Methods of Effective Field Theory and Lattice Field Theory
Bad Honnef Physics Schoo
July 22/26 2021

Nuclear physics with applications to neutrino physics (EFT): II

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Plan for the lectures

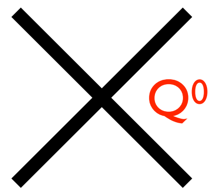
- Big picture:
  - Nuclear probes of physics beyond the Standard Model
  - Connecting scales using EFTs: from BSM to nuclear physics
- EFTs for low-energy QCD and nuclear physics
  - Chiral perturbation theory for mesons and baryons (π, N)
  - Chiral EFT for multi-nucleon systems
- Application to neutrino physics (neutrinoless double beta decay)
  - Lepton Number Violation (LNV) in the Standard Model EFT
  - Nuclear scale realizations
  - Where are we?

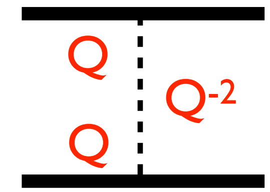
1st lecture

2nd lecture

Chiral NN potential (recap)

- Leading order: pion exchange + contact (in s-waves only)

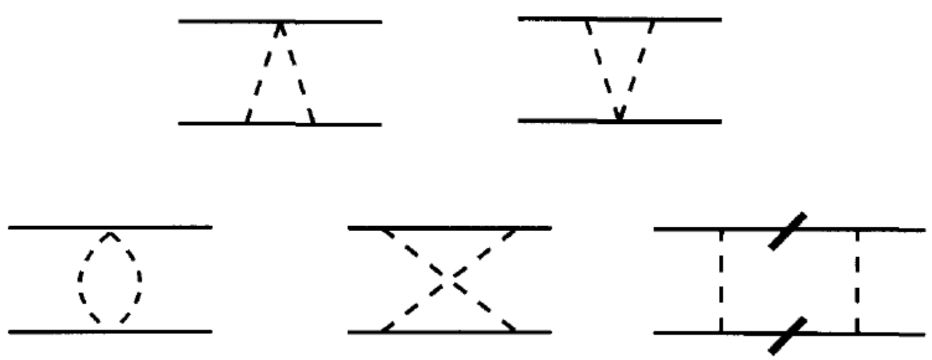




$O(Q^0)$

$$V_0^{(2)} = C_S + C_T\,\vec{\sigma}_1\cdot\vec{\sigma}_2 - \frac{g_A^2}{4F_\pi^2}\frac{(\vec{q}\cdot\vec{\sigma}_1\,\vec{q}\cdot\vec{\sigma}_2)(\vec{\tau}_1\cdot\vec{\tau}_2)}{\vec{q}^2+m_\pi^2}$$

- Multi pion exchange + (many) contact terms with derivatives and m_q



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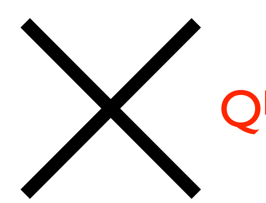

k= #derivatives
+ 2 #m_q

- Scaling of contact interactions in Weinberg's counting

$$C_k \sim \frac{4\pi}{m_N M_{NN}}\frac{1}{\Lambda^k}$$

$$M_{NN} = \frac{16\pi F_\pi^2}{g_A^2 m_N} \sim F_\pi$$

$\Lambda \sim 4\pi F_\pi$

Qualitative successes

- This counting explains order of magnitude of nuclear binding energies

$$T \sim \frac{4\pi}{m_N M_{NN}} \left[1 + \mathcal{O}\left(\frac{Q}{M_{NN}} \right) + \ldots \right]$$

- Shallow bound states: poles at $Q \sim M_{NN} \sim F_\pi \rightarrow$

$$B \sim \frac{M_{NN}^2}{m_N} \sim \frac{F_\pi}{4\pi} \sim 10 \text{ MeV}$$

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- Scattering lengths: $|a| \sim \frac{1}{M_{NN}} \sim \frac{1}{F_\pi} \quad \Leftarrow \quad T = \frac{4\pi}{m_N} \frac{1}{\left(-\frac{1}{a} + \frac{r_{0s}}{2} k^2 + \ldots. - ik\right)}$

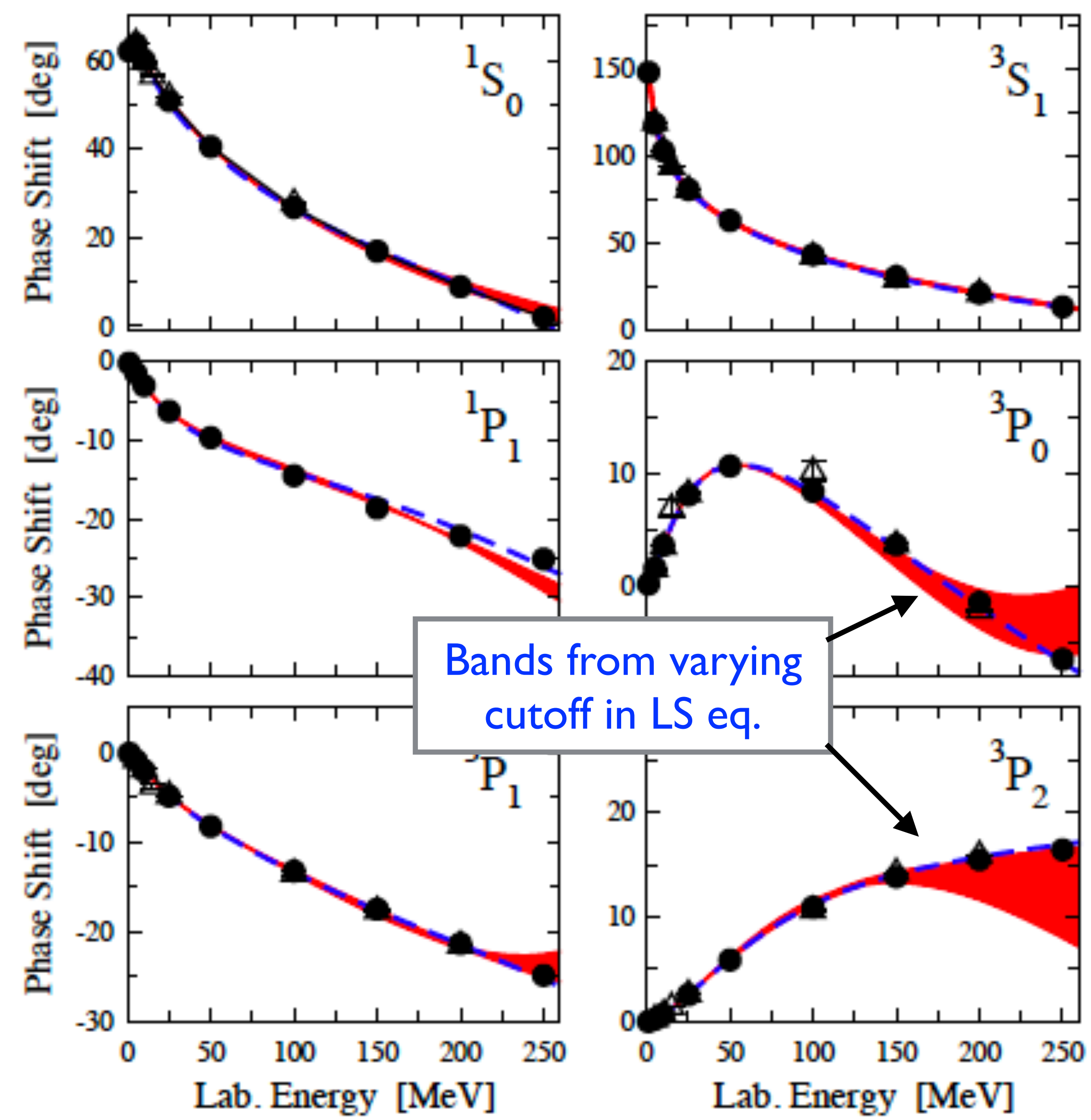
a^{-1} ($^1S_0$) = 8.3 MeV
a^{-1} ($^3S_1$) = 36 MeV

S-wave scattering lengths larger by a factor of few, require fine tuning of contact interactions

Quantitative successes

- Potentials developed up $O(Q/\Lambda)^6$ (Δ-less) and $O(Q/\Lambda)^4$ (Δ-ful)
- LECs fitted to NN scattering data, generally good quality

N3LO (Q^4) Δ-less



[Plots from 0811.1338, see also 1906.12122]

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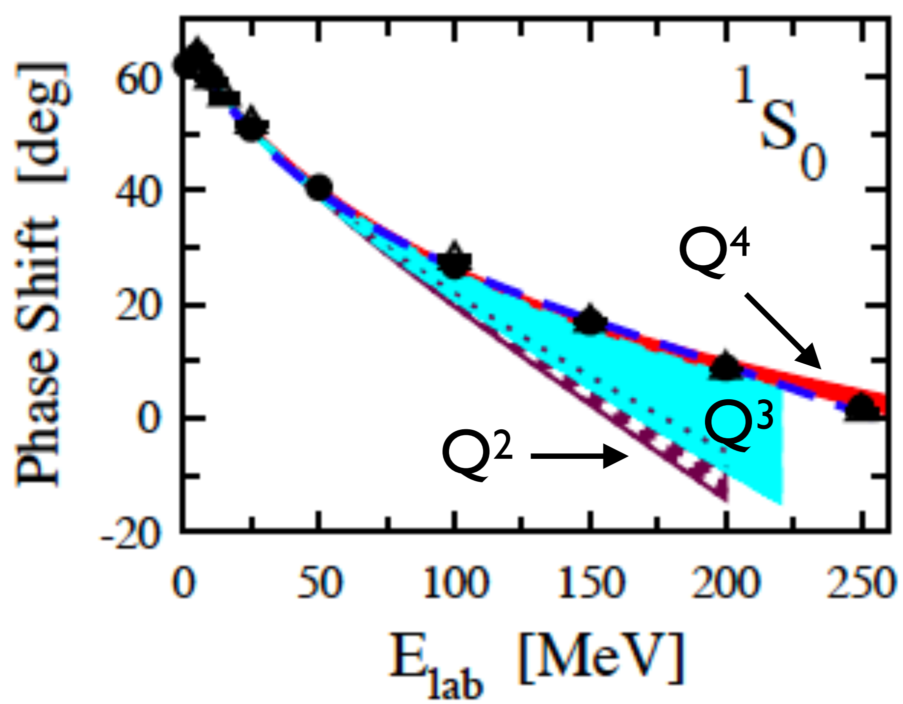

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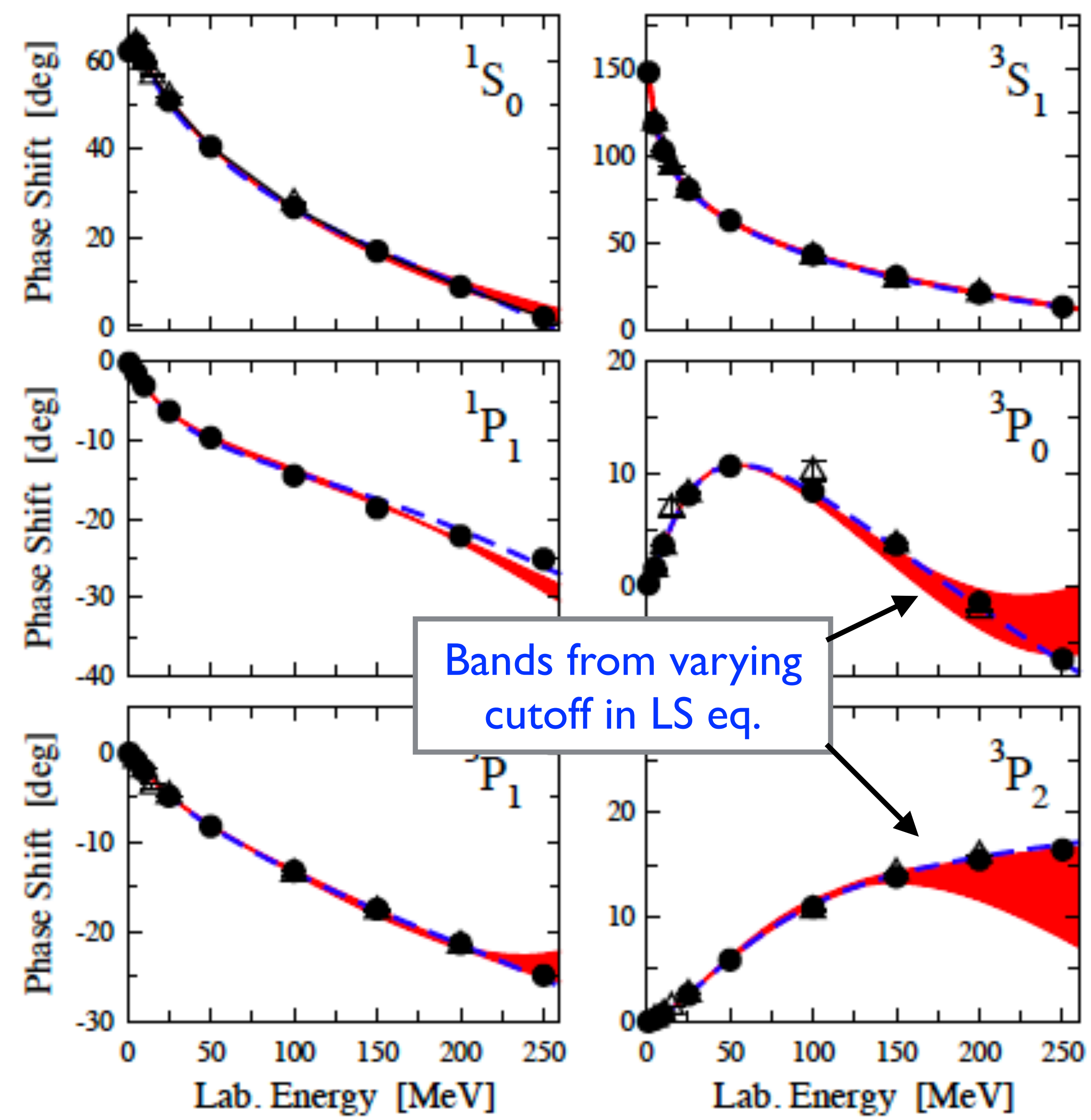

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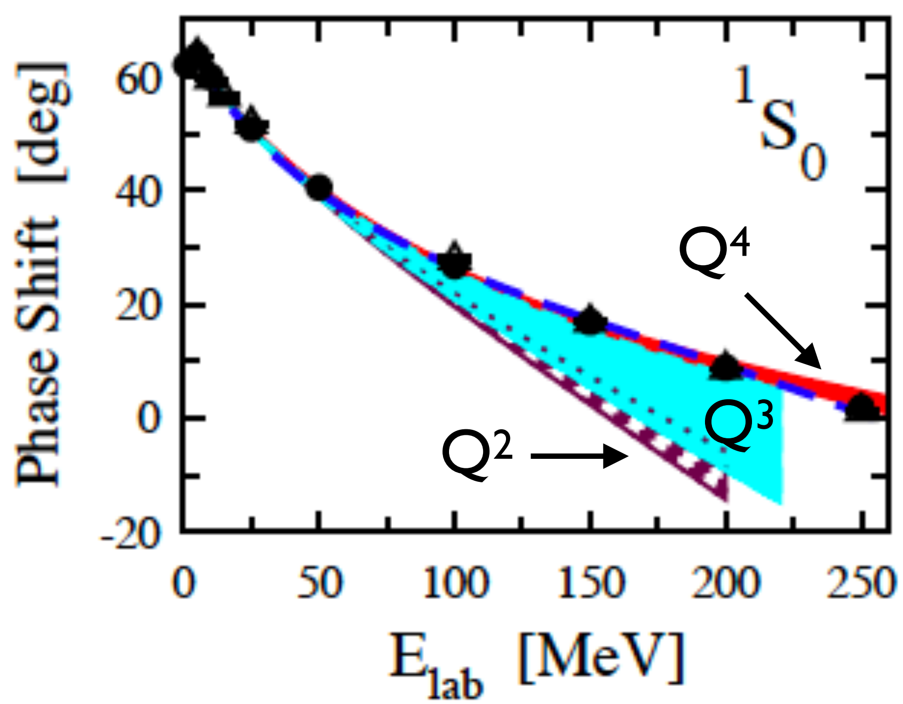

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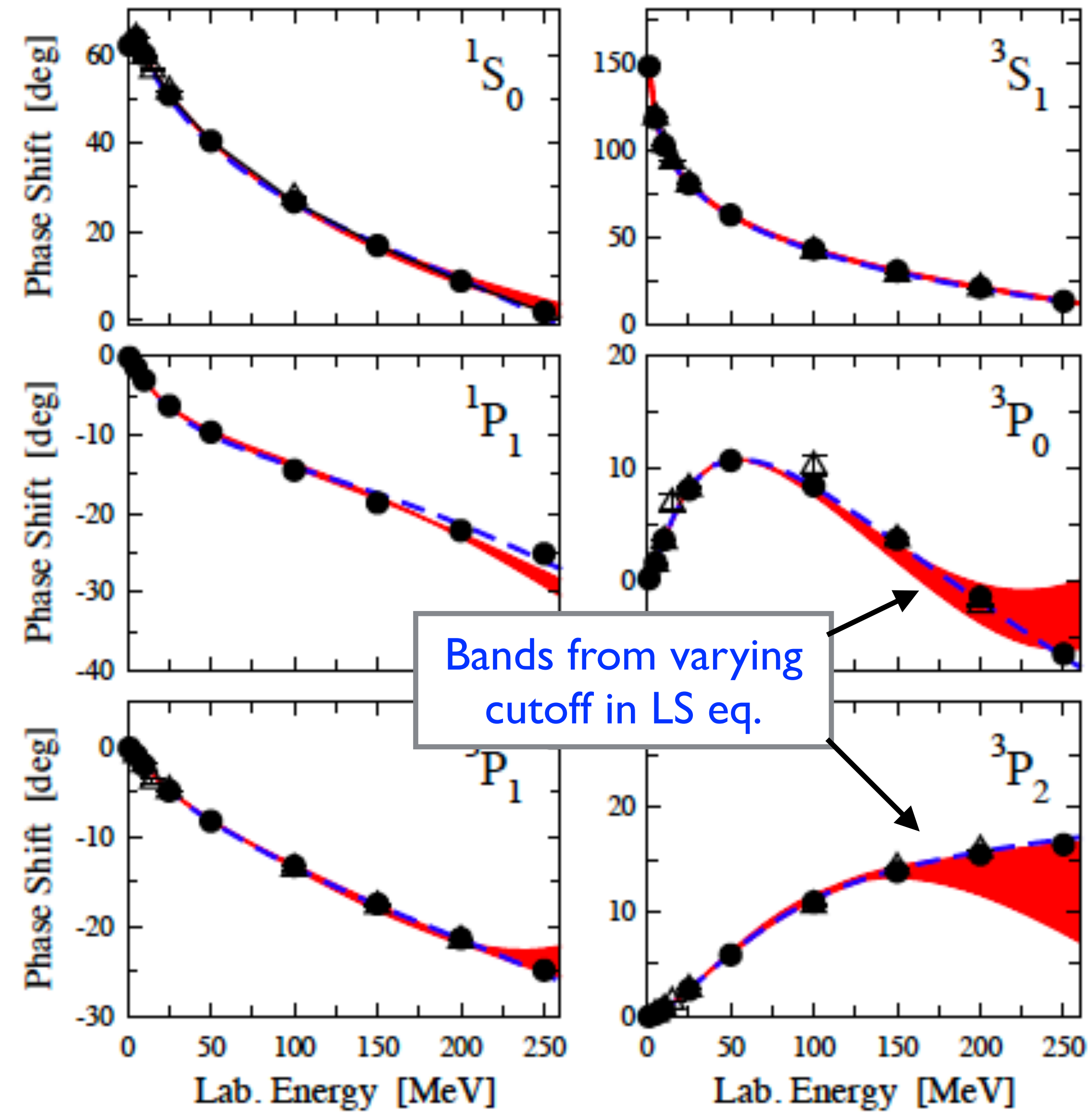

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- Chiral potentials now routinely used in "ab-initio" nuclear structure calculations (rapidly expanding applicability across nuclear chart)

Issues & fixes

Kaplan-Savage-Wise ’96, Mehen-Stewart ’99, Beane et al’ 02, Nogga-Timmermans-van Kolck ’05 , …

- UV divergences appear via iteration of the potentials (reducible loops). This could upset up the NDA scaling for the contact terms C_k
- No problem if at any given order one can reabsorb the ‘iteration divergences’ in the parameters of the potential at that order. Unfortunately this is not the case: there are formal inconsistencies.

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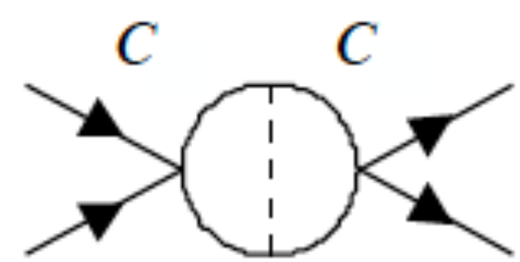
  - $^1S_0$ channel: iteration of the LO contact and one-pion exchange. Need contact term ~ m_π^2 at leading order



$$\sim m_\pi^2\, C^2 \left(\frac{1}{4-d} + \log \mu^2 \right)$$

$$\mathcal{L} = -C\ \bar{N}N\bar{N}N - \frac{m_\pi^2}{(4\pi F_\pi)^2}\, D_2\ \bar{N}N\bar{N}N$$

$C, D_2 \sim 1/F_\pi^2$

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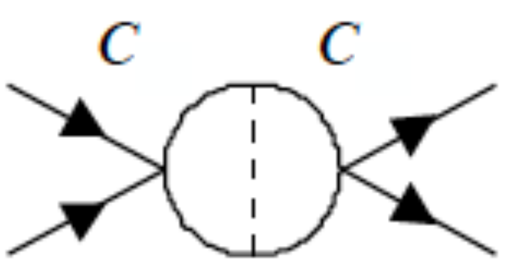

  - Spin triplet waves with attractive 1π-exchange tensor potential (singular potential ~$-1/r^3$) : $^3P_0$, … Need derivative contact terms at leading order

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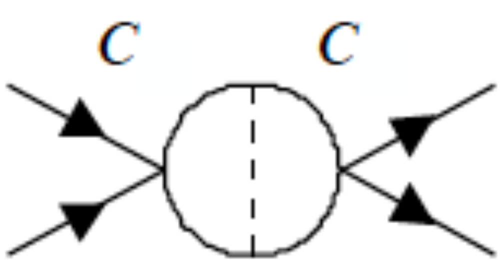

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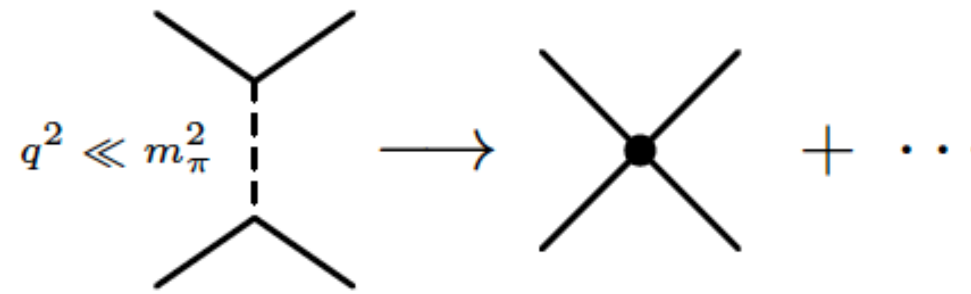
- *Working solution*: promote these LECs to lower order compared to NDA

Pionless EFT

- If $q^2 << m_\pi^2$, the EFT without pions is more appropriate



$$\mathcal{L} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m_N} \right) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2 + \frac{C_2}{16} \left[(\psi\psi)^\dagger \left(\psi \overleftrightarrow{\nabla}^2 \psi \right) + \mathrm{H.c.} \right] + \cdots$$

- Simpler EFT: historically provided insights on NN and 3N interactions
- Power counting well understood, fully renormalized results
- Applicable beyond nuclear physics
- Quite useful for making contact with Lattice QCD at large pion mass

See lectures by Zohreh Davoudi

Neutrinoless double beta decay and Lepton Number Violation

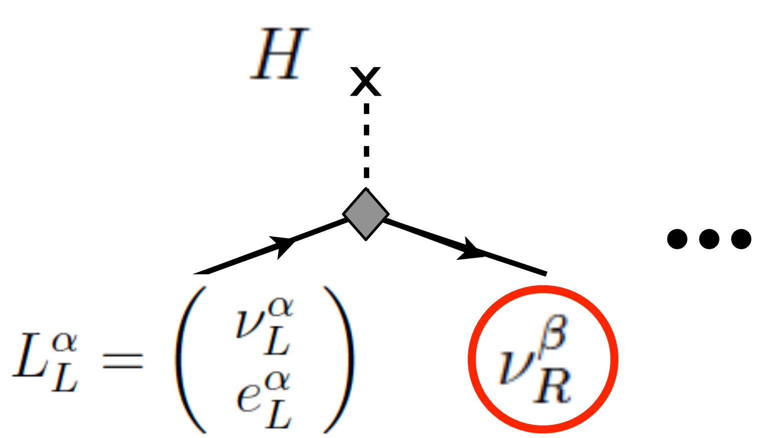
Neutrino mass and new physics

- ν mass requires introducing new degrees of freedom

Dirac mass

$$m\,\bar{\nu}_L \nu_R \;+\; \text{h.c.} = m\,\bar{\nu}\nu$$

$$\nu = \nu_L + \nu_R$$



- Violates $L_{e,\mu,\tau}$, conserves L
  $L = L_e + L_\mu + L_\tau$

Neutrino mass and new physics

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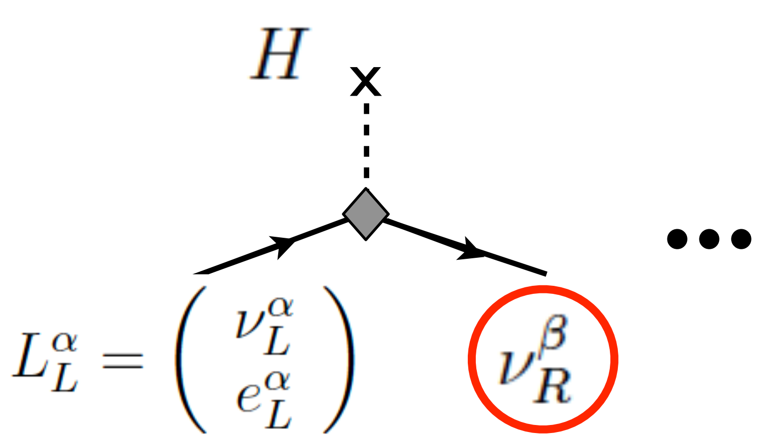
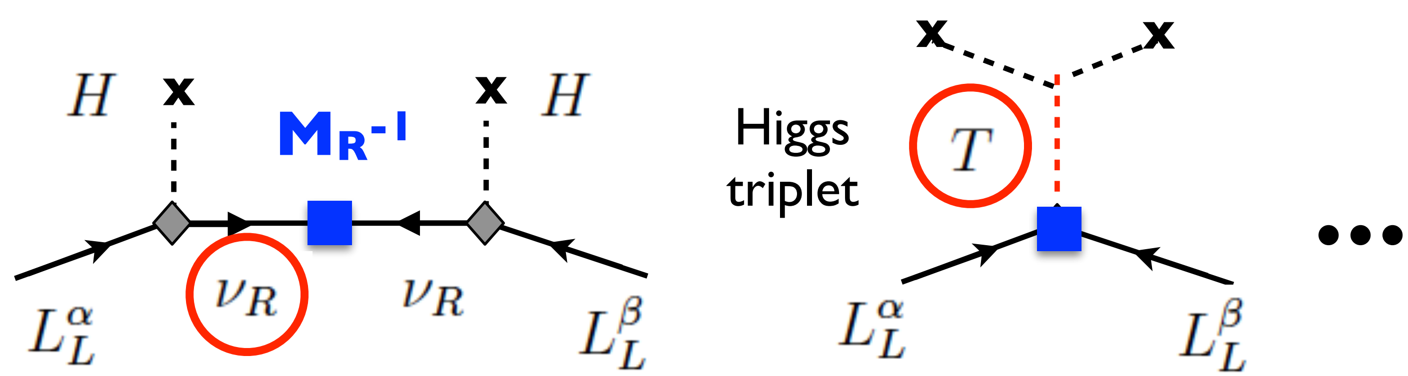


Majorana mass

$$m\,\nu_L^T C \nu_L \;+\; \text{h.c.} = m\,\bar{\nu}\nu$$

$$\nu = \nu_L + \nu_L^c = \nu^c$$



- Violates $L_{e,\mu,\tau}$, conserves L
  $L = L_e + L_\mu + L_\tau$

- Violates $L_{e,\mu,\tau}$ *and* L ($\Delta L=2$)

Neutrino mass and new physics

- ν mass requires introducing new degrees of freedom

$$\mathcal{L}_{\nu\mathrm{SM}} = \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_{\nu-\mathrm{mass}} + \ldots$$

- Key questions:
  - Is Lepton Number a good symmetry of the new dynamics?
  - What are the sources and mediators of lepton family violation?

