EXERCISE 1

According to the naive counting, how many contractions are required for a nucleus at the source and sink with atomic numbers A = 4, 8, 12, 16? How many contractions are there with the use of the efficient algorithm described (ignore the pre-factor $M_w \cdot N_w$)?

EXERCISE 2

With a given amount of computational resources, you have achieved a 1% statistical uncertainty on the extracted mass of the nucleon from your lattice QCD calculation. By what factor should you increase your computing resources (your statistics) to also achieve a 1% statistical uncertainty on the binding energy of the deuteron?

EXERCISE 3

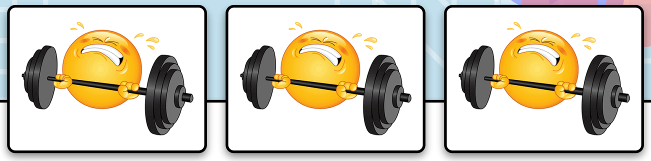
Consider a simple two-state model in the spectral decomposition of a Euclidean two-point function. Demonstrate that the time scale to reach the ground state of the model with a finite statistical precision can depend highly on the corresponding overlap factor for the state. It is sufficient to show this numerically and for a set of chosen energies and overlap factors.

EXERCISE 4

Consider the simpler case of SU(2) symmetry and only the proton and neutron. This exercise helps you classify two-nucleon systems. and hopefully allows you to also make sense of SU(3) classifications.

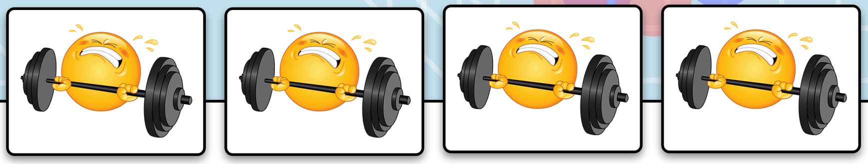
Isospin symmetry, that is a SU(2) symmetry in the space of up and down quark flavors, is a good approximate symmetry of strong interactions. Consider two-nucleon (*NN*) systems in nature under this symmetry. Classify positive-parity *NN* systems according to their spin, isospin, and angular momentum. Recall that fermionic wavefunctions must be fully antisymmetric under exchange of the fermions. Each nucleon has spin 1/2 and isospin 1/2. Proton (p) and neutron (n) have $I_3 = 1/2$ and $I_3 = -1/2$, respectively, where I_3 is the third component of isospin. Write down the states explicitly according to their spin and flavor content. Which one of these channels exhibit a bound state in nature?

EXERCISE 5



Starting from the leading-order SU(3) flavor-symmetric Lagrangian for interactions of two octet baryons, derive the relation between the scattering length and Savage-Wise coefficients in the 27 irreducible representation. You can express the scattering amplitude in terms of a leading-order effective range expansion and set μ equal to zero (assuming natural interactions).

BONUS EXERCISE 1



Repeat the same exercise for all other irreducible representations of SU(3). If you have already automated this procedure using Mathematica or other programs in the above exercise, all relations can be obtained at the same time.

EXERCISE 6

If the computational resources do not allow large source, operator, and sink time separations to be achieved, one should worry about the effect of excited states. One way to have more confidence over the extracted ground state to ground state matrix element is to perform a multi-exponential fits to the ratio of 3pt to 2pt functions as a function of both the source-sink and the source-operator separations. Assume that both the ground state and the first excited states contribute significantly to such a ratio. Write down a generic form for such a multi-exponential function.

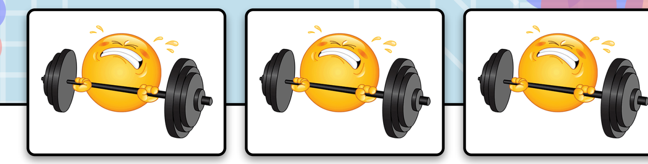
BONUS EXERCISE 2

In the above exercise, sum over the time insertions of the operator and write down a new form for the ratio of 3pt to 2pt functions, which now is only a function of the source-sink time separation. This is referred to as the summation method in literature.

EXERCISE 7

In each of the four examples in the previous slide, what is the dependence of the modified correlation functions plotted to the parameter λ_q?

EXERCISE 8



Suppose the desired quantity in the lattice QCD calculation is the axial charge radius of the nucleon. This is a quantity that is important for constraining various BSM EFTs. How would you obtain this quantity from the calculation of the axial nucleon matrix elements? [Compare with the case of electric charge radius.] What uncertainties do you expect in such a determination?