Homework 1: m=1, l=2, V(r)=-20 Exp[-r^2], E=0.5 : determine and plot u(r) using Mathematica **NDSolve**

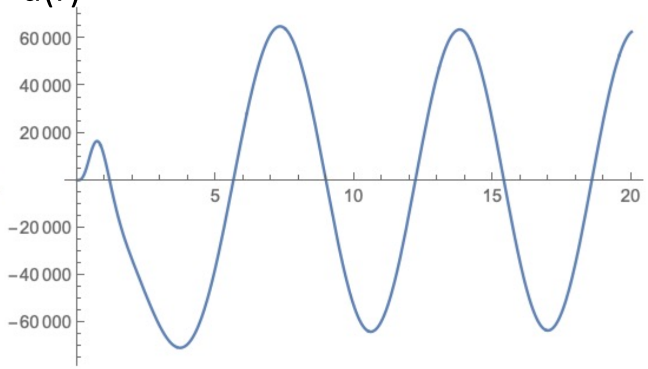
take boundary condition: u(0)=0, u'(0)=1; caution: 1/r^2 -> 1/(r^2+eps) to avoid singularity

```
m = 1; l = 2; en = 0.5; V[r_] := -20 * Exp[-r^2]; eps = 10^(-5);
rep = NDSolve[
  {-u''[r] / (2 * m) + l * (l + 1) / (2 * m * r^2 + eps) * u[r] + V[r] * u[r] == en * u[r],
   u[0] == 0, u'[0] == 1}, u, {r, 0, 20}]
Plot[u[r] /. rep, {r, 0, 20}]
```

Out[31]= {{u → InterpolatingFunction[Domain: {{0., 20.}} Output: scalar]}}

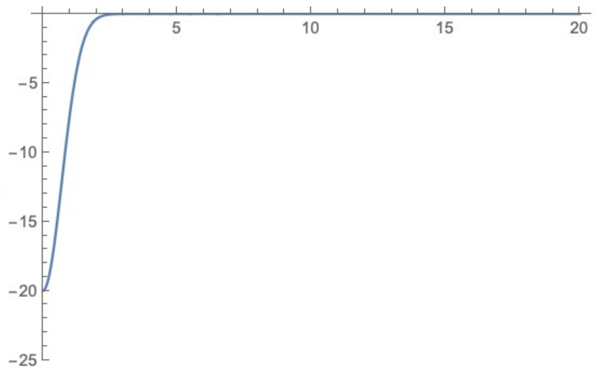
u(r)

Out[32]=



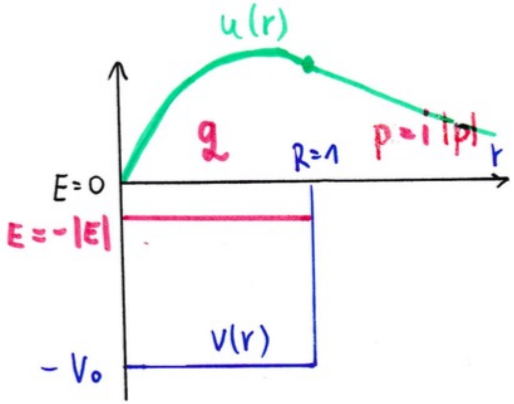
V(r)

Out[33]=



Homework 3a

Spherical well in 3D : bound state (l=0)



$A\ \sin qr \qquad B\ e^{-|p|r}$

$$\psi(r) = R(r) = \frac{u(r)}{r} \quad (l=0)$$

$$\left.\begin{aligned} u(R) &= A\ \sin qR = B\ e^{-|p|R} \\ u'(R) &= qA \cos qR = -|p|\,B\ e^{-|p|R} \end{aligned}\right\} \quad \frac{1}{q}\tan qR = -\frac{1}{|p|} \longrightarrow$$

example: deuterium (pn, J=1)

