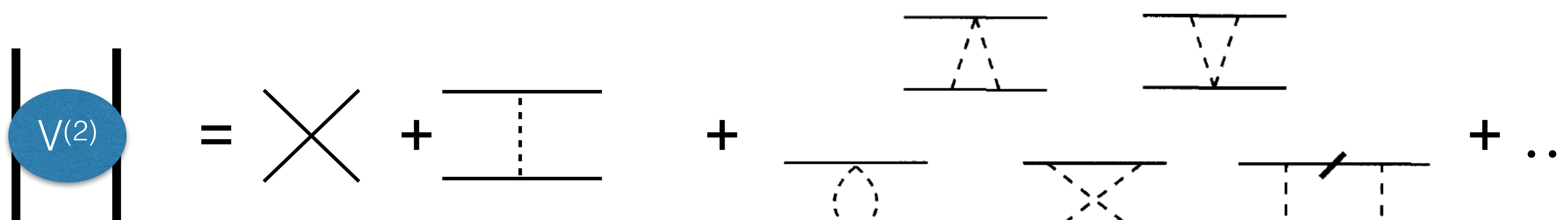
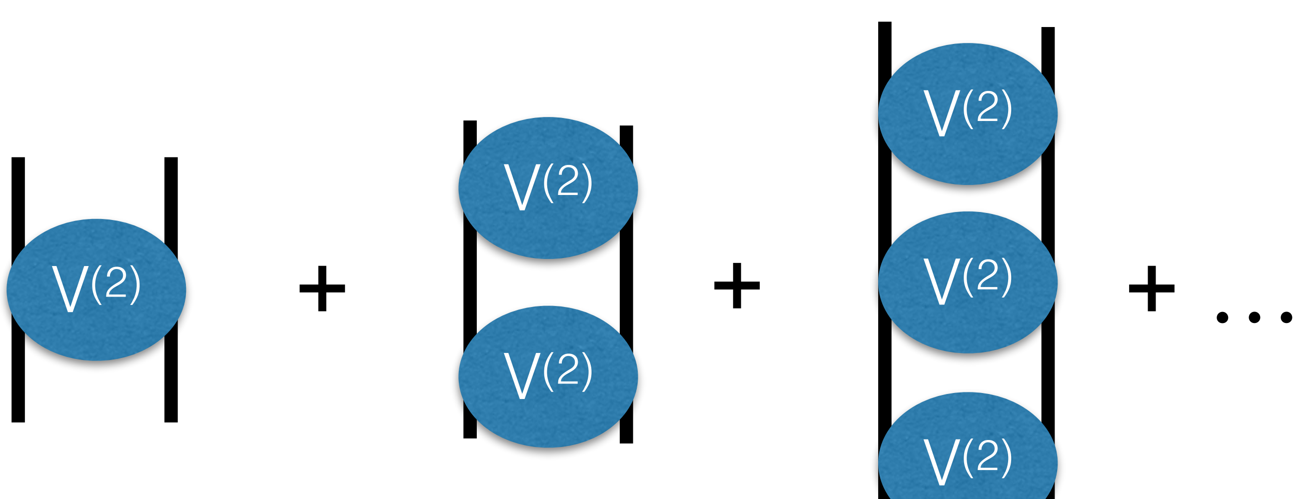
A=2



T =



$$T = V + VG_0V + VG_0VG_0V + \ldots$$
$$= V + VG_0T,$$

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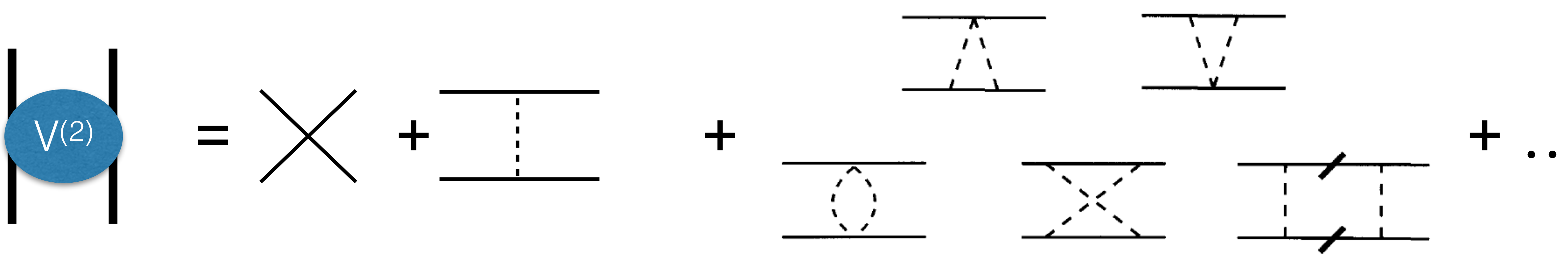

