EFT for dark matter

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July 27/29, 2021

Methods of Effective Field Theory and Lattice Field Theory

Bad Honnef Physics School

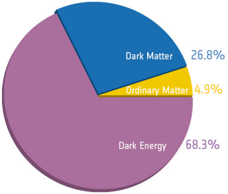
Outline

1. **Big picture**
   - Search strategies for dark matter and status
   - Basics of EFT approach: from SMEFT to nuclear physics see also lectures by V. Cirigliano
2. **From the TeV to the GeV scale**
   - BSM operator basis
   - Electroweak running
3. **Nuclear EFTs** see also lectures by V. Cirigliano
   - Chiral perturbation theory and its pitfalls: hadronic matrix elements, two-body currents
   - Non-relativistic EFT
4. **Applications**
   - Dark-matter direct detection
   - Coherent elastic neutrino–nucleus scattering (CEνNS)

Disclaimers

- These lectures are not a review, but a personal selection
- Referencing will be incomplete, no complete bibliography intended or implied
- **Please do not hesitate to stop me at any point to ask questions!**

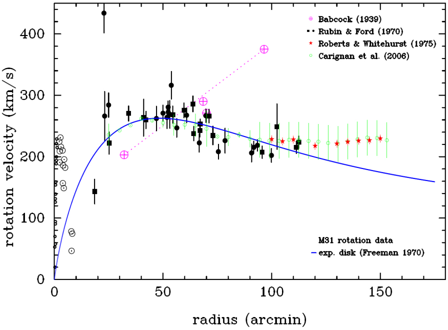
Evidence for dark matter I



Planck 2013

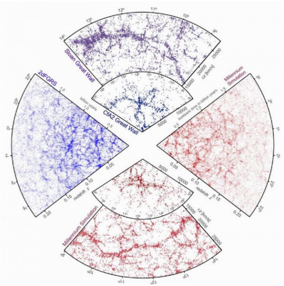
- In hindsight: early evidence from **rotation curves** of galaxies
- Expect $1/\sqrt{R}$ behavior

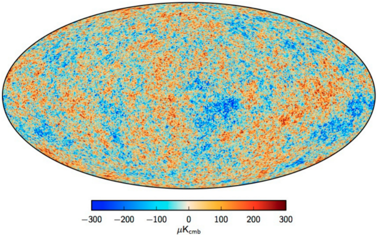
$$v = \sqrt{\frac{GM(R)}{R}}$$



Rotation curve of Andromeda, Bertone, Hooper 2016

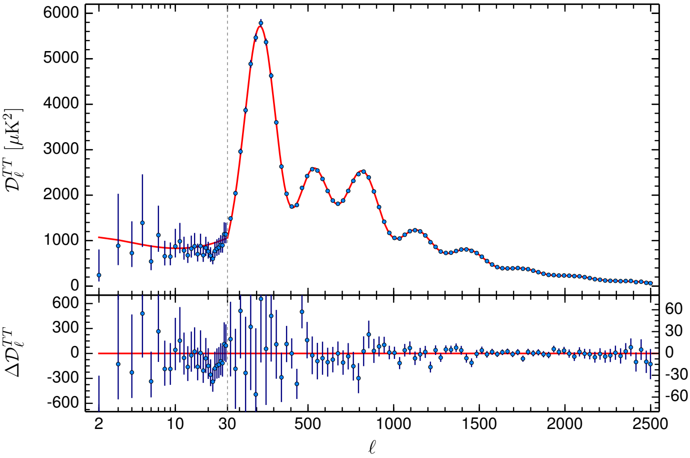
Evidence for dark matter II





- **Large scale structure**: *N*-body simulations only match galaxy surveys with a sufficient amount of cold dark matter 1980s
- **Cosmic microwave background**: temperature variations $\Delta T/T \sim 10^{-5}$ too small for a purely baryonic universe 1990s
- **Microlensing searches**: dark matter cannot be in form of compact objects 1990s
- **Baryon content of the Universe**: light element abundances imply $\Omega_b \lesssim 5\%$ 1990s

Evidence for dark matter III

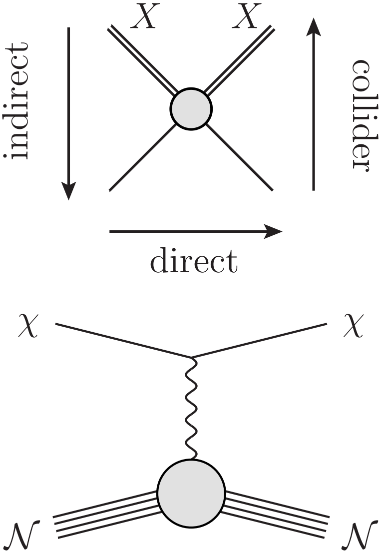


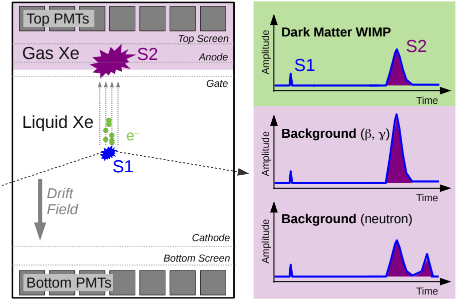
Planck 2015

- **Cosmic microwave background**: third peak of the **power spectrum** extremely difficult to get with alternatives to dark matter

How to search for dark matter?

- Assume dark matter exists and is a **weakly interacting massive particle** (WIMP)
- Search strategies: direct, indirect, collider
- **Direct detection**: search for **WIMPs** scattering off nuclei in the large-scale detectors
- Ingredients for interpretation:
  - **Dark matter halo**: velocity distribution
  - **Nucleon matrix elements**: WIMP–nucleon couplings
  - **Nuclear structure factors**: embedding into target nucleus

