EFT for dark matter

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FOR FUNDAMENTAL PHYSICS

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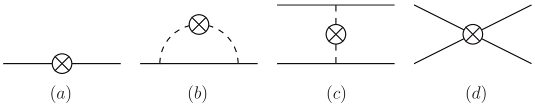
Methods of Effective Field Theory and Lattice Field Theory

Bad Honnef Physics School

Nucleon matrix elements



- So far: **single-nucleon matrix elements** (*a*), (*b*)

  $\hookrightarrow$ vector, axial vector, scalar, pseudoscalar

- Others not covered:
  - **Tensor operators**: $\bar{q}\sigma^{\mu\nu}q$

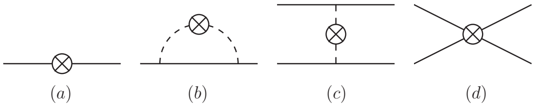
    $\hookrightarrow$ lattice QCD, unitarity arguments
  - **Spin-2 operators**: $\bar{\theta}_q^{\mu\nu} = \frac{1}{2}\bar{q}\Big(\gamma^{\{\mu}iD_-^{\nu\}} - \frac{m_q}{2}g^{\mu\nu}\Big)q$, $\bar{\theta}_g^{\mu\nu} = \frac{g^{\mu\nu}}{4}G^a_{\lambda\sigma}G_a^{\lambda\sigma} - G_a^{\mu\lambda}G^{\nu}_{a\lambda}$

    $\hookrightarrow$ nuclear PDFs

- Next: **few-body effects** (*c*), (*d*)

  $\hookrightarrow$ hierarchy predicted from chiral EFT

Two-body currents: scalar operators



- Recall counting of $\sigma_{\pi N} = -4c_1 M_\pi^2 + \mathcal{O}(M_\pi^3)$

  $\hookrightarrow$ (a) scales like $\mathcal{O}(p^2)$ in the chiral counting

- In contrast to the nucleon, the pion has a **leading-order scalar coupling**

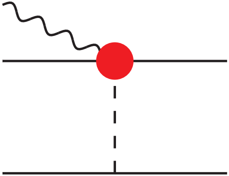
$$\mathcal{M}_{(b)} \simeq \underbrace{p \times \frac{1}{p} \times p}_{\text{nucleon line}} \times \underbrace{\frac{1}{p^2} \times p^2 \times \frac{1}{p^2}}_{\text{pion line}} \times \underbrace{p^4}_{\text{loop}} = \mathcal{O}(p^3)$$

$$\mathcal{M}_{(c)} \simeq \underbrace{p \times \frac{1}{p^2} \times p^2 \times \frac{1}{p^2} \times p}_{\text{pion line}} \times \underbrace{p^3}_{\delta\text{-function for spectator nucleon}} = \mathcal{O}(p^3)$$

$\hookrightarrow$ two-body effects as important as loop corrections

- Chiral corrections only **suppressed by a single chiral order**

Two-body currents: axial-vector operators



- Axial-vector current can couple via pion pole

  $\hookrightarrow$ two-body effects proportional to c_i, including $\Delta(1232)$ enhancement
- **One-body current**

$$\mathbf{J}^3_{i,1\mathrm{b}} = \frac{1}{2}\,\tau_i^3\left(G_A^3(\mathbf{q}^2)\,\boldsymbol{\sigma}_i - \frac{G_P^3(\mathbf{q}^2)}{4m_N^2}\,(\mathbf{q}\cdot\boldsymbol{\sigma}_i)\,\mathbf{q}\right)$$

- **Two-body current**

$$\begin{aligned}\mathbf{J}^3_{12,2\mathrm{b}} = &-\frac{g_A}{2F_\pi^2}\,[\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2]^3\left[c_4\left(1-\frac{\mathbf{q}}{\mathbf{q}^2+M_\pi^2}\mathbf{q}\cdot\right)(\boldsymbol{\sigma}_1\times\mathbf{k}_2)+\frac{c_6}{4}(\boldsymbol{\sigma}_1\times\mathbf{q})+i\,\frac{\mathbf{p}_1+\mathbf{p}_1'}{4m_N}\right]\frac{\boldsymbol{\sigma}_2\cdot\mathbf{k}_2}{M_\pi^2+k_2^2}\\ &-\frac{g_A}{F_\pi^2}\,\tau_2^3\left[c_3\left(1-\frac{\mathbf{q}}{\mathbf{q}^2+M_\pi^2}\mathbf{q}\cdot\right)\mathbf{k}_2+2c_1\,M_\pi^2\,\frac{\mathbf{q}}{\mathbf{q}^2+M_\pi^2}\right]\frac{\boldsymbol{\sigma}_2\cdot\mathbf{k}_2}{M_\pi^2+k_2^2}+\text{contact terms}\end{aligned}$$

Integrating out the pions: NREFT

- **Non-relativistic EFT for dark matter** Fan et al. 2010, Fitzpatrick et al. 2012, Anand et al. 2013

  $\hookrightarrow$ integrate out the pions

- Similar to **pionless EFT** for nuclear physics see lectures by V. Cirigliano, Z. Davoudi