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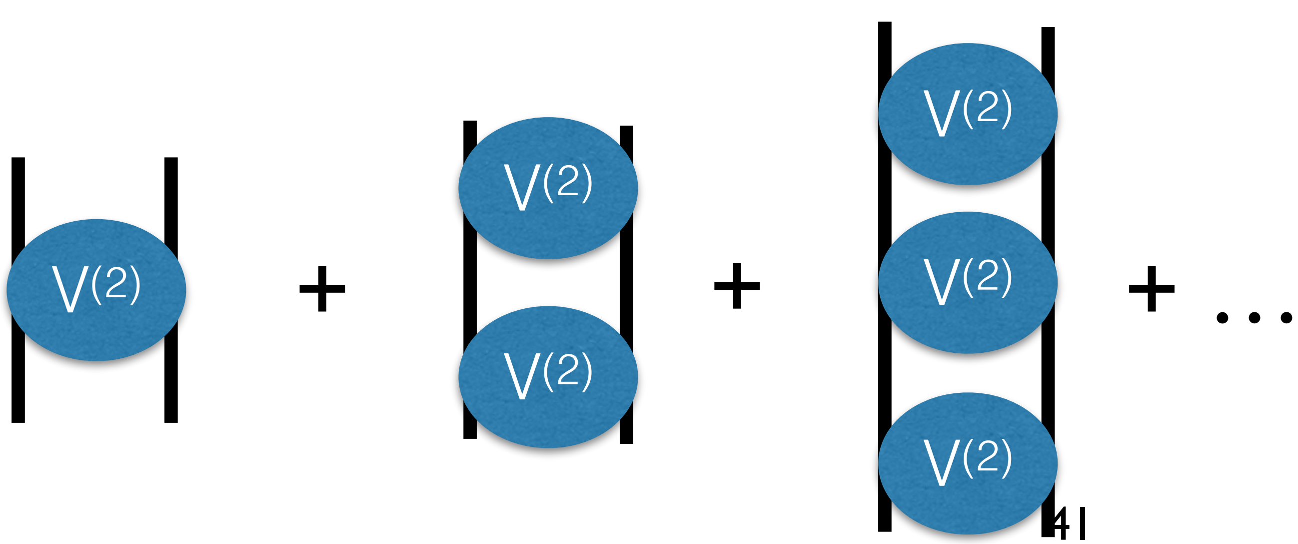
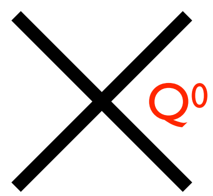
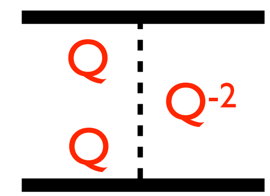


Both parts of the calculation require appropriate regularization and renormalization.
RG invariance?

Leading order NN potential

- One pion exchange + contact (in s-waves only)





$O(Q^0)$

$$V_0^{(2)} = C_S + C_T\, \vec{\sigma}_1 \cdot \vec{\sigma}_2 - \frac{g_A^2}{4F_\pi^2} \frac{(\vec{q} \cdot \vec{\sigma}_1\, \vec{q} \cdot \vec{\sigma}_2)(\vec{\tau}_1 \cdot \vec{\tau}_2)}{\vec{q}^2 + m_\pi^2}$$

For the dimension-ful constants $C_{S,T}$ assume $C_{S,T} \sim 1/(F_\pi)^2$,

as the contact term induced by one pion exchange

Leading order NN potential

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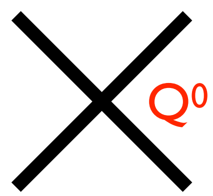

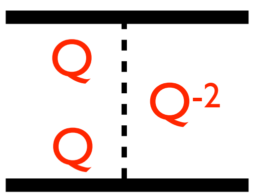

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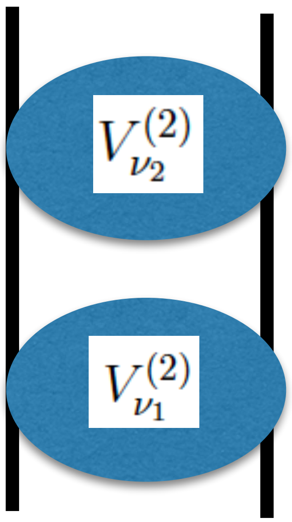
- Is IR enhancement sufficient to justify iteration of $V_0^{(2)}$?



- Term with L reducible loops** ~ $Q^{\nu_1+\ldots+\nu_{L+1}}\ (Q m_N)^L$
- Iteration requires booking $m_N \sim \Lambda^2/Q$

** NN loop: $\int dq_0 d\vec{q} \sim Q^5/m_N$ —— $\sim\ m_N/Q^2$

Leading order NN potential

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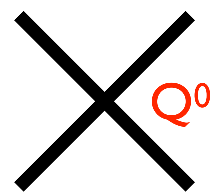

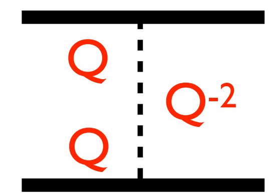

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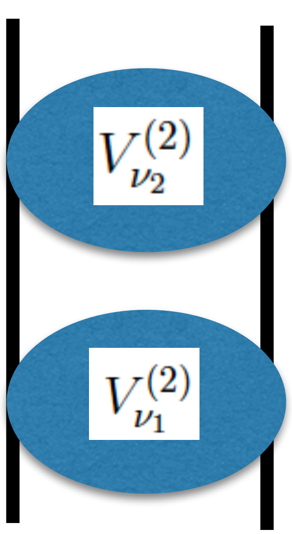

  - What scale divides Q^L ?

$$\frac{V_0^{(2)} G_0 V_0^{(2)}}{V_0^{(2)}} \sim \frac{Q}{M_{NN}} \qquad M_{NN} = \frac{16\pi F_\pi^2}{g_A^2 m_N} \sim F_\pi$$

  - Weinberg's power counting: book $M_{NN} \sim Q \sim m_\pi$.