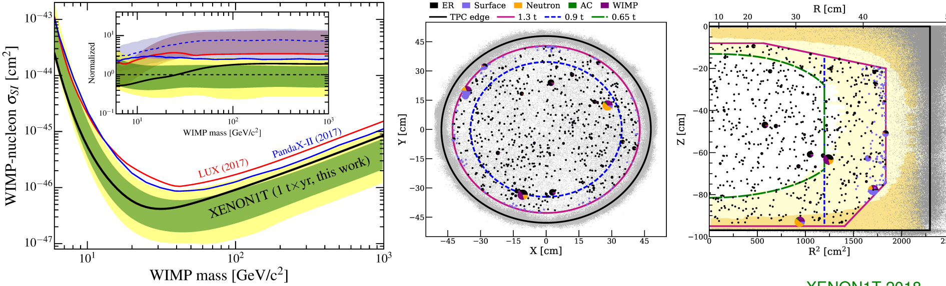




XENON1T, 1708.07051

- **Dual-phase liquid Xe detectors**
  - Primary scintillation signal S1, light observed by PMTs
  - Secondary scintillation signal S2 from drift electrons
- Background sources
  - Electron recoil (β, γ), neutron recoil
  - Surface events (radioactivity), accidental coincidence
  - Coherent elastic neutrino–nucleus scattering (CEνNS)

Direct detection of dark matter: schematics



XENON1T 2018

- Rate for WIMP–nucleus scattering

$$\frac{dR}{dE_r} = \underbrace{\frac{\sigma^{SI}_{\chi N}}{m_\chi \mu_N^2}}_{\text{particle + hadronic physics}} \times \underbrace{\left|\mathcal{F}_+^M(q^2)\right|^2}_{\text{nuclear physics}} \times \underbrace{\rho_0 \int_{v_{\min}}^{v_{\rm esc}} \frac{f(\mathbf{v},t)}{v} d^3v}_{\text{astrophysics}}$$

- **Decomposition into the three terms follows from EFT**

Direct detection of dark matter: astrophysical uncertainties

- Master formula for the rate

$$\frac{\mathrm{d}R}{\mathrm{d}E_\mathrm{r}} = \frac{\rho_0 \sigma^{\mathrm{SI}}_{\chi N}}{m_\chi \mu_N^2} \times |\mathcal{F}_+^M(q^2)|^2 \times \int_{v_\mathrm{min}}^{v_\mathrm{esc}} \frac{f(\mathbf{v}, t)}{v} \mathrm{d}^3 v$$

- **Standard halo model**
  - Local dark matter density: $\rho_0 = 0.3\,\mathrm{GeV}\,\mathrm{cm}^{-3}$
  - Boltzmann distribution: $f(\mathbf{v}) = f_0 e^{-\mathbf{v}^2/v_0^2}$

but large (unquantified) uncertainties

Direct detection of dark matter: astrophysical uncertainties

- Master formula for the rate

$$\frac{dR}{dE_r} = \frac{\rho_0 \sigma^{SI}_{\chi N}}{m_\chi \mu_N^2} \times |\mathcal{F}_+^M(q^2)|^2 \times \int_{v_{min}}^{v_{esc}} \frac{f(\mathbf{v},t)}{v} d^3 v$$

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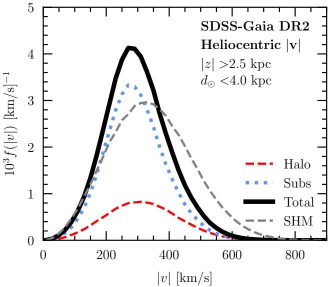
  but large (unquantified) uncertainties

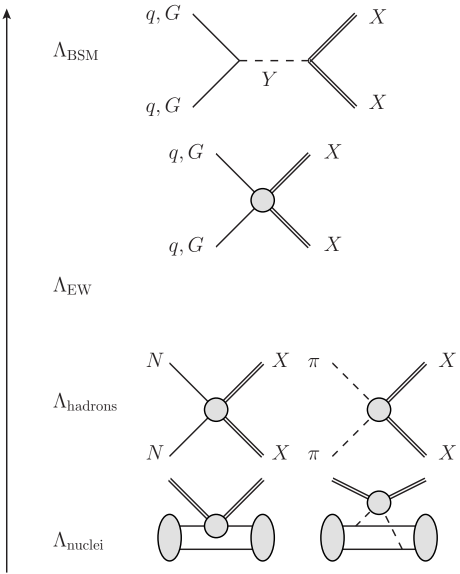
- New developments thanks to **Gaia**
  - Better determination of the density, e.g. $\rho_0 = 0.30(3)\,\text{GeV}\,\text{cm}^{-3}$ Eilers et al. 1810.09466
  - Velocity distribution Necib et al. 1807.02519, Evans et al. 1810.11468

$\hookrightarrow$ astrophysical uncertainties soon to be under much better control than ever before



Necib et al. 2018

Direct detection of dark matter: scales



1. **BSM scale** Λ_{BSM}: $\mathcal{L}_{\text{BSM}}$
2. **Effective Operators**: $\mathcal{L}_{\text{SM}} + \sum_{i,k} \frac{1}{\Lambda^i_{\text{BSM}}} \mathcal{O}_{i,k}$

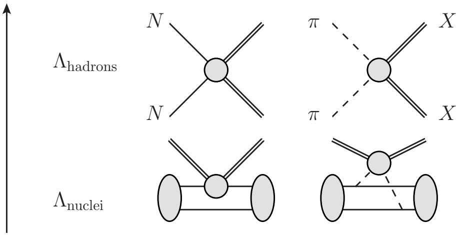
   SMEFT
3. Integrate out **EW physics**
4. **Hadronic scale**: nucleons and pions

   $\hookrightarrow$ effective interaction Hamiltonian H_I

   Chiral EFT
5. **Nuclear scale**: $\langle \mathcal{N} | H_I | \mathcal{N} \rangle$

   $\hookrightarrow$ nuclear wave function Chiral EFT, NREFT

Direct detection of dark matter: scales



4. **Hadronic scale**: nucleons and pions

   ↪ effective interaction Hamiltonian H_I

   Chiral EFT

5. **Nuclear scale**: $\langle \mathcal{N}|H_I|\mathcal{N}\rangle$

   ↪ nuclear wave function Chiral EFT, NREFT

- Typical WIMP–nucleon **momentum transfer**

$$|\mathbf{q}_{\rm max}| = 2\mu_{\mathcal{N}\chi}|\mathbf{v}_{\rm rel}| \sim 200\,\text{MeV} \qquad |\mathbf{v}_{\rm rel}| \sim 10^{-3} \qquad \mu_{\mathcal{N}\chi} \sim 100\,\text{GeV}$$

- **Chiral EFT**: pions, nucleons, and WIMPs as degrees of freedom

  Prézeau et al. 2003, Cirigliano et al. 2012, 2013, Menéndez et al. 2012, Klos et al. 2013, MH et al. 2015, Bishara et al. 2017