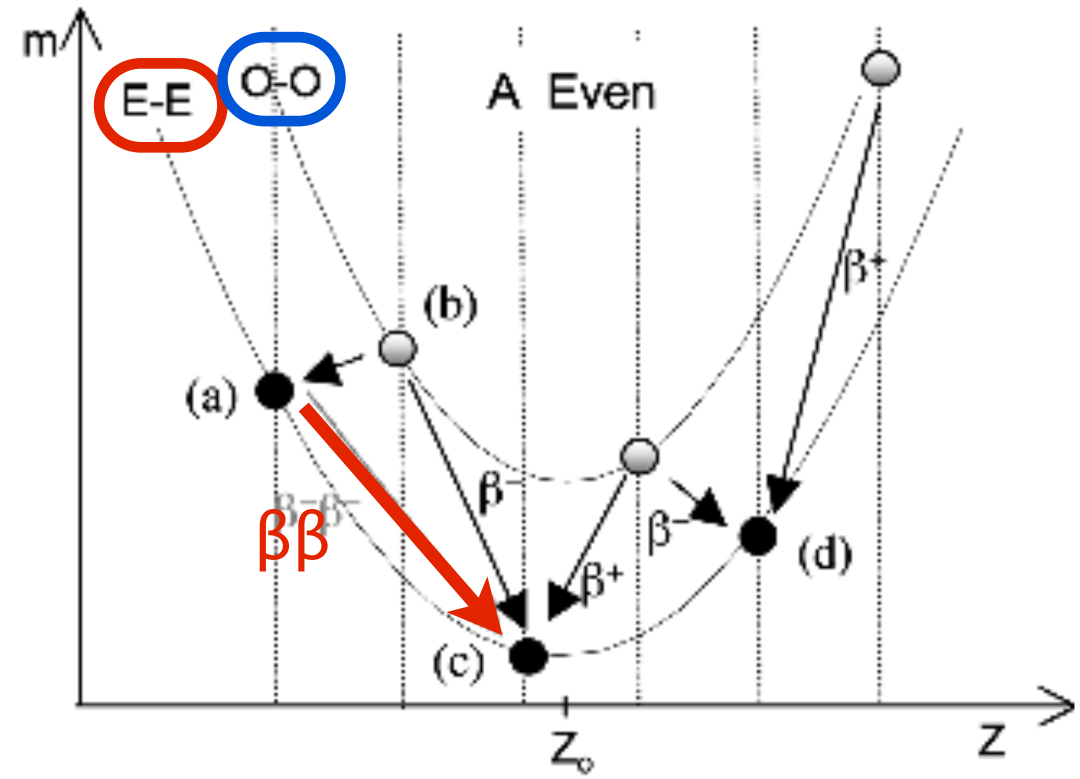
- Nuclear probes can address these questions:
  - Neutrinoless double beta decay is the strongest probe of LNV
  - $\mu \rightarrow e$ in nuclei and ep $\rightarrow \tau X$ at EIC are powerful probes of LFV

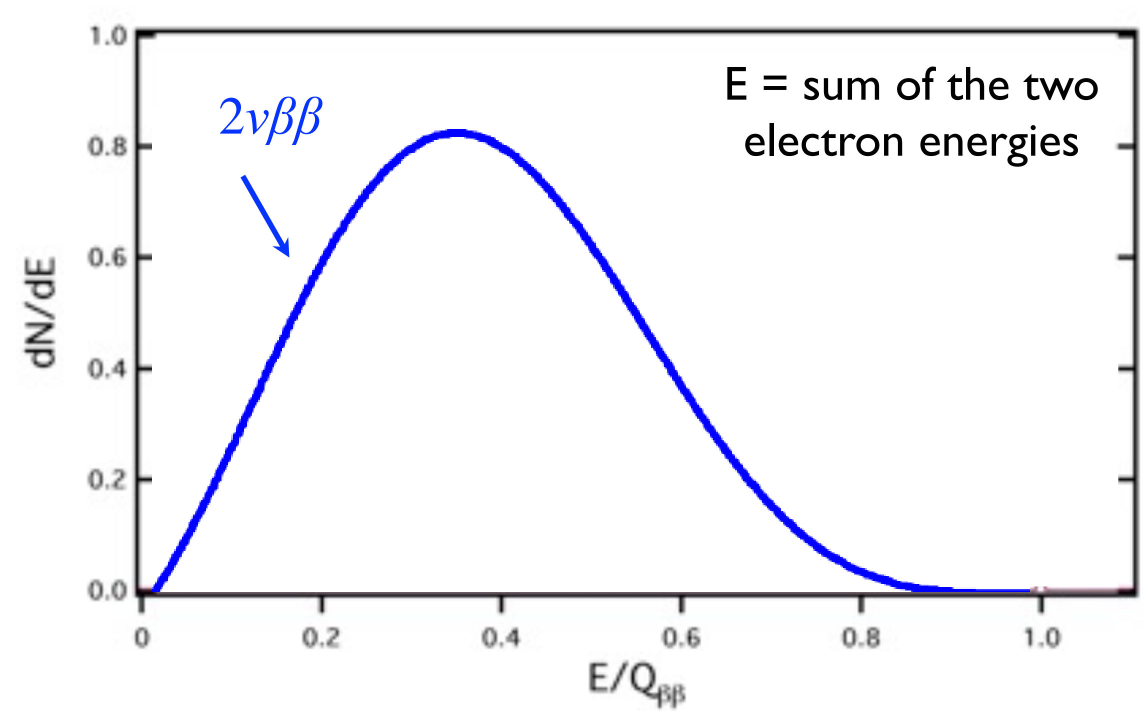
Double beta decay

- For certain even-even nuclei (^{48}Ca, ^{76}Ge, ^{136}Xe, …), single beta decay is energetically forbidden

- 2νββ is a (very rare) 2nd order weak process, expected in the Standard Model and observed

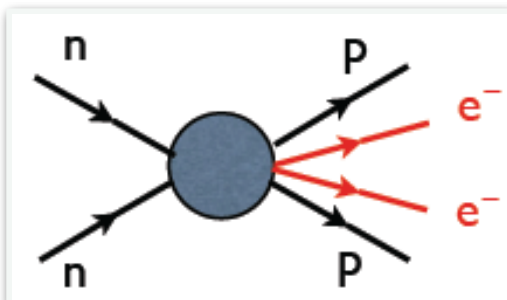
- 0νββ is quite special



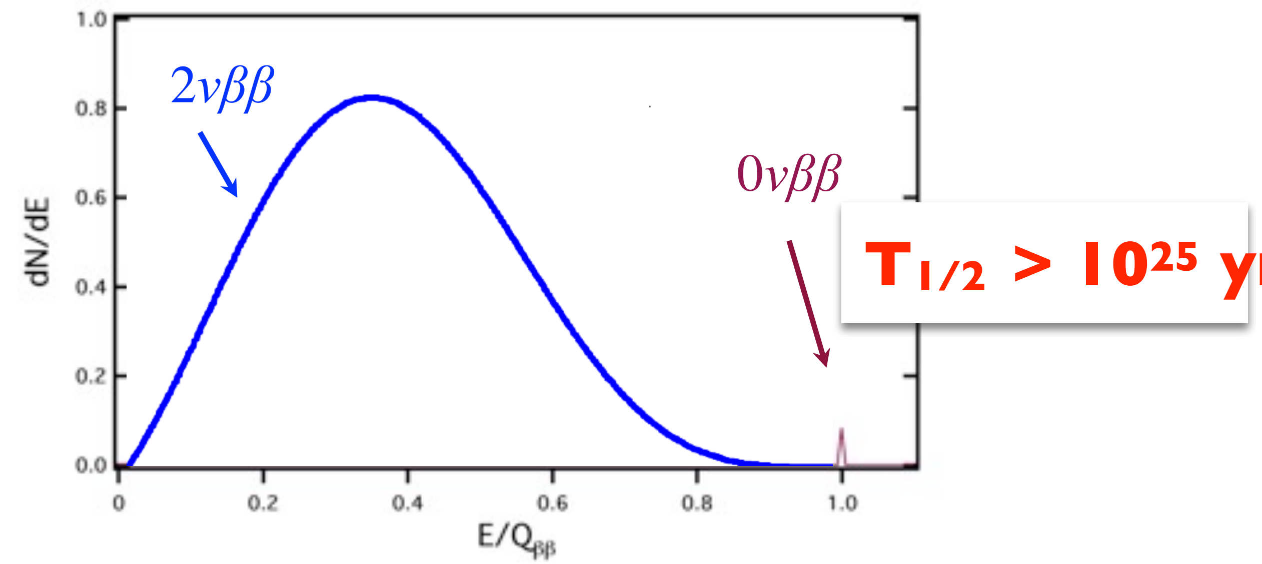


0νββ and Lepton Number Violation

$$(N, Z) \rightarrow (N-2, Z+2) + e^- + e^-$$



Lepton number changes by two units: ΔL=2



- B-L conserved in SM → 0νββ observation would signal new physics

0νββ and Lepton Number Violation

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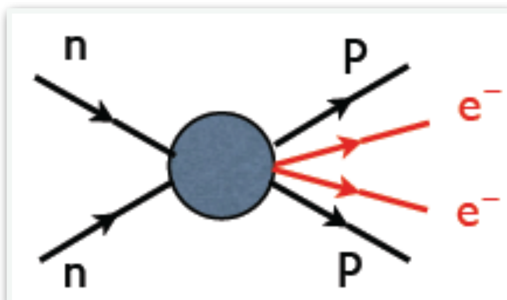
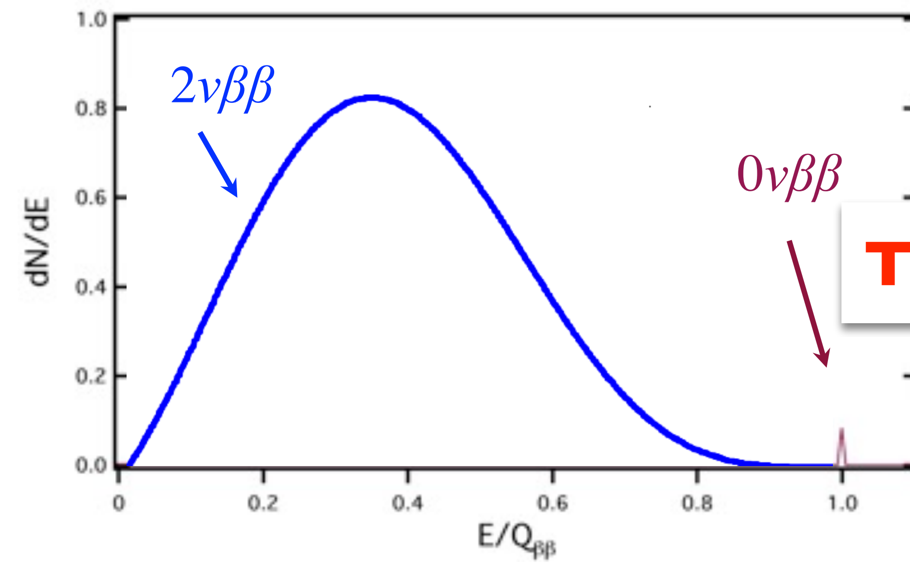
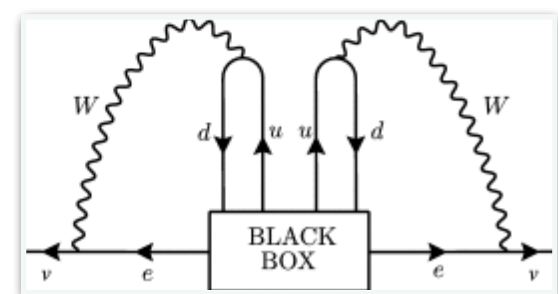


Lepton number changes by two units: $\Delta L=2$

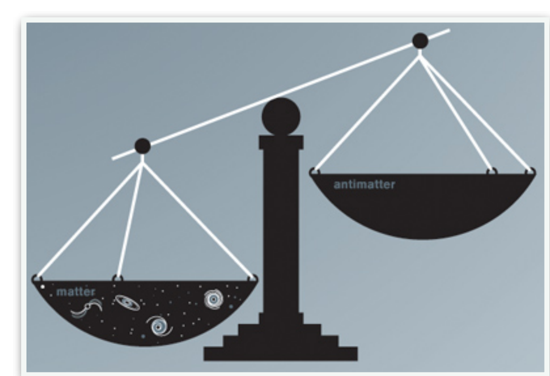


$T_{1/2} > 10^{25}$ yr

- B-L conserved in SM → 0νββ observation would signal new physics
  - Demonstrate that neutrinos are Majorana fermions
  - Establish a key ingredient to generate the baryon asymmetry via leptogenesis



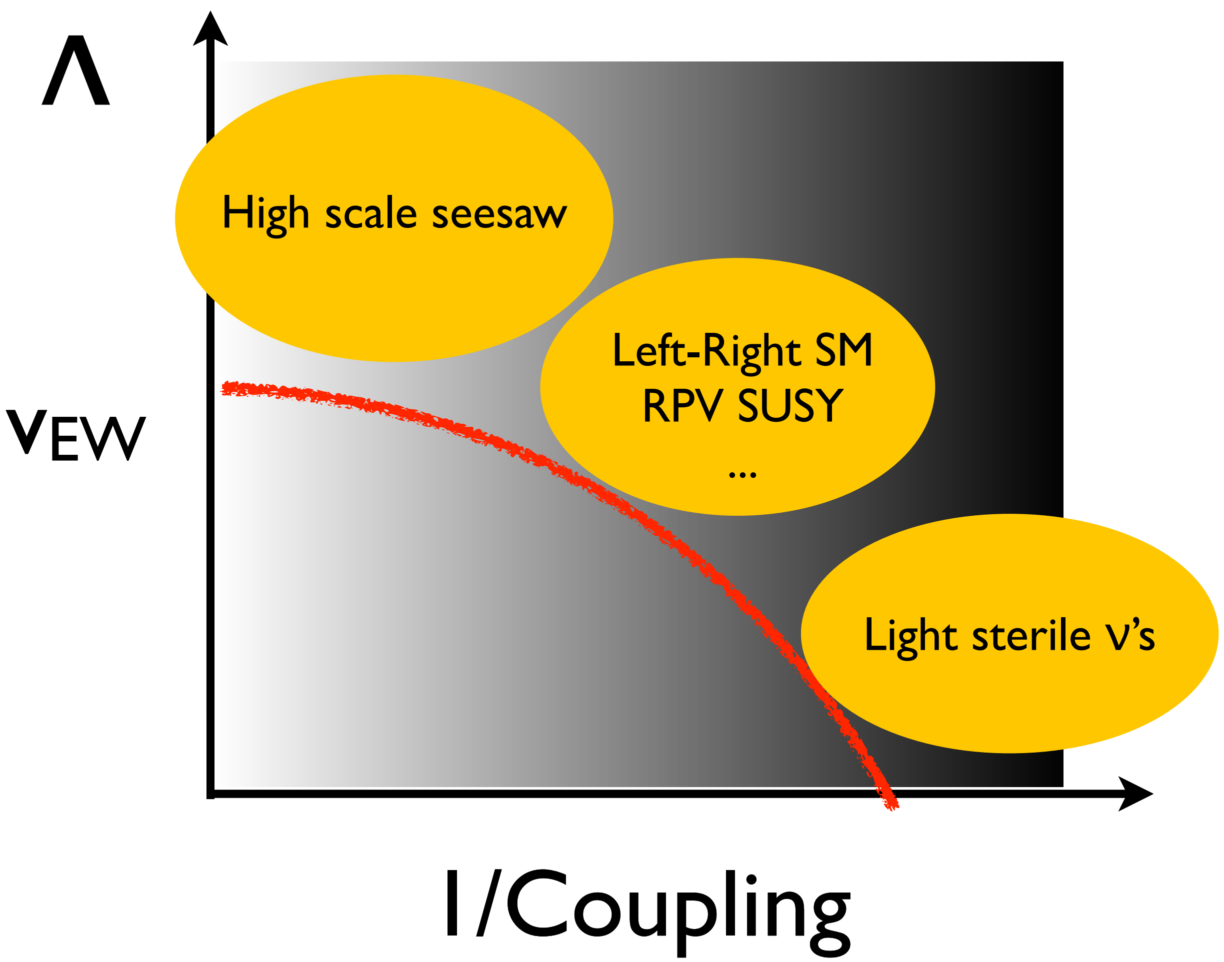
Shechter-Valle 1982



Fukujgita-Yanagida 1987

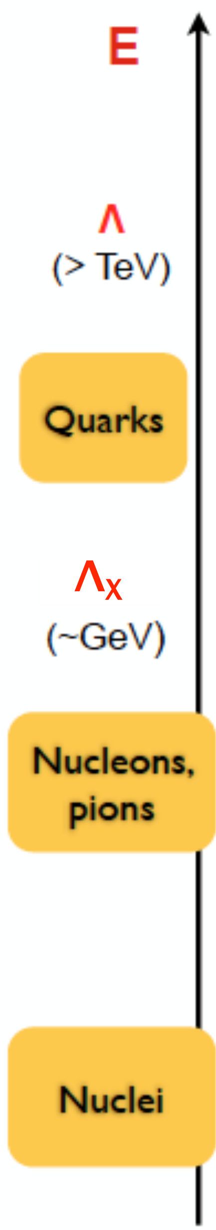
0νββ physics reach

- Ton-scale 0νββ searches ($T_{1/2}$ >$10^{27\text{-}28}$ yr) will probe at unprecedented levels LNV from a broad range of mechanisms



Impact of 0νββ searches most efficiently analyzed within an "end-to-end" EFT framework: connecting new physics at the scale Λ to nuclear scales, with controllable uncertainties

\`End-to-end' EFT framework for 0νββ



$\mathcal{L}_{\rm BSM}$

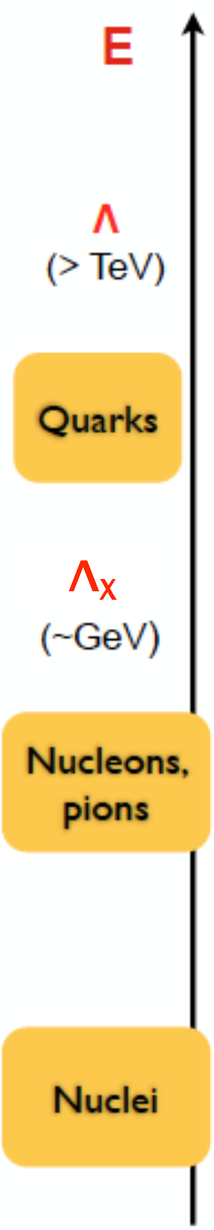
↓ Integrate out heavy particles

$$\mathcal{L}_{\rm eff} = \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(7)}}{\Lambda^3} O_i^{(7)} + \sum_i \frac{C_i^{(9)}}{\Lambda^5} O_i^{(9)}$$

ΔL=2 in the "Standard Model EFT": only dim=5,7,9,…

Addition of light ν_R systematically studied in Dekens et al. 2002.07182

`End-to-end’ EFT framework for 0νββ



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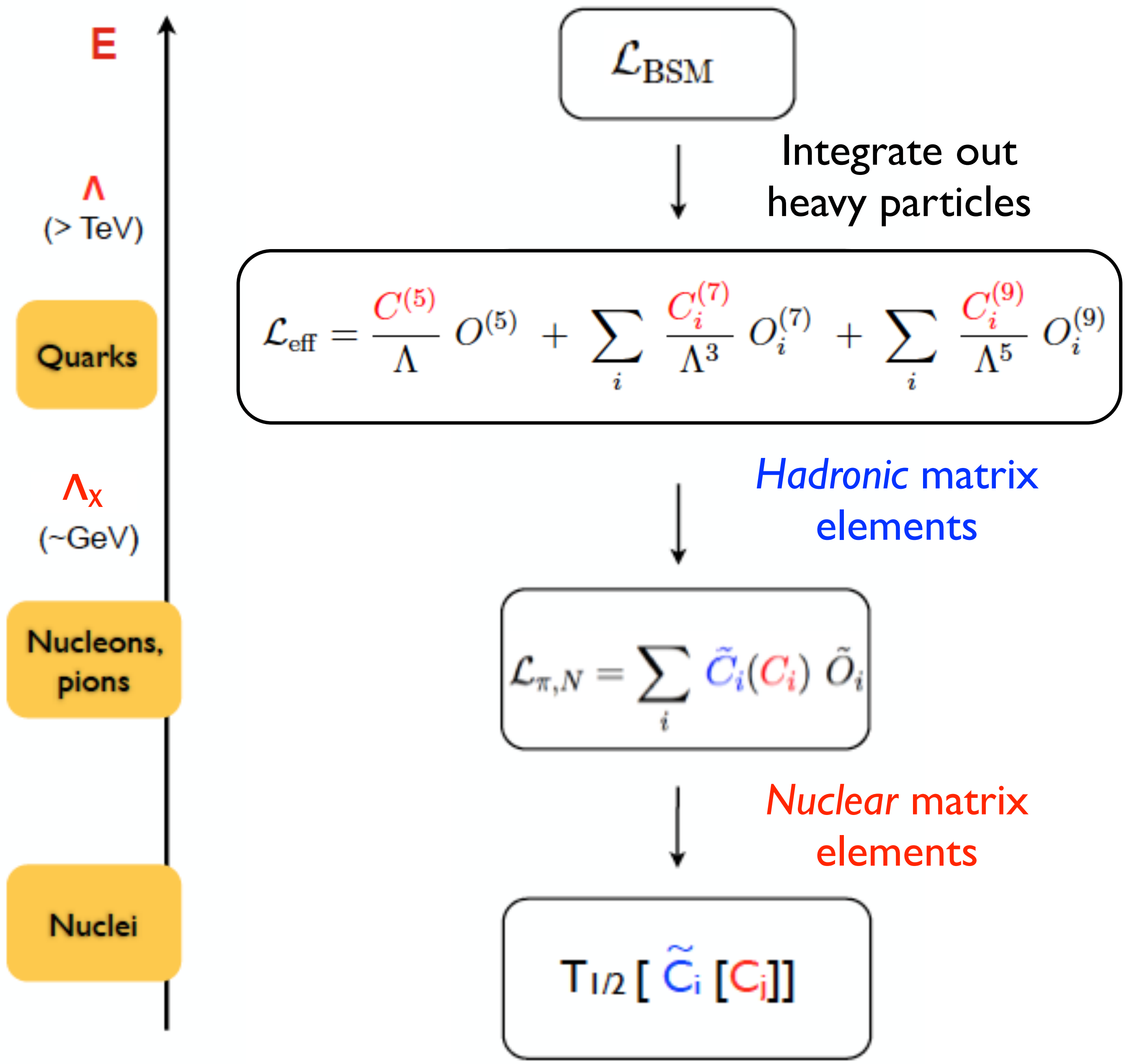
↓ *Hadronic* matrix elements

$$\mathcal{L}_{\pi,N} = \sum_i \tilde{C}_i(C_i)\, \tilde{O}_i$$

Chiral EFT: Map ΔL=2 interactions onto π, N operators, organized according to power-counting in Q/Λ_χ ($Q \sim k_F \sim m_\pi$)

Derive two-nucleon `potentials’

`End-to-end' EFT framework for $0\nu\beta\beta$



Connect LNV sources to nuclear scale with controllable uncertainty

Chain of EFTs +
Hadronic matrix elements (LQCD,…)
& many-body methods

$\rightarrow$ $T_{1/2}[\, \tilde{C}_i\, [C_j]\,] \sim (m_W/\Lambda)^A\ (\Lambda_\chi/m_W)^B\ (Q/\Lambda_\chi)^C$

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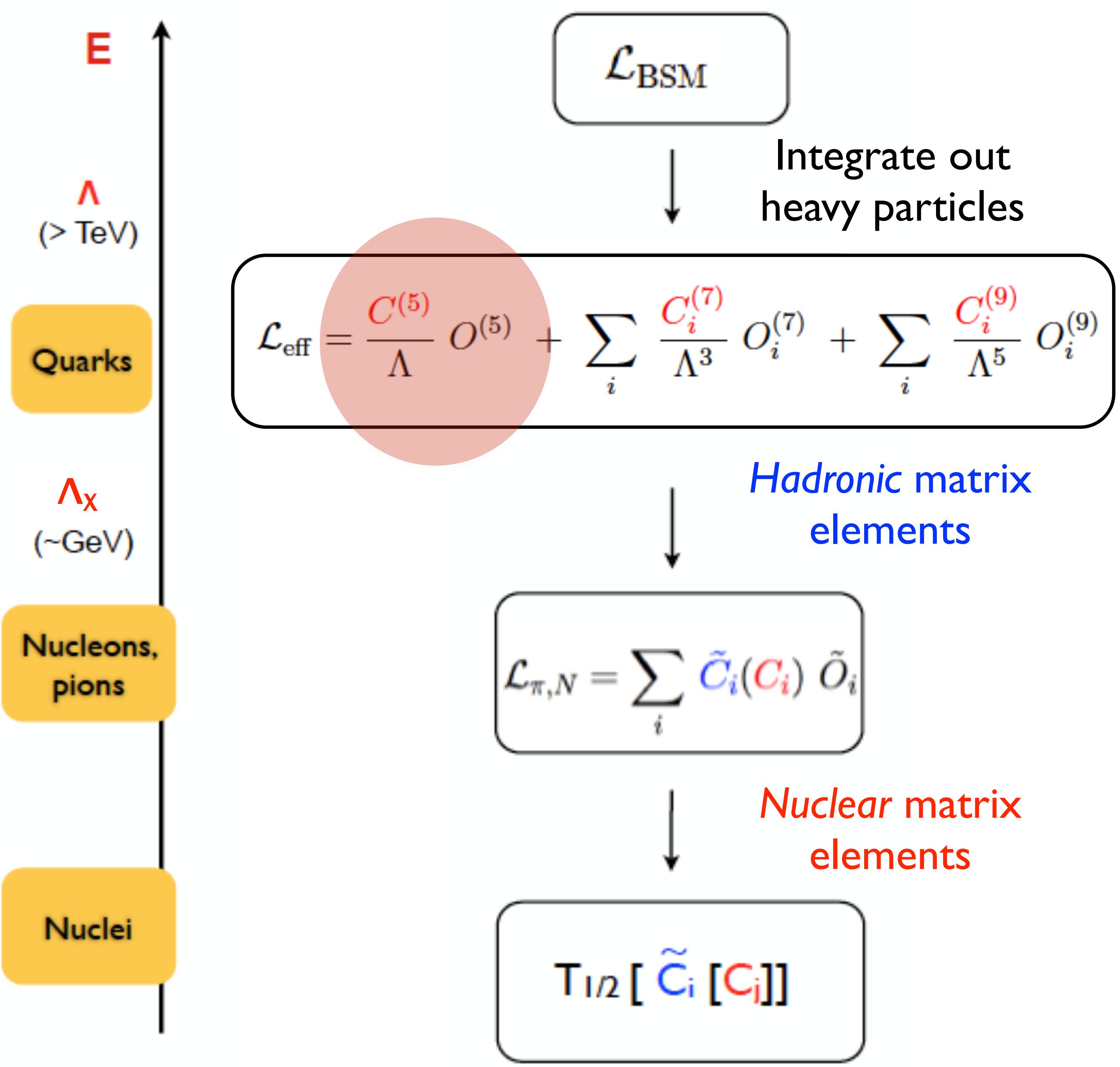

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LNV @ dimension 5

[LNV @ dimension 7 and 9 in the ‘bonus’ slides]