Leading order NN potential

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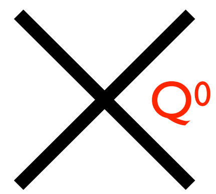

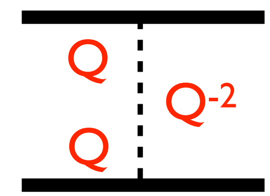

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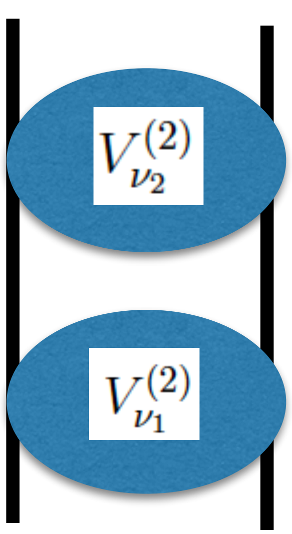

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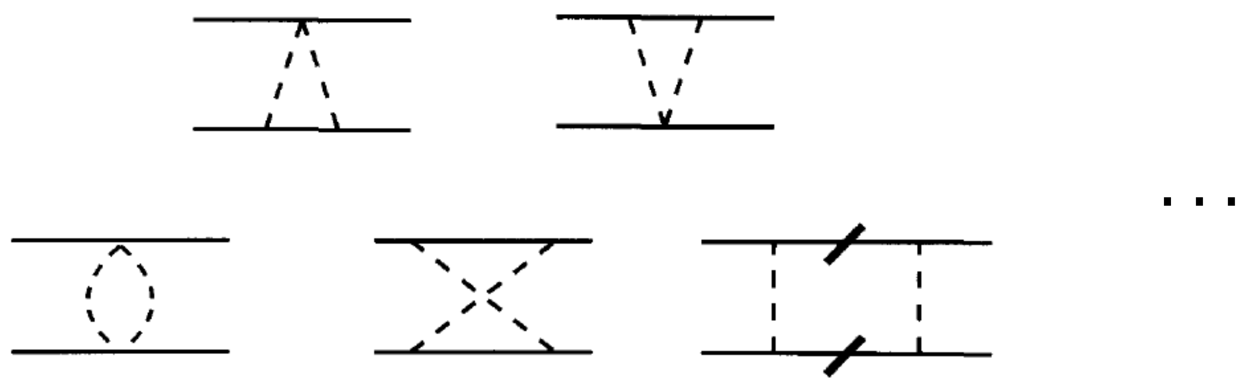
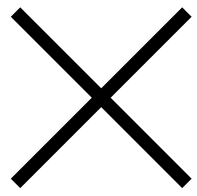
  - Alternative (KSW): $Q \sim m_\pi < M_{NN}$ ⇒ perturbative π's

Kaplan-Savage-Wise '98

Higher orders NN potential

- Multi pion exchange + (many) contact terms with derivatives and m_q





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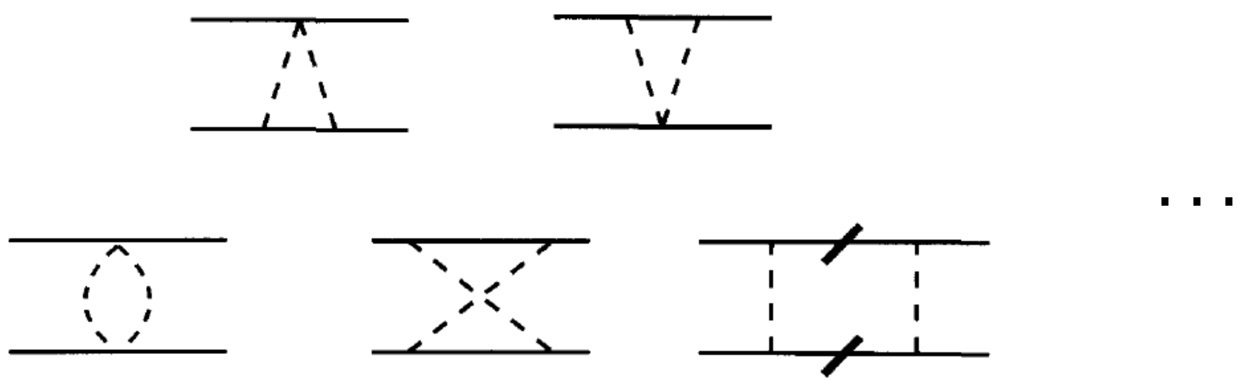