- **NREFT**: all degrees of freedom integrated out but nucleons and WIMPs

  Fan et al. 2010, Fitzpatrick et al. 2012, Anand et al. 2013

Operator basis

- The operator basis depends on
  - **WIMP spin**: 0, $1/2$, 1
  - **WIMP charges**: SM singlet?
  - **WIMP mass**: heavy compared to EW scale?
- Start with SM singlet and spin $1/2$ for m_χ not too heavy

  $\hookrightarrow$ can use a **derivative expansion** Goodman et al. 2011
- Possible building blocks
  - Bilinears

$$\bar{q}\Gamma q\,\bar{\chi}\Gamma\chi \qquad \Gamma = \{\mathbb{1}, \gamma_5, \gamma^\mu, \gamma^\mu\gamma_5, \sigma^{\mu\nu}\}$$

  - Terms with (dual) gluon field strength

$$\bar{\chi}\Gamma\chi\, G^a_{\mu\nu} G^{\mu\nu}_a \qquad \tilde{G}^a_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta} G^{\alpha\beta}_a$$

  - Dipole interactions

$$\bar{\chi}\sigma^{\mu\nu}\chi F_{\mu\nu} \qquad \bar{\chi}\sigma^{\mu\nu}\gamma_5\chi F_{\mu\nu}$$

Operator basis

Name	Operator	Coefficient
D1	$\bar{\chi}\chi\,\bar{q}q$	m_q/Λ^3
D2	$\bar{\chi}\gamma_5\chi\,\bar{q}q$	im_q/Λ^3
D3	$\bar{\chi}\chi\,\bar{q}\gamma_5 q$	im_q/Λ^3
D4	$\bar{\chi}\gamma_5\chi\,\bar{q}\gamma_5 q$	m_q/Λ^3
D5	$\bar{\chi}\gamma^\mu\chi\,\bar{q}\gamma_\mu q$	$1/\Lambda^2$
D6	$\bar{\chi}\gamma^\mu\gamma_5\chi\,\bar{q}\gamma_\mu q$	$1/\Lambda^2$
D7	$\bar{\chi}\gamma^\mu\chi\,\bar{q}\gamma_\mu\gamma_5 q$	$1/\Lambda^2$
D8	$\bar{\chi}\gamma^\mu\gamma_5\chi\,\bar{q}\gamma_\mu\gamma_5 q$	$1/\Lambda^2$
D9	$\bar{\chi}\sigma^{\mu\nu}\chi\,\bar{q}\sigma_{\mu\nu} q$	$1/\Lambda^2$
D10	$\bar{\chi}\sigma^{\mu\nu}\gamma_5\chi\,\bar{q}\sigma_{\mu\nu} q$	i/Λ^2
D11	$\bar{\chi}\chi\,G^a_{\mu\nu}G_a^{\mu\nu}$	$\alpha_s/(4\Lambda^3)$
D12	$\bar{\chi}\gamma_5\chi\,G^a_{\mu\nu}G_a^{\mu\nu}$	$i\alpha_s/(4\Lambda^3)$
D13	$\bar{\chi}\chi\,G^a_{\mu\nu}\tilde{G}_a^{\mu\nu}$	$i\alpha_s/(4\Lambda^3)$
D14	$\bar{\chi}\gamma_5\chi\,G^a_{\mu\nu}\tilde{G}_a^{\mu\nu}$	$\alpha_s/(4\Lambda^3)$

Name	Operator	Coefficient
C1	$\chi^\dagger\chi\,\bar{q}q$	m_q/Λ^2
C2	$\chi^\dagger\chi\,\bar{q}\gamma_5 q$	im_q/Λ^2
C3	$\chi^\dagger\partial_\mu\chi\,\bar{q}\gamma^\mu q$	$1/\Lambda^2$
C4	$\chi^\dagger\partial_\mu\chi\,\bar{q}\gamma^\mu\gamma_5 q$	$1/\Lambda^2$
C5	$\chi^\dagger\chi\,G^a_{\mu\nu}G_a^{\mu\nu}$	$\alpha_s/4\Lambda^2$
C6	$\chi^\dagger\chi\,G^a_{\mu\nu}\tilde{G}_a^{\mu\nu}$	$i\alpha_s/4\Lambda^2$
R1	$\chi^2\,\bar{q}q$	$m_q/2\Lambda^2$
R2	$\chi^2\,\bar{q}\gamma_5 q$	$im_q/2\Lambda^2$
R3	$\chi^2\,G^a_{\mu\nu}G_a^{\mu\nu}$	$\alpha_s/8\Lambda^2$
R4	$\chi^2\,G^a_{\mu\nu}\tilde{G}_a^{\mu\nu}$	$i\alpha_s/8\Lambda^2$

- Di: Dirac, Ci: charged scalar, Ri: real scalar
- Dimension of the operator depends on how one counts masses!

An example: supersymmetric dark matter

- SUSY DM candidates are **Majorana fermions**: wino, bino, higgsino

  $\hookrightarrow$ tensor and vector currents vanish $\bar{\chi}\gamma^{\mu}\chi = \bar{\chi}\sigma^{\mu\nu}\chi = 0$
- Two classes of nuclear responses Engel, Pittel, Vogel 1992
  - **Spin-independent scattering**: D1, D11, non-relativistic expansion becomes $\mathbb{1}$

    $\hookrightarrow$ cross section scales with A^2 (**coherent enhancement**)
  - **Spin-dependent scattering**: D8, leads to $\mathbf{S}_N = \frac{\boldsymbol{\sigma}}{2}$ on the nucleon side

    $\hookrightarrow$ scales with nuclear spin expectation values (vanishes for even-A nuclei)
- SI and SD cross section traditional framework for dark matter searches

  $\hookrightarrow$ EFT analysis allows one to **systematically derive corrections**
- For **collider searches**: if the center-of-mass energy becomes too large, EFT breaks down (since mediator can no longer be integrated out)

  $\hookrightarrow$ **simplified models** Simplified Models for LHC New Physics Searches 1105.2838

Heavy-WIMP EFT

- If $m_\chi \gg \Lambda_{\text{EW}}$, one should count $m_\chi \sim \Lambda$

  $\hookrightarrow$ **heavy-WIMP EFT** Hill, Solon 2014

- Derivatives no longer suppressed, example:

$$\frac{1}{\Lambda^4}\bar{\chi}\gamma_\mu i\partial_\nu\chi\,\theta^{\mu\nu}$$

  with energy-momentum tensor $\theta^{\mu\nu}$, formally dimension 8

- However, since $\partial_0 \sim m_\chi$ and $m_\chi \sim \Lambda$, this operators scales as dimension 7