LNV@ dim-5 in the SMEFT

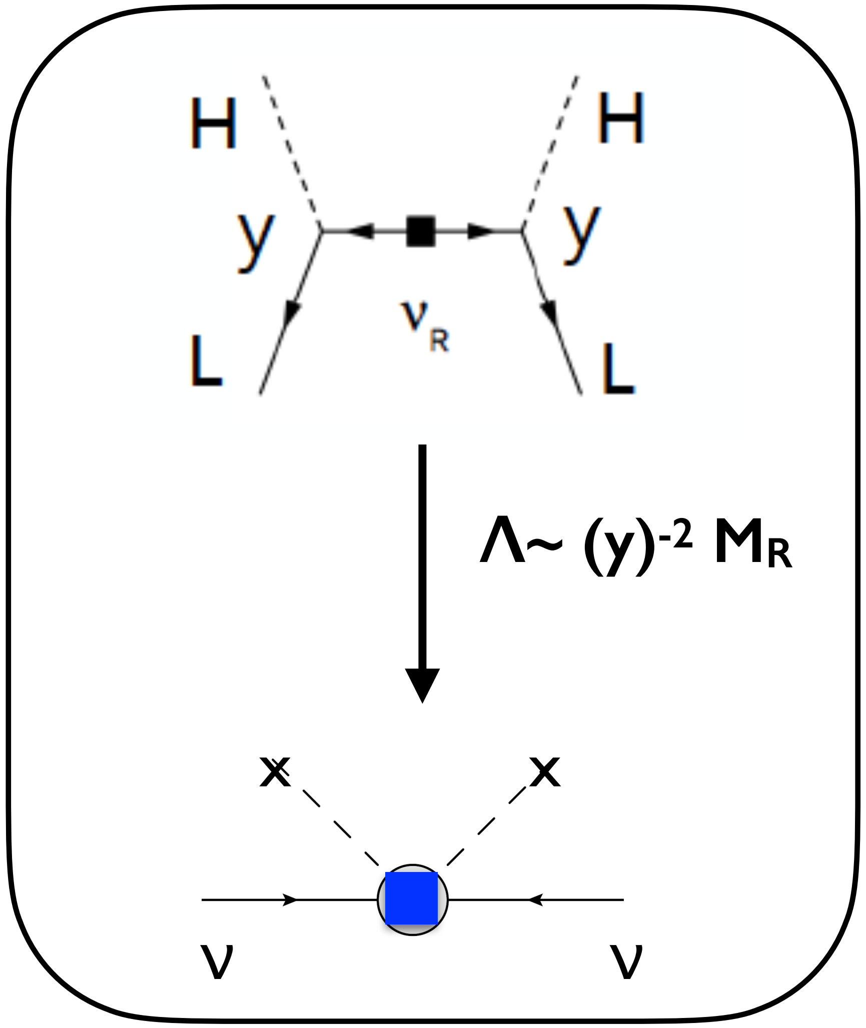
- LNV originates at very high scale ($\Lambda >> v$) → dominant low-energy remnant is Weinberg's dim-5 operator:

$$\mathcal{L}_5 = \frac{w_{\alpha\alpha'}}{\Lambda} L_\alpha^T C \epsilon H \, H^T \epsilon L_{\alpha'}$$

$$H = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \qquad L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

- Below the weak scale this is just the neutrino Majorana mass ($m_{\beta\beta} \sim w_{ee}\, v^2/\Lambda$)

Ex: Type I see-saw with heavy ν_R



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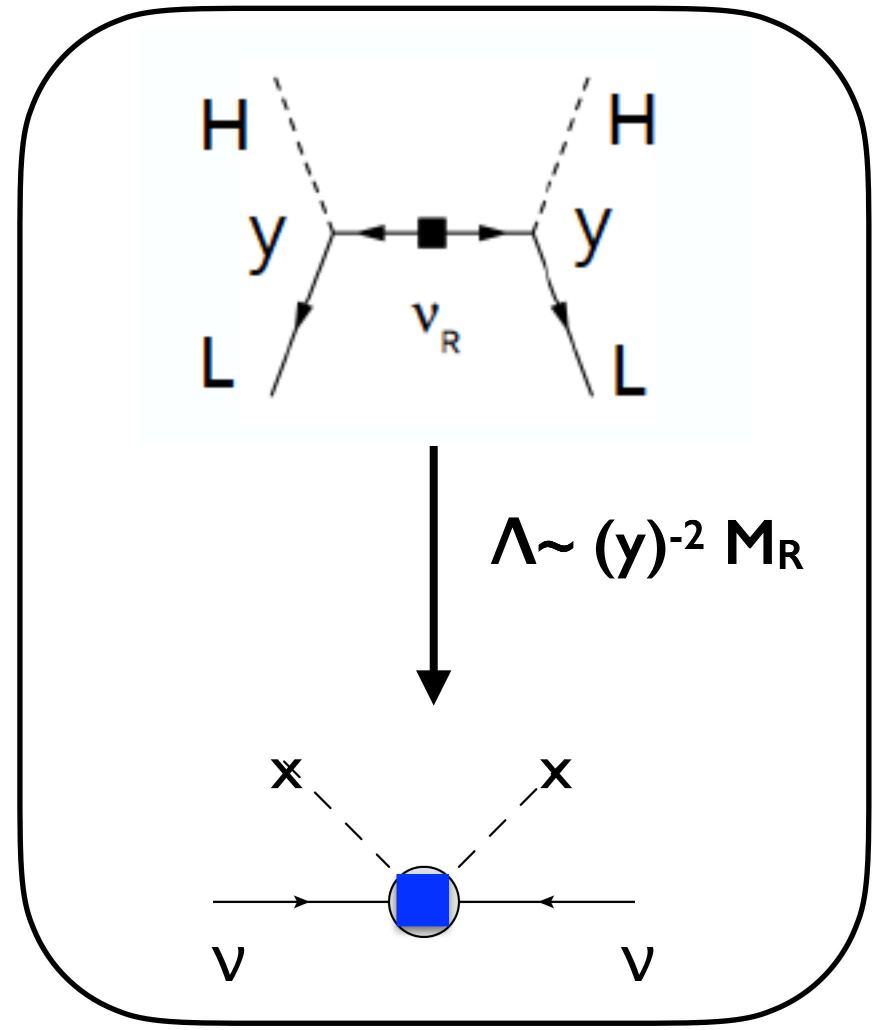

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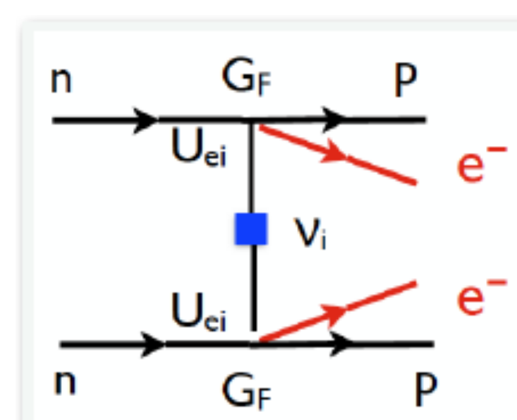
- $0\nu\beta\beta$ mediated by *active* ν_M with amplitude proportional to $m_{\beta\beta}$



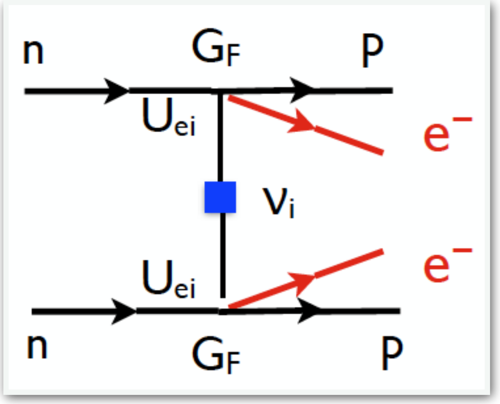
$\propto m_{\beta\beta}$

Phenomenological interest

- In this case $0\nu\beta\beta$ is a *direct* probe of ν Majorana mass: $\Gamma \propto |M_{0\nu}|^2 \, (m_{\beta\beta})^2$

Strong correlation with oscillation parameters

$$\langle m_{\beta\beta} \rangle^2 = |\sum U_{ei}^2 m_{\nu i}|^2$$



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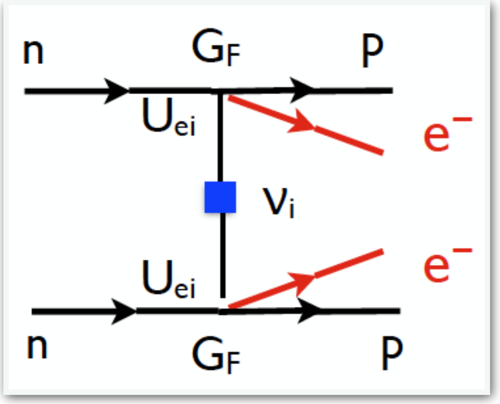

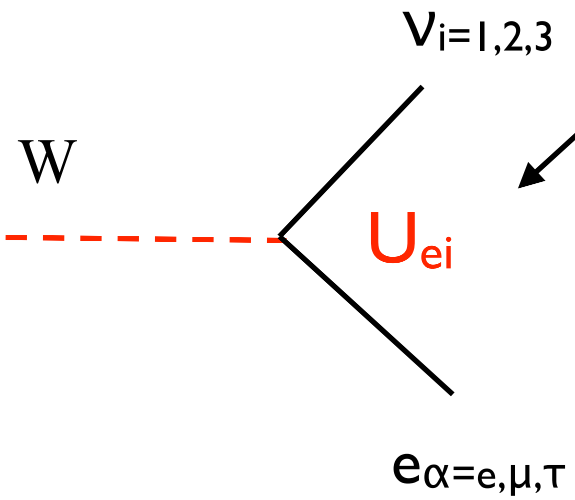

$$\frac{g}{\sqrt{2}} W_\mu^- \, \bar{e}_L^\alpha \, \gamma^\mu U^{\alpha i} \, \nu_L^i$$

Unitary mixing in CC vertex:
3 angles (known), 1+2 phases (unknown)

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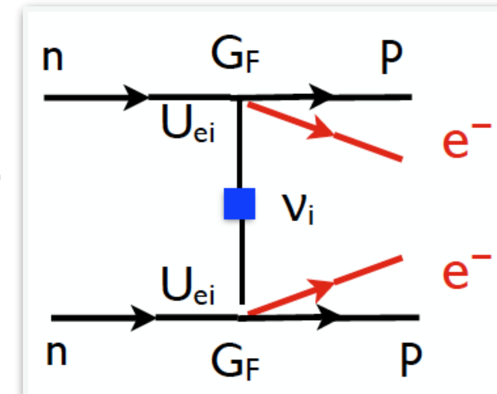
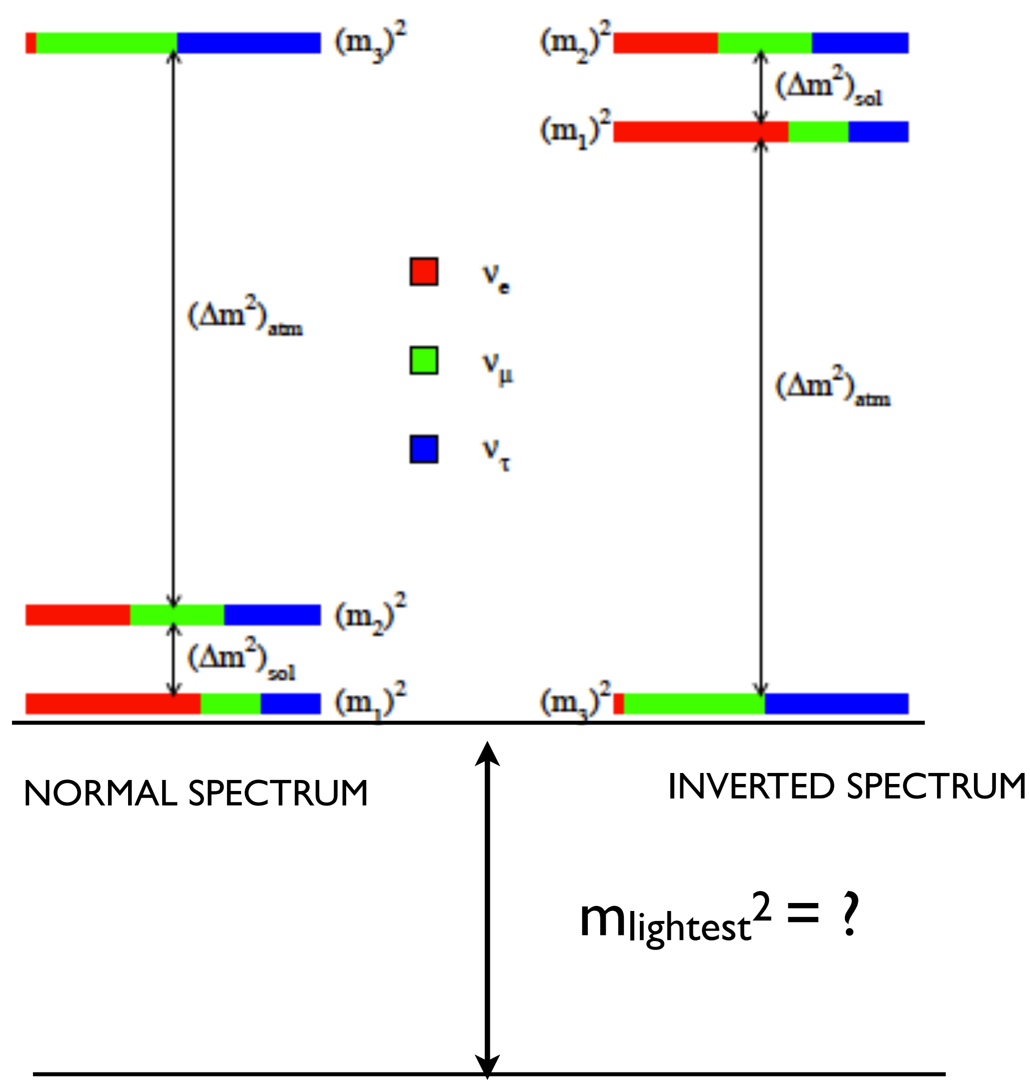
$$\langle m_{\beta\beta} \rangle^2 = |\sum U_{ei}^2 m_{\nu i}|^2$$





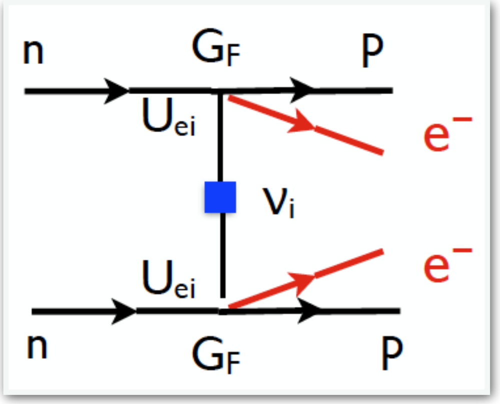
Mass ordering still not fixed by oscillation data

Phenomenological interest

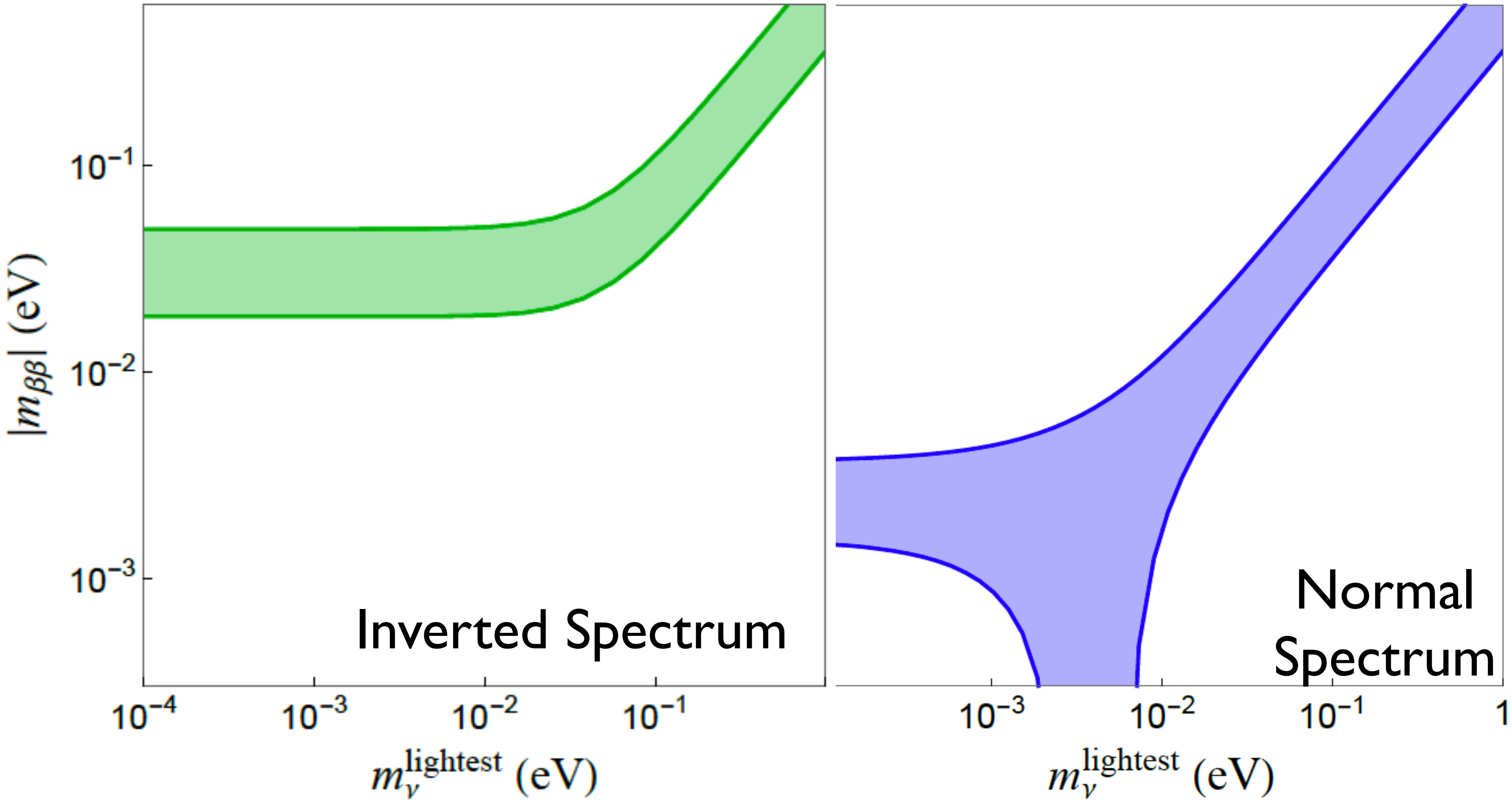
- In this case $0\nu\beta\beta$ is a *direct* probe of ν Majorana mass: $\Gamma \propto |M_{0\nu}|^2 \, (m_{\beta\beta})^2$

Strong correlation with oscillation parameters

$$\langle m_{\beta\beta}\rangle^2 = |\sum U_{ei}^2 m_{\nu i}|^2$$



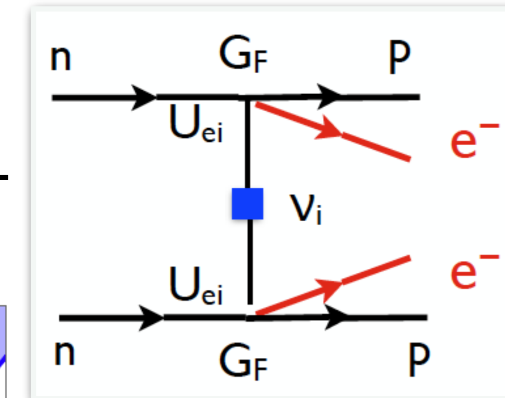
Bands: unknown phases

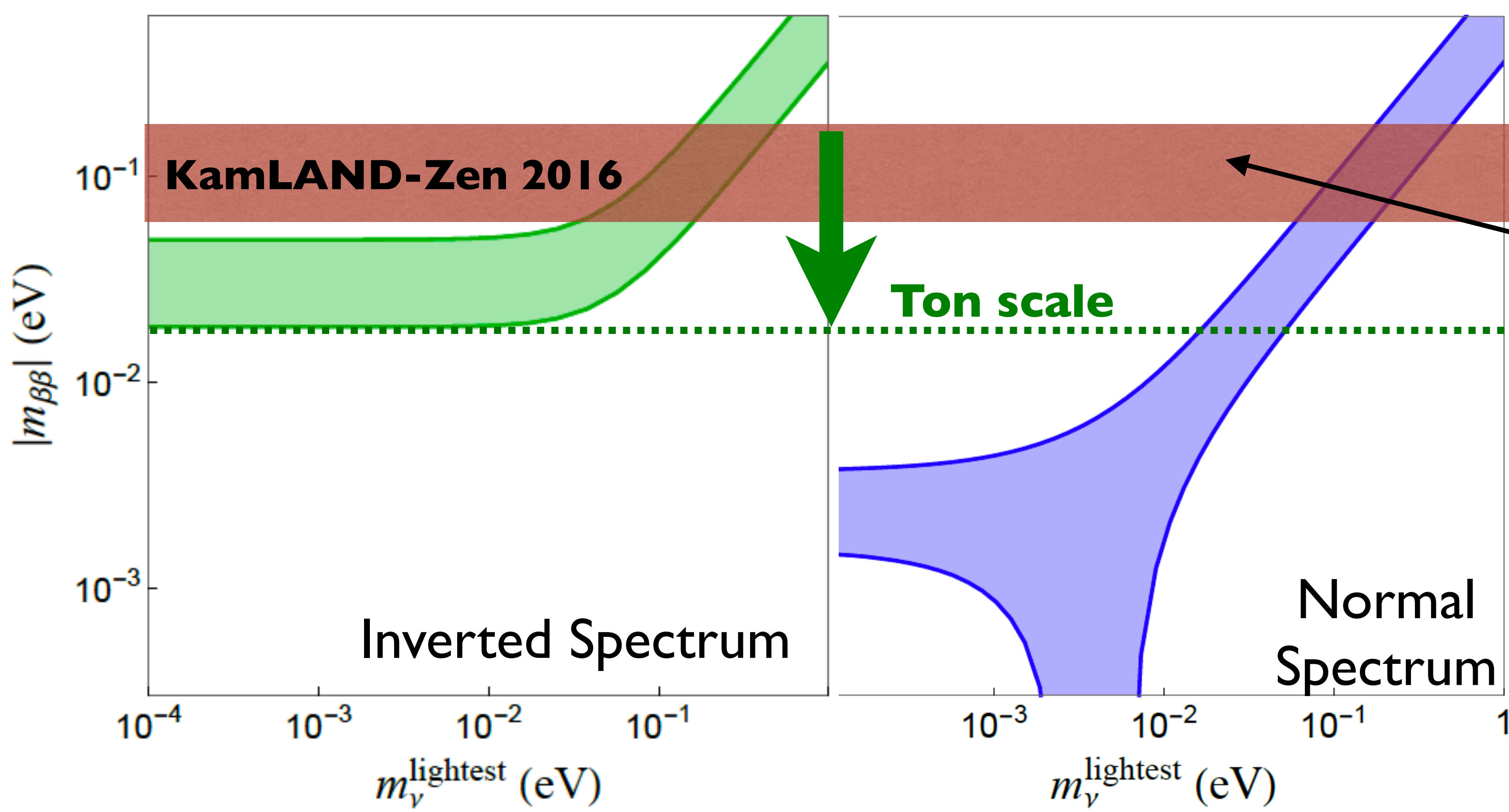


Phenomenological interest

- In this case $0\nu\beta\beta$ is a *direct* probe of ν Majorana mass: $\Gamma \propto |M_{0\nu}|^2 \, (m_{\beta\beta})^2$

$$\langle m_{\beta\beta} \rangle^2 = |\sum_i U_{ei}^2 m_{\nu i}|^2$$





Bands: unknown phases

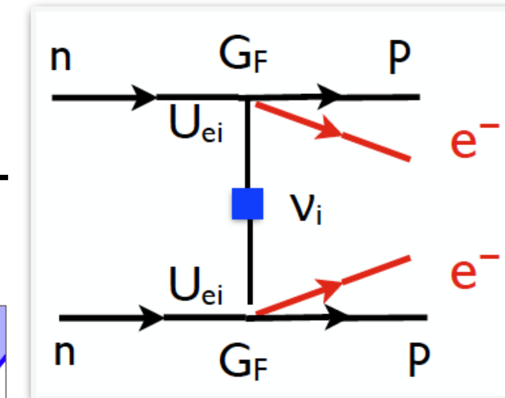
Assume range for nuclear matrix elements from different many-body methods

Assuming current range for matrix elements, discovery @ ton-scale *possible* for inverted spectrum or $m_{lightest}$ > 50 meV