  $\hookrightarrow$ only remaining degrees of freedom nucleons and WIMPs

- Calculation organized in terms of

$$\mathbf{q} \qquad \mathbf{v}^\perp = \mathbf{v} + \frac{\mathbf{q}}{2\mu_N} \qquad \mathbf{S}_\chi \qquad \mathbf{S}_N$$

$\hookrightarrow$ expand in $\mathbf{q}$, $\mathbf{v}^\perp$ (using $\mathbf{v}^\perp \cdot \mathbf{q} = 0$)

Matching to NREFT

- Operator basis for **WIMP and nucleon fields** Fan et al. 2010, Fitzpatrick et al. 2012

$$\mathcal{O}_1 = \mathbb{1} \qquad \mathcal{O}_2 = (\mathbf{v}^\perp)^2 \qquad \mathcal{O}_3 = i\mathbf{S}_N \cdot (\mathbf{q} \times \mathbf{v}^\perp) \qquad \mathcal{O}_4 = \mathbf{S}_\chi \cdot \mathbf{S}_N$$
$$\mathcal{O}_5 = i\mathbf{S}_\chi \cdot (\mathbf{q} \times \mathbf{v}^\perp) \qquad \mathcal{O}_6 = \mathbf{S}_\chi \cdot \mathbf{q}\, \mathbf{S}_N \cdot \mathbf{q} \qquad \mathcal{O}_7 = \mathbf{S}_N \cdot \mathbf{v}^\perp \qquad \mathcal{O}_8 = \mathbf{S}_\chi \cdot \mathbf{v}^\perp$$
$$\mathcal{O}_9 = i\mathbf{S}_\chi \cdot (\mathbf{S}_N \times \mathbf{q}) \qquad \mathcal{O}_{10} = i\mathbf{S}_N \cdot \mathbf{q} \qquad \mathcal{O}_{11} = i\mathbf{S}_\chi \cdot \mathbf{q} \qquad \ldots$$

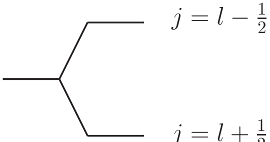
- Matching to relativistic amplitudes

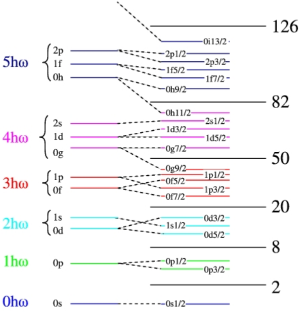
$$\mathcal{M}^{SS}_{1,\text{NR}} = \mathcal{O}_1 f_N(t) \qquad \mathcal{M}^{SP}_{1,\text{NR}} = \mathcal{O}_{10} g_5^N(t) \qquad \mathcal{M}^{PP}_{1,\text{NR}} = \frac{1}{m_\chi}\mathcal{O}_6 h_5^N(t)$$
$$\mathcal{M}^{VV}_{1,\text{NR}} = \mathcal{O}_1\Big(f_1^{V,N}(t) + \frac{t}{4m_N^2}f_2^{V,N}(t)\Big) + \frac{1}{m_N}\mathcal{O}_3 f_2^{V,N}(t) + \frac{1}{m_N m_\chi}\Big(t\mathcal{O}_4 + \mathcal{O}_6\Big)f_2^{V,N}(t)$$
$$\mathcal{M}^{AV}_{1,\text{NR}} = 2\mathcal{O}_8 f_1^{V,N}(t) + \frac{2}{m_N}\mathcal{O}_9\Big(f_1^{V,N}(t) + f_2^{V,N}(t)\Big)$$
$$\mathcal{M}^{AA}_{1,\text{NR}} = -4\mathcal{O}_4 g_A^N(t) + \frac{1}{m_N^2}\mathcal{O}_6 g_P^N(t) \qquad \mathcal{M}^{VA}_{1,\text{NR}} = \Big\{-2\mathcal{O}_7 + \frac{2}{m_\chi}\mathcal{O}_9\Big\}h_A^N(t)$$

- Observations
  - SI: $\mathcal{O}_1$, SD: combination of $\mathcal{O}_4$ and $\mathcal{O}_6$
  - Not all the $\mathcal{O}_i$ equally important, QCD implies relations among them

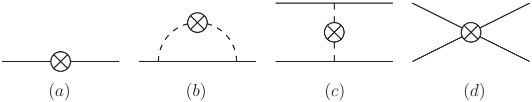
Coherence effects

- NREFT useful to analyze **coherence effects**
- Six distinct nuclear responses
  Fitzpatrick et al. 2012, Anand et al. 2013
  - $M \leftrightarrow \mathcal{O}_1 \leftrightarrow$ SI
  - $\Sigma', \Sigma'' \leftrightarrow \mathcal{O}_4, \mathcal{O}_6 \leftrightarrow$ SD
  - $\Phi'' \leftrightarrow \mathcal{O}_3 \leftrightarrow$ quasi-coherent, spin-orbit operator
  - $\Delta, \tilde{\Phi}'$: not coherent
- **Quasi-coherence** of Φ''
  - Spin-orbit splitting
  - Coherence until mid-shell
  - About 20 coherent nucleons in Xe
  - Interference M–$\Phi'' \leftrightarrow \mathcal{O}_1$–$\mathcal{O}_3$
- Further coherent M-responses from $\mathcal{O}_5$, $\mathcal{O}_8$, $\mathcal{O}_{11}$, but no interference with $\mathcal{O}_1$ due to sum over $\mathbf{S}_\chi$