  $\hookrightarrow$ **same order as scalar operator D1**

- Phenomenologically relevant: cancellation between spin-0 and spin-2 terms
- Systematic approach in heavy-WIMP EFT analogous to heavy-quark EFT and heavy-baryon ChPT

$$\mathcal{L}_{\text{heavy-WIMP}} = \bar{h}_v\big[iv\cdot D - \delta m_\chi - f(H)\big]h_v + \mathcal{O}(1/m_\chi)$$

  with residual mass matrix δm_χ and $f(H)$ describing couplings to the Higgs field

RG corrections I

- So far, we have ignored operators such as $\bar\chi\gamma^\mu\chi\,\bar q\gamma_\mu\gamma_5 q$, as they vanish at threshold
- Suppose, RG corrections produced a mixing with $\bar\chi\gamma^\mu\chi\,\bar q\gamma_\mu q$

  $\hookrightarrow$ **indirect limit via the RG** Crivellin, D'Eramo, Procura 2014, ...
- Let us rewrite the operator basis up to dimension 6 in the **unbroken phase**

$$\mathcal{L} = \underbrace{\frac{C_D}{\Lambda}\bar\chi\sigma^{\mu\nu}\chi\,B_{\mu\nu} + \frac{i\tilde C_D}{\Lambda}\bar\chi\sigma^{\mu\nu}\gamma_5\chi\,B_{\mu\nu}}_{\text{dipole operators}} + \underbrace{\frac{C_{HH}}{\Lambda}\bar\chi\chi\,H^\dagger H + \frac{i\tilde C_{HH}}{\Lambda}\bar\chi\gamma_5\chi\,H^\dagger H}_{\text{scalar operators}}$$

$$+ \underbrace{\sum_{I,J=V,A}\frac{C_q^{IJ}}{\Lambda^2}\bar\chi\Gamma_I^\mu\chi\,\bar q\Gamma_{\mu,J}q}_{\text{vector operators}} + \sum_{I=V,A}\frac{iC_{HHD}^I}{\Lambda^2}\bar\chi\Gamma_I^\mu\chi\left[H^\dagger D_\mu H - (D_\mu H)^\dagger H\right]$$

with $\Gamma_V^\mu = \gamma^\mu$, $\Gamma_A^\mu = \gamma^\mu\gamma_5$ and covariant derivative $D_\mu = \partial_\mu + ig' Y_H B_\mu + ig\frac{\tau^I}{2}W_\mu^I$

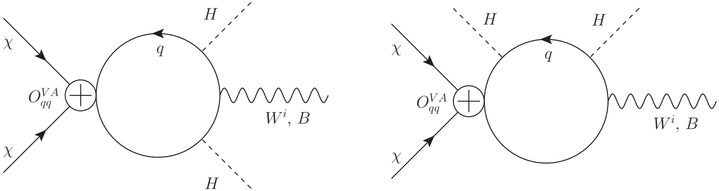
$(e = g\sin\theta_W,\ g' = g\tan\theta_W)$

RG corrections II

- After EW symmetry breaking:
  - **Dipole operators** give contribution to $\bar{\chi}\sigma^{\mu\nu}\chi F_{\mu\nu}$ and $\bar{\chi}\sigma^{\mu\nu}\gamma_5\chi F_{\mu\nu}$
  - **Scalar operators** contribute to $\bar{\chi}\chi\,\bar{q}q$ etc. (one Higgs gets a vev, the other couples to quark pair)
  - Also contribution to $\bar{\chi}\chi\, G^a_{\mu\nu} G_a^{\mu\nu}$ by integrating out heavy quarks (see later)
  - C^V_{HHD} generates a **threshold correction** to C^{VV}_q (integrating out the Z boson coupling to the quark pair), i.e.

$$C_u^{VV} \to C_u^{VV} + \frac{1}{2}\Big(1 - \frac{8}{3}\sin^2\theta_W\Big) C^V_{HHD}$$
$$C_d^{VV} \to C_d^{VV} - \frac{1}{2}\Big(1 - \frac{4}{3}\sin^2\theta_W\Big) C^V_{HHD}$$

RG corrections III



- O_q^{VA} mixes into O_{HHD}^{V}, with **RG solution**

$$C_{HHD}^{V}(\mu) = C_{HHD}^{V}(\Lambda) - \frac{\alpha_t N_c}{\pi} C_t^{VA}(\Lambda) \log \frac{\mu}{\Lambda} - (t \leftrightarrow b)$$

where $\alpha_t = Y_t^2/4\pi$ (Y_t: Yukawa coupling of top quark)

$\hookrightarrow$ this generates a contribution to C_q^{VV}, leading to an indirect limit on $C_t^{VA}(\Lambda)$

- Further mixing effects at dimension 7 Crivellin, Haisch 2014 and if WIMP is charged under $SU(2)_L$ Bishara et al. 2018

Chiral Perturbation Theory

- Effective theory of QCD based on **chiral symmetry**

$$\mathcal{L}_{\text{QCD}} = \bar{q}_L i\not{D} q_L + \bar{q}_R i\not{D} q_R - \bar{q}_L \mathcal{M} q_R - \bar{q}_R \mathcal{M} q_L - \frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu}$$

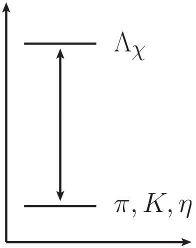
- Expansion in momenta p/Λ_χ and quark masses $m_q \sim p^2$

  $\hookrightarrow$ **scale separation**

- Breakdown scale: $\Lambda_\chi = M_\rho \ldots 4\pi F_\pi \sim 1\,\text{GeV}$
- Two variants
  - ***SU*(2)**: *u*- and *d*-quark **dynamical**, m_s fixed at **physical value**

    $\hookrightarrow$ expansion in M_π/Λ_χ, usually nice convergence
  - ***SU*(3)**: *u*-, *d*-, and *s*-quark dynamical

