Solution Exercises. Part 1

July 21, 2021

1 Lecture 1

1.1 Exercise 1

Using the Vlasov equation discussed in the lecture, compute the electric field created by a static point-like charge inside of a medium. Show that it is related with the Yukawa potential.

First, we write the Electric field in terms of the electric potential

$$\mathbf{E} = \nabla V \tag{1}$$

Now we consider the equation

$$\nabla \mathbf{D} = \rho \tag{2}$$

and we perform its Fourier transform to momentum space

$$i\epsilon_L(k)\mathbf{k}\mathbf{D}(k) = \rho(k) \tag{3}$$

which implies that

$$-|\mathbf{k}|^2\epsilon_L(k)V(k) = \rho(k) \tag{4}$$

A static point-like charge can be characterized in momentum space with the following formula

$$\rho(k) = 2\pi\delta(k_0)Q \tag{5}$$

We also note that

$$\lim_{k_0\to 0} |\mathbf{k}|^2\epsilon_L(k) = |\mathbf{k}|^2 + m_D^2 \tag{6}$$

Therefore,

$$V(k) = -\frac{2\pi\delta(k_0)Q}{|\mathbf{k}|^2 + m_D^2} \tag{7}$$

Now, we only need to do the Fourier transform to coordinate space to obtain the solution

$$V(\mathbf{x}) = -Q\frac{d^3k}{(2\pi)^3}\frac{\mathrm{e}^{i\mathbf{k}\mathbf{x}}}{k^2 + m_D^2} = -\frac{Q}{2\pi x}\int_0^\infty \frac{dk k\sin(kx)}{k^2 + m_D^2} = -\frac{Q\mathrm{e}^{-m_D x}}{4\pi x} \tag{8}$$

So, we get

$$\mathbf{E} = -\frac{Q}{4\pi}\nabla\left(\frac{\mathrm{e}^{-m_D x}}{x}\right) \tag{9}$$

1.2 Exercise 2

Compute the temporal component of the photon propagator in the HTL approximation using as a guide what we discussed in the lecture.

You can find a complete description of this exercise in the review of Thoma Thoma, New developments and applications of thermal field theory, ArXiv:hep-ph/0010164