Constraining the strength of $U(1)_A$ symmetry breaking using a non-local chiral quark model

Chowdhury Aminul Islam

University of Chinese Academy of Sciences

Based on **arXiv:2009.13563**

Collaborators: **Mahammad Sabir Ali & Rishi Sharma**

Methods of Effective Field Theory and Lattice Field Theory
Bad Honnef Physics School

July 23, 2021

Introduction

- We will be using NJL model as chiral quark model.

- With the help of the NJL model one can study the chiral properties of QCD. [Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345(1961); 124, 246(1961)]

- In the NJL model the chiral condensate which breaks the chiral symmetry spontaneously, increases with magnetic field(eB) for all temperature(T), termed as magnetic catalysis(MC). Whereas lattice QCD simulation obtained decrease in chiral condensate around the crossover T as eB increases which is termed as inverse magnetic catalysis(IMC). [G. S. Bali et al., Phys. Rev. D 86, 071502]

- One can consider non-local interaction in NJL model which is more realistic as it captures some aspects of the asymptotic freedom of QCD through the non-local form factor. [V. P. Pagura et al., Phys. Rev. D 95, 034013]

Motivation

- Standard NJL model assume the strength of the axial U(1) symmetry breaking ’t Hooft determinant term to be equal to that of the axial U(1) symmetric term, even in the non-local one.

- Our main goals:

  a) To constrain the strength of the $U(1)_A$ symmetry breaking interactions using lattice QCD results.

  b) And once it is fixed, study the effect of T and eB on it.

NJL model with different $U(1)_A$ breaking strength

- The NJL Lagrangian [M. Frank, M Buballa, M. Oertel, Phys. Lett. B 562 (2003) 221-226]

$$\mathcal{L}_{\mathrm{NJL}} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2$$

$$\mathcal{L}_0 = \bar{\psi}\left(i\partial\!\!\!/ - m\right)\psi$$

$$\mathcal{L}_1 = G_1\left\{(\bar{\psi}\psi)^2 + (\bar{\psi}\vec{\tau}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right\}$$

$$\mathcal{L}_2 = G_2\left\{(\bar{\psi}\psi)^2 - (\bar{\psi}\vec{\tau}\psi)^2 - (\bar{\psi}i\gamma_5\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right\}$$

- $\mathcal{L}_2$ explicitly breaks $U(1)_A$.
- Symmetry only allows the $\langle\bar{\psi}\psi\rangle$ condensate, which depends on $(G_1 + G_2)$.
- With μ_I or magnetic field as the $SU(2)$ symmetry is broken one can have $\langle\bar{\psi}\tau_3\psi\rangle$ which has a $(G_1 - G_2)$ dependence.
- G_1 and G_2 can be parametrized as $G_1 = (1-c)G_0/2$ and $G_2 = cG_0/2$
- $c = 1/2$ corresponds to the standard NJL model.

Non-local NJL model with arbitrary $U(1)_A$ breaking strength

- The NJL Lagrangian with non-local interaction

$$\mathcal{L} = \bar{\psi}\left(i\partial\!\!\!/ - m\right)\psi + \mathcal{L}_1 + \mathcal{L}_2.$$

- $\mathcal{L}_1$ respects $U(1)_A$ but $\mathcal{L}_2$ does not,

$$\mathcal{L}_1 = G_1\left\{j_a(x)j_a(x) + j_b(x)j_b(x)\right\}$$

$$\mathcal{L}_2 = G_2\left\{j_a(x)j_a(x) - j_b(x)j_b(x)\right\}$$

  with the above definition of $j_{a/b}(x)$ where $\Gamma_a = (\mathbb{I}, i\gamma_5\vec{\tau})$ and $\Gamma_b = (i\gamma_5, \vec{\tau})$

- $j_{a/b}(x)$ are the non-local currents, given by

$$j_{a/b}(x) = \int d^4z\ \mathcal{G}(z)\bar{\psi}\left(x + \tfrac{z}{2}\right)\Gamma_{a/b}\psi(x - \tfrac{z}{2}),$$

$$\Gamma_a = (\mathbb{I}, i\gamma_5\vec{\tau})\ \text{and}\ \Gamma_b = (i\gamma_5, \vec{\tau})$$