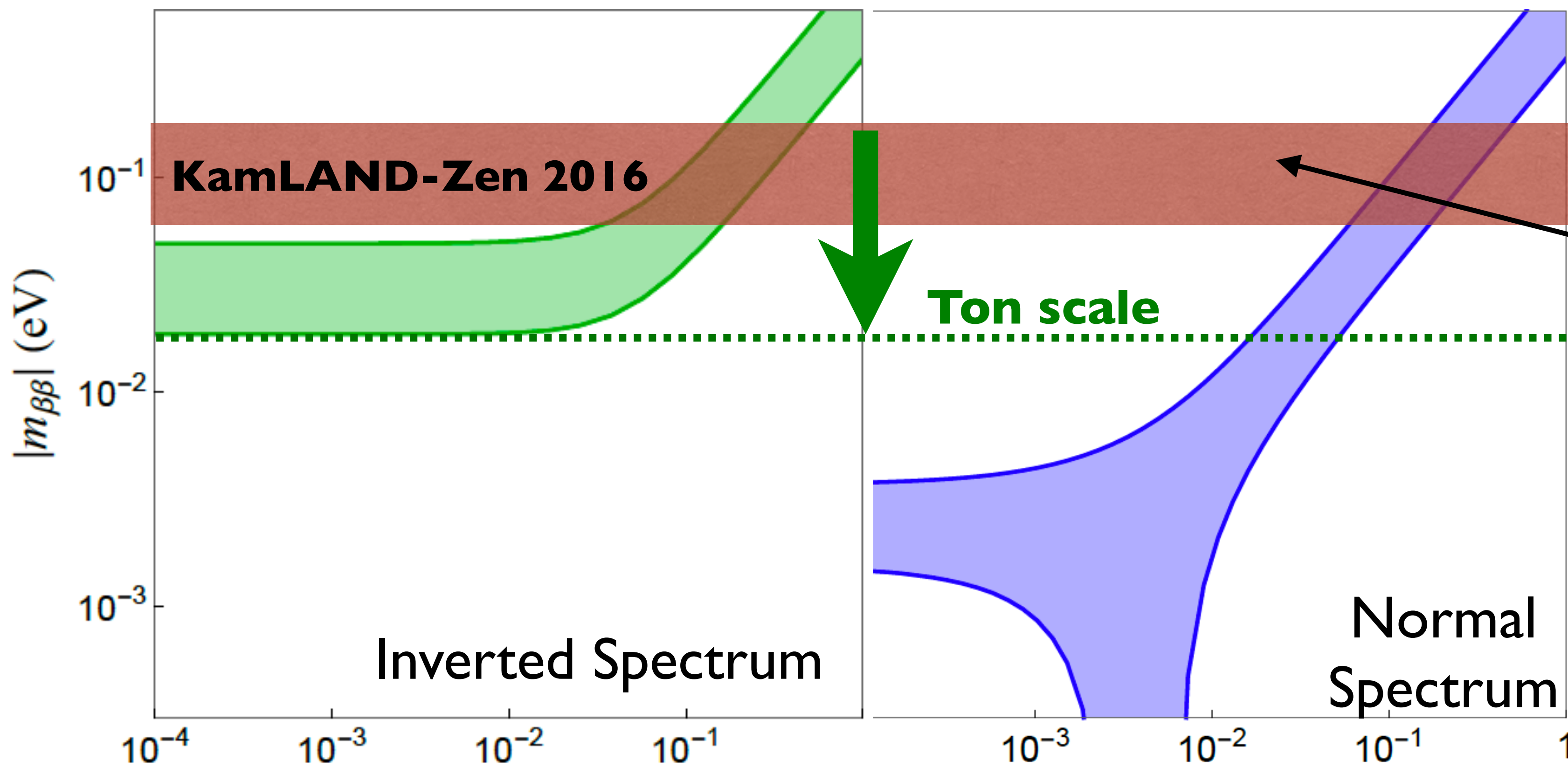
Phenomenological interest

- In this case 0νββ is a *direct* probe of ν Majorana mass: $\Gamma \propto |M_{0\nu}|^2 (m_{\beta\beta})^2$

$$\langle m_{\beta\beta}\rangle^2 = |\sum U_{ei}^2 m_{\nu i}|^2$$



Bands: unknown phases



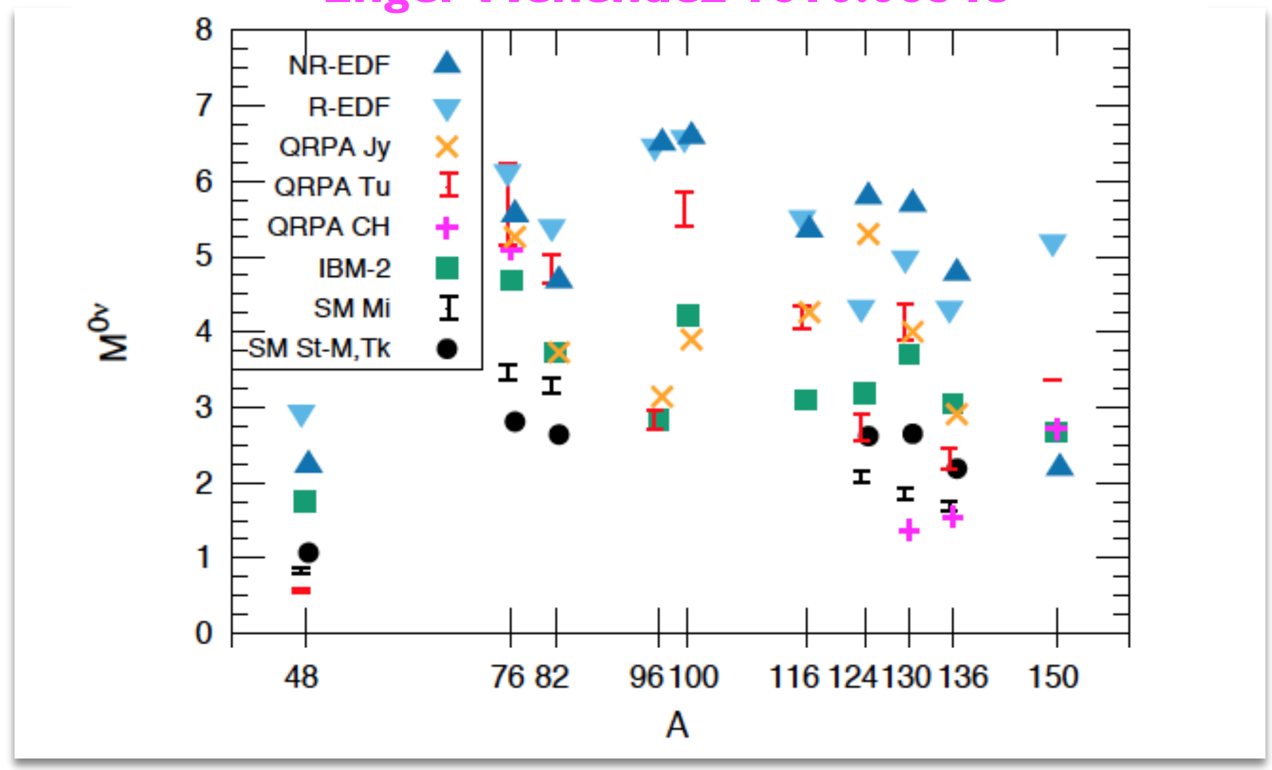
Assume range for nuclear matrix elements from different many-body methods

Correlation with other ν mass probes can test high-scale seesaw and possibly unravel new sources of LNV or physics beyond "ΛCDM + m_ν". But these interesting connections require robust matrix elements!

c

Theory status / developments

Engel-Menendez 1610.06548



- Snapshot as of a few years ago [recall $\Gamma \propto |M_{0\nu}|^2 \, (m_{\beta\beta})^2$]

- Steps towards controlled uncertainties in matrix elements:
  - Use chiral EFT as guiding principle: construct consistently the strong potential and 'weak' transition operators
  - Compute *hadronic* matrix elements with QCD-based methods
  - Ab initio nuclear structure calculations: light nuclei, ^{48}Ca , ^{76}Ge, …

Pastore et al., 1710.05026; Yao et al., 1908.05424;
Belley et al.; 2008.06588; Novario et al., 2008.09696

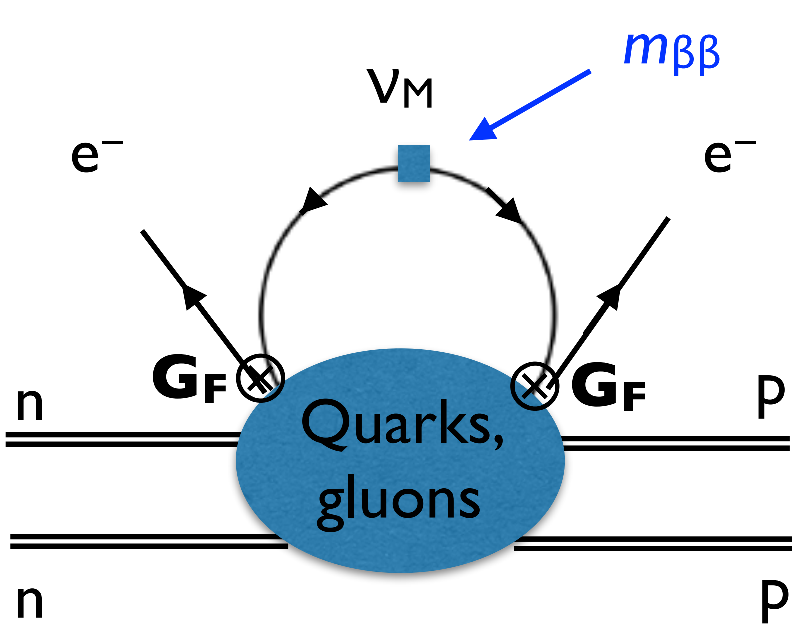
ΔL=2 amplitudes in EFT

- ΔL=2 amplitudes controlled by neutrino-less effective action

$$S_{\rm eff}^{\Delta L=2} = \frac{8 G_F^2 V_{ud}^2 m_{\beta\beta}}{2!} \int d^4x d^4y \; S(x-y) \times \bar{e}_L(x) \gamma^\mu \gamma^\nu e_L^c(y) \times T\Big(\bar{u}_L \gamma_\mu d_L(x) \; \bar{u}_L \gamma_\nu d_L(y) \Big)$$

Scalar massless propagator

To all orders in QCD



ΔL=2 amplitudes in EFT

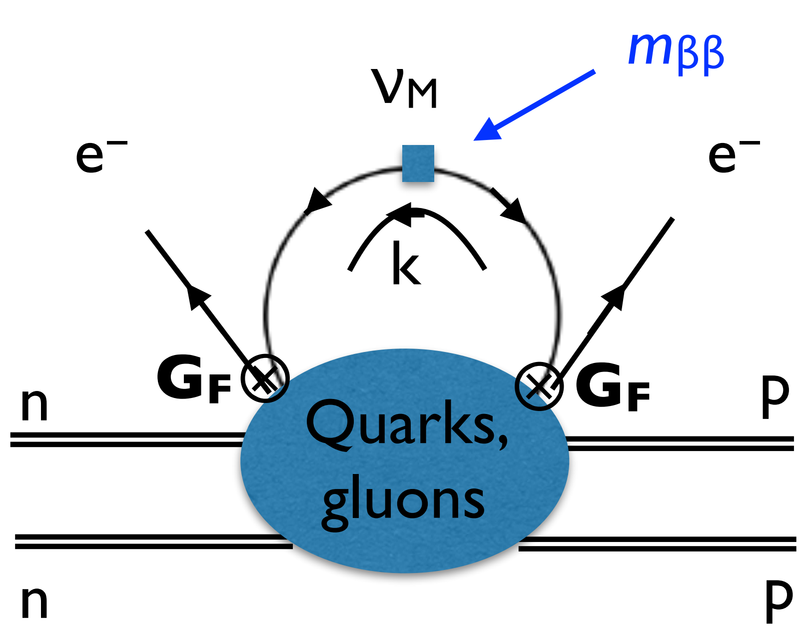
- ΔL=2 amplitudes controlled by neutrino-less effective action

$$\mathcal{A}_{i\to fee} = \frac{8G_F^2 V_{ud}^2 m_{\beta\beta}}{2!} \int d^4x \, L^{\mu\nu}(x) \int \frac{d^4k}{(2\pi)^4} \frac{\langle f|\hat{\Pi}^{LL}_{\mu\nu}(k)|i\rangle}{k^2 + i\epsilon}$$

$$\hat{\Pi}^{LL}_{\mu\nu}(k) = \int d^4r \, e^{ik\cdot r} \, T\Big(\bar{u}_L\gamma_\mu d_L(r/2) \; u_L\gamma_\nu d_L(-r/2)\Big)$$

Momentum space representation

LNV hadronic amplitudes such as nn → ppee receive contributions from all neutrino virtual momenta (k)

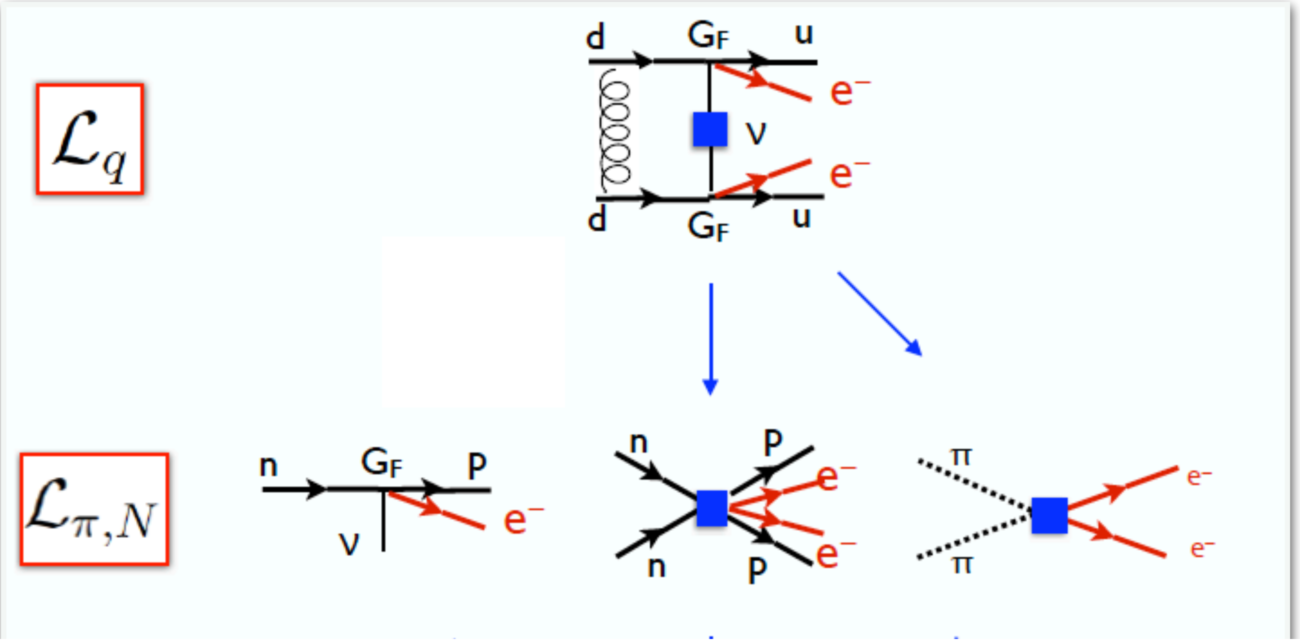


Chiral EFT captures contributions from all relevant momentum regions

From quarks to nuclei using EFT

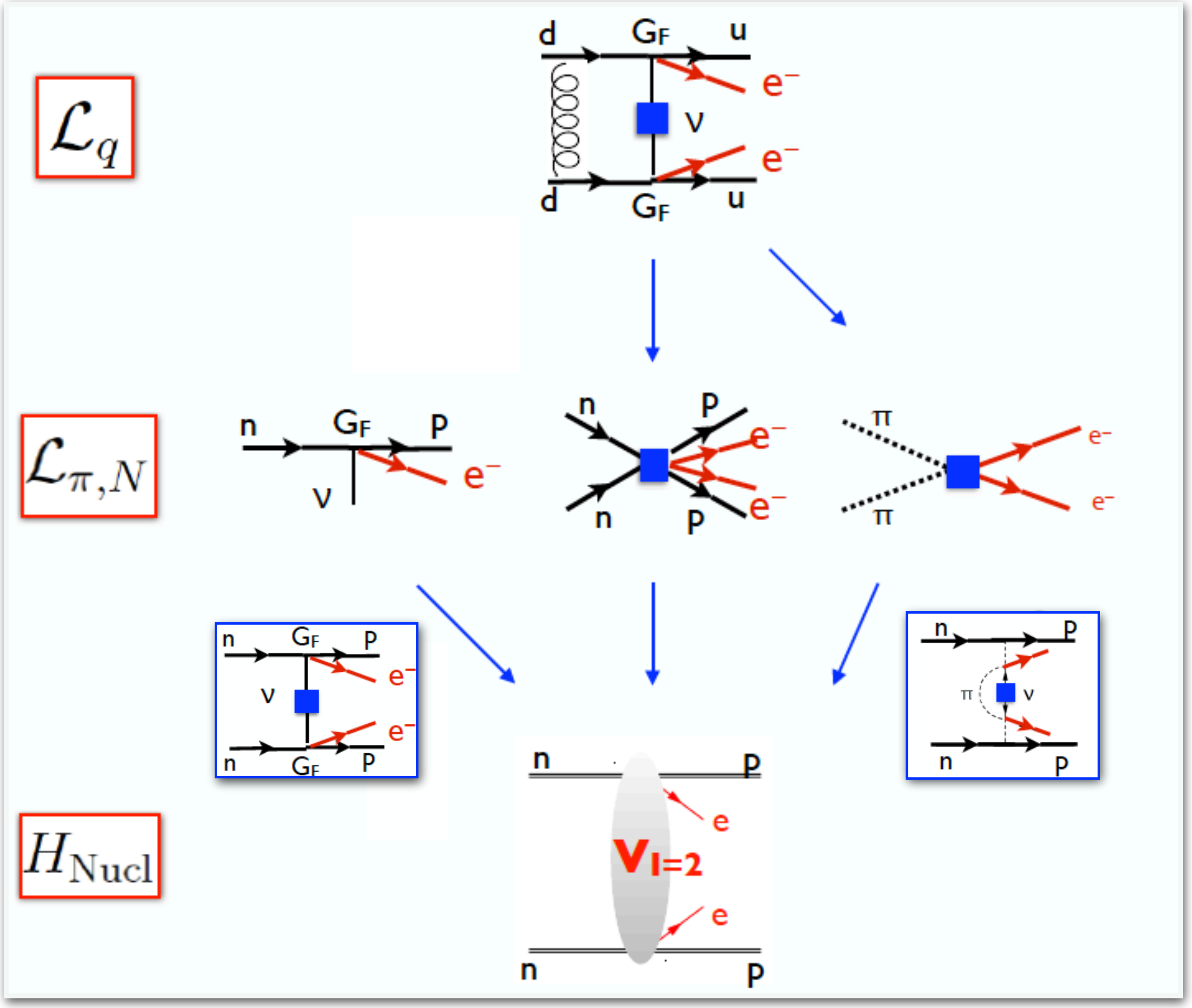
- At E ~ Λ_X ~ m_N ~ GeV integrate out hard ν's and gluons (E, |**k**| > Λ_X)

- Map ΔL=2 Lagrangian onto π, N operators, organized according to power-counting in Q/Λ_X (Q ~ k_F ~ m_π)



From quarks to nuclei using EFT

- At $E \sim \Lambda_X \sim m_N \sim$ GeV integrate out hard ν's and gluons (E, |**k**| > Λ_X)
- Map ΔL=2 Lagrangian onto π, N operators, organized according to power-counting in Q/Λ_X ($Q \sim k_F \sim m_\pi$)
- Integrate out soft and potential ν's and π's with (E,|**k**|)~Q and (E,|**k**|)~(Q^2/m_N, Q) → obtain nuclear Hamiltonian

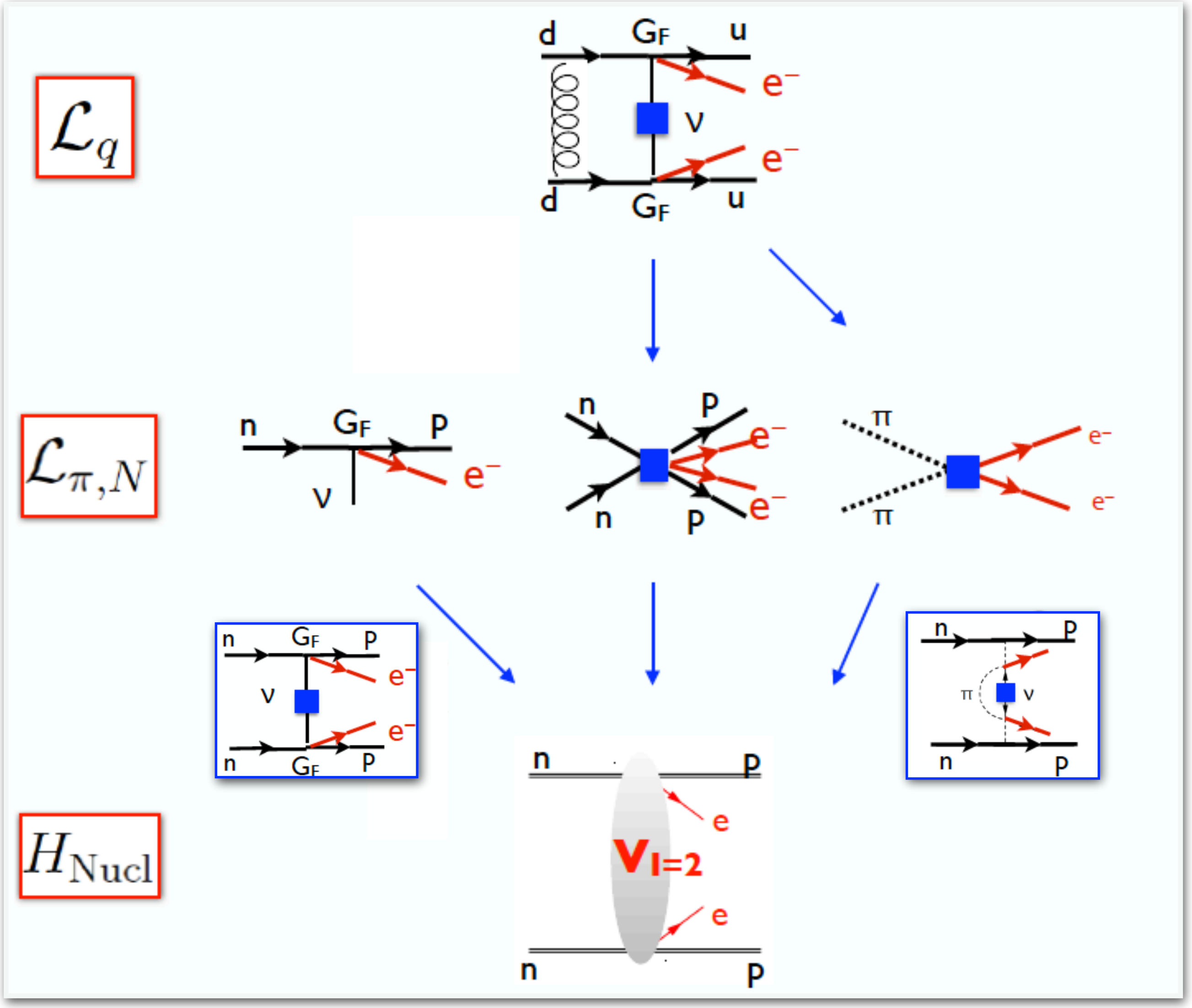


From quarks to nuclei using EFT

- At $E \sim \Lambda_\chi \sim m_N \sim$ GeV integrate out hard ν's and gluons ($E, |\mathbf{k}| > \Lambda_\chi$)

- Map ΔL=2 Lagrangian onto π, N operators, organized according to power-counting in Q/Λ_χ ($Q \sim k_F \sim m_\pi$)

- Integrate out soft and potential ν's and π's with $(E,|\mathbf{k}|) \sim Q$ and $(E,|\mathbf{k}|) \sim (Q^2/m_N, Q)$ → obtain nuclear Hamiltonian



$$H_{\rm Nucl} = H_0 + \sqrt{2} G_F V_{ud} \bar{N} \left(g_V \delta^{\mu 0} - g_A \delta^{\mu i} \sigma^i \right) \tau^+ N \, \bar{e}_L \gamma_\mu \nu_L + 2 G_F^2 V_{ud}^2 m_{\beta\beta} \, \bar{e}_L e_L^c \, V_{I=2}$$

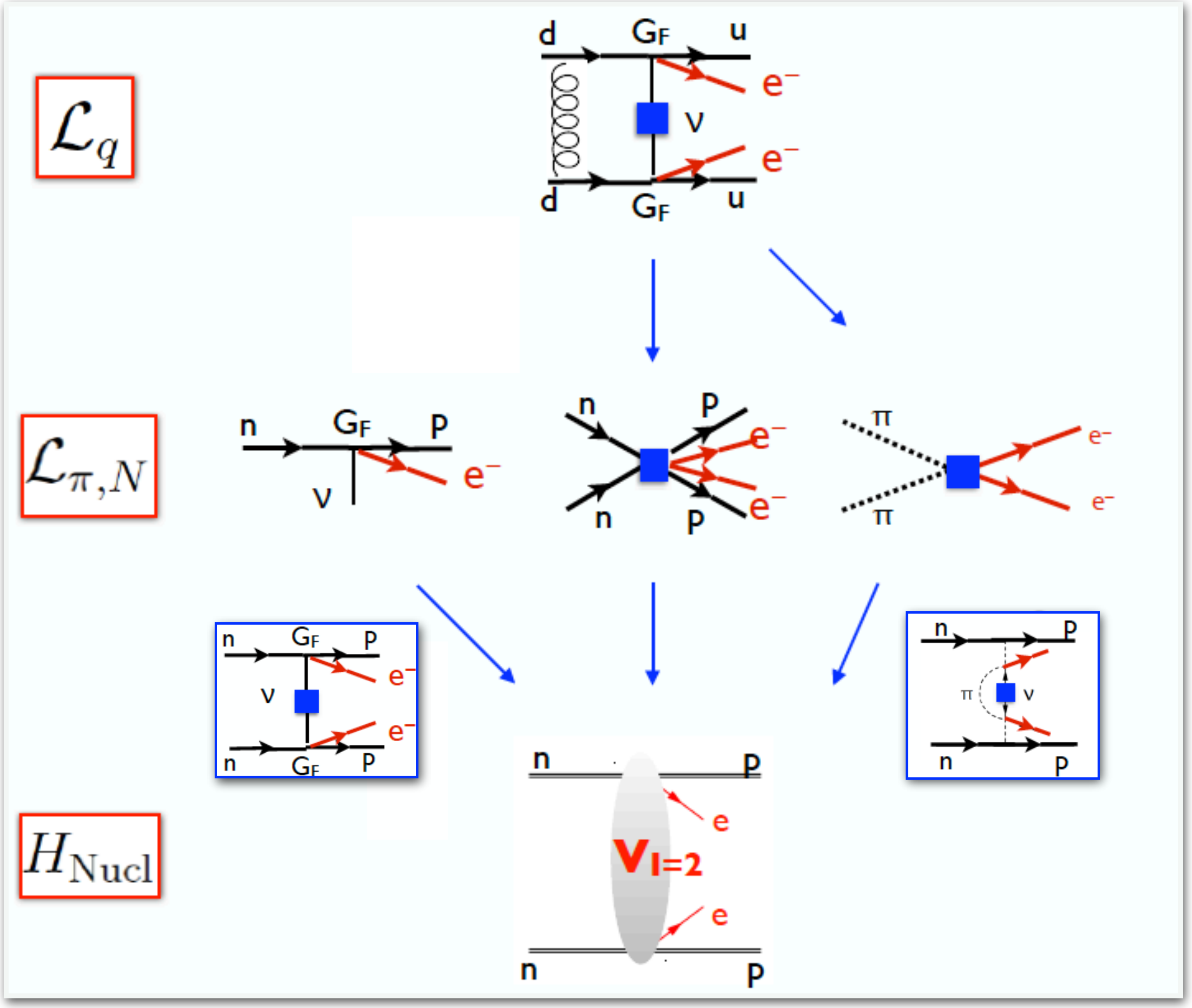
Kinetic term and strong NN potential

From quarks to nuclei using EFT

- At E ~ Λ_χ ~ m_N ~ GeV integrate out hard ν's and gluons (E, |**k**| > Λ_χ)

- Map ΔL=2 Lagrangian onto π, N operators, organized according to power-counting in Q/Λ_χ (Q ~ k_F ~ m_π)

- Integrate out soft and potential ν's and π's with (E,|**k**|)~Q and (E,|**k**|)~(Q^2/m_N, Q) → obtain nuclear Hamiltonian



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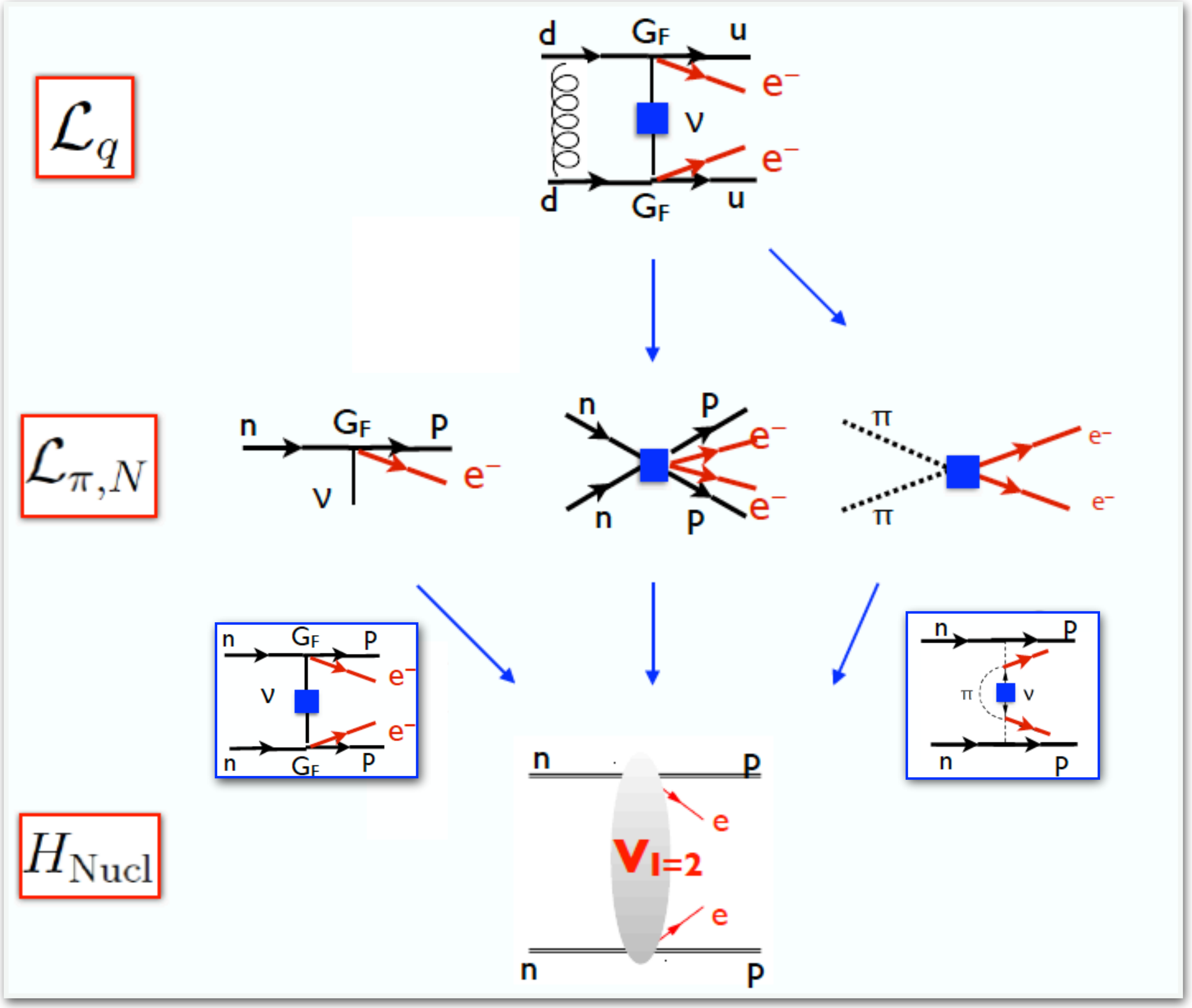
"Ultra-soft" (e, ν) with |**k**|, E << k_F cannot be integrated out