V0=36 MeV, R=2 fm, $m=m_N/2$

$E = E_B = -2\ MeV$

$q^2/2m = E + V0$

$p^2/2m = E < 0$

$p=+\,i\,|p|$: we will call this Riemann sheet 1, Im(p) > 0

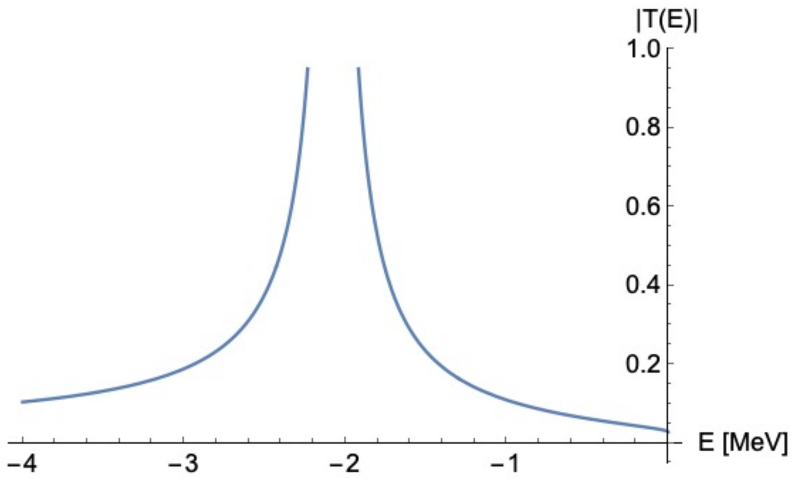
```
mN = 940;  mr = mN / 2;  V0 = 36;  R = 2 / hbarc;  hbarc = 197;
(* all in MeV *)
q[en_] := Sqrt[2 * mr * (en + V0)];
FindRoot[Tan[ q[en] * R] / q[en] == -1 / Sqrt[2 * mr * (-en)], {en, -2}]
Out[•]= {en → -2.07166}
```

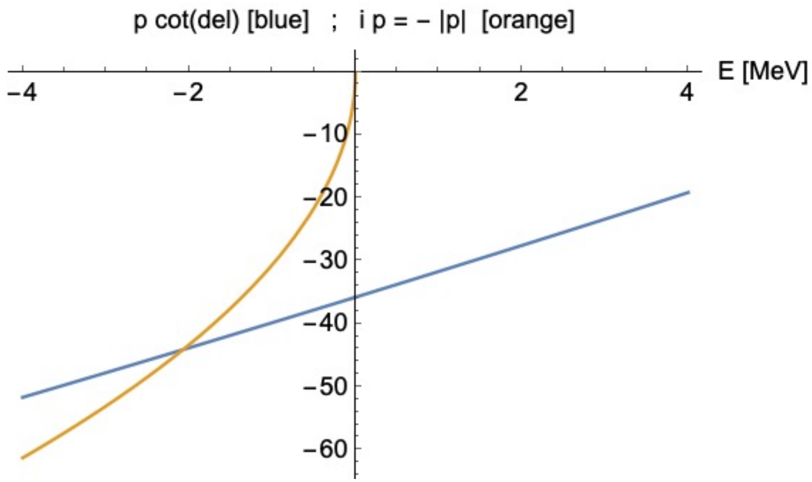
Homework 3b,c (continuation of 3a)

```
p[en_] := Sqrt[2 * mr * en]; q[en_] := Sqrt[2 * mr * (en + V0)];
del[en_] := ArcTan[p[en] / q[en] * Tan[q[en] * R]] - p[en] * R;
T[en_] := 1 / (p[en] * (Cot[del[en]] - I) );

Plot[Abs[T[en]] , {en, -4, 0}, AxesLabel → {"E  [MeV]", "|T(E)|"},
 AxesStyle → Directive[Black, Medium]]

Plot[{p[en] * Cot[del[en]] , I * p[en]}, {en, -4, 4},
 AxesLabel → {"E  [MeV]", "p  cot(del)  [blue]    ;    i  p  =  -  |p|   [orange]"},
 AxesStyle → Directive[Black,
   Medium]]
```