Cross section and nuclear structure factors



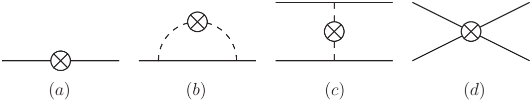
$$\frac{d\sigma_{\chi\mathcal{N}}}{dq^2} = \frac{1}{4\pi v^2}\bigg|\sum_{I=\pm}\Big(c_I^M - \frac{q^2}{m_N^2}\dot{c}_I^M\Big)\mathcal{F}_I^M(q^2) + c_\pi \mathcal{F}_\pi(q^2) + c_b\mathcal{F}_b(q^2) + \frac{q^2}{2m_N^2}\sum_{I=\pm} c_I^{\Phi''}\mathcal{F}_I^{\Phi''}(q^2)\bigg|^2$$

$$+ \frac{1}{4\pi v^2}\sum_{i=5,8,11}\bigg|\sum_{I=\pm}\xi_i(q, v_T^\perp)c_I^{M,i}\mathcal{F}_I^M(q^2)\bigg|^2$$

$$+ \frac{1}{v^2(2J+1)}\Big(|a_+|^2 S_{00}(q^2) + \mathrm{Re}\,(a_+ a_-^*)S_{01}(q^2) + |a_-|^2 S_{11}(q^2)\Big)$$

- Decomposition into **nuclear structure factors** $\mathcal{F}$, S_{ij} and coefficients c, a
- Three classes of contributions:
  - **(Sub-) Leading 1b responses** (a): $c_I^M\mathcal{F}_I^M(q^2)$, $c_I^{\Phi''}\mathcal{F}_I^{\Phi''}(q^2)$, $|a_\pm|^2 S_{ij}(q^2)$
  - **Radius corrections** (b): $\dot{c}_I^M\mathcal{F}_I^M(q^2)$
  - **Two-body currents** (c), (d): $c_\pi\mathcal{F}_\pi(q^2)$, $c_b\mathcal{F}_b(q^2)$
- (a)+(b) essentially **nucleon form factors**, but (c)+(d) genuinely new effects

Cross section and nuclear structure factors



$$\frac{d\sigma_{\chi\mathcal{N}}}{dq^2} = \frac{1}{4\pi v^2}\left|\sum_{I=\pm}\left(c_I^M - \frac{q^2}{m_N^2}\dot{c}_I^M\right)\mathcal{F}_I^M(q^2) + c_\pi \mathcal{F}_\pi(q^2) + c_b \mathcal{F}_b(q^2) + \frac{q^2}{2m_N^2}\sum_{I=\pm} c_I^{\Phi''}\mathcal{F}_I^{\Phi''}(q^2)\right|^2$$

$$+ \frac{1}{4\pi v^2}\sum_{i=5,8,11}\left|\sum_{I=\pm}\xi_i(q, v_T^\perp) c_I^{M,i}\mathcal{F}_I^M(q^2)\right|^2$$

$$+ \frac{1}{v^2(2J+1)}\Big(|a_+|^2 S_{00}(q^2) + \text{Re}\,(a_+ a_-^*) S_{01}(q^2) + |a_-|^2 S_{11}(q^2)\Big)$$

- Nuclear structure interpretation:
  - $\mathcal{F}_I^M(q^2)$: $\mathbb{1} \Rightarrow$ charge distribution (coherent), $\mathcal{F}_\pm^M(0) = Z \pm N$
  - $\mathcal{F}_I^{\Phi''}(q^2)$: $i\mathbf{S}_N \cdot (\mathbf{q} \times \mathbf{v}^\perp) \Rightarrow$ spin-orbit interaction (quasi-coherent)
  - $S_{ij}(q^2)$: $\mathbf{S}_\chi \cdot \mathbf{S}_N$, $\mathbf{S}_\chi \cdot \mathbf{q}\,\mathbf{S}_N \cdot \mathbf{q} \Rightarrow$ spin average (not coherent), $S_{00}(0) \pm S_{01}(0) + S_{11}(0) = \frac{(2J+1)(J+1)}{4\pi J}|\langle \mathbf{S}_{p/n}\rangle|^2$
- Coefficients: convolution of **Wilson coefficients** and **nucleon matrix elements**

$$c_\pm^M = \frac{\zeta}{2}(f_p \pm f_n) + \cdots \qquad f_N = \frac{m_N}{\Lambda^3}\sum_{q=u,d,s} C_q^{SS} f_q^N + \cdots \qquad \langle N|m_q\bar{q}q|N\rangle = m_N f_q^N$$

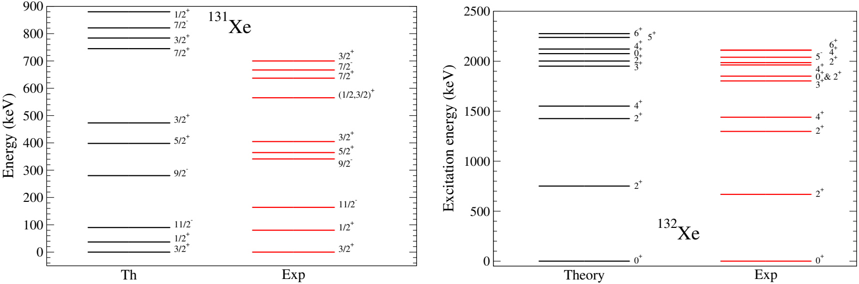
ChiralEFT4DM: a PYTHON package for dark matter