From quarks to nuclei using EFT

- At $E \sim \Lambda_\chi \sim m_N \sim$ GeV integrate out hard ν's and gluons (E, |**k**| > Λ_χ)
- Map ΔL=2 Lagrangian onto π, N operators, organized according to power-counting in Q/Λ_χ ($Q \sim k_F \sim m_\pi$)
- Integrate out soft and potential ν's and π's with (E,|**k**|)~Q and (E,|**k**|)~(Q^2/m_N, Q) → obtain nuclear Hamiltonian



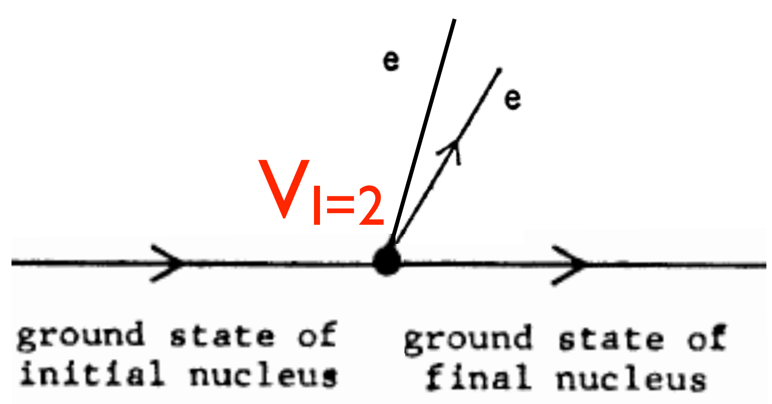
$$H_{\rm Nucl} = H_0 + \sqrt{2} G_F V_{ud}\, \bar{N} \left(g_V \delta^{\mu 0} - g_A \delta^{\mu i} \sigma^i \right) \tau^+ N\, \bar{e}_L \gamma_\mu \nu_L + 2 G_F^2 V_{ud}^2 m_{\beta\beta}\, \bar{e}_L e_L^c\, V_{I=2}$$

"Ultra-soft" (e, ν) with |**k**|, E << k_F cannot be integrated out

"Neutrino potential" that mediates nn→pp.
It can be identified to a given order in Q/Λ_χ by computing 2-nucleon amplitudes

Anatomy of 0νββ amplitude in EFT

Figure adapted from Primakoff-Rosen 1969



Hard, soft, and potential ν

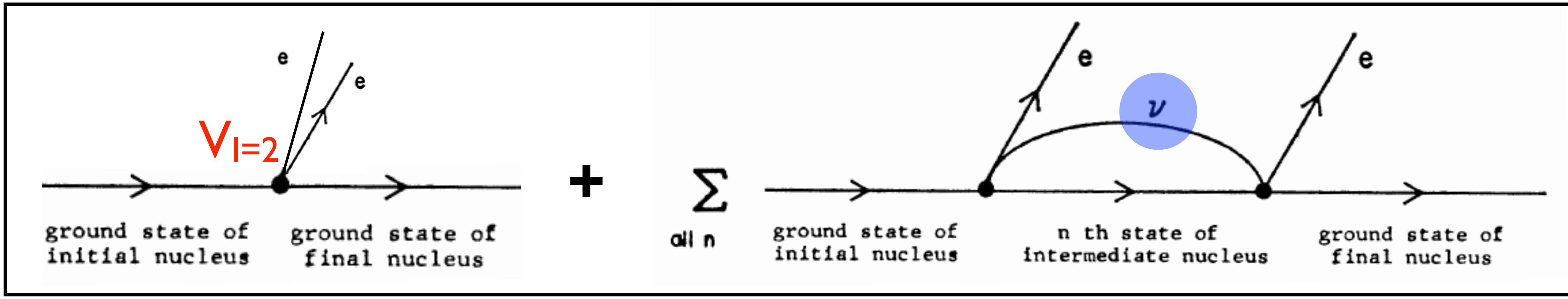
$$V_{I=2} = \sum_{a \neq b} \left(V_{\nu,0}^{(a,b)} + V_{\nu,2}^{(a,b)} + \ldots \right)$$

$V_\nu \sim 1/Q^2\ ,\ 1/(Q\Lambda_\chi)\ ,\ 1/(\Lambda_\chi)^2, \ldots$

LO NLO N²LO

Anatomy of 0νββ amplitude in EFT

Figure adapted from Primakoff-Rosen 1969



Hard, soft, and potential ν

Ultrasoft ν

$$V_{I=2} = \sum_{a\neq b} \left(V_{\nu,0}^{(a,b)} + V_{\nu,2}^{(a,b)} + \ldots \right)$$

Calculable in terms of $E_n - E_i$ and $\langle f|J_\mu|n\rangle\langle n|J^\mu|i\rangle$, that also control 2νββ. Contributes to the amplitude at N²LO

$V_\nu \sim 1/Q^2$, $1/(Q\Lambda_\chi)$, $1/(\Lambda_\chi)^2$, …

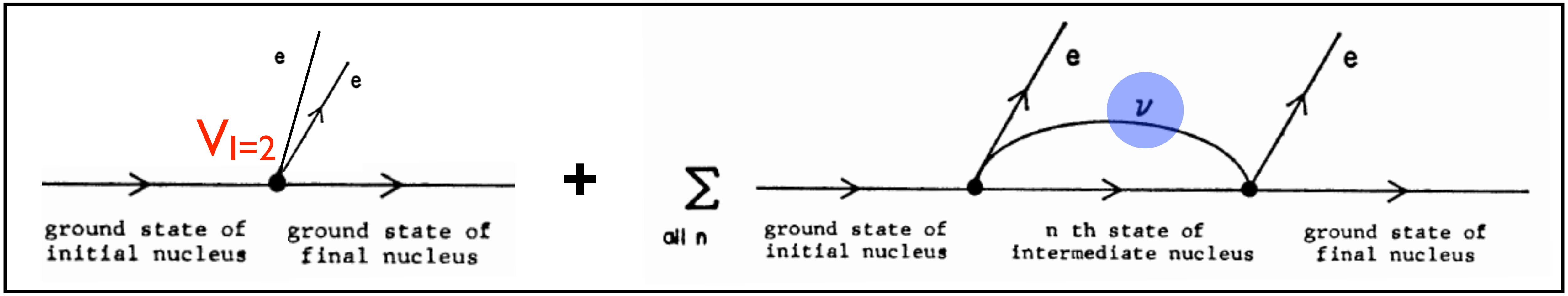
↑ LO

↑ NLO

↑ N²LO

Anatomy of 0νββ amplitude in EFT

Figure adapted from Primakoff-Rosen 1969



Hard, soft, and potential ν

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$V_\nu \sim 1/Q^2$, $1/(Q\Lambda_\chi)$, $1/(\Lambda_\chi)^2, \dots$

LO ($1/Q^2$) — NLO ($1/(Q\Lambda_\chi)$) — N²LO ($1/(\Lambda_\chi)^2$)

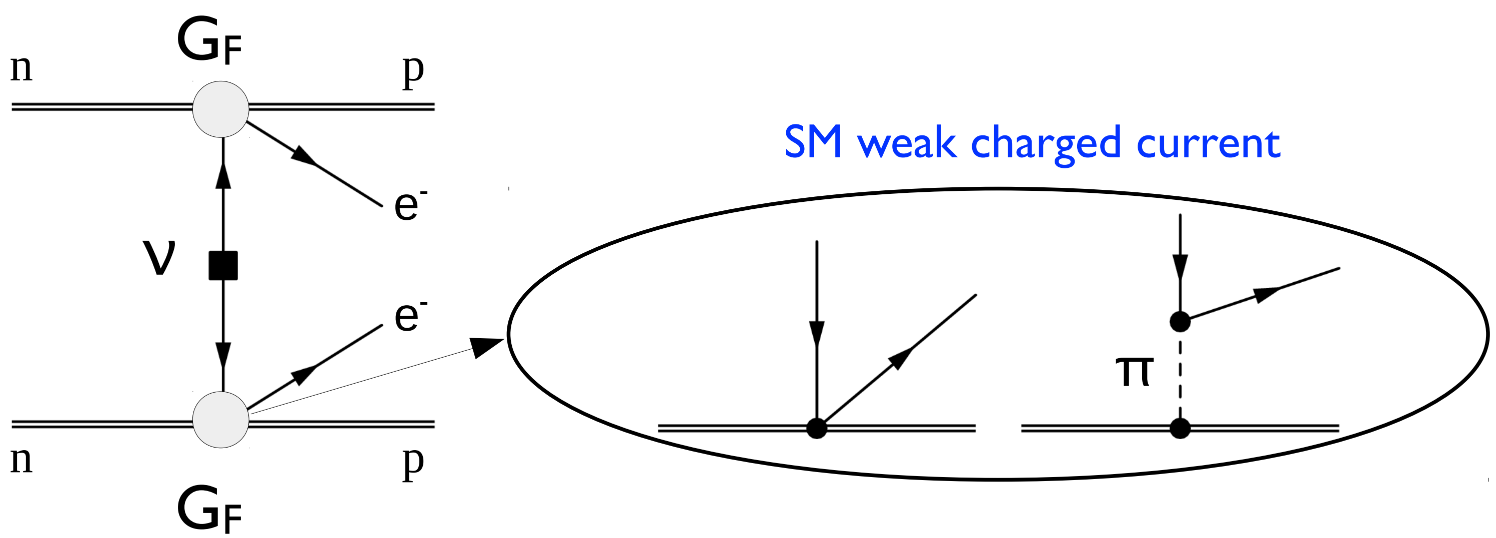
Ultrasoft ν

Calculable in terms of $E_n - E_i$ and $\langle f\,|J_\mu|\,n\rangle\langle n|J^\mu|i\rangle$, that also control 2νββ.

Contributes to the amplitude at N²LO

For details and analysis beyond LO see 1710.01729, 1802.10097, 1907.11254

Leading order 0νββ potential

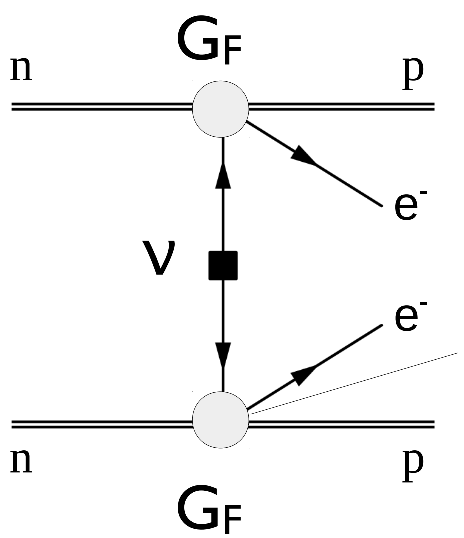


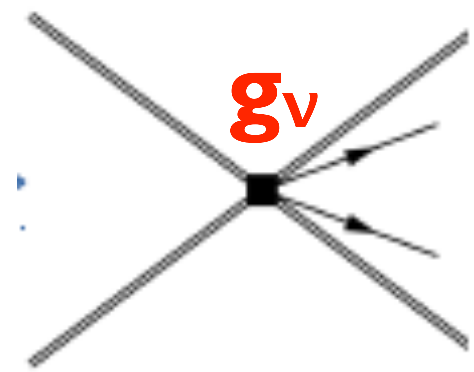
- Tree-level exchange of Majorana neutrinos

$$V_{\nu,0}^{(a,b)} = \tau^{(a)+}\tau^{(b)+}\frac{1}{\mathbf{q}^2}\left\{1 - g_A^2\left[\boldsymbol{\sigma}^{(a)}\cdot\boldsymbol{\sigma}^{(b)} - \boldsymbol{\sigma}^{(a)}\cdot\mathbf{q}\,\boldsymbol{\sigma}^{(b)}\cdot\mathbf{q}\,\frac{2m_\pi^2+\mathbf{q}^2}{(\mathbf{q}^2+m_\pi^2)^2}\right]\right\}$$

Hadronic input: g_A

Leading order 0νββ potential





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Hadronic input: g_A

- Chiral symmetry allow us to write down a contact term

$$V_{\nu,CT}^{(a,b)} = -2\,g_\nu\;\tau^{(a)+}\tau^{(b)+}$$

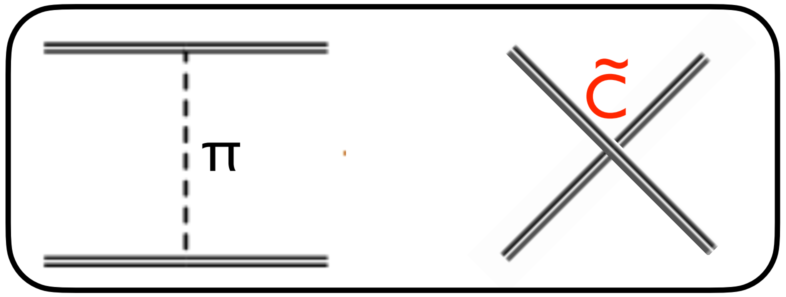
$g_\nu \sim 1/(4\pi F_\pi)^2$ in NDA / Weinberg counting (and hence sub-leading)

But is it?

Scaling of the contact term

Weinberg 1991, Kaplan-Savage-Wise 1996

- Study nn→ppee amplitude (in $^1S_0$ channel) to LO, including strong potential



$\tilde{C} \sim 4\pi/(m_N Q) \sim 1/F_\pi^2$ from fit to a_{NN}

Scaling of the contact term

Weinberg 1991, Kaplan-Savage-Wise 1996

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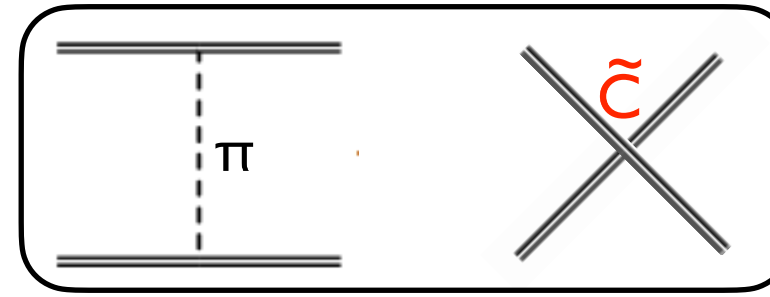


$\tilde{C} \sim 4\pi/(m_N Q) \sim 1/F_\pi^2$ from fit to a_{NN}

$$\mathcal{A}_\nu(E,E') = -\langle \Psi_{pp}(E') | V_{\nu\,\mathrm{L}}^{^1S_0} | \Psi_{nn}(E) \rangle$$

$$V_{\nu\,\mathrm{L}}^{^1S_0}(\mathbf{q}) = \frac{\tau^{(1)+}\tau^{(2)+}}{\mathbf{q}^2}\left[1 + 2g_A^2 + \frac{g_A^2 m_\pi^4}{(\mathbf{q}^2+m_\pi^2)^2}\right]$$

$|\Psi_{nn}(E)\rangle$

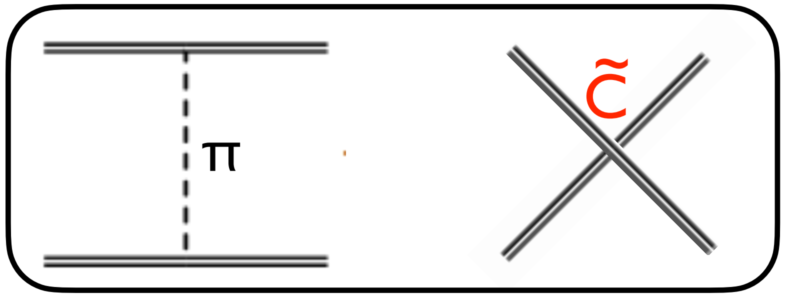
$|\Psi_{pp}(E')\rangle$

$$V_{NN}^{^1S_0} = \tilde{C} + V_\pi^{^1S_0}(\mathbf{q}) \qquad V_\pi^{^1S_0}(\mathbf{q}) = -\frac{g_A^2}{4F_\pi^2}\frac{m_\pi^2}{\mathbf{q}^2+m_\pi^2}$$

Scaling of the contact term

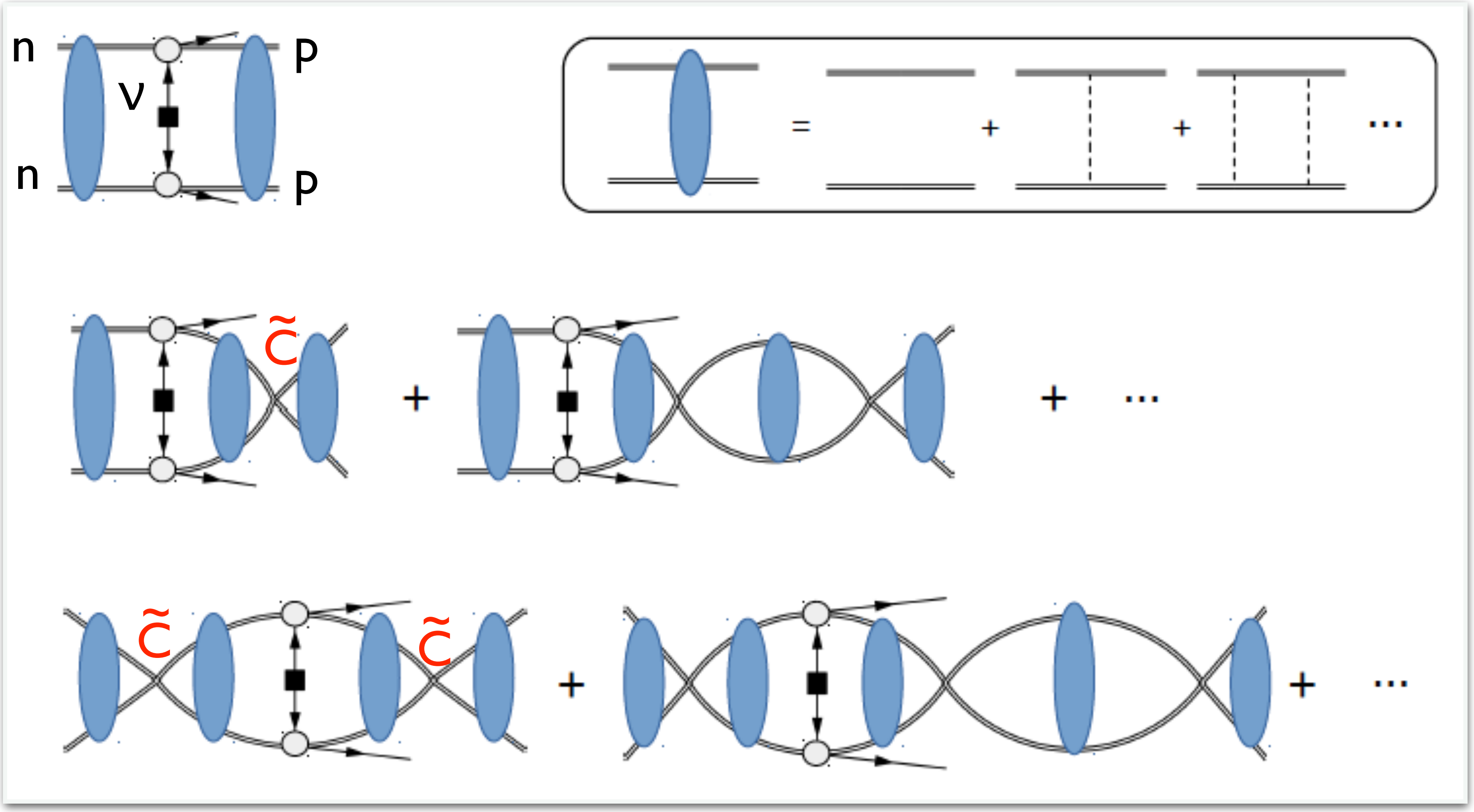
Weinberg 1991, Kaplan-Savage-Wise 1996

- Study nn→ppee amplitude (in $^1S_0$ channel) to LO, including strong potential



$\tilde{C} \sim 4\pi/(m_N Q) \sim 1/F_\pi^2$ from fit to a_{NN}

LO ~ $1/Q^2$: three classes of diagrams (recall discussion on pp. 42-43 of Lecture 1)



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Weinberg 1991, Kaplan-Savage-Wise 1996

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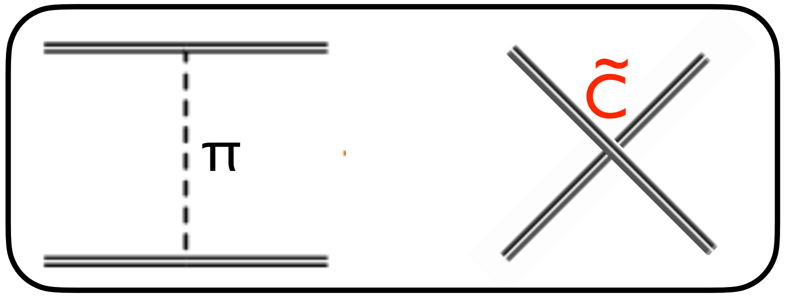

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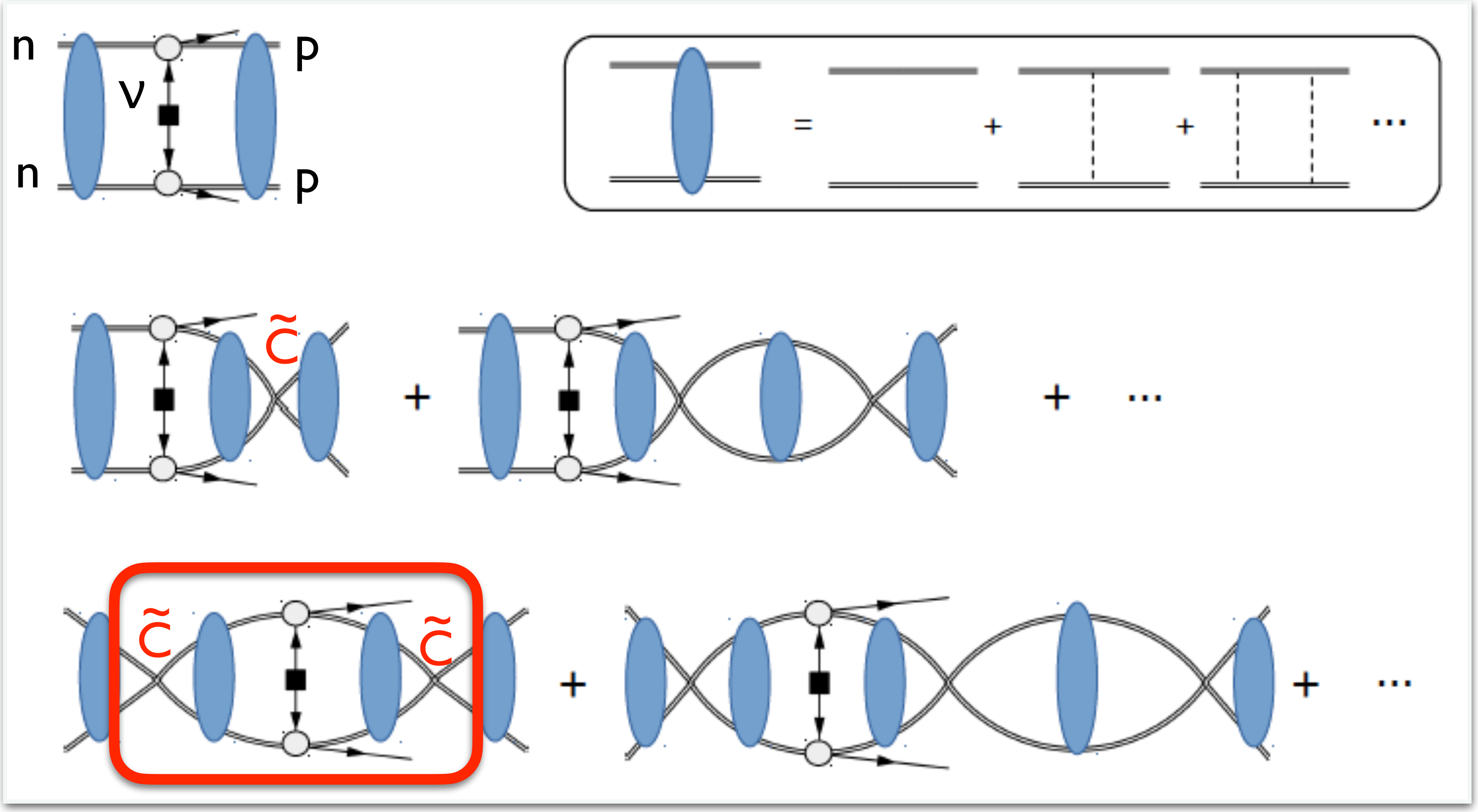


UV finite ($V_\pi \sim m_\pi^2/k^2$)

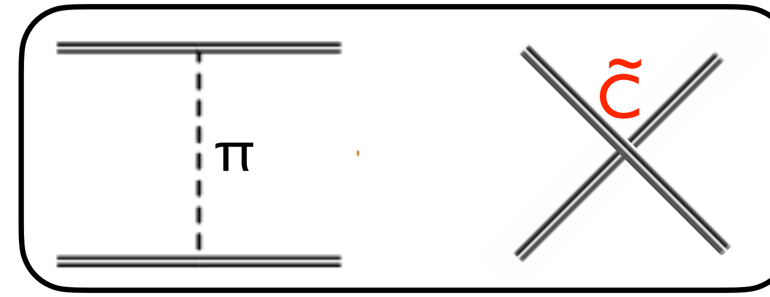
UV finite

2-loop diagram is UV divergent!