- $\mathcal{G}(z)$ is the non-locality form factor.
- Symmetries

$$SU(2)_V \times SU(2)_A \times U(1)_V$$

Non-local NJL model with arbitrary $U(1)_A$ breaking strength

- With non-zero mean field $\langle\bar{\psi}\psi\rangle$, the free energy becomes

$$\frac{\Omega_{\rm MF}}{V^{(4)}} = \frac{\sigma^2}{2G_0} - 2N_c \sum_f \int \frac{d^4p}{(2\pi)^4} \log\left[p^2 + M_f^2(p)\right]$$

with $M(p) = m + g(p^2)\sigma$

- With Lorentz symmetry the Fourier transformation $\mathcal{G}(z)$ can only depends on p^2, which is written as $g(p^2)$.

- We have consider $g(p^2)$ to be Gaussian in nature

$$g(p^2) = \exp[-p^2/\Lambda^2]$$

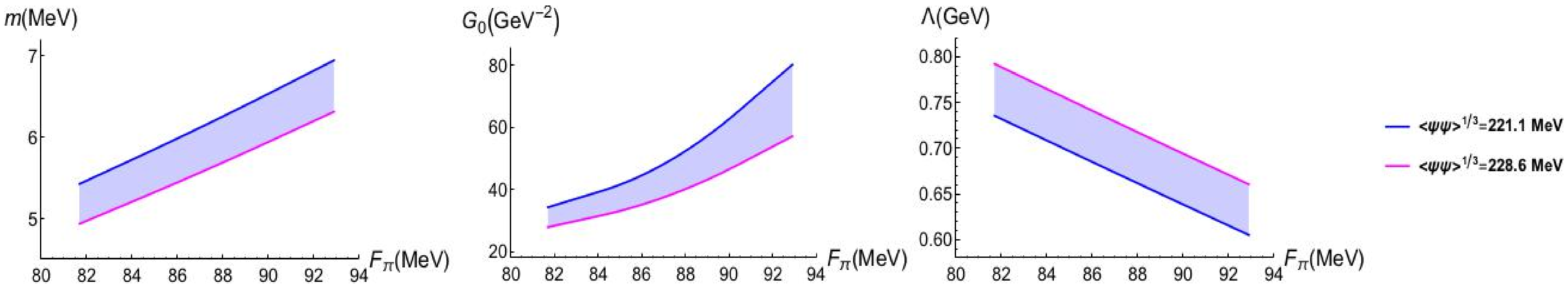
Parameter dependence

- Condensate and F_{π} taken from LQCD calculation with $m_{\pi} = 135$ MeV. [H. Fukaya et al.(JLQCD), Phys. Rev. D 77, 074503 (2008)]

$$\langle \bar{\psi}\psi \rangle^{1/3}|_{\mu=2\ \mathrm{GeV}} = 240(4)\ \mathrm{MeV\ and}$$
$$F_{\pi} = 87.3(5.6)\ \mathrm{MeV}.$$

- We have used perturbative RG running to obtain the condensate at 1 GeV following the L. Giusti's work. [L.Giusti et al., Nuclear Physics B 538 (1999) 249-277]

$$\langle \bar{\psi}\psi \rangle^{1/3}|_{\mu=1\ \mathrm{GeV}} = 224.8(3.7)\ \mathrm{MeV}.$$



Parameter dependence

- Central and the four corner parameter sets:

	$\langle\bar{\psi}_f\psi_f\rangle^{1/3}$(MeV)	m_π(MeV)	F_π(MeV)	$F_{\pi,0}$(MeV)	m(MeV)	G_0(GeV^{-2})	Λ(MeV)
Parameter Set CC	224.8	135	87.3	84.25	5.87	43.34	697.22
Parameter Set HH	228.6	135	92.9	90.63	6.31	57.15	660.46
Parameter Set HL	228.6	135	81.7	77.04	4.94	27.90	792.22
Parameter Set LH	221.1	135	92.9	91.00	6.94	80.26	605.05
Parameter Set LL	221.1	135	81.7	77.61	5.42	34.32	735.38