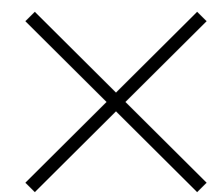

Q^k

k= #derivatives + 2 #m_q

- Scaling of contact interactions in Weinberg's counting

This is also known as naive dimensional analysis (NDA): LECs scale in the same way as loops for which they absorb divergences (in this case loops with pions)

$$C_k \sim \frac{4\pi}{m_N M_{NN}} \frac{1}{\Lambda^k}$$

$$M_{NN} = \frac{16\pi F_\pi^2}{g_A^2 m_N} \sim F_\pi$$

$$\Lambda \sim 4\pi F_\pi$$

Note that this is consistent with $C_{S,T} \sim 4\pi/(m_N M_{NN})$

Qualitative successes (1)

- This counting explains order of magnitude of nuclear binding energies

$$T \sim \frac{4\pi}{m_N M_{NN}} \left[1 + \mathcal{O}\left(\frac{Q}{M_{NN}} \right) + \ldots \right]$$

- Shallow bound states: poles at $Q \sim M_{NN} \sim F_\pi \rightarrow$

$$B \sim \frac{M_{NN}^2}{m_N} \sim \frac{F_\pi}{4\pi} \sim 10 \text{ MeV}$$

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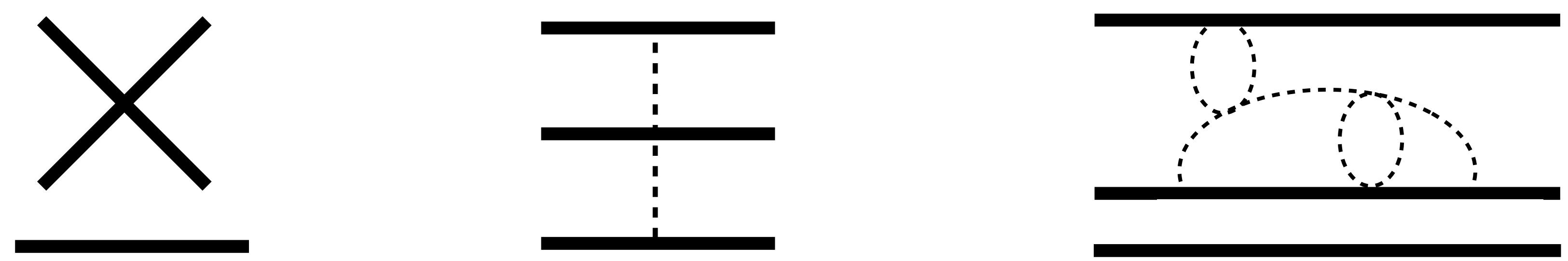
- Scattering lengths: $|a| \sim \frac{1}{M_{NN}} \sim \frac{1}{F_\pi} \quad \Leftarrow \quad T = \frac{4\pi}{m_N} \frac{1}{\left(-\frac{1}{a} + \frac{r_{0s}}{2} k^2 + \ldots - ik\right)}$

a^{-1} ($^1S_0$) = 8.3 MeV
a^{-1} ($^3S_1$) = 36 MeV

S-wave scattering lengths larger by a factor of few,
require fine tuning of contact interactions

Qualitative successes (2)

- Chiral counting ⇒ hierarchy of multi-nucleon forces
- Scaling of general A→A nucleon amplitudes