- ChiralEFT4DM: all results shown in the following available as PYTHON package at

  https://theorie.ikp.physik.tu-darmstadt.de/strongint/ChiralEFT4DM.html

- Includes:
  - (Quasi-) Coherent structure factors for F, Si, Ar, Ge, Xe
  - Nucleon matrix elements and matching relations for spin-$1/2$ and spin-0 WIMP
  - 1b and 2b responses up to third chiral order
  - S, P, V, A, T, θ^{μ}_{μ}, and spin-2 effective operators that can lead to a coherent response
  - Convolution with Standard Halo Model

Spectra and shell-model calculation



- **Shell-model diagonalization** for Xe isotopes with ^{100}Sn core
- **Uncertainty estimates**: currently phenomenological shell-model interaction

  ↪ chiral-EFT-based interactions in the future

  ↪ **ab-initio calculations for light nuclei**

Charge radii and neutron skin

	^{19}F	^{28}Si	^{40}Ar	^{74}Ge	^{132}Xe
$\sqrt{\langle r^2\rangle_{\text{ch}}}$ [fm] (th)	2.83	3.19	3.43	4.08	4.77
(exp)	2.898(2)	3.122(2)	3.427(3)	4.0742(12)	4.7808(49)
$\sqrt{\langle r_n^2\rangle}-\sqrt{\langle r_p^2\rangle}$ [fm]	0.02	0	0.11	0.17	0.28
shell-model interaction	USDB	USDB	SDPF.SM	RG	GCN

- Excellent agreement for charge radii

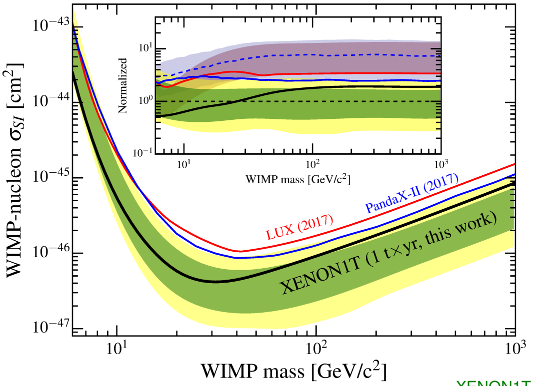
$$\langle r^2_{\text{ch}}\rangle = \langle r_p^2\rangle + \langle r^2_{E,p}\rangle + \frac{N}{Z}\langle r^2_{E,n}\rangle + \langle r^2_{\text{rel}}\rangle + \langle r^2_{\text{spin-orbit}}\rangle$$

- Point-neutron radii more uncertain
- Related to structure factors by

$$\langle r_p^2\rangle = -\frac{3}{Z}\frac{\mathrm{d}}{\mathrm{d}q^2}\big(\mathcal{F}_+^M(q^2)+\mathcal{F}_-^M(q^2)\big)\big|_{q^2=0} \qquad \langle r_n^2\rangle = -\frac{3}{N}\frac{\mathrm{d}}{\mathrm{d}q^2}\big(\mathcal{F}_+^M(q^2)-\mathcal{F}_-^M(q^2)\big)\big|_{q^2=0}$$

- Further cross checks: electric quadrupole and magnetic dipole moments, transition matrix elements

Case 1: spin-independent scattering

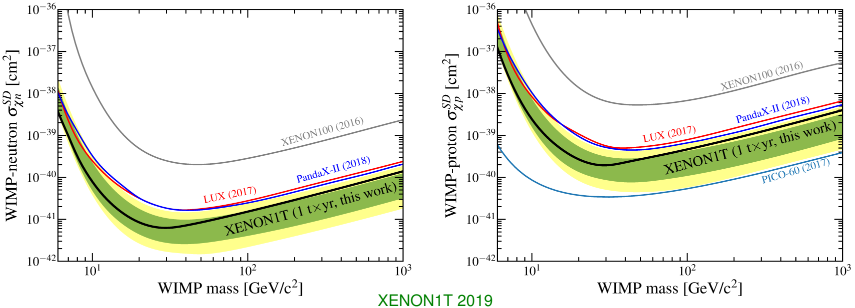


XENON1T 2018

- All $c = 0$, $a = 0$ except for c_+^M: **spin-independent scattering**

$$\frac{d\sigma_{\chi\mathcal{N}}}{dq^2} = \frac{\sigma_{\chi N}^{\text{SI}}}{4\mu_N^2 v^2}\left|\mathcal{F}_+^M(q^2)\right|^2 \qquad \sigma_{\chi N}^{\text{SI}} = \frac{\mu_N^2}{\pi}\left|c_+^M\right|^2 \qquad \mu_N = \frac{m_N m_\chi}{m_N + m_\chi}$$