    $\hookrightarrow$ expansion in M_K/Λ_χ, sometimes tricky



Gell-Mann–Oakes–Renner relation

- Leading order in $SU(2)$ **meson ChPT**

$$\mathcal{L}_{\text{ChPT}} = \frac{F_\pi^2}{4}\text{Tr}\Big(D^\mu U^\dagger D_\mu U + 2B\mathcal{M}\big(U+U^\dagger\big)\Big) + \cdots$$
$$= (m_u+m_d)BF_\pi^2 - \frac{1}{2}(m_u+m_d)B\,(\pi^0)^2 - (m_u+m_d)B\,\pi^+\pi^- + \cdots$$

- Comparison with **QCD** Lagrangian

$$\big\langle \mathcal{L}_{\text{QCD}}\big\rangle = -m_u\langle\bar{u}u\rangle - m_d\langle\bar{d}d\rangle + \cdots \quad\Rightarrow\quad BF_\pi^2 = -\langle\bar{q}q\rangle \qquad \langle\bar{q}q\rangle = \langle\bar{u}u\rangle = \langle\bar{d}d\rangle$$

Gell-Mann–Oakes–Renner relation

$$M_\pi^2 = (m_u+m_d)B + \mathcal{O}\big(m_q^2\big) \qquad B = -\frac{\langle\bar{q}q\rangle}{F_\pi^2}$$

Gell-Mann–Oakes–Renner relation

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- Comparison with **QCD** Lagrangian

$$\big\langle \mathcal{L}_{\text{QCD}} \big\rangle = -m_u\langle \bar{u}u\rangle - m_d\langle \bar{d}d\rangle + \cdots \quad \Rightarrow \quad BF_\pi^2 = -\langle \bar{q}q\rangle \qquad \langle \bar{q}q\rangle = \langle \bar{u}u\rangle = \langle \bar{d}d\rangle$$

- Mass difference entirely due to **electromagnetism**

$$M_{\pi^\pm}^2 = M_{\pi^0}^2 + 2e^2F_\pi^2 Z + \mathcal{O}\big(m_d - m_u\big)^2$$

Gell-Mann–Oakes–Renner relation

$$M_{\pi^0}^2 = 2\hat{m}B + \mathcal{O}\big(m_q^2\big) \qquad \hat{m} = \frac{m_\text{u} + m_\text{d}}{2} \qquad B = -\frac{\langle \bar{q}q\rangle}{F_\pi^2}$$

Chiral EFT: a modern approach to nuclear forces

- Traditionally: meson-exchange potentials
- Chiral effective field theory
  - Based on **chiral symmetry** of QCD
  - **Power counting**
  - **Low-energy constants**
  - Hierarchy of multi-nucleon forces
  - Consistency of *NN* and 3*N*

↪ modern theory of nuclear forces

- Long-range part related to **pion–nucleon scattering**

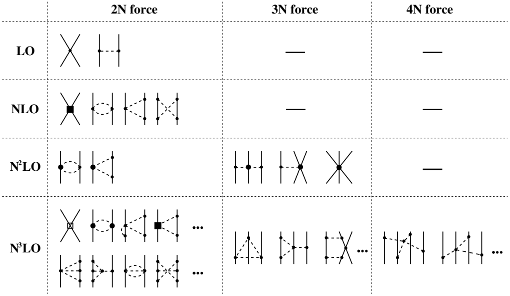
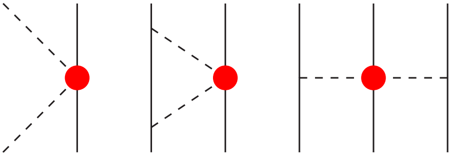


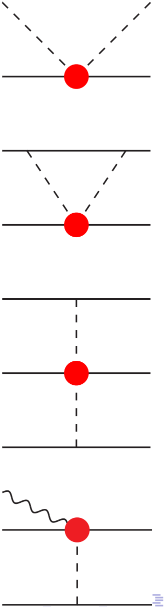
Figure taken from 1011.1343



Chiral EFT: currents

- Coupling to **external sources** $\mathcal{L}(v_\mu, a_\mu, s, p)$
- Same LECs appear in **axial current**

  $\hookrightarrow$ β decay, neutrino interactions, dark matter
- Vast literature for v_μ and a_μ, up to one-loop level
  - With unitary transformations: Kölling et al. 2009, 2011, Krebs et al. 2016, 2019
  - Without unitary transformations: Park et al. 2003, Pastore et al. 2008, Baroni et al. 2015
- For **dark matter** further currents: s, p, tensor, spin-2, θ^μ_μ



- **Effective WIMP Lagrangian** for spin-$1/2$ SM singlet χ Goodman et al. 2010

$$
\begin{aligned}
\mathcal{L}_\chi = \frac{1}{\Lambda^3}\sum_q \Big[& C_q^{SS}\bar\chi\chi\, m_q\bar q q + C_q^{PS}\bar\chi i\gamma_5\chi\, m_q\bar q q + C_q^{SP}\bar\chi\chi\, m_q\bar q i\gamma_5 q + C_q^{PP}\bar\chi i\gamma_5\chi\, m_q\bar q i\gamma_5 q\Big]\\
+\frac{1}{\Lambda^2}\sum_q \Big[& C_q^{VV}\bar\chi\gamma^\mu\chi\,\bar q\gamma_\mu q + C_q^{AV}\bar\chi\gamma^\mu\gamma_5\chi\,\bar q\gamma_\mu q + C_q^{VA}\bar\chi\gamma^\mu\chi\,\bar q\gamma_\mu\gamma_5 q + C_q^{AA}\bar\chi\gamma^\mu\gamma_5\chi\,\bar q\gamma_\mu\gamma_5 q\Big]\\
+\frac{1}{\Lambda^2}\sum_q \Big[& C_q^{TT}\bar\chi\sigma^{\mu\nu}\chi\,\bar q\sigma_{\mu\nu} q + \tilde C_q^{TT}\bar\chi\sigma^{\mu\nu}\gamma_5\chi\,\bar q\sigma_{\mu\nu} q\Big] + \frac{1}{\Lambda^3}\Big[C_g^S\bar\chi\chi\,\alpha_s G^a_{\mu\nu}G_a^{\mu\nu} + \cdots\Big]
\end{aligned}
$$

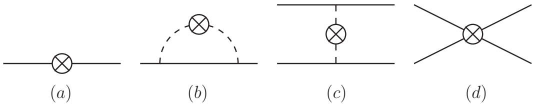
- **Chiral power counting**

$$
\partial = \mathcal{O}(p), \qquad m_q = \mathcal{O}(p^2) = \mathcal{O}(M_\pi^2), \qquad a_\mu, v_\mu = \mathcal{O}(p), \qquad \frac{\partial}{m_N} = \mathcal{O}(p^2)
$$