$$\mathcal{A} \sim Q^{\nu}$$

$$Q \sim m_{\pi}$$

$$\nu = 4 - 2C - A + 2L + \sum_i V_i \Delta_i$$

C = # of connected pieces

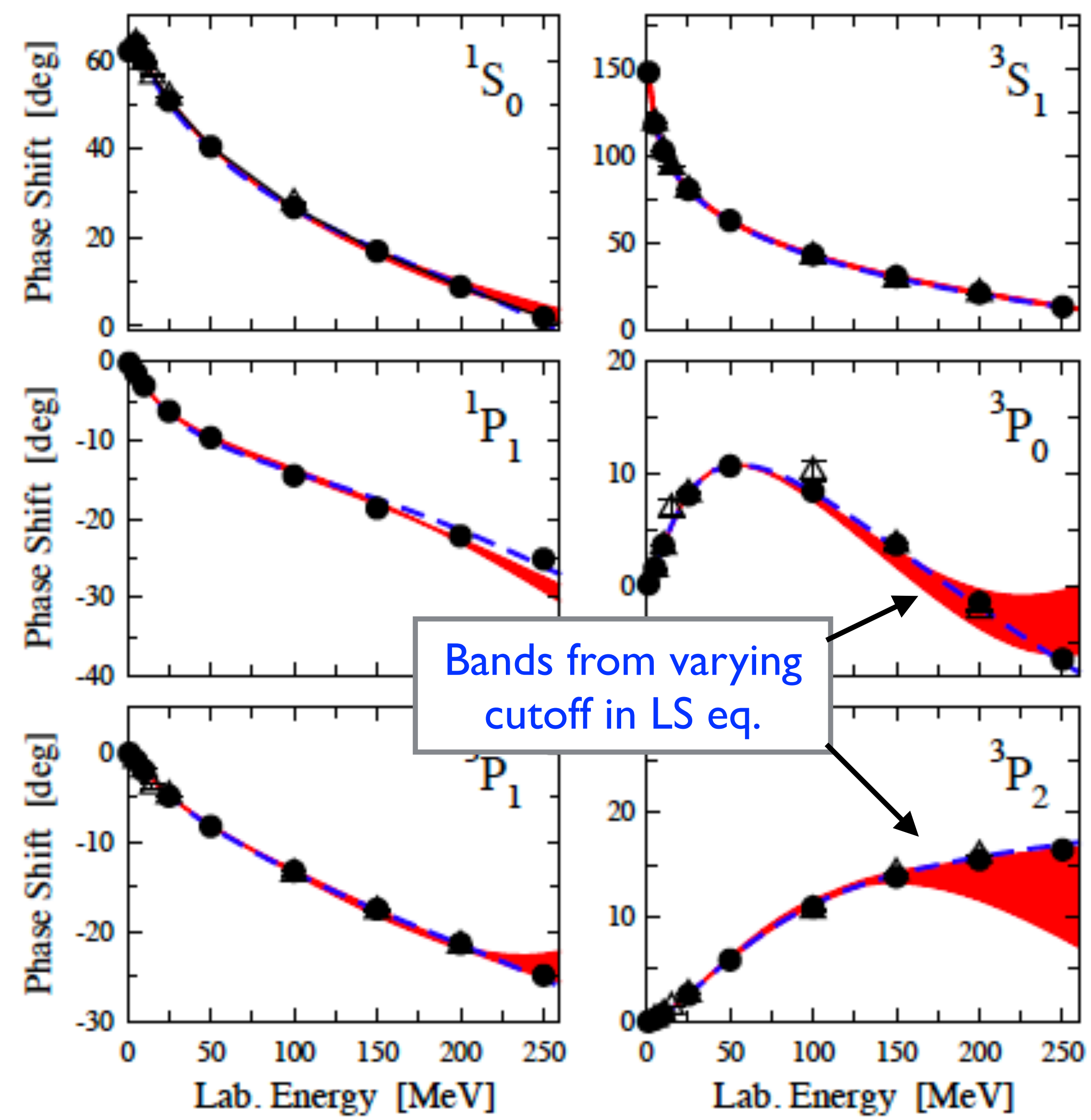
$$\Delta_i = d_i + n_i/2 - 2$$

- A-body amplitude dominated by 2-body force:
  - C=A-1 ⇒ ν_{min} = 6 - 3A
  - Irreducible A-body force starts at [C=1, L=0] $\nu = \nu_{min} + 2A - 4$

Quantitative successes

- Potentials developed up $O(Q/\Lambda)^6$ (Δ-less) and $O(Q/\Lambda)^4$ (Δ-ful)
- LECs fitted to NN scattering data, generally good quality

N3LO (Q^4) Δ-less



[Plots from 0811.1338, see also 1906.12122]

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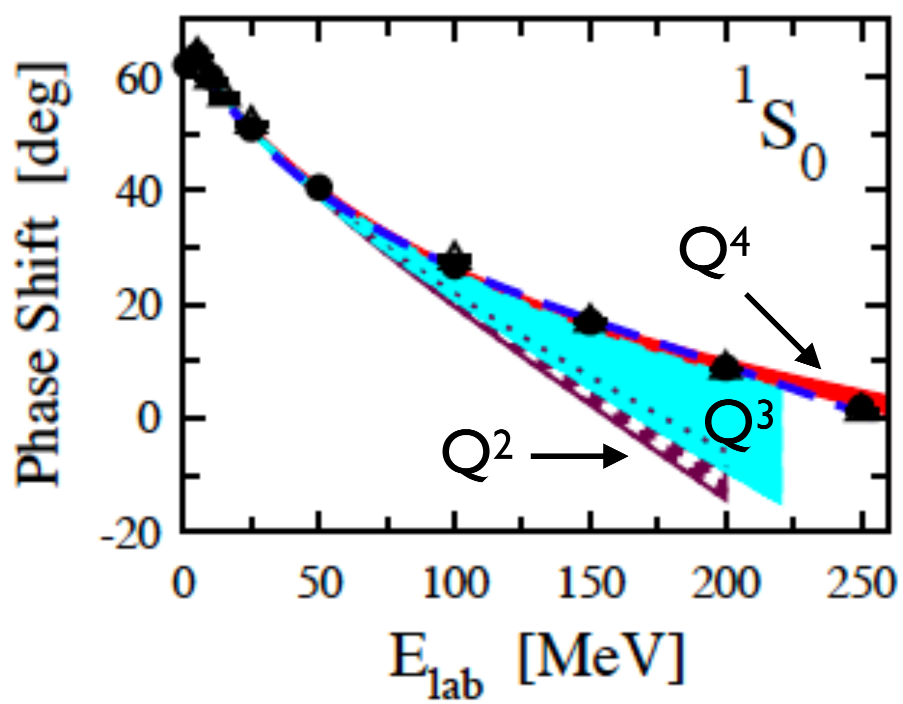