Scaling of the contact term

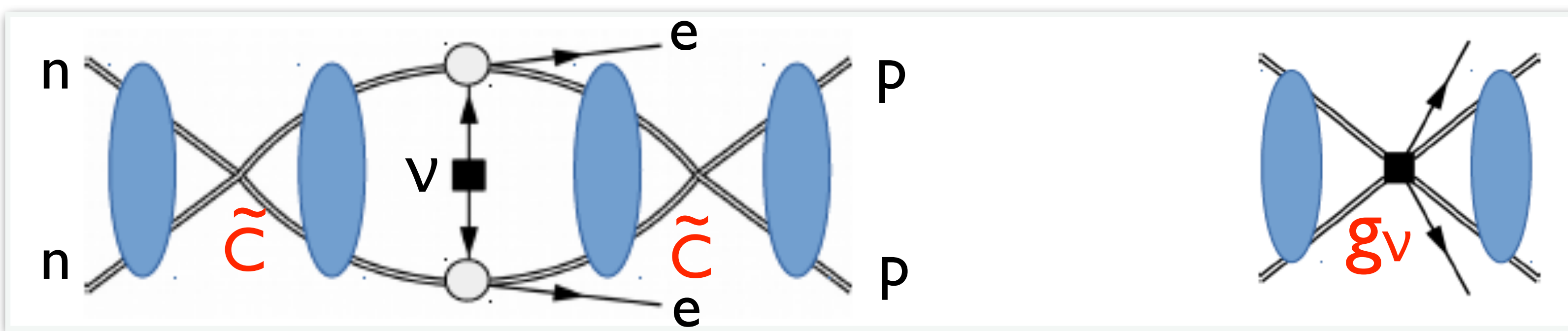
Weinberg 1991, Kaplan-Savage-Wise 1996

- Study nn→ppee amplitude (in $^1S_0$ channel) to LO, including strong potential



$\tilde{C} \sim 4\pi/(m_N Q) \sim 1/F_\pi^2$ from fit to a_{NN}

- Renormalization requires contact operator at LO



UV divergence

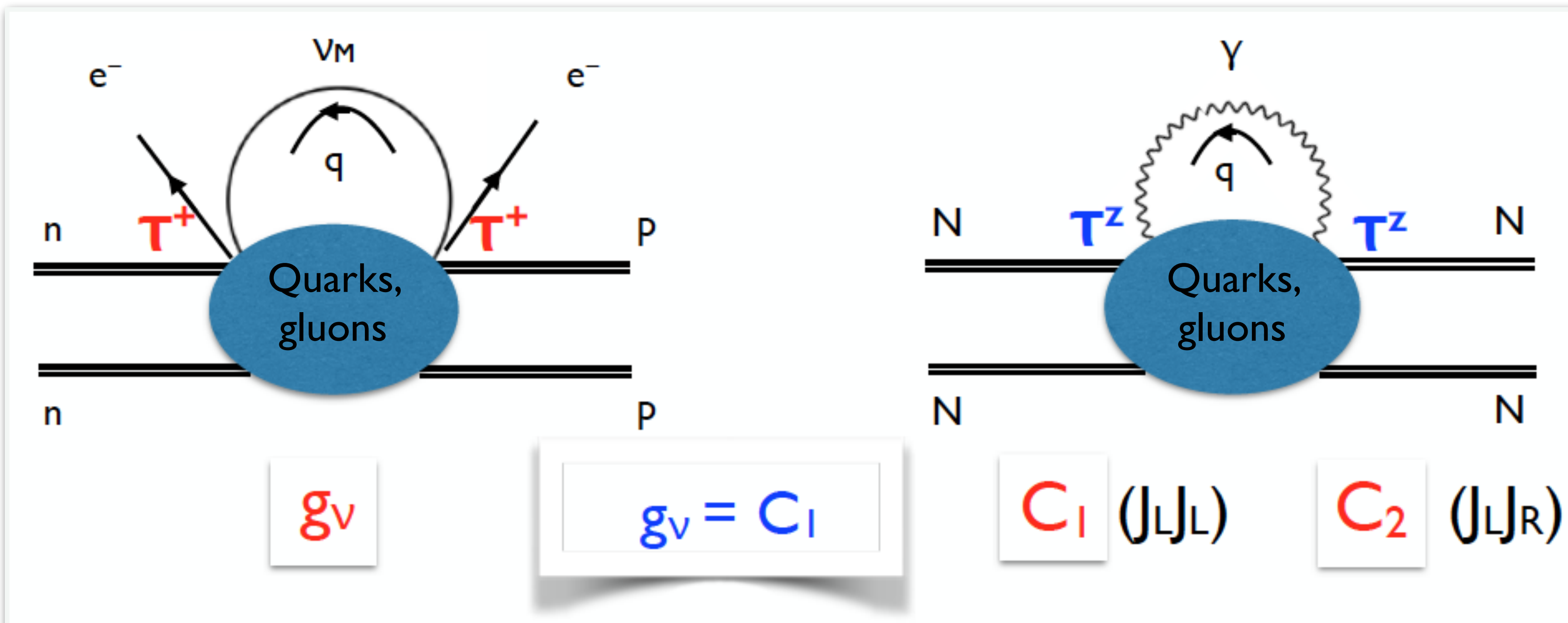
$$\sim \frac{1}{2}(1+2g_A^2)\left(\frac{m_N\tilde{C}}{4\pi}\right)^2\left(\frac{1}{4-d}+\log\mu^2\right)$$

$\sim 1/(F_\pi)^2 \;\log \mu$

- Coupling flows to $g_\nu \sim 1/F_\pi^2 \;>>\; 1/(4\pi F_\pi)^2$, same order as $1/q^2$ from tree-level neutrino exchange!

Connection with data?

- Chiral+isospin symmetry relates g_ν to I=2 e.m. couplings (hard γ's & ν's)

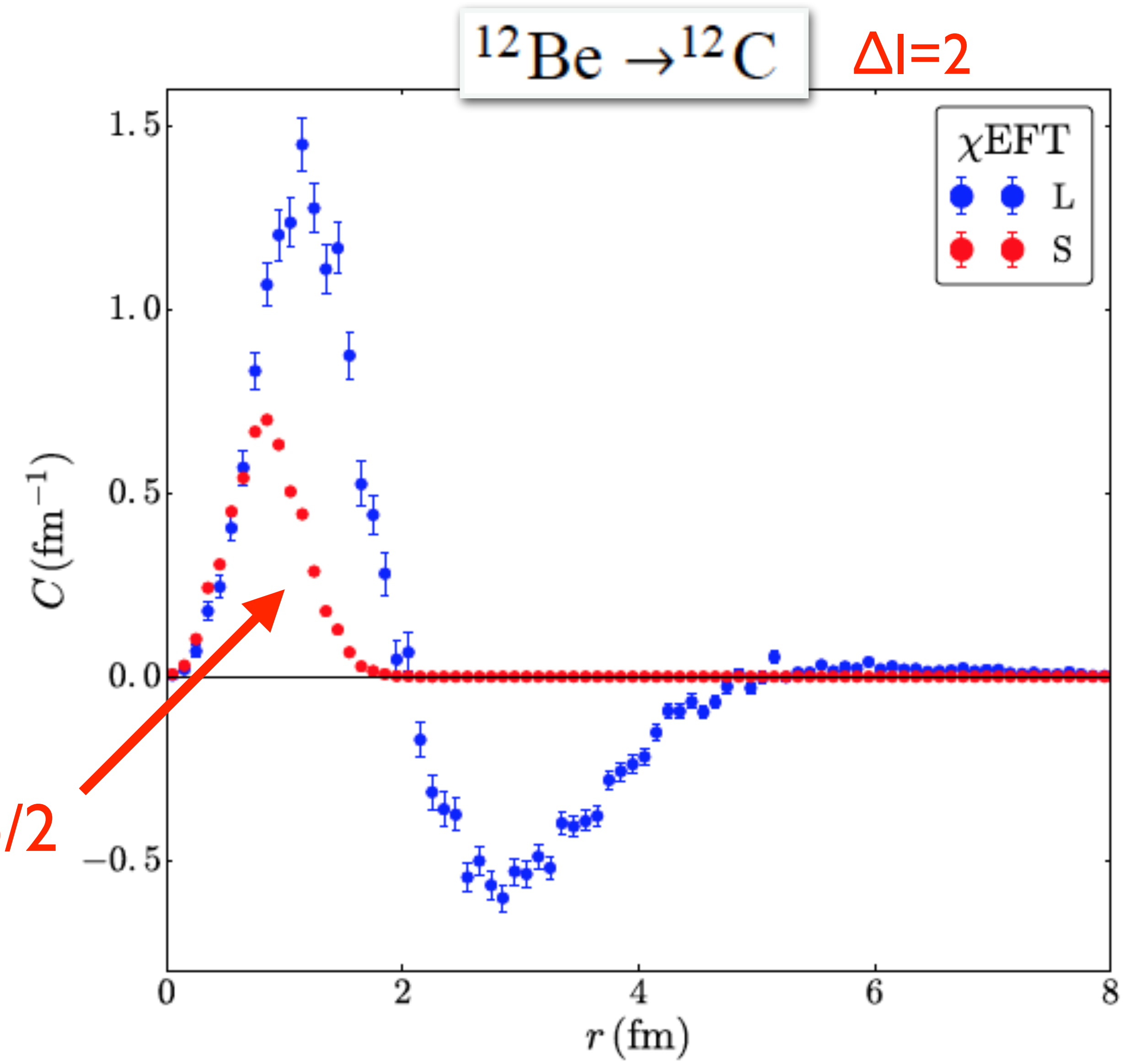


- NN data ($a_{nn}+a_{pp}-2a_{np}$) determine C_1+C_2, confirming LO scaling!
- Assuming $g_\nu \sim (C_1+C_2)/2$, what is the impact on $m_{\beta\beta}$ extraction?

Impact on nuclear matrix elements

VC, W. Dekens, J. de Vries, M. Graesser, E. Mereghetti, S. Pastore, M. Piarulli, U. van Kolck, R. Wiringa, 1907.11254

Amplitude = $\int dr\, C(r)$



$g_\nu \sim (C_1 + C_2)/2$

Light nuclei with Norfolk chiral potential [1606.06335]

g_ν contribution large in $\Delta I=2$ transition: for A=12, A_S/A_L = 0.75

Transitions of experimental interest (^{76}Ge→ ^{76}Se, …) have node ($\Delta I=2$)
⇒ expect significant effect!

Challenge: determination of g_ν

- Large-N_C arguments point to $g_\nu \sim (C_1+C_2)/2 + O(1/N_C)$

Richardson, Shindler, Pastore, Springer, 2102.02814

- Compute nn→ppee in full theory and match to EFT expression

Lattice QCD

- $\pi^- \to \pi^+ e^- e^-$ precisely known

Tuo et al. 1909.13525; Detmold, Murphy 2004.07404

- Formalism for NN developed

Davoudi, Kadam, 2012.02083

Analytic approach

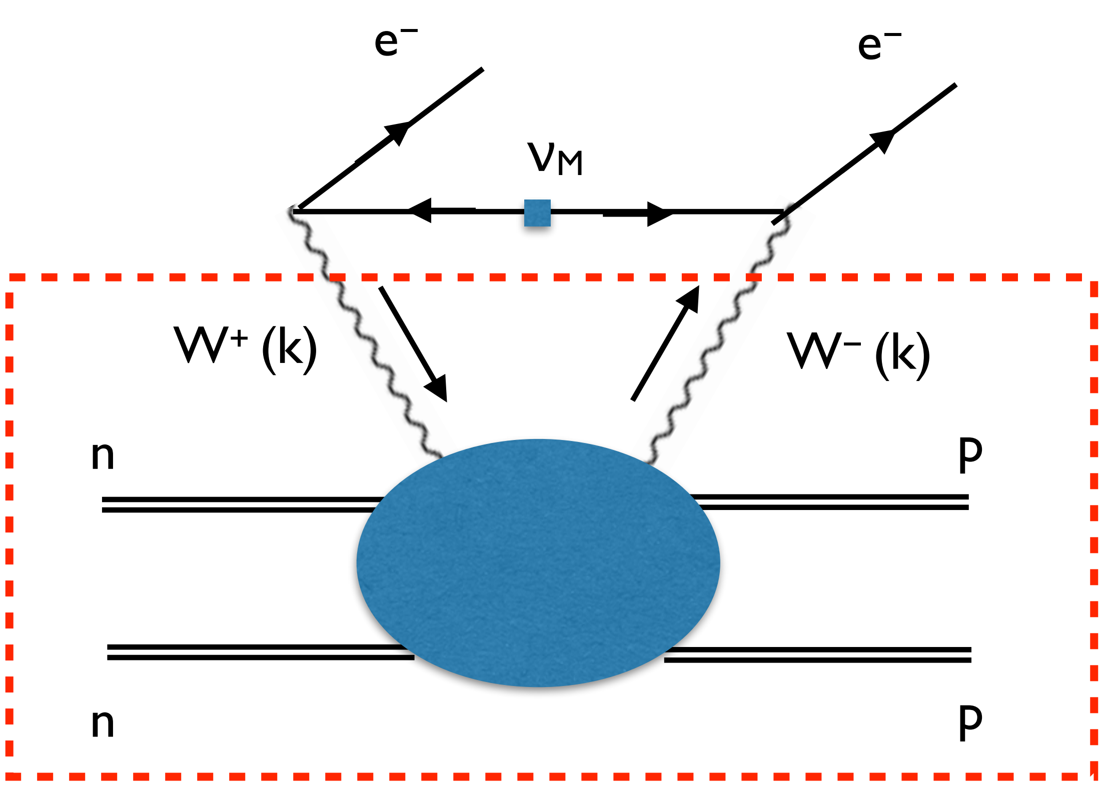
Inspired by the Cottingham approach to electromagnetic nucleon mass splitting

VC, Dekens, deVries, Hoferichter, Mereghetti, 2012.11602, 2102.03371

Estimating the contact term

- Integral representation of the amplitude

$$\mathcal{A}_\nu \propto \int \frac{d^4k}{(2\pi)^4} \frac{g_{\alpha\beta}}{k^2 + i\epsilon} \int d^4x \, e^{ik\cdot x} \langle pp | T\{ j_{\rm w}^{\alpha}(x) j_{\rm w}^{\beta}(0)\} | nn \rangle$$



Forward
“Compton” amplitude

Estimating the contact term

- Integral representation of the amplitude

$$\mathcal{A}_\nu \propto \int \frac{d^4k}{(2\pi)^4} \frac{g_{\alpha\beta}}{k^2 + i\epsilon} \int d^4x \, e^{ik\cdot x} \langle pp | T\{ j_{\rm w}^{\alpha}(x) j_{\rm w}^{\beta}(0)\} | nn \rangle$$

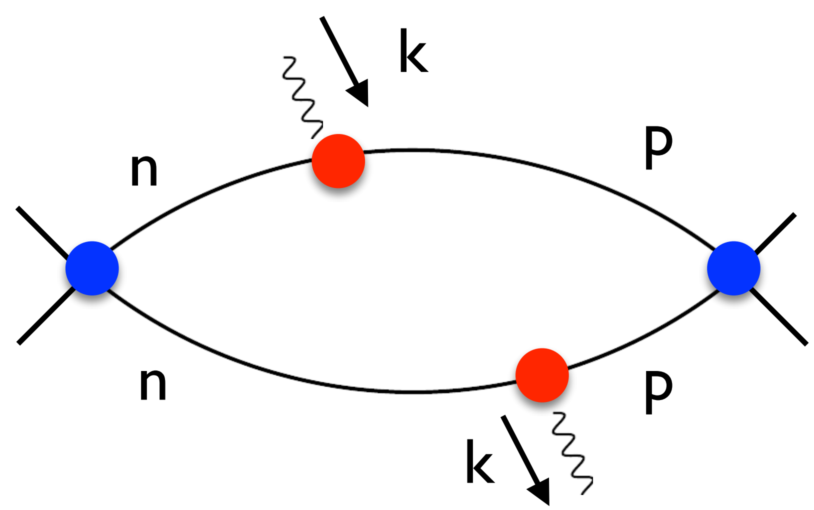
- From ChPT and the QCD operator product expansion, we know the behavior of the integrand at low and high momentum, respectively
- Introduce separation scale Λ ~ 1-2 GeV, corresponding to onset of QCD asymptotic behavior
- Split `full theory' amplitude into "<" and ">" components, corresponding to |**k**| < Λ and |**k**| > Λ
- Use appropriate degrees of freedom in each region

Estimating the contact term

- Integral representation of the amplitude

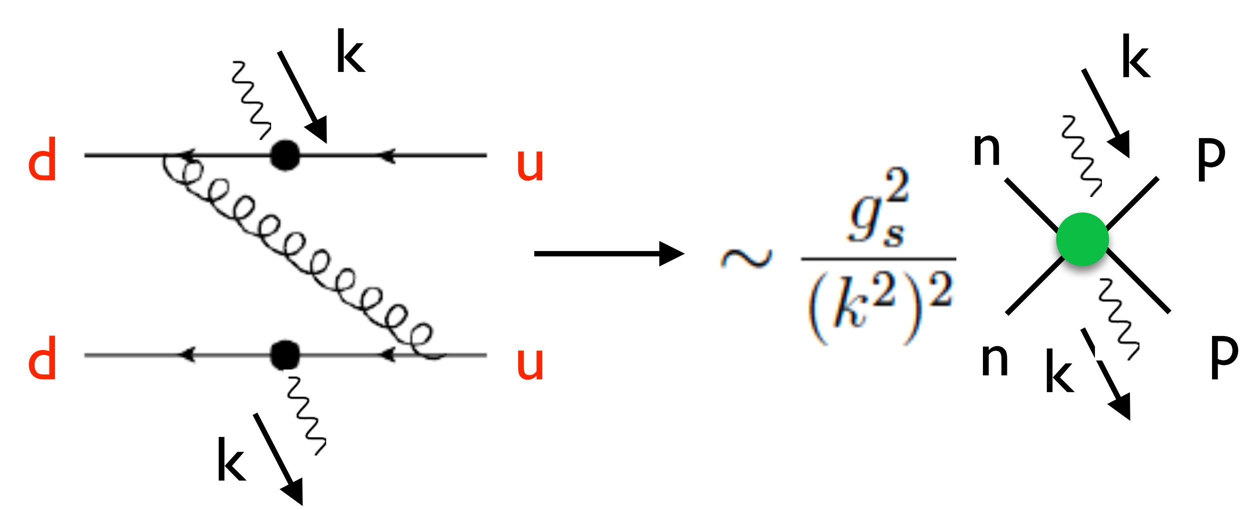
$$\mathcal{A}_\nu \propto \int \frac{d^4k}{(2\pi)^4} \frac{g_{\alpha\beta}}{k^2 + i\epsilon} \int d^4x \, e^{ik\cdot x} \langle pp | T\{ j_{\rm w}^{\alpha}(x) j_{\rm w}^{\beta}(0) \} | nn \rangle$$

- Low momentum: chiral EFT up to NLO



- Extend to intermediate momentum: resonance contributions to nucleon weak form factors and $^1S_0$ NN scattering

- High momentum: QCD operator product expansion



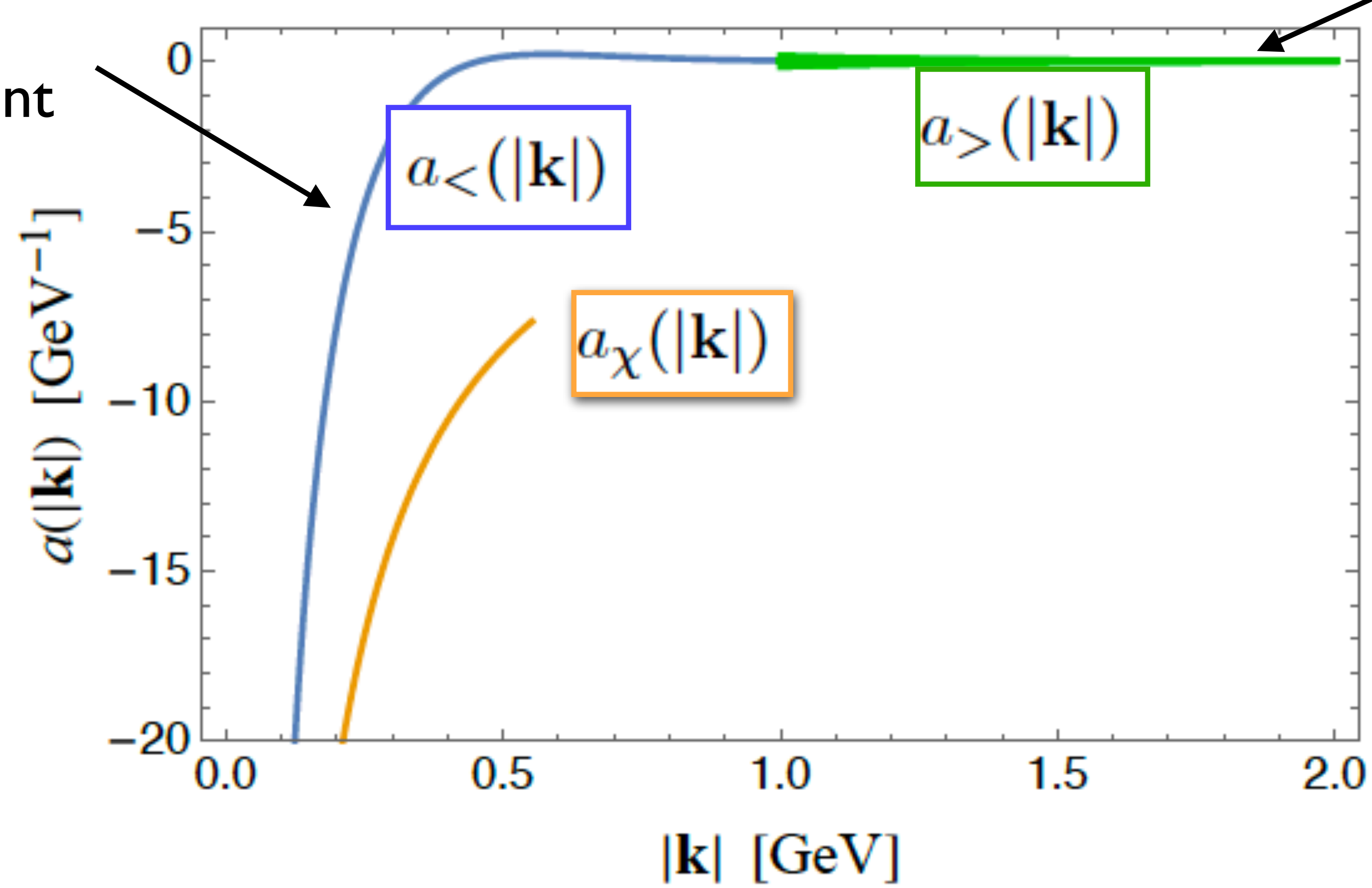
Estimating the contact term

- Integral representation of the amplitude

$$\mathcal{A}_\nu \propto \int_0^\Lambda d|\mathbf{k}|\ a_<(|\mathbf{k}|) + \int_\Lambda^\infty d|\mathbf{k}|\ a_>(|\mathbf{k}|)$$

Steep falloff controlled by the $^1S_0$ effective range: model-independent

Uncertainty due to unknown local operator matrix element is negligible



Estimating the contact term

- Integral representation of the amplitude

$$\mathcal{A}_\nu \propto \int_0^\Lambda d|\mathbf{k}|\; a_<(|\mathbf{k}|) + \int_\Lambda^\infty d|\mathbf{k}|\; a_>(|\mathbf{k}|)$$

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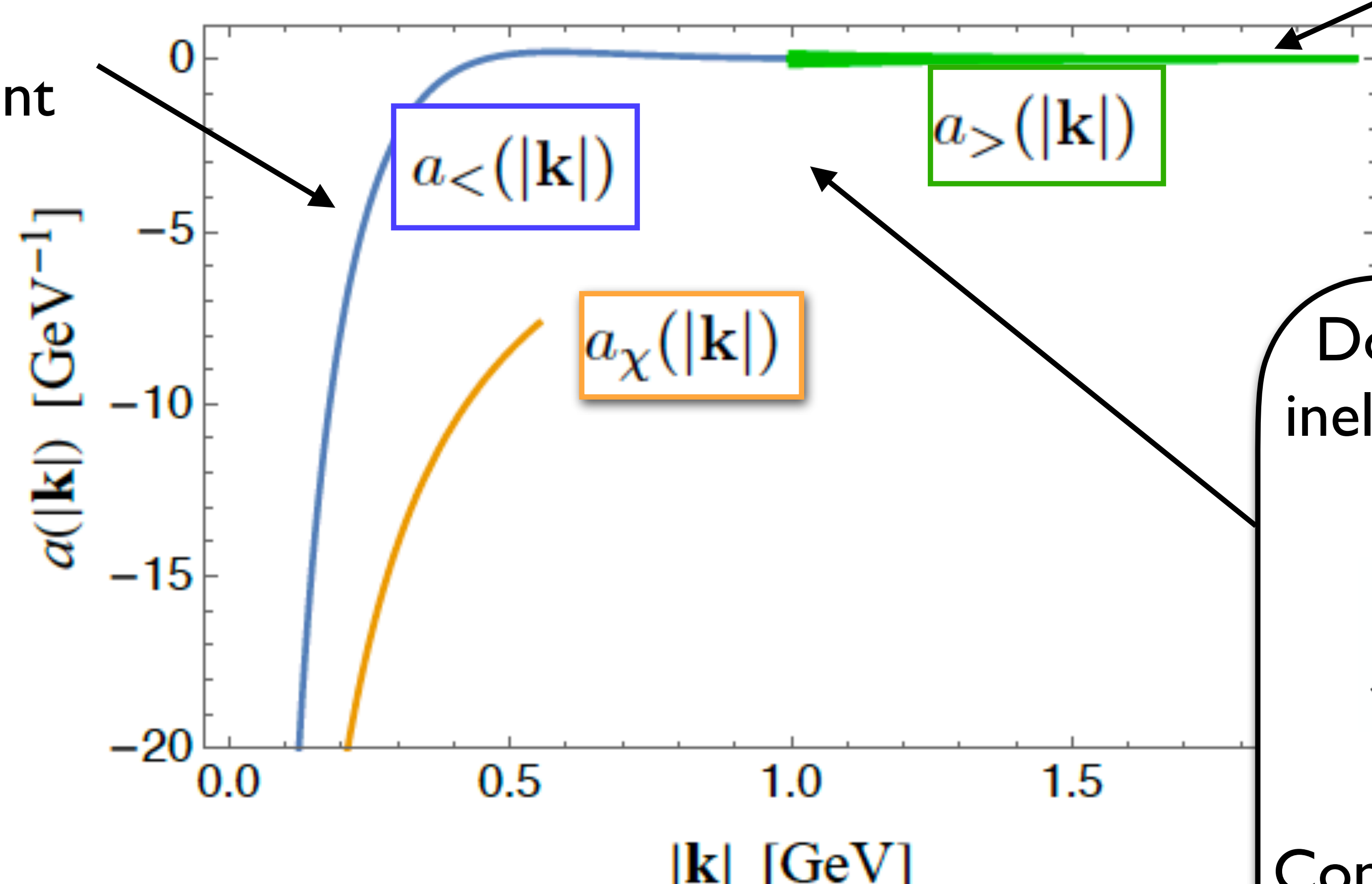
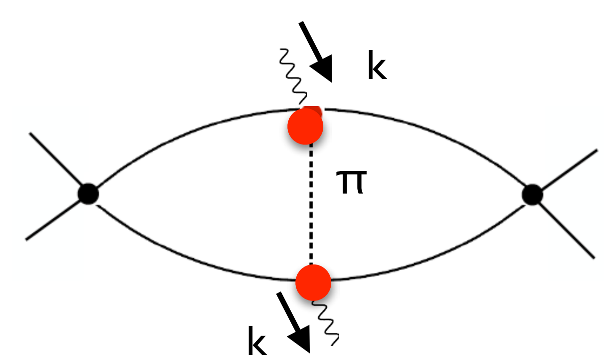


Dominant uncertainty from inelastic channels (NNπ , …):



Consistent with <30% effect in Cottingham approach to π,N EM mass splittings

Results & validation

- LECs in dim. reg. with modified minimal subtraction

$$\tilde{\mathcal{C}}_1(\mu_\chi = M_\pi) = 1.3(6)$$

$$(\tilde{\mathcal{C}}_1 + \tilde{\mathcal{C}}_2)(\mu_\chi = M_\pi) = 2.9(1.2)$$

$$C_{1,2} = \left(\frac{m_N C_{^1S_0}}{4\pi}\right)^2 \tilde{\mathcal{C}}_{1,2}$$

$$g_\nu = C_1$$

- Validation: use C_1+C_2 to predict CIB scattering lengths to LO in χEFT

$$a_{\rm CIB} = \frac{a_{nn} + a_{pp}^C}{2} - a_{np} = 15.5^{+4.5}_{-4.0}\ \text{fm} \quad \text{vs} \quad 10.4(2)\ \text{fm, from data}$$

Fairly good agreement.

Note: $(C_1+C_2)(M_\pi)=0$ → $a_{\rm CIB} \sim 30$ fm: contact term pushes result in the right direction.

Uncertainty estimate is realistic

Connecting to nuclear structure

- Provided 'synthetic data' for the nn→pp amplitude to be used to fit g_ν with regulators suitable for many-body nuclear calculations

$$|\mathbf{p}| = 25\,\mathrm{MeV} \qquad |\mathbf{p}'| = 30\,\mathrm{MeV}$$

$$\mathcal{A}_\nu(|\mathbf{p}|, |\mathbf{p}'|)e^{-i(\delta_{^1S_0}(|\mathbf{p}|)+\delta_{^1S_0}(|\mathbf{p}'|))} = -0.0195(5)\,\mathrm{MeV}^{-2}$$

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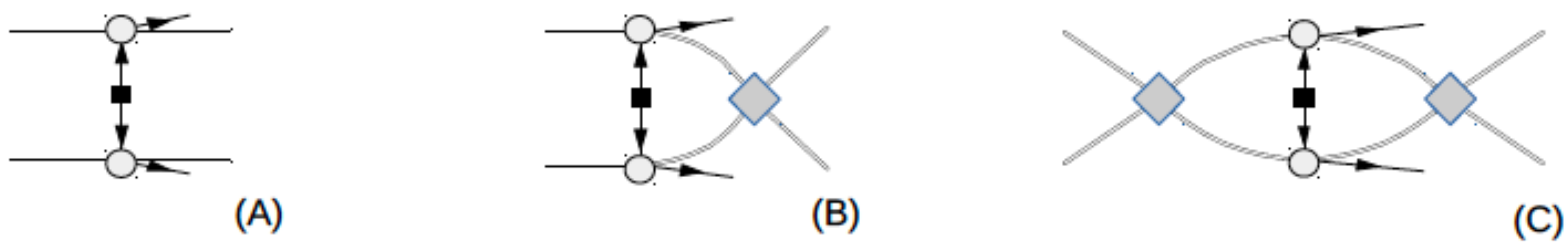
Why the small uncertainty?