$\hookrightarrow$ construction of effective Lagrangian for nucleon and pion fields

$\hookrightarrow$ organize in terms of **chiral order ν**, $\mathcal{M} = \mathcal{O}(p^\nu)$

Classes of contributions



- Three classes of corrections:
  - **Subleading one-body responses** (*a*)
  - **Radius corrections** (*b*)
  - **Two-body currents** (*c*), (*d*)
- (*a*)+(*b*) just **ChPT for nucleon matrix elements**, but (*c*)+(*d*) genuinely new effects
- In the following: nucleon matrix elements, ChPT, and its limitations
  - Rescattering and unitarity corrections
  - Chiral convergence: covariant formulations and Δ degrees of freedom
  - NN EFT and power counting

Vector and axial-vector matrix elements: form factors

Vector and axial-vector form factors

$$\langle N(p')|\bar{q}\gamma^\mu q|N(p)\rangle = \bar{u}(p')\Big[F_1^{q,N}(t)\gamma^\mu + F_2^{q,N}(t)\frac{i\sigma^{\mu\nu}q_\nu}{2m_N}\Big]u(p) \qquad q = p' - p$$

$$\langle N(p')|\bar{q}\gamma^\mu\gamma_5 q|N(p)\rangle = \bar{u}(p')\Big[\gamma^\mu\gamma_5 G_A^{q,N}(t) + \gamma_5\frac{q^\mu}{2m_N}G_P^{q,N}(t) + \frac{i\sigma^{\mu\nu}}{2m_N}q_\nu\gamma_5 G_T^{q,N}(t)\Big]u(p)$$

- Decomposition from Lorentz invariance, gauge invariance, parity (and C/G parity)
- Single-nucleon form factors can be probed in SM with γ and Z exchange

  $\hookrightarrow$ **charge radii**, **magnetic moments**, etc.
- Main benefit from EFT in multi-nucleon observables

Vector and axial-vector matrix elements: flavor decomposition

- **Electromagnetic form factors** of the nucleon measure matrix elements of

$$j_\mu^{\text{EM}} = e\left(\frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s\right)$$

- For **flavor decomposition**:
  - Isospin symmetry: $F_1^{u,p} = F_1^{d,n}$ etc.
  - Parity-violating electron scattering ($c_W = \cos\theta_W$, $s_W = \sin\theta_W$)

$$j_\mu^Z = \frac{g}{4c_W}\left[\bar{u}\gamma_\mu\left(1 - \frac{8}{3}s_W^2 - \gamma_5\right)u - \bar{d}\gamma_\mu\left(1 - \frac{4}{3}s_W^2 - \gamma_5\right)d - \bar{s}\gamma_\mu\left(1 - \frac{4}{3}s_W^2 - \gamma_5\right)s\right]$$

$\hookrightarrow$ **weak current** involves different flavor structure

  - Strangeness form factors from lattice QCD

Ward identities

- Powerful method for other matrix elements from **conservation of vector current (CVC)** and **partial conservation of the axial current (PCAC)**

$$\partial_\mu(\bar{q}_f\gamma^\mu q_i) = i(m_f - m_i)\bar{q}_f q_i$$

$$\partial_\mu(\bar{q}_f\gamma^\mu\gamma_5 q_i) = i(m_f + m_i)\bar{q}_f\gamma_5 q_i - \delta_{fi}\frac{\alpha_s}{4\pi}G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a$$

$\hookrightarrow$ operator identities determine some (pseudo-)scalar matrix elements

- Does not work for singlet components
  - RHS of CVC vanishes for $f = i$
  - Singlet PCAC affected by $U(1)_A$ anomaly

$\hookrightarrow$ need another method for **scalar matrix elements**

Integrating out heavy quarks

Trace anomaly

$$\theta^\mu_\mu = \sum_q m_q \bar{q} q + \frac{\beta_{\text{QCD}}}{2g_s} G^a_{\mu\nu} G_a^{\mu\nu} \qquad \frac{\beta_{\text{QCD}}}{2g_s} = -\left(11 - \frac{2N_f}{3}\right)\frac{\alpha_s}{8\pi} + \mathcal{O}(\alpha_s^2)$$

- $M_Q \bar{Q} Q$, $Q = c, b, t$, can contribute indirectly via **gluon loops**
- Consider $\Delta N_f = 1$

$$\langle N | m_Q \bar{Q} Q | N \rangle = -\frac{\alpha_s}{12\pi} \langle N | G^a_{\mu\nu} G_a^{\mu\nu} | N \rangle \equiv m_N f_Q^N$$

- For $N_f = 3$ (with $\langle N | \theta^\mu_\mu | N \rangle = m_N$)

$$\langle N | \theta^\mu_\mu | N \rangle = m_N \sum_{q=u,d,s} f_q^N - \frac{9\alpha_s}{8\pi} \langle N | G^a_{\mu\nu} G_a^{\mu\nu} | N \rangle$$

$$f_Q^N = -\frac{\alpha_s}{12\pi m_N} \langle N | G^a_{\mu\nu} G_a^{\mu\nu} | N \rangle = \frac{2}{27}\left(1 - \sum_{q=u,d,s} f_q^N\right)$$

Chiral counting in scalar channel

- Leading pion–nucleon Lagrangian

$$\mathcal{L}^{(1)}_{\pi N} = \bar{\Psi}\Big[i\gamma_\mu\big(\partial^\mu - iv^\mu\big) - m_N + \frac{g_A}{2}\gamma_\mu\gamma_5\Big(2a^\mu - \frac{\partial^\mu \boldsymbol{\pi}}{F_\pi}\Big) + \cdots\Big]\Psi$$

$\hookrightarrow$ **no scalar source**!