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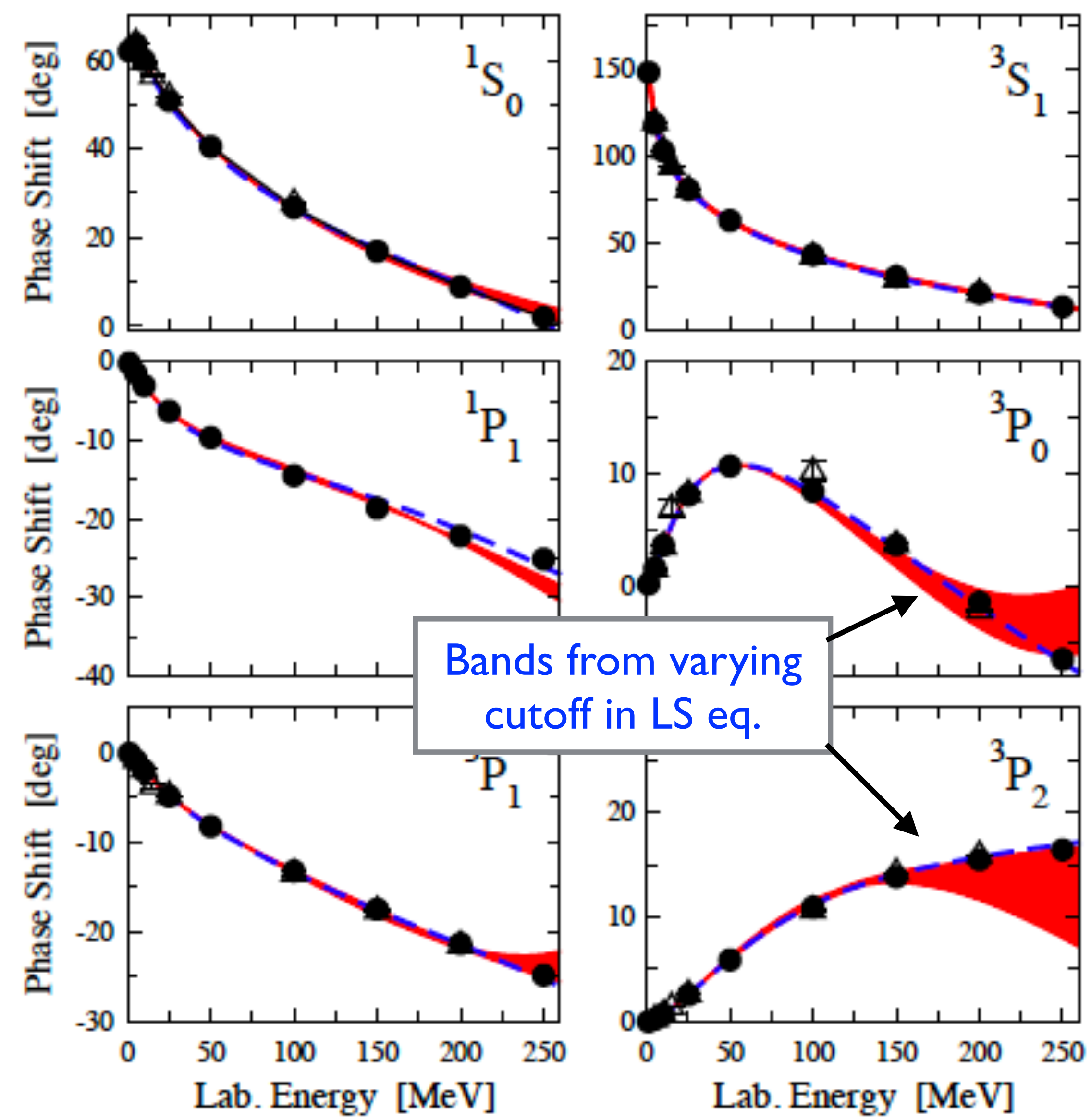

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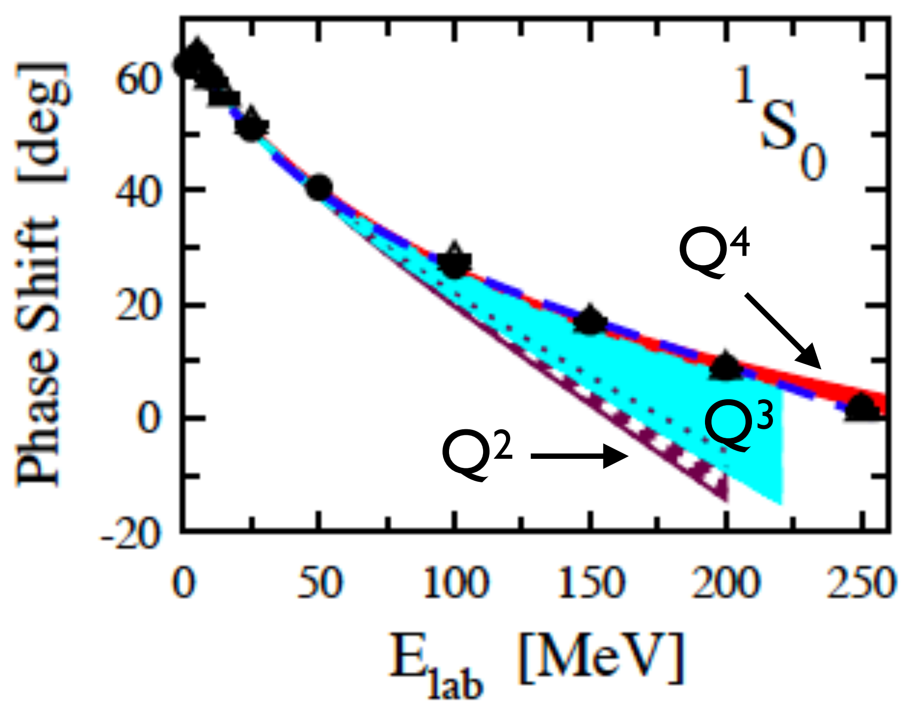