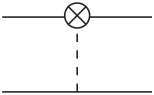
Case 2: spin-dependent scattering



XENON1T 2019

- All $c = 0$, $a_+ = \pm a_-$: **spin-dependent scattering**

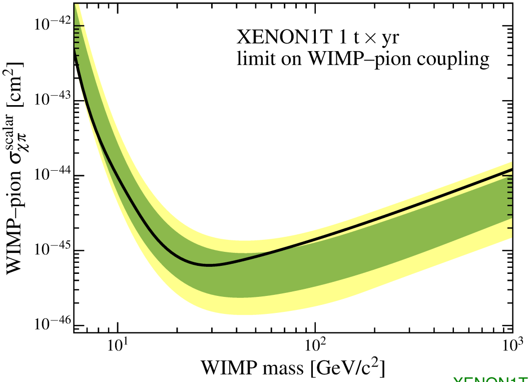
$$\frac{d\sigma_{\chi\mathcal{N}}}{dq^2} = \frac{\sigma^{\text{SD}}_{\chi N}}{3\mu_N^2 v^2}\frac{\pi}{2J+1}S_N(q^2) \qquad \sigma^{\text{SD}}_{\chi N} = \frac{3\mu_N^2}{\pi}\big|a_+\big|^2$$

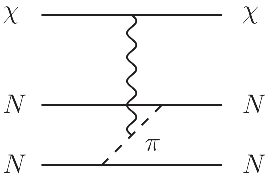


- Xe sensitive to proton spin due to **two-body currents**

  Klos, Menéndez, Gazit, Schwenk 2013

Case 3: WIMP–pion scattering





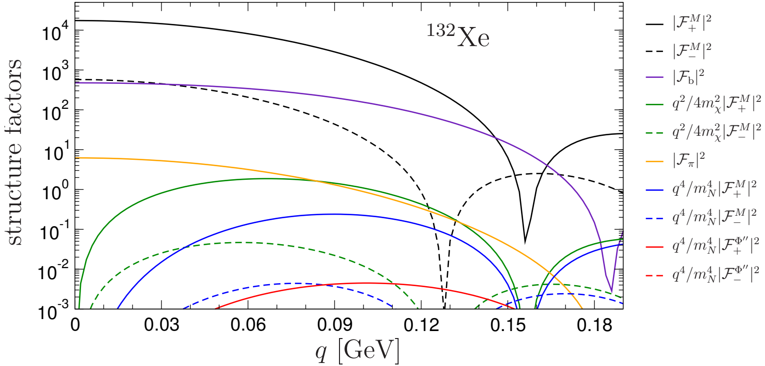
XENON1T + MH, Klos, Menéndez, Schwenk 2019

- Only c_π nonzero: **WIMP–pion scattering**

$$\frac{d\sigma_{\chi\mathcal{N}}}{dq^2} = \frac{\sigma_{\chi\pi}^{\text{scalar}}}{\mu_\pi^2 v^2}\big|\mathcal{F}_\pi(q^2)\big|^2 \qquad \sigma_{\chi\pi}^{\text{scalar}} = \frac{\mu_\pi^2}{4\pi}\big|c_\pi\big|^2 \qquad \mu_\pi = \frac{m_\chi M_\pi}{m_\chi + M_\pi}$$

- Expression in terms of cross section depends on underlying operator, here for a scalar $\bar{\chi}\chi\bar{q}q$

Full set of coherent contributions



$$\frac{d\sigma_{\chi\mathcal{N}}}{dq^2} = \frac{1}{4\pi v^2}\bigg|\sum_{I=\pm}\Big(c_I^M - \frac{q^2}{m_N^2}\dot{c}_I^M\Big)\mathcal{F}_I^M(q^2) + c_\pi\mathcal{F}_\pi(q^2) + c_\mathrm{b}\mathcal{F}_\mathrm{b}(q^2) + \frac{q^2}{2m_N^2}\sum_{I=\pm} c_I^{\Phi''}\mathcal{F}_I^{\Phi''}(q^2)\bigg|^2$$

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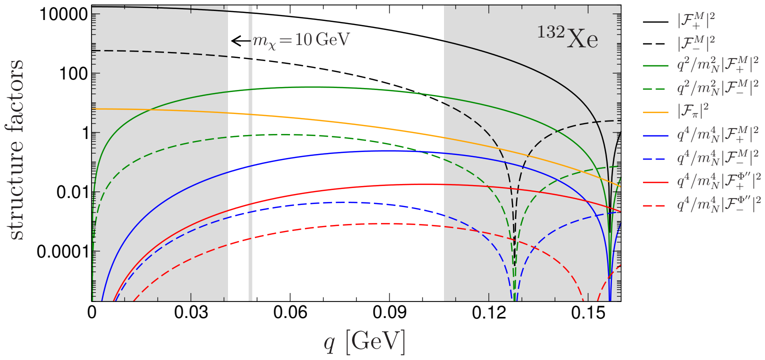
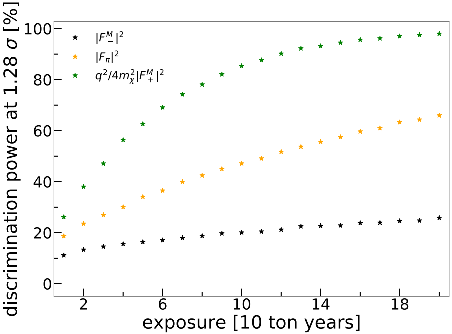
Discriminating different response functions