- Next order:

$$\mathcal{L}^{(2)}_{\pi N} = \bar{\Psi}\sum_{i=1}^{7} c_i O^i \Psi \qquad O_1 = \text{Tr}(\chi_+) \qquad O_5 = \hat{\chi}_+$$

$$\chi_\pm = u^\dagger \chi u^\dagger \pm u\chi^\dagger u \qquad \hat{\chi}_+ = \chi_+ - \frac{1}{2}\text{Tr}(\chi_+) \qquad \chi = 2B(s + ip) = 2B\mathcal{M} + \cdots$$

- **Pion–nucleon σ term**

$$\sigma_{\pi N} = m_N(f_u^N + f_d^N) = -4c_1 M_\pi^2 - \frac{9g_A^2 M_\pi^3}{64\pi F_\pi^2} + \mathcal{O}(M_\pi^4)$$

$\hookrightarrow$ loop corrections enhanced

Some more context for dark matter

- Go back to **quark-level operators**
- For a Majorana WIMP χ: **scalar quark current**

$$\mathcal{L} = \sum_q C_q^{SS} \bar{\chi}\chi\, m_q \bar{q}q$$

leading contribution to responses with coherent enhancement

$\hookrightarrow$ **spin-independent scattering**

- Need nucleon matrix elements

$$\langle N|m_q\bar{q}q|N\rangle = f_q^N m_N$$

to extract BSM information from cross section

$$\sigma_{\text{SI}} = \frac{4\mu_N^2}{\pi}\left|m_N \sum_q C_q^{SS} f_q^N + \cdots\right|^2 \qquad \mu_N = \frac{m_N m_\chi}{m_N + m_\chi}$$

- $\sigma_{\pi N}$ critical for interpretation of direct-detection searches, especially in SUSY models Bottino et al. 2000, Ellis et al. 2008

The σ term from phenomenology and lattice QCD

- No scalar probe, but still relation to experiment! How?

  $\hookrightarrow$ **low-energy theorem**

- Goes back to Cheng, Dashen; Brown, Pardee, Peccei 1971

- Relates $\sigma_{\pi N}$ to πN scattering amplitude, but at **unphysical kinematics**

  $\hookrightarrow$ analytic continuation to the Cheng–Dashen point

- **No chiral logs** at one-loop order! Bernard, Kaiser, Meißner 1996

- **Protected by $SU(2)$**

  $\hookrightarrow$ expected correction: $\sigma_{\pi N} M_\pi^2 / m_N^2 \sim 1\,\text{MeV}$

- Rare opportunity to check lattice BSM nucleon matrix elements against experiment

Extracting $\sigma_{\pi N}$ from πN scattering: strategy

- **Scalar form factor** of the nucleon

$$\sigma(t) = \langle N(p')|\hat{m}(\bar{u}u + \bar{d}d)|N(p)\rangle \qquad t = (p' - p)^2 \qquad \sigma_{\pi N} = \sigma(0)$$

- **Low-energy theorem** Cheng, Dashen 1971

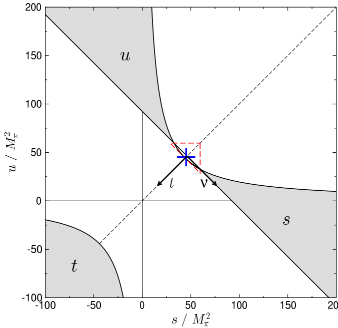
$$\underbrace{F_\pi^2 \bar{D}^+(\nu = 0, t = 2M_\pi^2)}_{F_\pi^2\left(d_{00}^+ + 2M_\pi^2 d_{01}^+\right) + \Delta_D} = \underbrace{\sigma(2M_\pi^2)}_{\sigma_{\pi N} + \Delta_\sigma} + \Delta_R$$

- Remainder $|\Delta_R| \lesssim 2\,\text{MeV}$ small Bernard, Kaiser, Meißner 1996

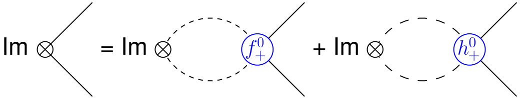
- **Dispersive approach** Gasser, Leutwyler, Sainio 1991, update MH, Ditsche, Kubis, Meißner 2012

$$\Delta_D - \Delta_\sigma = (-1.8 \pm 0.2)\,\text{MeV}$$

- Need to determine subthreshold parameters d_{00}^+, d_{01}^+



ChPT and rescattering



- Combined correction $\Delta_D - \Delta_\sigma$ small, but let's look at the individual terms
- Chiral expansion of Δ_σ and Δ_D Gasser, Leutwyler, Sainio 1992

$$\Delta_\sigma^{\text{ChPT}} = \frac{3g_A^2 M_\pi^3}{64\pi F_\pi^2} + \mathcal{O}(M_\pi^4) \simeq 7.7\,\text{MeV} \qquad \Delta_D^{\text{ChPT}} = \frac{23g_A^2 M_\pi^3}{384\pi F_\pi^2} + \mathcal{O}(M_\pi^4) \simeq 9.8\,\text{MeV}$$

$$\Delta_\sigma^{\text{disp}} = 13.9(3)\,\text{MeV} \qquad \Delta_D^{\text{disp}} = 12.1(3)\,\text{MeV}$$

- ChPT misses curvature due to strong $\pi\pi$ **rescattering**

  $\hookrightarrow$ unitarity only restored perturbatively, ultimately related to $f_0(500)$ resonance
- Another prominent example: isospin ($I = 0$), ***S*-wave $\pi\pi$ scattering lengths**

$$a_0^0\big|_{\text{ChPT}} = \frac{7M_\pi^2}{32\pi F_\pi^2} + \mathcal{O}(M_\pi^4) \simeq 0.16 \qquad a_0^0\big|_{\text{disp}} = 0.220(5)$$

Making use of the Cheng–Dashen low-energy theorem

- Can we use ChPT for the extrapolation to the Cheng–Dashen point?
- Need to determine low-energy constants c_i from somewhere

  $\hookrightarrow$ result for $\sigma_{\pi N}$ **depends on the πN partial-wave analysis**
- Systematics of the low-energy theorem
  - Relations only fulfilled perturbatively, known to fail for Δ_D, Δ_σ
  - Effect from t-channel D-waves missing
  - Relating the subthreshold and the physical region problematic
  - Isospin breaking

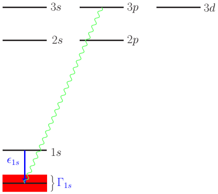
  $\hookrightarrow$ **1-loop ChPT not sufficient for rigorous extraction**
- Instead: solve system of **partial-wave dispersion relations** ("Roy–Steiner equations")
- Experimental input via: **πN scattering lengths**