Uncertainty dominated by topology C (which comes with fractional error of ~30-40%), but A and B give large contribution to the amplitude at this kinematic point

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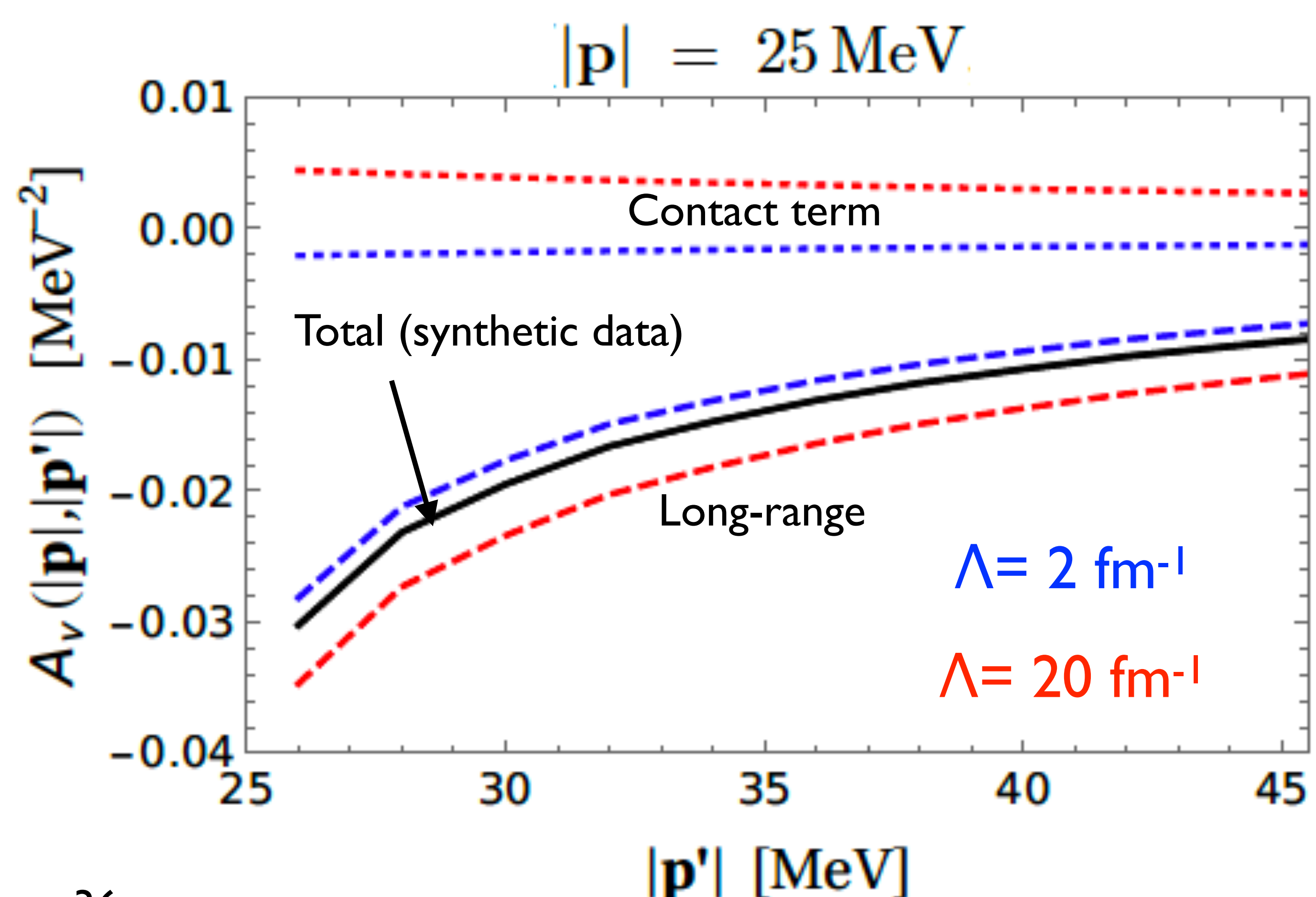
- Illustrated fitting procedure with various cutoffs

- Constructive or destructive?
  The sign of the interference is regulator dependent!



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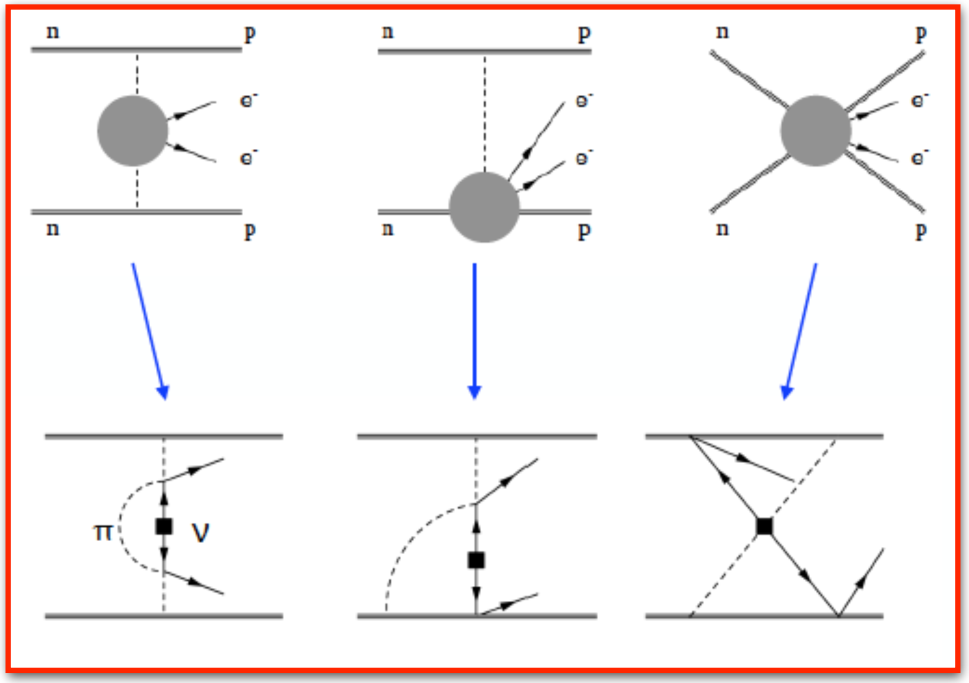
Wirth, Yao, Hergert, 2105.05415

- First calculation of $^{48}Ca \to {}^{48}Ti$ with contact fitted to synthetic datum
- Use Entem-Machleidt class of chiral potentials
- Contact term *enhances* nuclear matrix element by (43±7)%

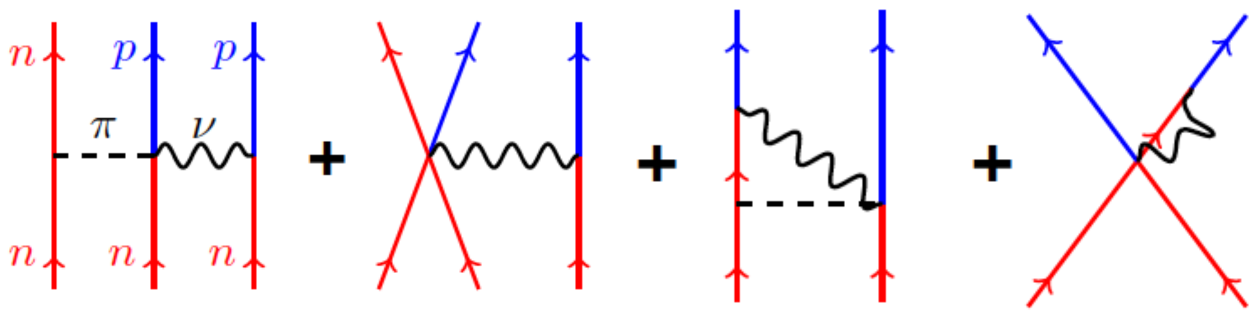
What about higher orders?

- New non-factorizable contributions to $V_{\nu,2} \sim V_{\nu,0}\,(k_F/4\pi F_\pi)^2$ [π-N loops and new contact terms]



VC, W. Dekens, E. Mereghetti, A. Walker-Loud, 1710.01729

- 2-body **x** 1-body current (and another contact…)



Wang-Engel-Yao 1805.10276

Calculations in light and heavy nuclei indicate O(10%) corrections

S. Pastore, J. Carlson, V.C., W. Dekens, E. Mereghetti, R. Wiringa 1710.05026
V.C., J. Engel, X. Menendez, E. Mereghetti, in preparation

What about higher orders?

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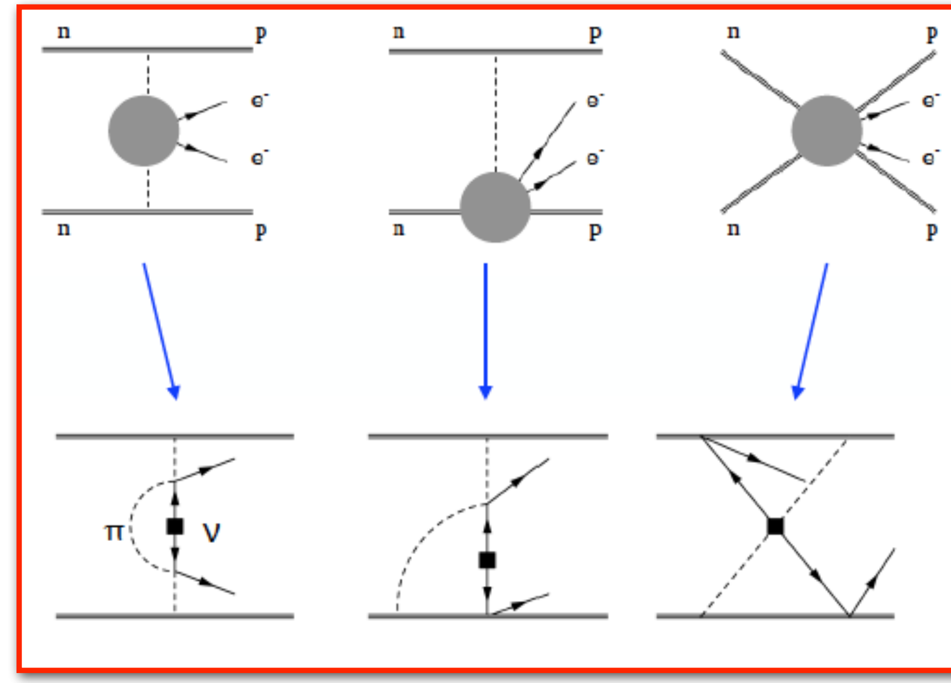

Chiral EFT + estimate of contact term + many-body →

Significant step towards reduction of matrix element uncertainty & robust interpretation of a positive or null result in terms of $m_{\beta\beta}$

Wang-Engel-Yao 1805.10276

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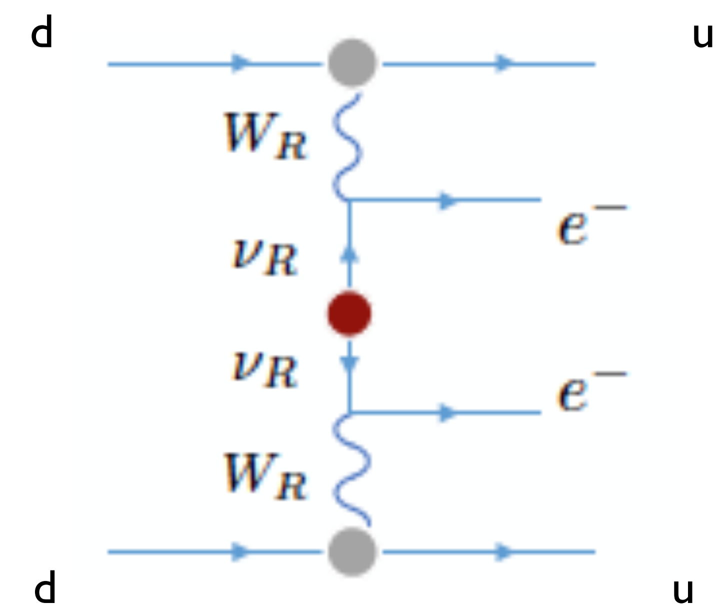
S. Pastore, J. Carlson, V.C., W. Dekens, E. Mereghetti, R. Wiringa 1710.05026
V.C., J. Engel, X. Menendez, E. Mereghetti, in preparation

LNV from multi-TeV scale physics



- Induce contributions to 0νββ *not directly related to the exchange of light neutrinos,* within reach of current experiments

- May lead to correlated (or precursor!) signal at LHC: pp → ee jj

- EFT framework: dim-7 and 9 in the SMEFT, leads to dim-6 and 9 operators at the GeV scale. Several unknown leading-order NN contact interactions*. All needed to probe physics beyond high scale seesaw and assess the complementarity to collider searches

* As in the case of dim-5 operator, opportunity for LQCD

V. Cirigliano, W. Dekens, J. de Vries, M. Graesser, E. Mereghetti, JHEP 1812 (2018) 097 [1806.02780]

$0\nu\beta\beta$ outlook

- 'End-to-end' EFT framework for $0\nu\beta\beta$ needed to interpret positive or null signal in next-generation experiments: (1) connect to underlying sources of LNV; (2) organize contributions to nuclear matrix elements according to chiral power counting: controllable errors

- Identified new leading order NN contact couplings in light ν exchange (discussed today) and TeV-scale mechanisms (dim-9 ops)

  - Estimated 'dim-5' contact, used in first many-body analysis, leads to (43±7)% increase in the amplitude in $^{48}Ca \rightarrow {}^{48}Ti$

  - Lattice QCD will ultimately provide more precise determination

- Good prospects to control theory uncertainties thanks to synergy of EFT, lattice QCD, and nuclear structure

Conclusions

- Beyond neutrinoless $\beta\beta$ decay, the pproach and techniques discussed in these lectures apply to other probes of new physics such as
    - Permanent electric dipole moments of hadrons and nuclei
    - Precision beta decay studies
    - …
- Common theme: construction of low energy realization of BSM operators order by order in chiral EFT, consistently with construction of strong-interaction potentials

Special thanks to collaborators:
W. Dekens, J. de Vries, M. Graesser, E. Mereghetti, S. Pastore, M. Piarulli, U. van Kolck, A. Walker-Loud, R. Wiringa

DBD Nuclear Theory Topical Collaboration (PI Jon Engel):
http://c51.lbl.gov/~0nubb/webhome/

Problems

1. Derive the SMEFT Lagrangian at dimension 5

   Hint: using the table given in the backup slides, construct the lowest dimensional invariants under SU(3)xSU(2)xU(1)

2. Derive the long-range potential mediating nn→pp to leading order in chiral EFT

   Hint: start from the weak current vertices mediating neutron decay and contract the neutrino propagator.

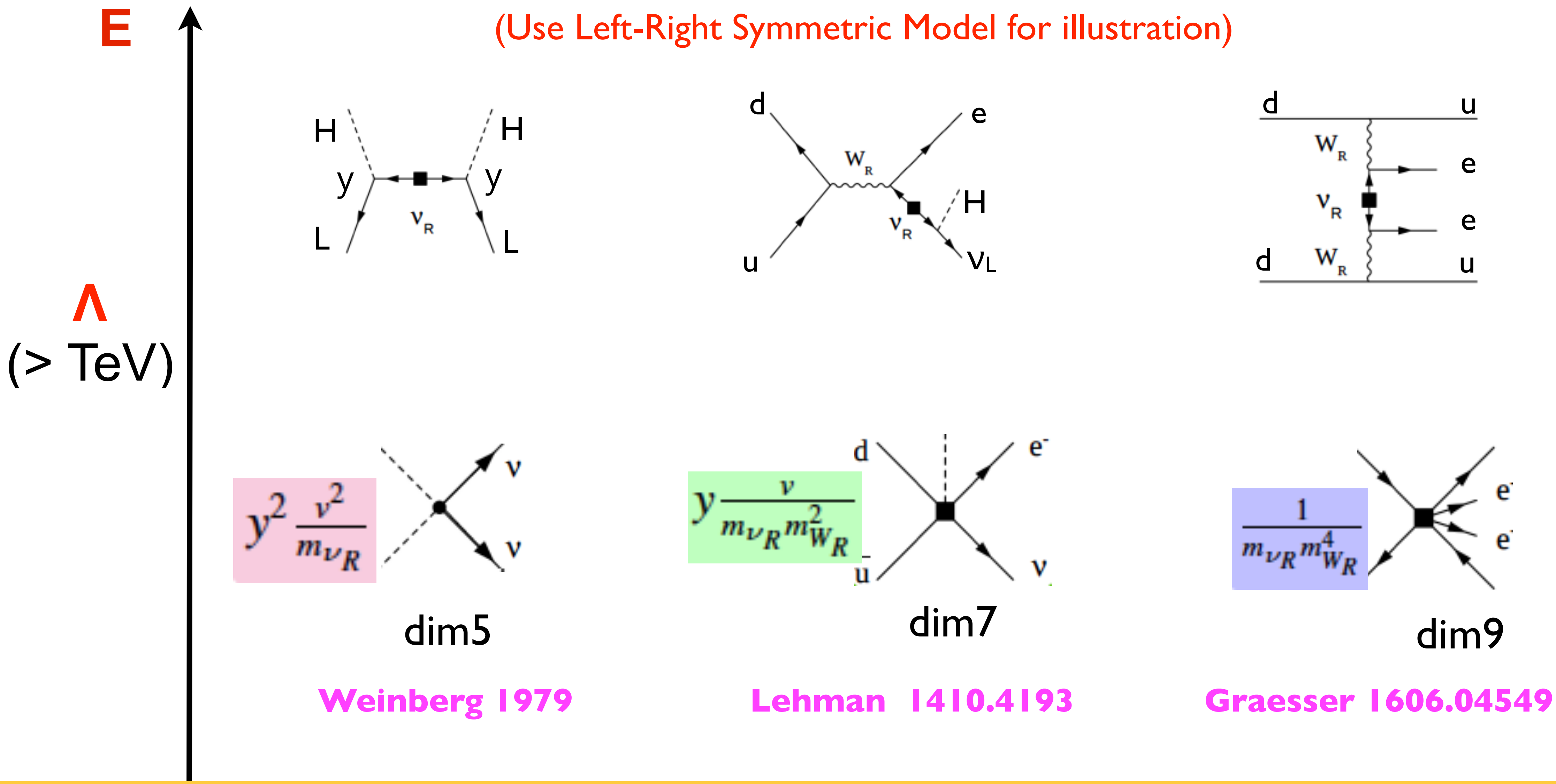
3. Convince yourself of the arguments about the scaling of contact term induced by light Majorana neutrino exchange

   Hint: power-count the various diagrams. Identify any possible UV divergence.

Bonus slides

LNV @ dimension 7,9, …

Why bother with higher dimension?



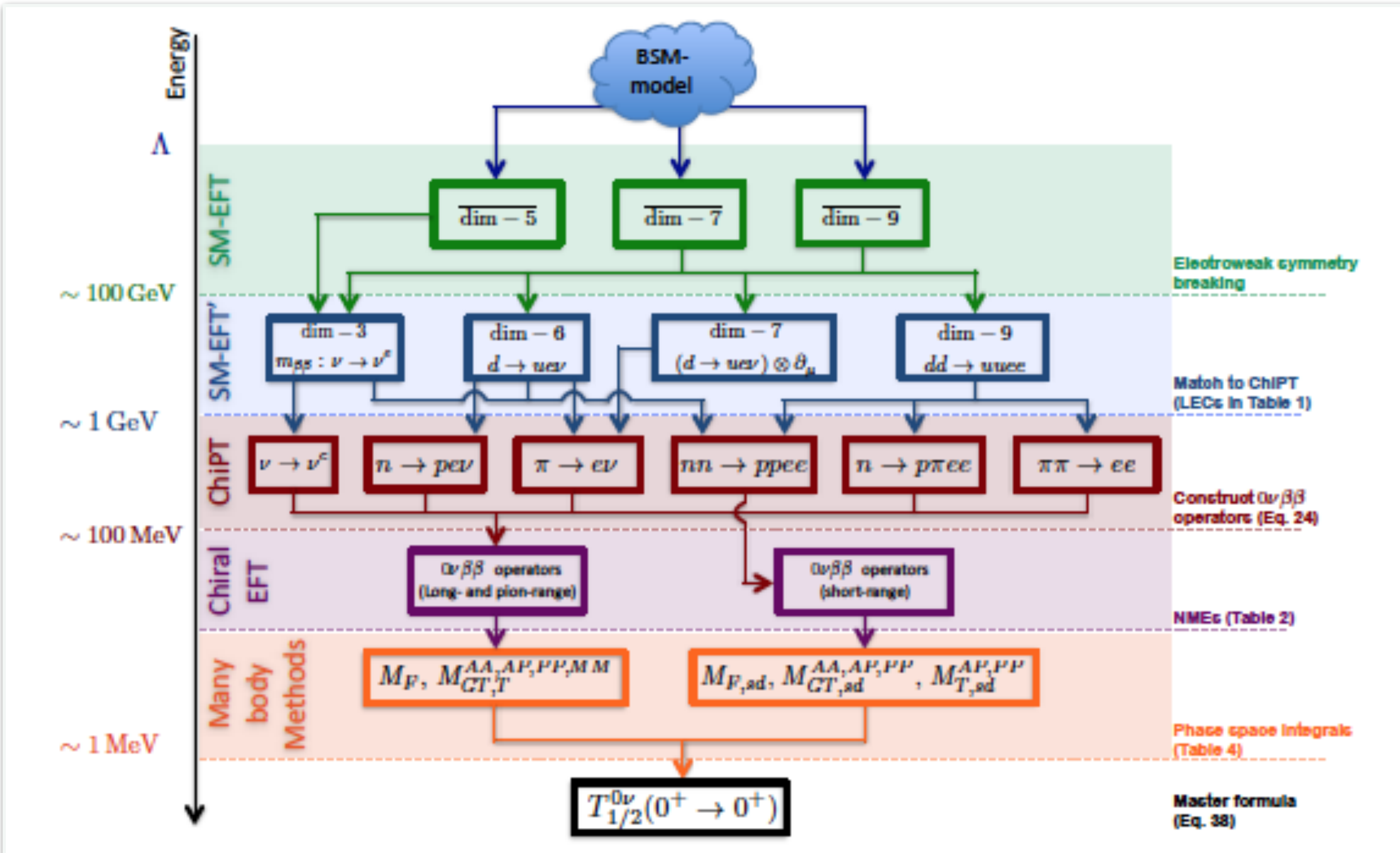
- For $\Lambda \sim$TeV (small Yukawas) dim=5,7,9 operators can be equally important, e.g.

  $m_{\beta\beta} G_F^2/Q^2 \sim 1/\Lambda^5$

- Give contributions to $0\nu\beta\beta$ amplitude ***not proportional to*** $m_{\beta\beta}$
- Can lead to observable effects at LHC (pp → eejj)

EFT-based master formula

- Framework to interpret experiments in terms of any UV source of LNV

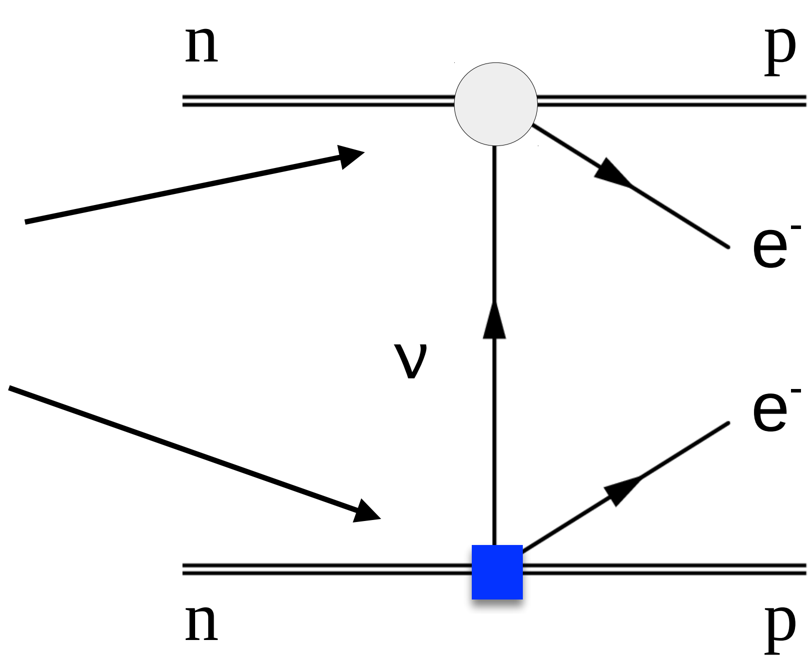


V. Cirigliano, W. Dekens, J. de Vries, M. Graesser, E. Mereghetti, JHEP 1812 (2018) 097 [1806.02780]

$0\nu\beta\beta$ from $\mathcal{L}^{(6)}_{\Delta L=2}$

SM weak charged current

LNV vertex from dim-7 SMEFT operators



V. Cirigliano, W. Dekens, J. de Vries, M. Graesser, E. Mereghetti, 1708.09390

Doi, Kotani, Takasugi 1985

Pas, Hirsch, Klapdor-Kleingrothaus, Kovalenko 1999

- Long range neutrino exchange without mass insertion
- Hadronic input in good shape: isovector nucleon charges V, A, S, P, T
- Nuclear m.e.: same as needed for light ν_M exchange

Horoi and Neacsu, 1706.05391 and refs therein

$0\nu\beta\beta$ from $\mathcal{L}^{(9)}_{\Delta L=2}$

- Example: scalar operators

VC, W. Dekens, M. Graesser, E. Mereghetti, J. de Vries 1806.02780

$$\mathcal{O}_1 = \bar{u}_L \gamma^\mu d_L \, \bar{u}_L \gamma_\mu d_L$$

$$\mathcal{O}_2 = \bar{u}_L d_R \, \bar{u}_L d_R, \qquad \mathcal{O}_3 = \bar{u}_L^\alpha d_R^\beta \, \bar{u}_L^\beta d_R^\alpha$$

$$\mathcal{O}_4 = \bar{u}_L \gamma^\mu d_L \, \bar{u}_R \gamma_\mu d_R, \qquad \mathcal{O}_5 = \bar{u}_L^\alpha \gamma^\mu d_L^\beta \, \bar{u}_R^\beta \gamma_\mu d_R^\alpha$$

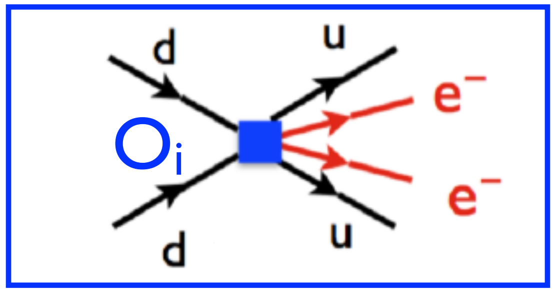
- Hadronic realization depends on O_i's chiral properties

$$\mathcal{L}_{NN} = \left(g_1^{NN} C_{1L}^{(9)} + g_2^{NN} C_{2L}^{(9)} + g_3^{NN} C_{3L}^{(9)} + g_4^{NN} C_{4L}^{(9)} + g_5^{NN} C_{5L}^{(9)} \right) (\bar{p}n)\, (\bar{p}n)\, \frac{\bar{e}_L C \bar{e}_L^T}{v^5}$$

$$\mathcal{L}_\pi = \frac{F_0^2}{2} \left[\frac{5}{3} g_1^{\pi\pi} C_{1L}^{(9)} \, \partial_\mu \pi^- \partial^\mu \pi^- + \left(g_4^{\pi\pi} C_{4L}^{(9)} + g_5^{\pi\pi} C_{5L}^{(9)} - g_2^{\pi\pi} C_{2L}^{(9)} - g_3^{\pi\pi} C_{3L}^{(9)} \right) \pi^- \pi^- \right] \times \frac{\bar{e}_L C \bar{e}_L^T}{v^5} + (L \leftrightarrow R) + \dots$$

$$g_1^{\pi\pi} \sim \mathcal{O}(1), \qquad g_{2,3,4,5}^{\pi\pi} \sim \mathcal{O}(\Lambda_\chi^2) \qquad\qquad g_1^{NN} \sim \mathcal{O}(1), \qquad g_{2,3,4,5}^{NN} \sim \mathcal{O}\left(\frac{\Lambda_\chi^2}{F_\pi^2} \right)$$