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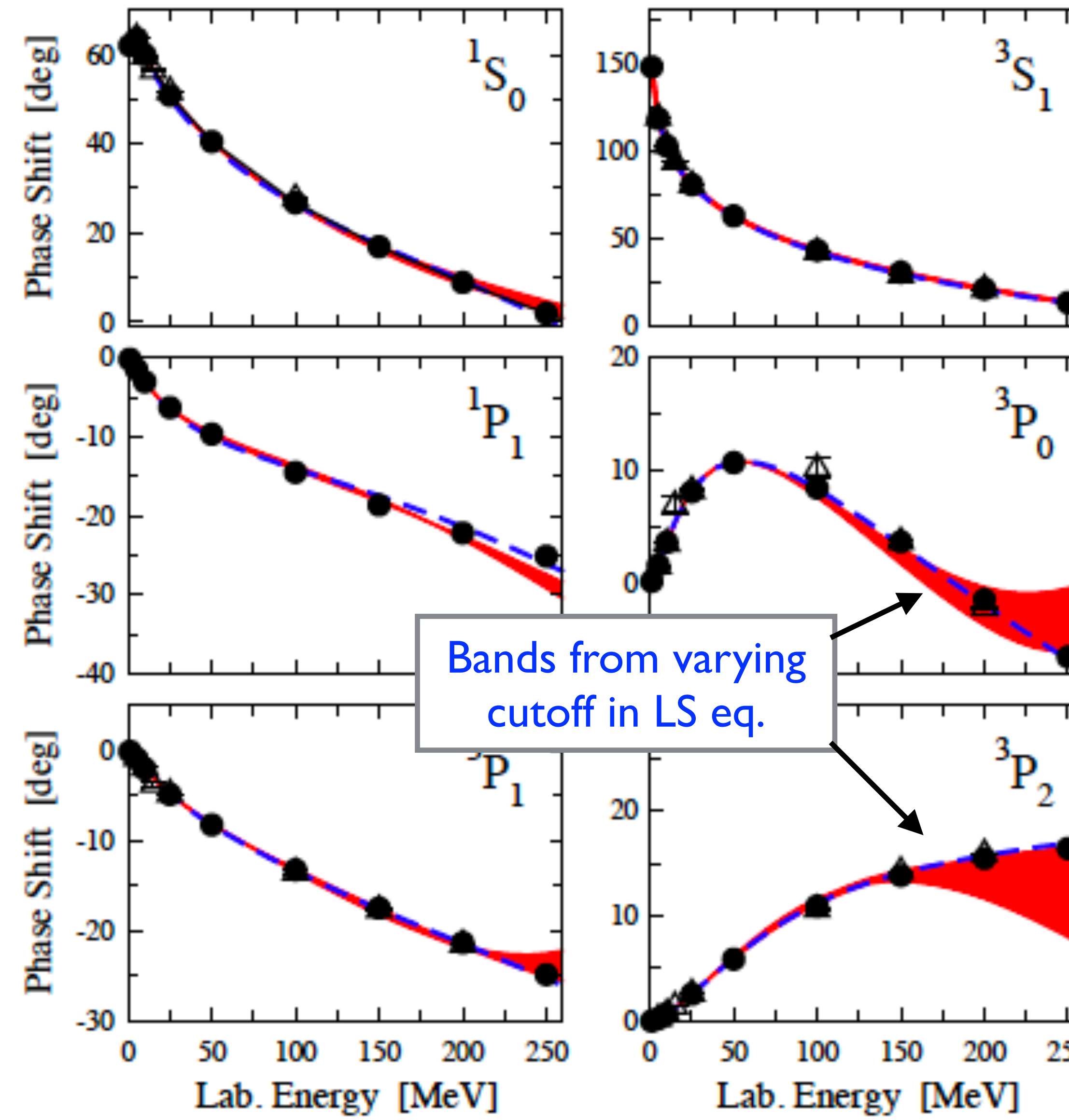

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- Chiral potentials now routinely used in "ab-initio" nuclear structure calculations (rapidly expanding applicability across nuclear chart)

Issues & fixes

Kaplan-Savage-Wise '96, Mehen-Stewart '99, Beane et al' 02, Nogga-Timmermans-van Kolck '05 , …

- UV divergences appear via iteration of the potentials (reducible loops). This could upset up the NDA scaling for the contact terms C_k
- No problem if at any given order one can reabsorb the 'iteration divergences' in the parameters of the potential at that order. Unfortunately this is not the case: there are formal inconsistencies.