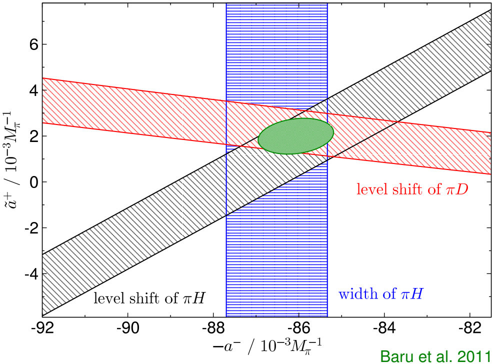
Constraints from pionic atoms

- $\pi H/\pi D$: bound state of π^- and p/d
  $\hookrightarrow$ spectrum sensitive to threshold πN scattering PSI 1995–2010
- **Combined analysis** of πH and πD

$$\bar{a}_{0+}^{1/2} = 169.8(2.0)\times 10^{-3} M_\pi^{-1}$$
$$\bar{a}_{0+}^{3/2} = -86.3(1.8)\times 10^{-3} M_\pi^{-1}$$

- Roy–Steiner solution by minimizing difference between LHS and RHS
  $\hookrightarrow$ impose $\bar{a}_{0+}^{1/2}$ and $\bar{a}_{0+}^{3/2}$





Baru et al. 2011

Solution of Roy–Steiner equations: πN σ-term

Central result

$$\sigma_{\pi N} = 59.1(3.1)\,\text{MeV} + \sum_{I_s} c_{I_s}\left(a_{0+}^{I_s} - \bar{a}_{0+}^{I_s}\right) = 59.1(3.5)\,\text{MeV}$$

$$c_{1/2} = 0.242\,\text{MeV} \times 10^3 M_\pi \qquad c_{3/2} = 0.874\,\text{MeV} \times 10^3 M_\pi$$

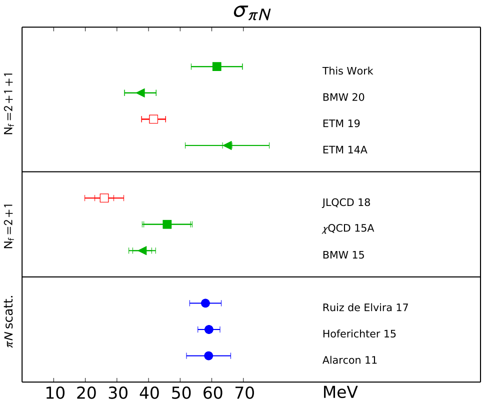
- In agreement with data on πN cross sections: $\sigma_{\pi N} = 58(5)\,\text{MeV}$
- Error budget:

Low-energy theorem				
Cheng–Dashen remainder	Isospin breaking	Scattering lengths	Roy–Steiner systematics	Total
2 MeV	2.2 MeV	1.6 MeV	0.9 MeV	3.5 MeV

A tension with lattice QCD

- Phenomenological extractions of the σ-term
  - **Pionic atoms**:
    $\sigma_{\pi N} = 59.1(3.5)$ MeV
  - **Low-energy cross sections**:
    $\sigma_{\pi N} = 58(5)$ MeV

  in tension with lattice QCD
- Evidence for large **excited-state contamination** Gupta et al. 2021
  - Lattice results in direct method differ significantly depending on fit strategy
  - Analysis in ChPT indeed suggests large effects (in the right direction)



Low-energy constants: subthreshold matching

- Convergence of heavy-baryon ChPT?
  - **Effect of Δ degrees of freedom**: $m_\Delta - m_N \simeq 2M_\pi$
  - **Threshold singularities**
- One-to-one correspondence between **subthreshold parameters** and **low-energy constants** (LECs)
  - Can solve for LECs analytically
  - Maximal distance from threshold singularities
  - πN amplitude can be expanded as a polynomial
  - Subthreshold region much closer to kinematics relevant for *NN* scattering
- Chiral convergence of threshold parameters: bad, even for isovector parameters
- N^3LO loops enhanced by $g_A^2(c_3 - c_4) \sim -16\,\text{GeV}^{-1}$
- Heavy-baryon amplitude does not converge equally well in whole low-energy region
- Should use subthreshold LECs for *NN* Entem, Machleidt, Nosyk 2017, Reinert, Krebs, Epelbaum 2017

Low-energy constants: Δ(1232) and relativistic corrections

a_{0+}^{-} $[10^{-3}M_\pi^{-1}]$	heavy-baryon-NN		heavy-baryon-πN		covariant	
	Δ-less	Δ-ful	Δ-less	Δ-ful	Δ-less	Δ-ful
LO	79.4	79.4	79.4	79.4	79.4	79.4
NLO	79.4	79.4(0)	79.4	79.4(0)	80.1	81.9(1)
N^2LO	92.2	92.7(10)	92.9	90.5(9)	89.9	81.7(1.2)
N^3LO	68.5	96.3(2.0)	58.6	69.1(1.2)	83.8	83.4(1.0)
Pionic atoms	85.4(9)	85.4(9)	85.4(9)	85.4(9)	85.4(9)	85.4(9)

- Comprehensive analysis of the **subthreshold-threshold comparison** with and without Δ(1232) and relativistic corrections
  - Including the Δ does reduce size of LECs and improve convergence
  - Further improvement in covariant formulation, but why?
  - Uncertainty of LECs c_i, d_i, e_i now totally dominated by scheme and chiral order

Problems

1. Why is $\bar{\chi}\sigma^{\mu\nu}\chi\,\bar{q}\sigma_{\mu\nu}\gamma_5 q$ not an independent operator for a spin-1/2 WIMP?
   Hint: you need to derive the relation $\sigma^{\mu\nu}\gamma_5 = \frac{i}{2}\epsilon^{\mu\nu\alpha\beta}\sigma_{\alpha\beta}$.
2. Show that the bilinears $\bar{\chi}\gamma^\mu\chi$ and $\bar{\chi}\sigma^{\mu\nu}\chi$ vanish for a Majorana WIMP.
   Hint: a Majorana particle fulfills $\chi = \chi^C = C(\bar{\chi})^T$, *with* $C = i\gamma_2\gamma_0$.
3. Derive the nucleon mass m_N and the σ-term $\sigma_{\pi N}$ at $\mathcal{O}(M_\pi^2)$ and verify the Feynman–Hellmann relation $\sigma_{\pi N} = m_q \frac{\partial m_N}{\partial m_q}$.
   Hint: use the Gell-Mann–Oakes–Renner relation to express M_π *in terms of* m_q.
   You can work in the isospin limit $m_q = m_u = m_d$.