- White region accessible to XENON-type experiment
- Can one tell these curves apart in a realistic experimental setting?
- Consider XENON1T-like, XENONnT-like, DARWIN-like settings

Discriminating different response functions

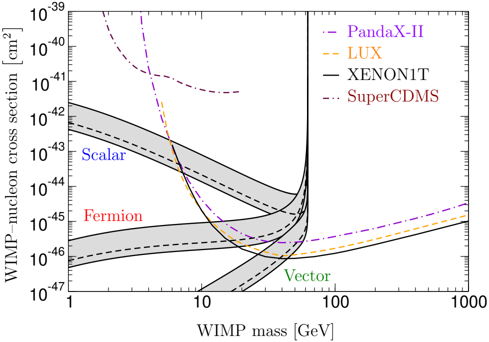


- DARWIN-like setting, $m_\chi = 100\,\text{GeV}$
- q-dependent responses more easily distinguishable
- If interaction not much weaker than current limits, DARWIN could discriminate most responses from standard SI structure factor

Higgs Portal dark matter

- **Higgs Portal**: WIMP interacts with SM via the Higgs
  - **Scalar**: $H^\dagger H S^2$
  - **Vector**: $H^\dagger H V_\mu V^\mu$
  - **Fermion**: $H^\dagger H \bar{f} f$
- If $m_h > 2m_\chi$, should happen at the LHC

  $\hookrightarrow$ limits on **invisible Higgs decays**



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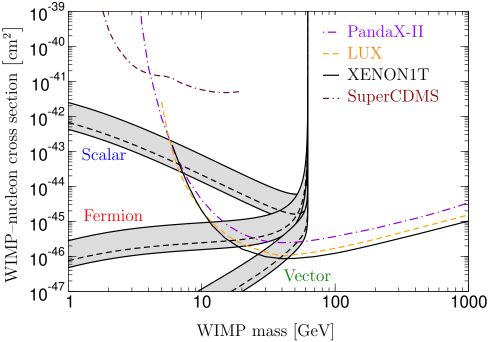

- Master formula for single-nucleon cross section

$$\sigma_{\chi N} = \Gamma_{\text{inv}} \frac{8 m_N^4 f_N{}^2}{v^2 \beta m_h^3 (m_\chi + m_N)^2} g_\chi\left(\frac{m_h}{m_\chi}\right) \qquad \beta = \sqrt{1 - 4m_\chi^2/m_h^2} \qquad v = 246\,\text{GeV}$$

$$g_S(x) = 1 \qquad g_V(x) = \frac{4}{12 - 4x^2 + x^4} \qquad g_f(x) = \frac{2}{x^2 - 4}$$

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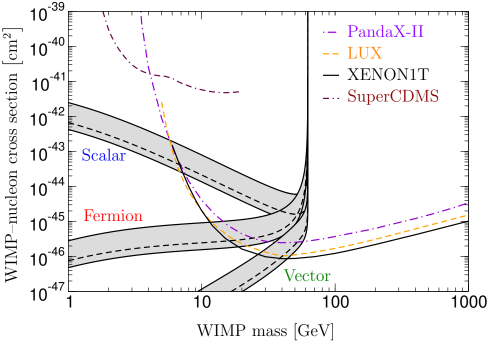
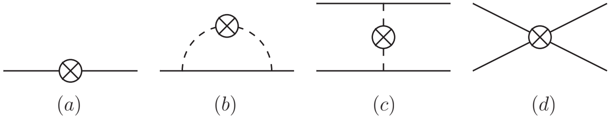


- Translation requires input for **Higgs–nucleon coupling**

$$f_N = \sum_{q=u,d,s,c,b,t} f_q^N = \frac{2}{9} + \frac{7}{9} \sum_{q=u,d,s} f_q^N + \mathcal{O}(\alpha_s) \qquad m_N f_q^N = \langle N | m_q \bar{q} q | N \rangle$$

- Issues: input for $f_N = 0.260 \ldots 0.629$ outdated, 2b currents missing



- **One-body contribution**

$$f_N^{1b} = 0.307(9)_{ud}(15)_s(5)_{\text{pert}} = 0.307(18)$$

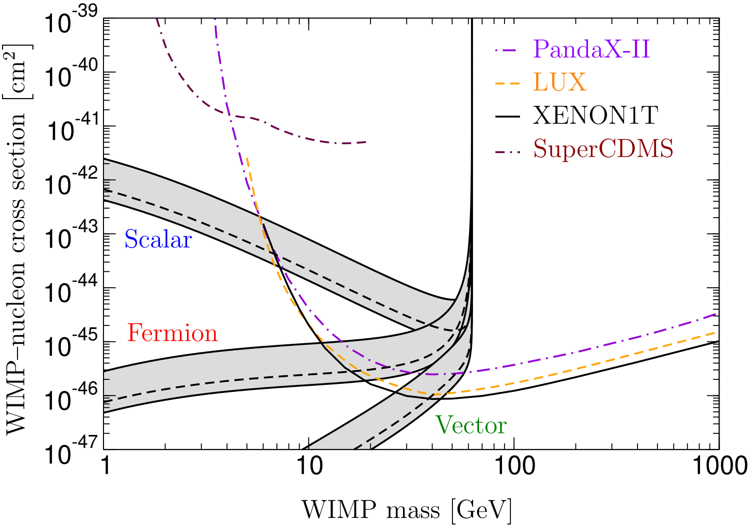
- Limits on WIMP–nucleon cross section subsume **2b effects**