Hadronic realization of dim-9 operators



Scalar operators (arising in most models)

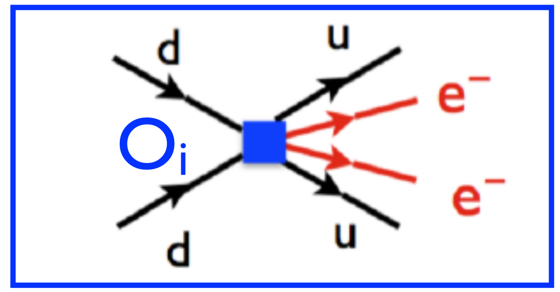
$$\mathcal{O}_1 = \bar{u}_L \gamma^\mu d_L \, \bar{u}_L \, \gamma_\mu d_L$$

$$\mathcal{O}_2 = \bar{u}_L d_R \, \bar{u}_L \, d_R, \qquad \mathcal{O}_3 = \bar{u}_L^\alpha d_R^\beta \, \bar{u}_L^\beta \, d_R^\alpha$$

$$\mathcal{O}_4 = \bar{u}_L \gamma^\mu d_L \, \bar{u}_R \, \gamma_\mu d_R, \qquad \mathcal{O}_5 = \bar{u}_L^\alpha \gamma^\mu d_L^\beta \, \bar{u}_R^\beta \, \gamma_\mu d_R^\alpha$$

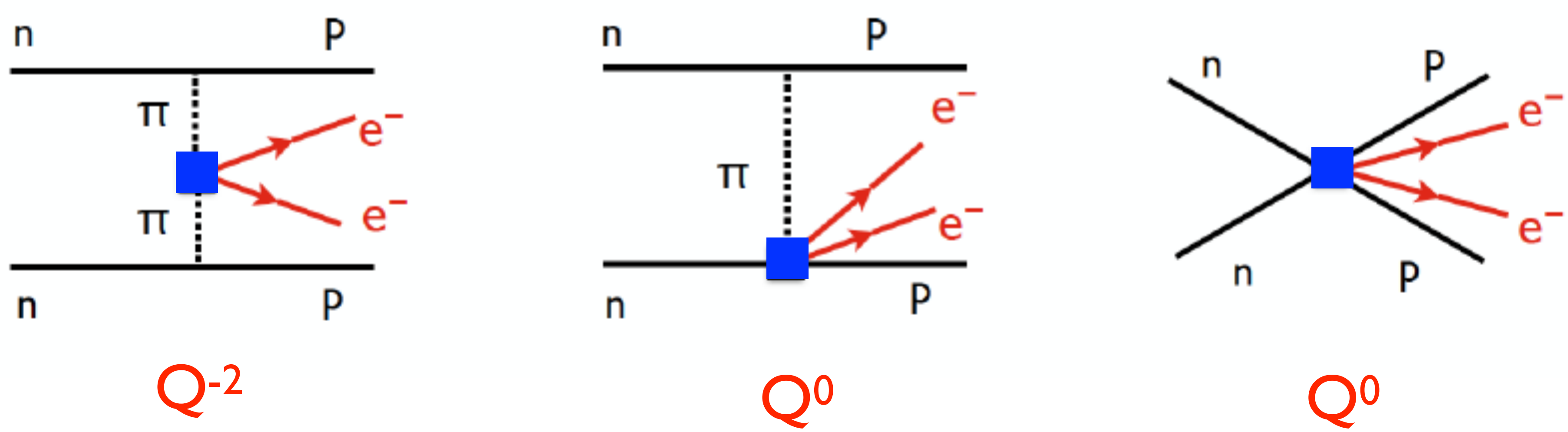
Prezeau, Ramsey-Musolf, Vogel hep-ph/0303205
M. Graesser 1606.04549

Hadronic realization of dim-9 operators



Pion-range effects

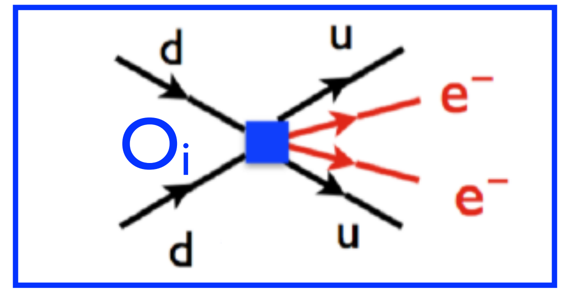
Short-range effects



Naive dimensional analysis → $V_{\pi\pi}$ dominates (except for O_1)

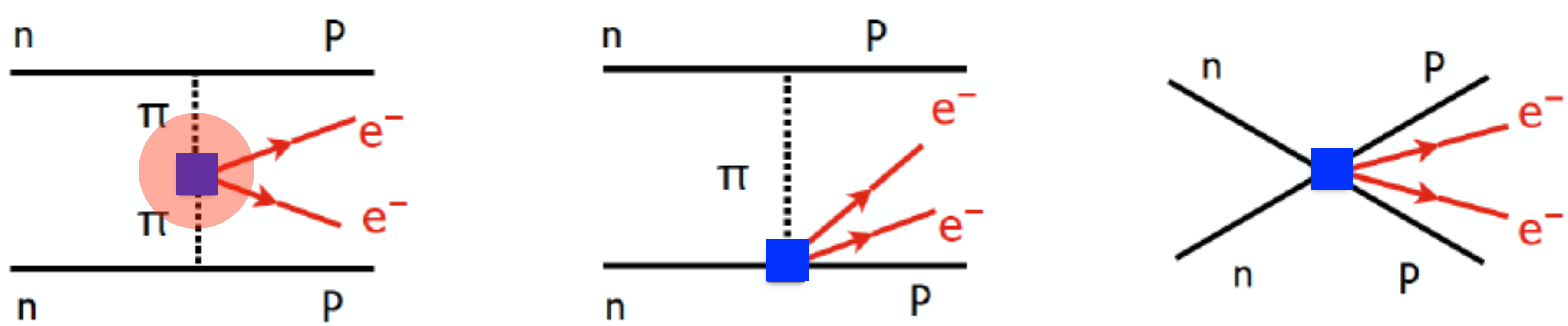
Vergados 1982, Faessler, Kovalenko, Simkovic, Schweiger 1996
Prezeau, Ramsey-Musolf, Vogel hep-ph/0303205

Hadronic realization of dim-9 operators

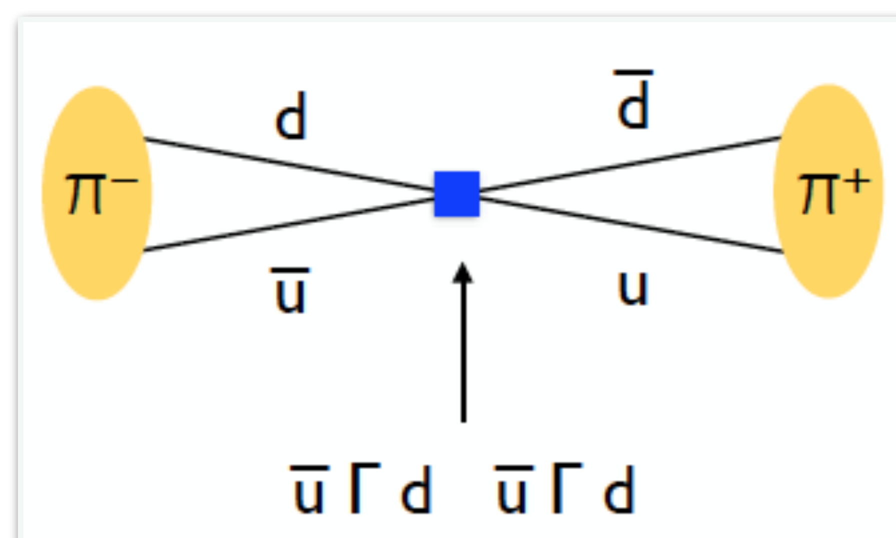


Pion-range effects

Short-range effects

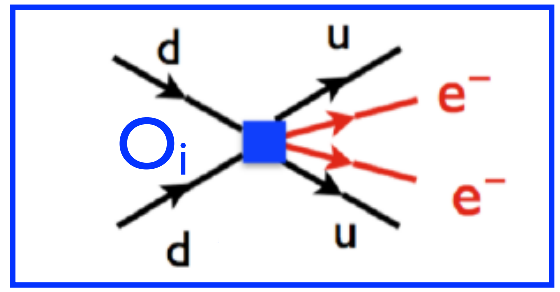


- Two recent developments:
    1. ππ matrix elements now precisely calculated in lattice QCD



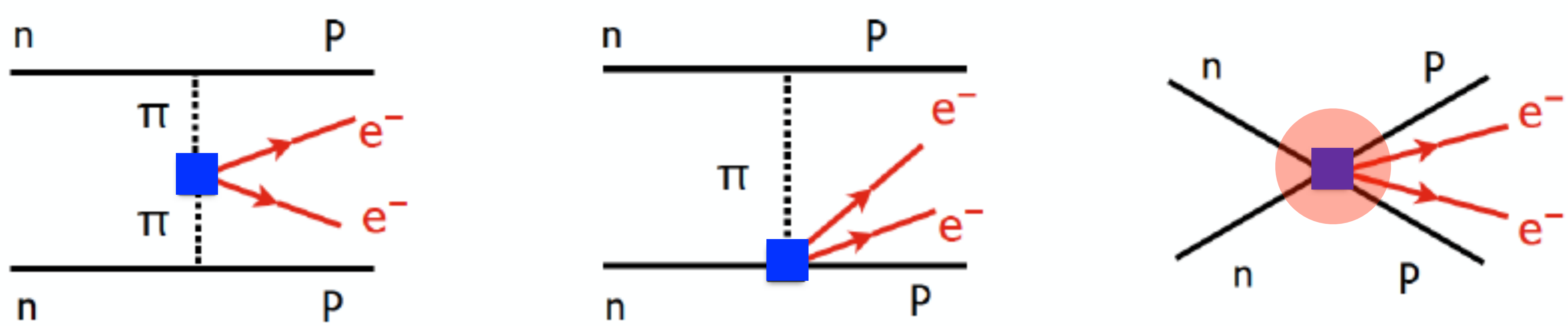
Nicholson et al (CalLat), 1805.02634, PRL

Hadronic realization of dim-9 operators



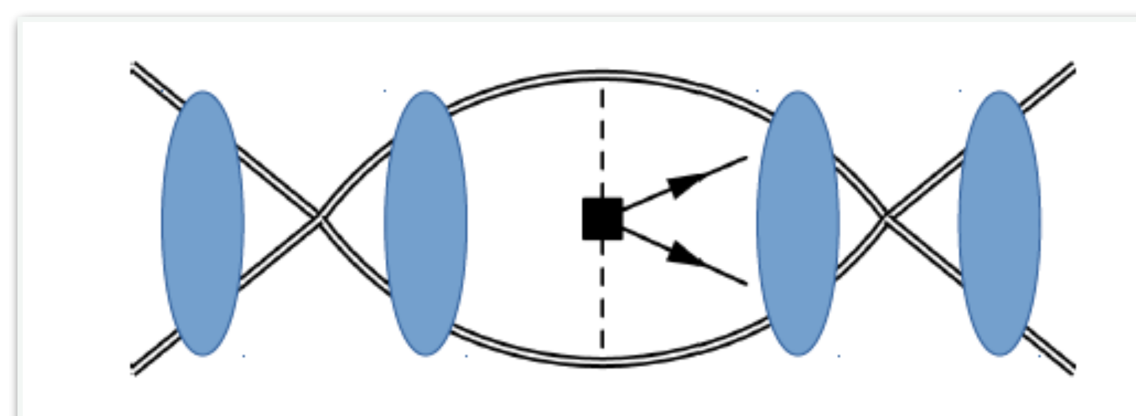
Pion-range effects

Short-range effects



- Two recent developments:

  2. Renormalization → $V_{\pi\pi}$ and V_{NN} are both leading order



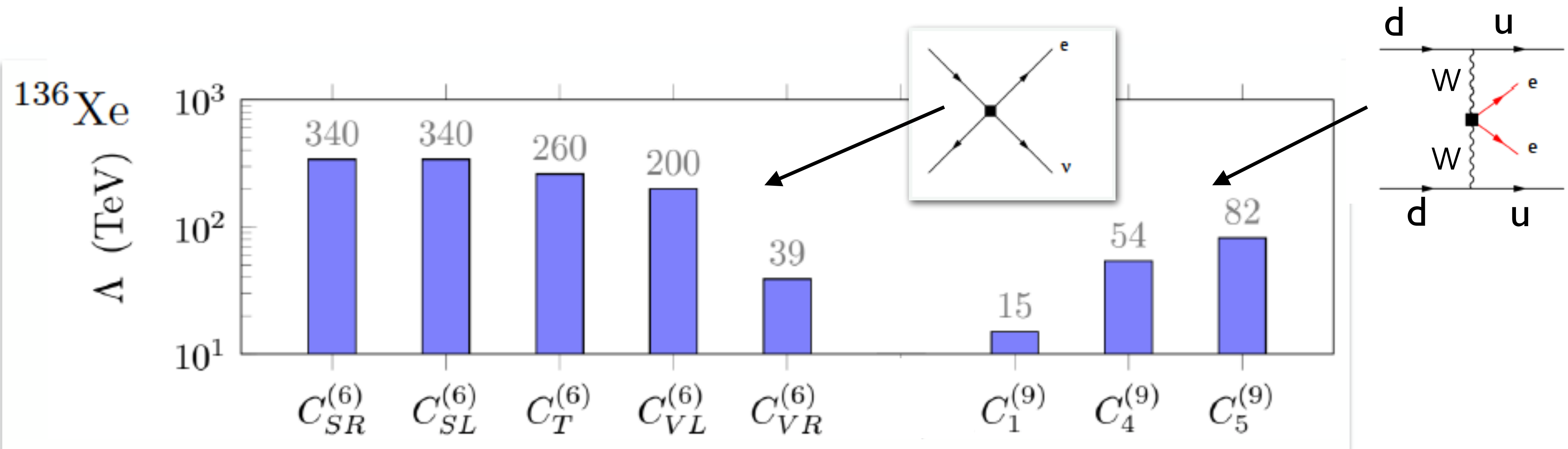
V.C, W. Dekens, J. de Vries, M. Graesser, E. Mereghetti [1806.02780]

What scales are we probing?

VC, W. Dekens, J. de Vries, M. Graesser, E. Mereghetti, 1806.02780

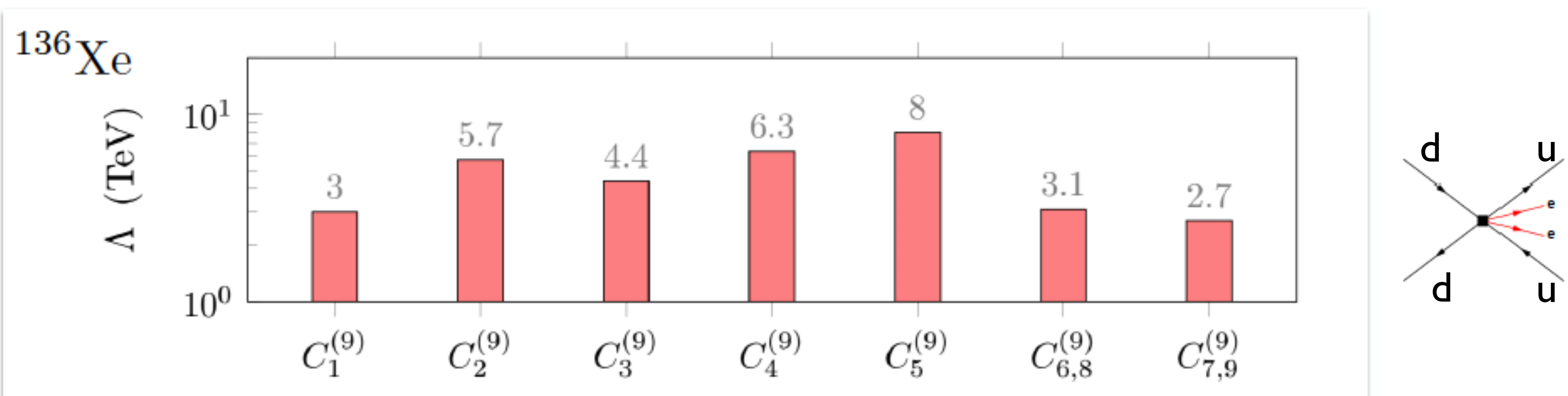
Dim 7 in SM-EFT

$(v/\Lambda)^3$



Dim 9 in SM-EFT

$(v/\Lambda)^5$



Bounds reflect dependence on Λ_X / Λ and Q/Λ_X

Backup

Building SMEFT operators

- Gauge group: $SU(3)_c \times SU(2)_W \times U(1)_Y$
- Building blocks

	$SU(3)_c \times SU(2)_W \times U(1)_Y$ representation: (dim[$SU(3)_c$], dim[$SU(2)_W$], Y)	$SU(2)_W$ transformation
$l = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	(1,2,-1/2)	$l \to V_{SU(2)}\, l$
$e = e_R$	(1,1,-1)	
$q^i = \begin{pmatrix} u^i_L \\ d^i_L \end{pmatrix}$	(3,2,1/6)	$q \to V_{SU(2)}\, q$
$u^i = u^i_R$	(3,1,2/3)	
$d^i = d^i_R$	(3,1,-1/3)	
$H = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$	(1,2,1/2)	$H \to V_{SU(2)} H$

$Q = T_3 + Y$

		$SU(3)_c \times SU(2)_W \times U(1)_Y$ representation
gluons:	$G^A_\mu, \quad A = 1 \cdots 8,$ $G^A_{\mu\nu} = \partial_\mu G^A_\nu - \partial_\nu G^A_\mu + g_s f_{ABC} G^B_\mu G^C_\nu$	(8,1,0)
W bosons:	$W^I_\mu, \quad I = 1 \cdots 3,$ $W^I_{\mu\nu} = \partial_\mu W^I_\nu - \partial_\nu W^I_\mu + g \epsilon_{IJK} W^J_\mu W^K_\nu$	(1,3,0)
B boson:	$B_\mu,$ $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu .$	(1,1,0)

Gauge transformation:

$$W^I_{\mu\nu} \frac{\sigma^I}{2} \longrightarrow V(x) \left[W^I_{\mu\nu} \frac{\sigma^I}{2} \right] V^\dagger(x)$$

$$V(x) = e^{ig\beta_a(x)\frac{\sigma_a}{2}}$$

- Write down all operators of a given dimension that respect gauge and Lorentz invariance

Diagnosing power

- High scale seesaw implies falsifiable correlation with other ν mass probes. Future data can unravel new LNV sources or physics beyond "ΛCDM + m_ν"

$$m_{\beta\beta} = \left| \sum_i U_{ei}^2 \, m_i \right|$$

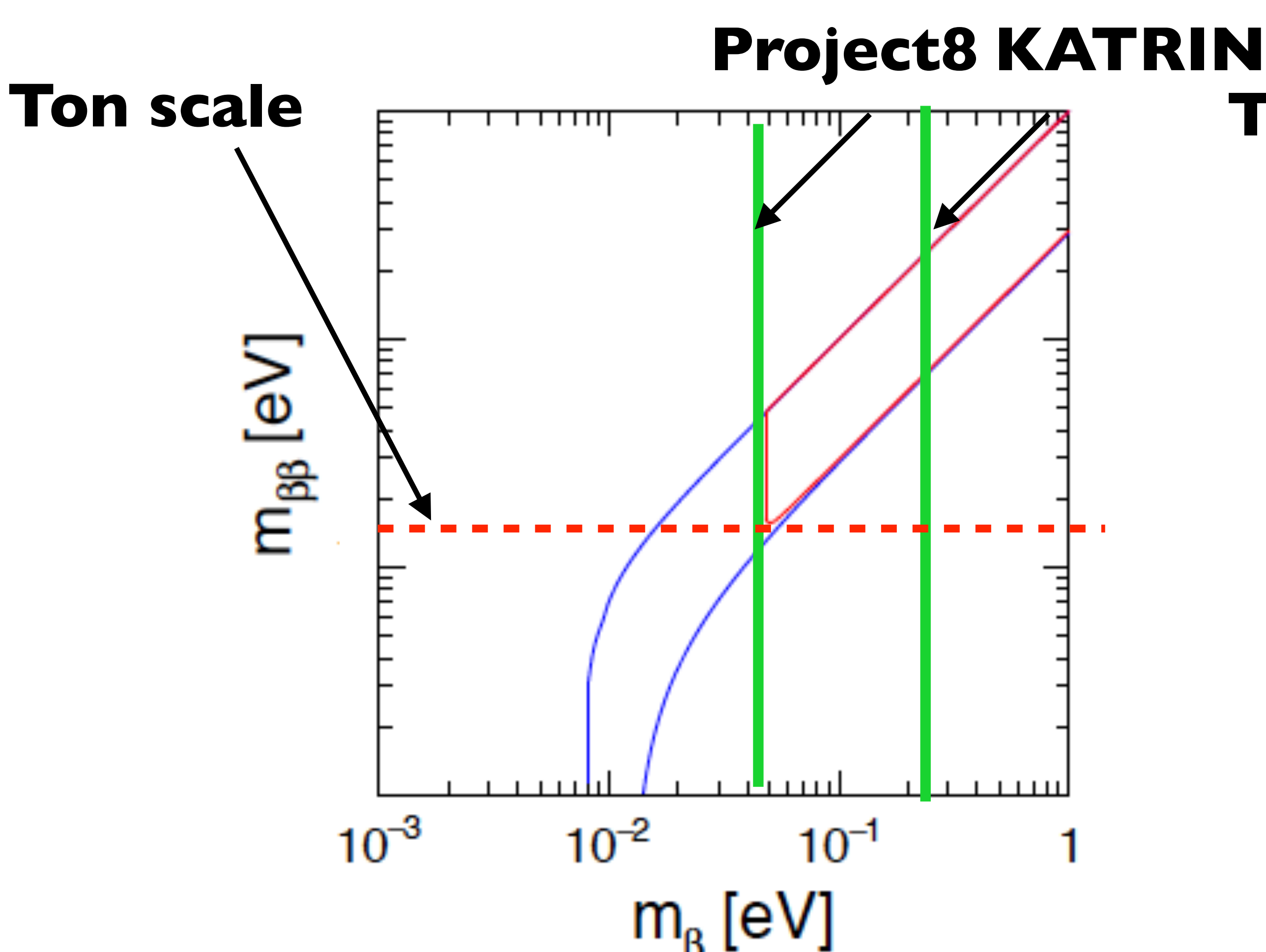
0νββ decay

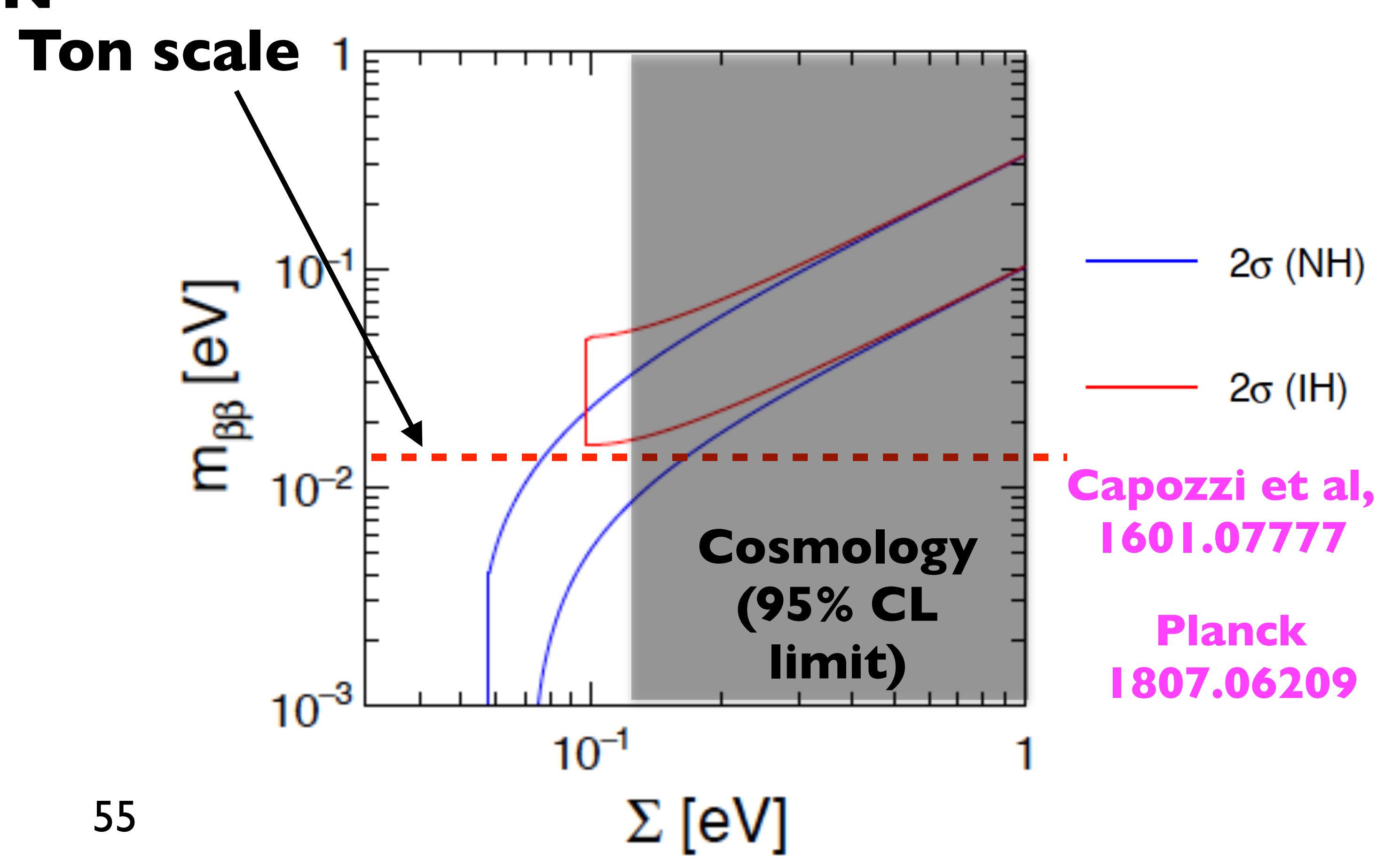
$$m_\beta = \sqrt{\sum_i |U_{ei}|^2 m_i^2}$$

Tritium β decay

$$\Sigma = \sum_i m_i$$

Cosmology





Using different regulators / schemes

- Same conclusion obtained by solving the Schroedinger equation
  - Use smeared delta function to regulate short range strong potential:
    $\tilde{C} \to \tilde{C}\,(R_S)$, fit to a_{NN}

$$\delta^{(3)}(\mathbf{r}) \to \frac{1}{\pi^{3/2} R_S^3} e^{-\frac{r^2}{R_S^2}}$$

  - Compute amplitude

$$\mathcal{A}_\nu = \int d^3\mathbf{r}\ \psi^-_{\mathbf{p}'}(\mathbf{r})\, V_{\nu,0}(\mathbf{r})\, \psi^+_{\mathbf{p}}(\mathbf{r})$$

Scattering states "fully correlated" according to the leading order strong potential in the $^1S_0$ channel

Using different regulators / schemes

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  - Compute amplitude

$$\mathcal{A}_\nu = \int d^3\mathbf{r}\ \psi^-_{\mathbf{p}'}(\mathbf{r})\, V_{\nu,0}(\mathbf{r})\, \psi^+_{\mathbf{p}}(\mathbf{r})$$

- Logarithmic dependence on R_S ⇒

  need LO counterterm $g_\nu(R_S) \sim 1/F_\pi^2 \log R_S$ to obtain physical, regulator-independent result