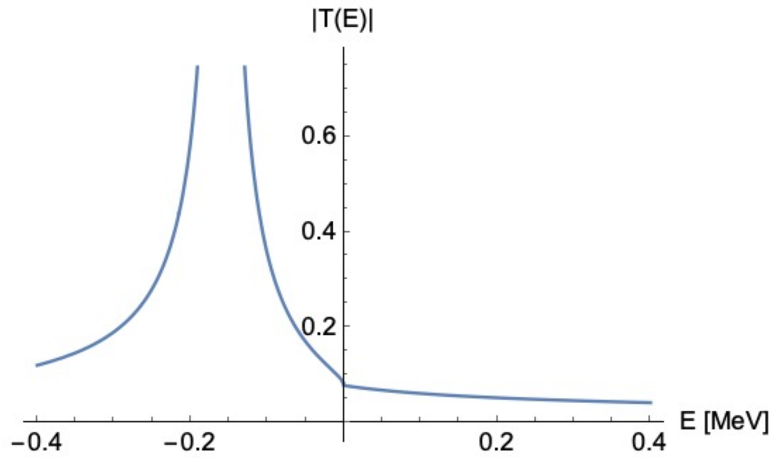
Homework 4

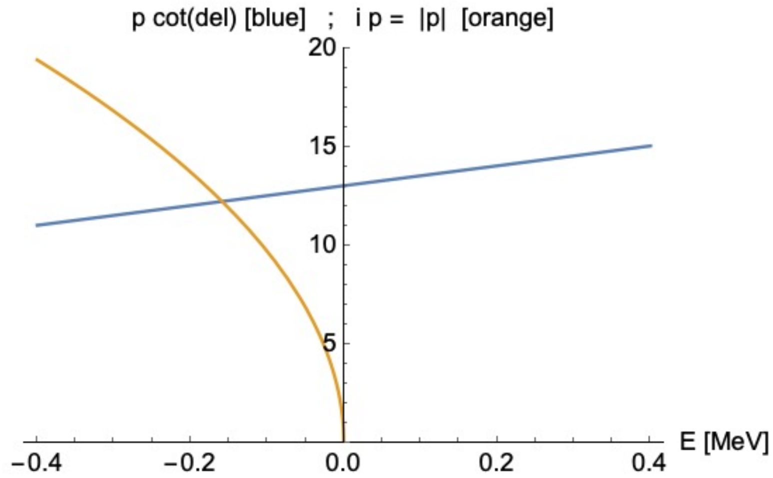
(same as 3, just V0 different and p=-i|p|)

```
mN = 940;  mr = mN / 2;  V0 = 23;  R = 2 / hbarc;  hbarc = 197;  (* all in MeV *)
p[en_] := -Sqrt[2 * mr * en];  q[en_] := Sqrt[2 * mr * (en + V0)];
del[en_] := ArcTan[p[en] / q[en] * Tan[q[en] * R]] - p[en] * R;
T[en_] := 1 / (p[en] * (Cot[del[en]] - I) );

Plot[Abs[T[en]] , {en, -0.4, 0.4}, AxesLabel → {"E [MeV]", "|T(E)|"},
 AxesStyle → Directive[Black, Medium]]

Plot[{p[en] * Cot[del[en]] , I * p[en]}, {en, -0.4, 0.4},
 AxesLabel → {"E [MeV]", "p cot(del) [blue]   ;   i p =  |p|  [orange]"},
 AxesStyle → Directive[Black, Medium], PlotRange → {0, 20}]
```





Homework 6: for square well potential with a given V0, R=1 and l=0:

Taylor expand p cot(δ) to order O(p^2) and extract a0 and r0 (using Mathematica)

l=0: $$p \cot\delta_0(p) = \frac{1}{a_0} + \frac{1}{2} r_0\, p^2$$ a_0: scattering length

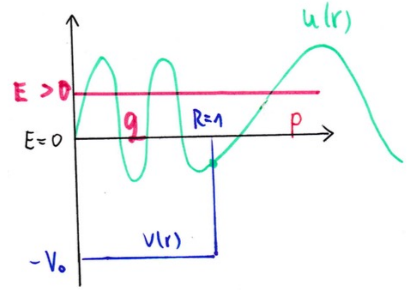
$$\delta_0(p) = \arctan\left(\frac{p}{q}\tan(qR)\right) - pR + n\pi$$

$$q = \sqrt{\underbrace{2\mu V_0}_{C^2} + p^2} = \sqrt{C^2 + p^2}$$

we defined constant C, which depends on reduced mass mu and V0

Taylor expanding with Mathematica

$$p \cot\delta_0(p) = \underbrace{\frac{C}{-C+\mathrm{Tan}[C]}}_{1/a_0} + \underbrace{\frac{1}{6}\left(3 - \frac{C^2}{(C-\mathrm{Tan}[C])^2} - \frac{3}{C^2 - C\,\mathrm{Tan}[C]}\right)}_{½\ r0} p^2 + O[p]^4 = \frac{1}{a_0} + \frac{1}{2} r_0\, p^2$$



Homework 9: this one is not suposed to be done independently by students

caution: this derivation is slopy concerning signs and factors of 1/2

$$F_{00,00} = F_S = \left(\frac{1}{L^3}\sum - \int\frac{d\vec{k}}{(2\pi)^3}\right)\frac{1}{2\omega_k}\,\frac{i}{E}\,\frac{1}{E-2\omega_k+i\varepsilon} \qquad 4\pi Y_{00}^2 = 1$$

$$\frac{E+2\omega_k}{E^2-(2\omega_k)^2} \xrightarrow{E=2\omega_k} \frac{4\omega_k}{4(p^2+m^2)-4(k^2-m^2)+i\varepsilon} = \frac{\omega_k}{p^2-\vec{k}^2+i\varepsilon}$$

$$= \left(\frac{1}{L^3}\sum - \int\frac{d\vec{k}}{(2\pi)^3}\right)\frac{i}{2E}\,\frac{1}{p^2-\vec{k}^2}\lim_{q\to 0} e^{-q(k^2-p^2)}$$