  $\hookrightarrow$ have to be included for meaningful comparison

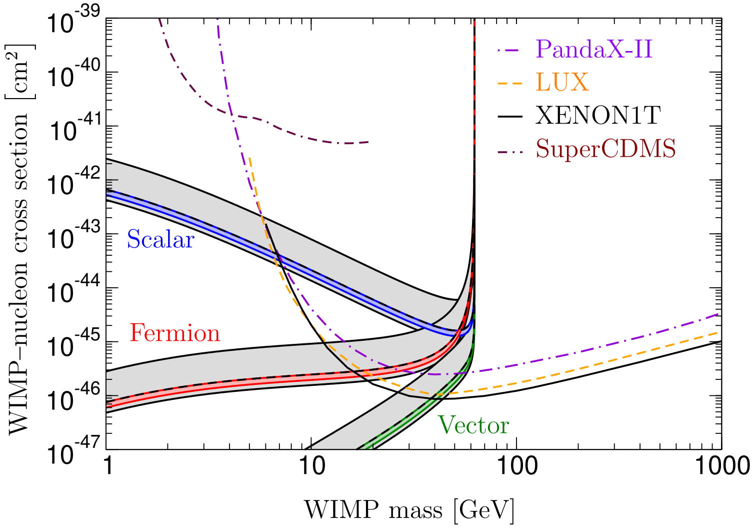
- **Two-body contribution**
  - Need s and θ^μ_μ currents
  - Treatment of θ^μ_μ tricky: several ill-defined terms combine to $\langle\Psi|T + V_{NN}|\Psi\rangle = E_b$
  - A cancellation makes the final result anomalously small

$$f_N^{2b} = \big[-3.2(0.2)_A(2.1)_{\text{ChEFT}} + 5.0(0.4)_A\big] \times 10^{-3} = 1.8(2.1) \times 10^{-3}$$

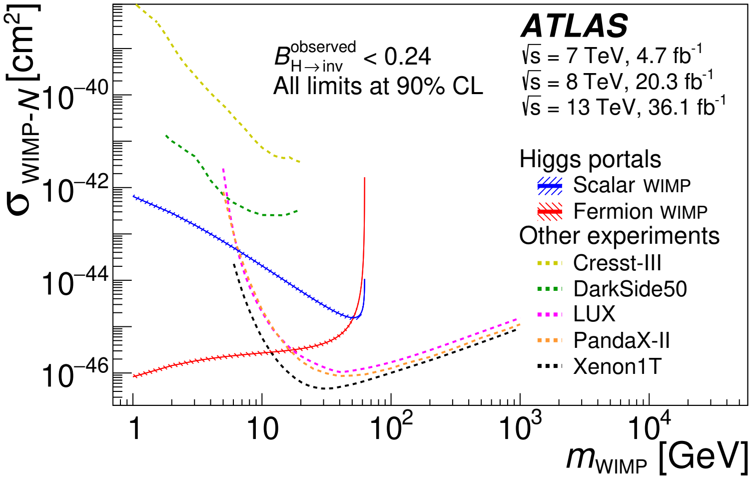
Improved limits for Higgs Portal dark matter



Improved limits for Higgs Portal dark matter



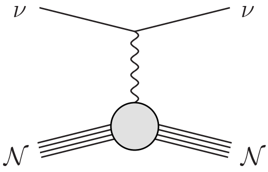
Improved limits for Higgs Portal dark matter



ATLAS 2019

Coherent neutrino–nucleus scattering

	neutrino–nucleus scattering
kinematics	elastic, ν relativistic
mediator	Z, BSM?
quantum numbers	$V-A$, others?
momentum transfer q	$\lesssim 50\,\text{MeV}$



- Sensitive probe of the **neutron distribution**
- Search for **non-standard neutrino interactions**
- Background process for dark matter experiments

$\hookrightarrow$ need short-distance couplings, hadronic matrix elements, and nuclear responses

Coherent neutrino–nucleus scattering: EFT approach

EFT approach to CEνNS

Rate $=$ BSM couplings $\otimes$ hadronic matrix elements $\otimes$ nuclear structure $\otimes$ neutrino flux

$\hookrightarrow$ most efficiently addressed in **effective field theory**

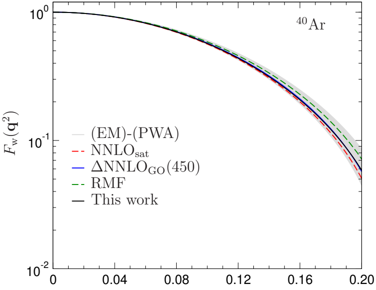
- Will assume a **heavy mediator**, e.g., Z in SM
- Light BSM physics can be added to set of BSM operators, e.g. light Z'
- **Same formalism as in dark matter case applies!**