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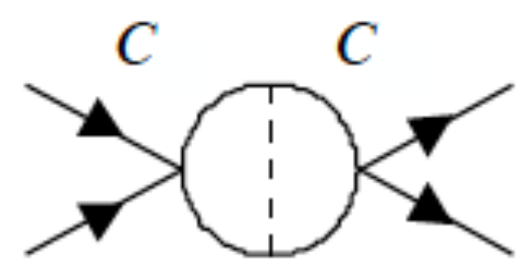

$$\sim m_\pi^2 \, C^2 \left(\frac{1}{4-d} + \log \mu^2 \right)$$

$$\mathcal{L} = -C \, \bar{N} N \bar{N} N - \frac{m_\pi^2}{(4\pi F_\pi)^2} D_2 \, \bar{N} N \bar{N} N$$

$C, D_2 \sim 1/F_\pi^2$

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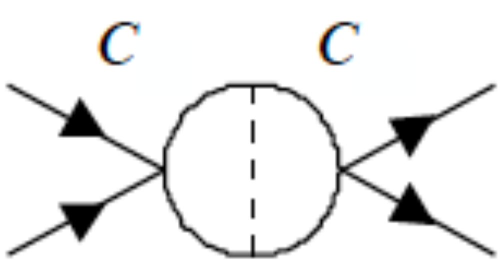

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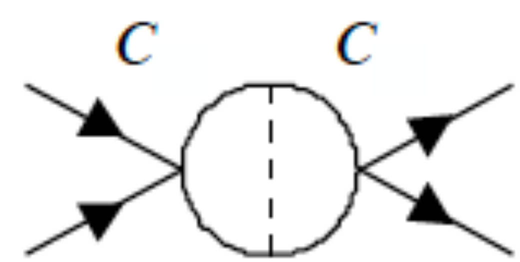
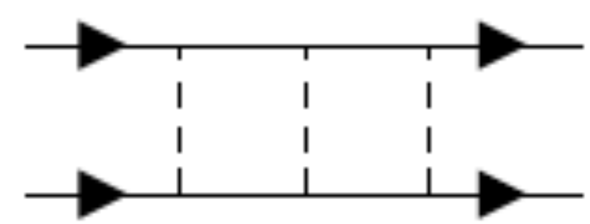


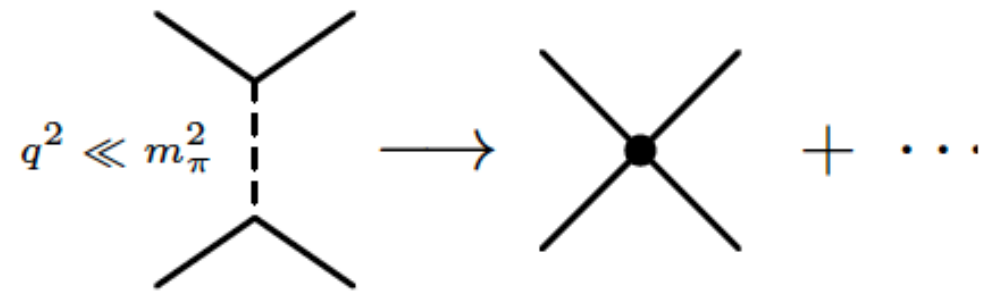
    - Spin triplet waves with attractive 1π-exchange tensor potential (singular potential ~-1/r^3) : $^3P_0$, … Need derivative contact terms at leading order



- *Working solution*: promote these LECs to lower order compared to NDA

Pionless EFT

- If $q^2 \ll m_\pi^2$, the EFT without pions is more appropriate



$$\mathcal{L} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m_N} \right) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2 + \frac{C_2}{16} \left[(\psi\psi)^\dagger \left(\psi \overleftrightarrow{\nabla}^2 \psi \right) + \text{H.c.} \right] + \cdots$$

- Simpler EFT: historically provided insights on NN and 3N interactions
- Power counting well understood, fully renormalized results
- Applicable beyond nuclear physics
- Quite useful for making contact with Lattice QCD at large pion mass

See lectures by Zohreh Davoudi

Problems

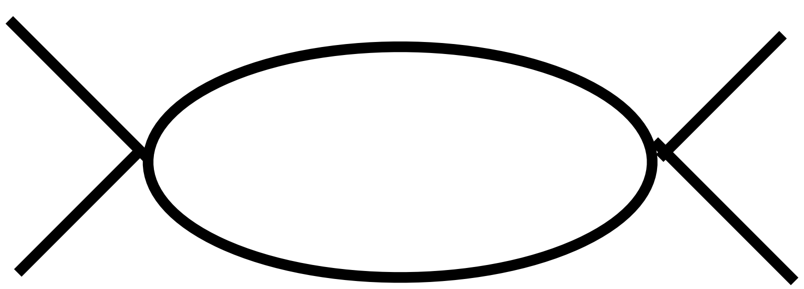
1. Derive the effective Lagrangian for pions to $O(e^2p^0)$ and $O(p^2)$

   Hint: start from the building blocks (U, $D_\mu U$, u_μ, $\chi_\pm$, $(Q^{em})_{L,R}$, …) and construct chiral invariants to the required order

2. Compute the amplitude for neutron decay ($n \to pe\nu$) to leading order in Heavy Baryon Chiral Perturbation theory

   Hint: identify the decay vertices (Vector and Axial currents) using the lowest order chiral Lagrangians in presence of external electroweak sources

3. Compute the bubble diagram (using C_0 vertices) in pionless EFT using dim. reg.



   Hint: perform first the integration over the energy (dq_0), identify the 'pinch' and IR enhanced result. Provide the regulated result in d spacetime dimensions. Identify poles at d=4, d=3.