$$= \lim_{q\to 0}\left(\frac{1}{L^3}\sum - \mathrm{P.V.}\int\frac{d\vec{k}}{(2\pi)^3}\right)\frac{i}{2E}\,\frac{e^{-q(k^2-p^2)}}{p^2-k^2} + \frac{p}{8\pi E}$$

$$\downarrow$$

$$\frac{1}{p^2-k^2+i\varepsilon} = \mathrm{P.V.}\,\frac{1}{p^2-k^2} - i\pi\,\underbrace{\delta(p^2-k^2)}_{\frac{1}{2p}\delta(p-k)}$$

this $F_{00,00} = F_S$ agrees with $F_S = \frac{p}{8\pi E} - \frac{i}{2E}c_p$ (53)

with c_p (69) from Kim, Sachrajda, Sharpe (2005)

$$c^P_{lm}(q^{*2}) = -\frac{\sqrt{4\pi}}{\gamma L^3}\left(\frac{2\pi}{L}\right)^{l-2} Z^d_{lm}[1;(q^*L/2\pi)^2] \quad (74)$$

$$F_s \equiv F_{00;00} = \frac{q^*}{8\pi E^*} - \frac{i}{2E^*}c^P(q^{*2})\,.$$

$$c^P(q^{*2}) = \frac{1}{\gamma L^3}\sum_{\vec{k}}\frac{e^{\alpha(q^{*2}-r^2)}}{q^{*2}-r^2} - \frac{1}{\gamma}\mathcal{P}\int\frac{d^3k}{(2\pi)^3}\frac{e^{\alpha(q^{*2}-r^2)}}{q^{*2}-r^2}$$

Homework 11: use a0 and r0 and

determine pB and binding energy of DK system using mD and mK from exp

	$m_\pi = 290$ MeV	$m_\pi = 150$ MeV	Expt.
a_0 [fm]	$-1.13(0.04)(+0.05)$	$-1.49(0.13)(-0.30)$	
r_0 [fm]	$0.08(0.03)(+0.08)$	$0.20(0.09)(+0.31)$	
$\lvert p_B \rvert$ [MeV]	$180(6)(0)$	$142(11)(-9)$	
Δm [MeV]	$40(3)(0)$	$26(4)(-3)$	$42.6(0.7)(2.0)$

```
hbarc = 0.197;
rep = Solve[-hbarc / 1.13 + 1 / 2 * 0.08 / hbarc * (-pB ^ 2) == -pB]
mD = 1.86; mK = 0.5;
Deltam = (mD + mK) - Sqrt[mD ^ 2 - pB ^ 2] - Sqrt[mK ^ 2 - pB ^ 2] /. rep
```

p=i|p| pB=|p|

```
= {{pB → 0.180987}, {pB → 4.74401}}

= {0.0427325, 2.36 - 9.08177 i}
```

binding energy (it does not match table exactly since exp mD and mK were used)

Homework 12

Example of 1D irrep: P=(1,1,0)

symmetry group C_{2v} or Dic_2

Leskovec, SP: 1202.2145

$C_n(v)$: rotation by angle 2*pi/n around v

sigma(v): reflection with respect to plane perpendicular to v

B3 is one-dimensional irrep: only character appears in projector

names of irreps depend on reference

respresent.	dim	Id	$C_2(e_x+e_y)$	$\sigma(e_x-e_y)$	$\sigma(e_z)$
irred. A_1	1	1	1	1	1
irred. A_2	1	1	1	-1	-1
irred. B_3	1	1	-1	1	-1
irred. B_2	1	1	-1	-1	1

$$O_{\vec{P},\Gamma} = \sum_{\tilde{R}\in\Gamma} \chi^{\Gamma}(\tilde{R})\; H_1(\tilde{R}\vec{p}_1) H_2(\tilde{R}\vec{p}_2)$$

This operator can be used to get info on scattering in partial waves l=1 and l=2

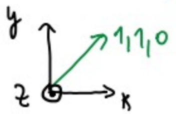
(unfortunately both mix in the quantization condition via non-diagonal G)

$$O_{P=110,B_3} = \sum_R \chi_{B_3}(R)\; H_1(R(1,0,1))\; H_2(R(0,1,-1)) =$$

$(1,0,1)+(0,1,-1)=(1,1,0)$

$$= H_1(1,0,1)H_2(0,1,-1) - H_1(0,1,-1)H_2(1,0,1)$$

$R=I,\ \chi=1$ $\qquad$ $R=C_2(1,1,0),\ \chi=-1$



inverts x,y
$z\to -z$

$$+ H_1(0,1,1)H_2(1,0,-1) - H_1(1,0,-1)H_2(0,1,1)$$

$R=\sigma(1,-1,0)\quad \chi=1$ $\qquad$ $R=\sigma(0,0,1)\quad \chi=-1$

inverts x,y
z : inv.

leaves x,y inv.
inverts z