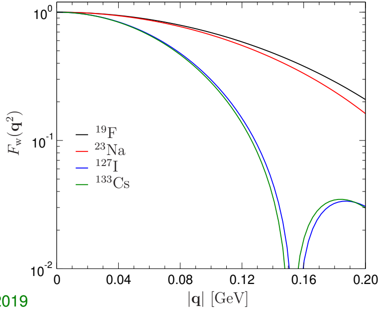
CEνNS cross section in the SM

$$\frac{d\sigma_A}{dT} = \frac{G_F^2 m_A}{4\pi}\left(1 - \frac{m_A T}{2E_\nu^2} - \frac{T}{E_\nu}\right) Q_w^2 \left|F_w(\mathbf{q}^2)\right|^2$$
$$+ \frac{G_F^2 m_A}{2J+1}\left(2 + \frac{m_A T}{E_\nu^2} - \frac{2T}{E_\nu}\right)\Big(\left(g_A^{s,N}\right)^2 S_{00}^{\mathcal{T}}(\mathbf{q}^2) - g_A g_A^{s,N} S_{01}^{\mathcal{T}}(\mathbf{q}^2) + (g_A)^2 S_{11}^{\mathcal{T}}(\mathbf{q}^2)\Big)$$

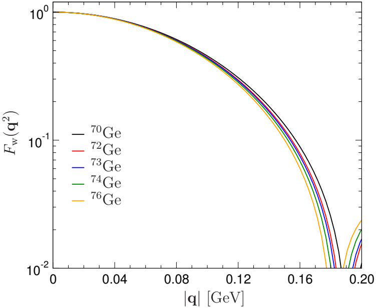
- Notation:
  - Incoming neutrino energy E_ν
  - Momentum transfer $t = -\mathbf{q}^2 = 2m_A T$ with target mass m_A, $T \in [0, 2E_\nu^2/(m_A + 2E_\nu)]$
  - $F_w(\mathbf{q}^2)$: **weak form factor**
  - Weak charge $Q_w = ZQ_w^p + NQ_w^n$
  - $S_{ij}^{\mathcal{T}}(\mathbf{q}^2)$: only transverse part $\mathcal{T}$ contributes in spin-dependent response
  - Nuclear spin J
  - Axial-vector couplings $g_A = \Delta u - \Delta d$, $g_A^{s,N} = \Delta s$

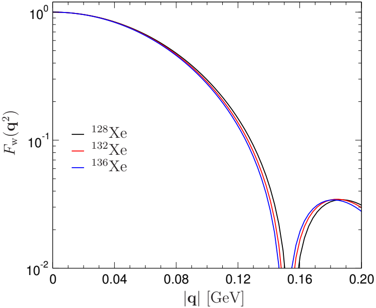
Results for the weak form factor



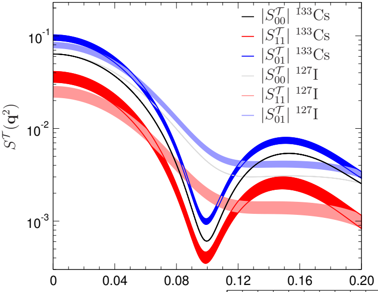


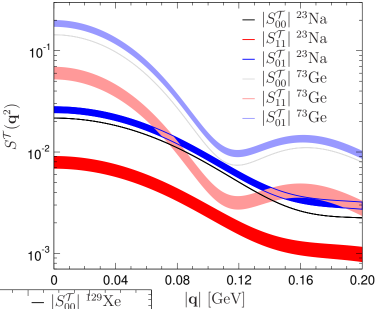
Coupled cluster: Payne et al. 2019 RMF: Yang et al. 2019

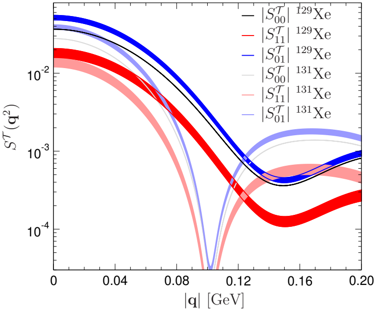




Axial-vector responses for CEνNS







Problems

1. Derive the WIMP–nucleon and WIMP–nucleus cross section for a scalar interaction $\mathcal{L} = \frac{1}{\Lambda^3} \sum_q C_q^{SS} \bar{\chi}\chi\, \bar{q}q$ for a Dirac spin-$1/2$ WIMP. What changes in the Majorana case?

   Hint: you can focus on the isoscalar part of the interaction, in which case you should find $\sigma_{\chi N} = \frac{\mu_N^2}{\pi} |c_+^M|^2$ *for the single-nucleon cross section at threshold (with* μ_N *the reduced WIMP–nucleon mass and* $c_+^M = \frac{m_N}{\Lambda^3} \sum_q C_q^{SS} f_q^N$*) and* $\frac{d\sigma_{\chi\mathcal{N}}}{dq^2} = \frac{\sigma_{\chi N}}{4\mu_N^2 v^2} |\mathcal{F}_+^M(q^2)|^2$ *for the differential WIMP–nucleus cross section.*

2. Verify the expression for $\sigma_{\chi N}$ in Higgs portal models given in the lecture.

   Hint: calculate the cross section and the invisible Higgs decay width Γ_{inv} *in terms of the Lagrangian coupling, and then replace the latter in* $\sigma_{\chi N}$ *in favor of* Γ_{inv}.