Non-local NJL model in presence

- For the non-local interaction the currents should transform as

$$j_{a/b}(x) \to$$
$$\int d^4z\ \mathcal{G}(z)\bar{\psi}\left(x+\frac{z}{2}\right) W^{\dagger}(x+z/2,x)\Gamma_a W(x,x-z/2)\psi(x-\frac{z}{2})$$

$$W(s,t) = P\ \exp\left[-iQ\int_s^t dr_\mu A_\mu(r)\right]$$

- The bosonized action

$$S_{\rm bos} = -\ln\det(\mathcal{D}) + \int \mathrm{d}^4\mathrm{x}\left[\frac{\sigma^2(\mathrm{x})}{2\mathrm{G}_0} + \frac{\Delta\sigma^2(\mathrm{x})}{2(1-2\mathrm{c})\mathrm{G}_0}\right]$$

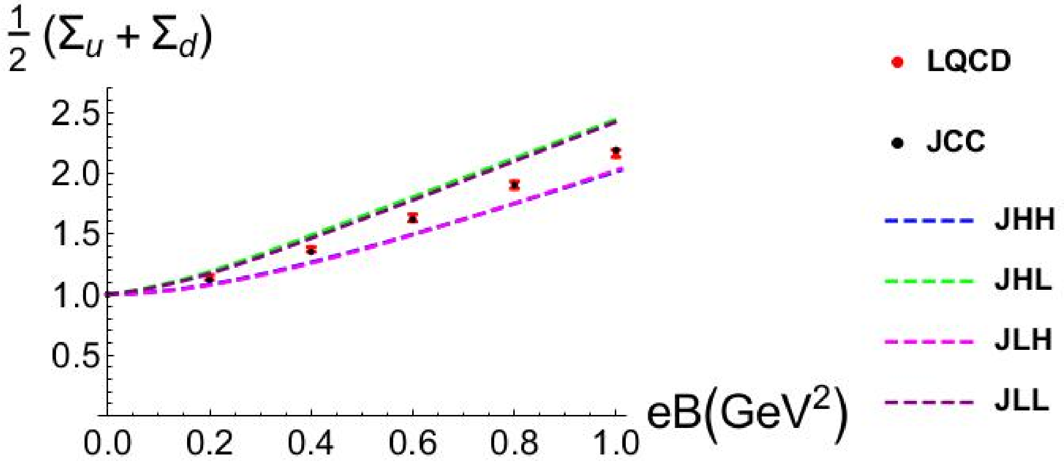
- In Lorentz gauge the inverse of fermionic propagator is given by

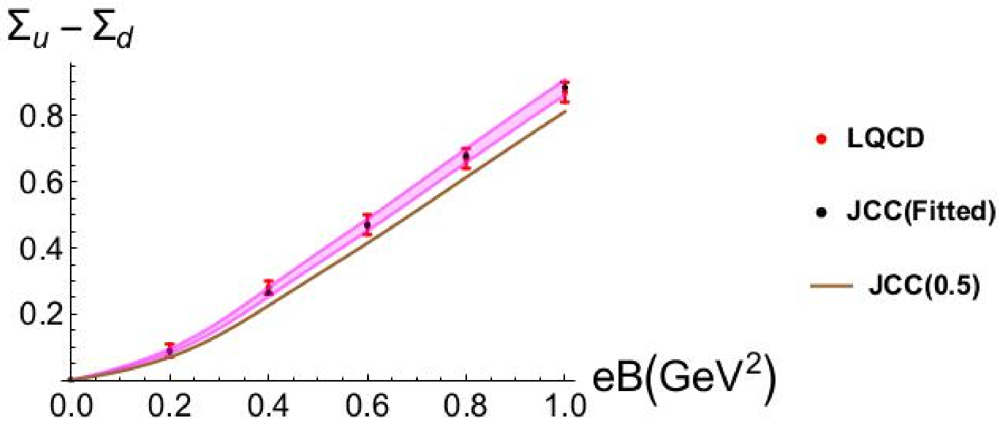
$$\begin{aligned}\mathcal{D}_{\rm MF}(x,x') &= \delta^{(4)}(x-x')(-i\partial\!\!\!/ - QBx_1\gamma_2 + m) + \mathcal{G}(x-x') \\ &\quad *(\sigma+\tau_3\Delta\sigma)\exp\left[\frac{i}{2}QB(x_2-x_2')(x_1+x_1')\right].\end{aligned}$$

- Using Ritus eigenfunction one obtain the Fourier transform of the above.

Fitting of c using LQCD data

- As mentioned earlier the average condensate does not depend on c.

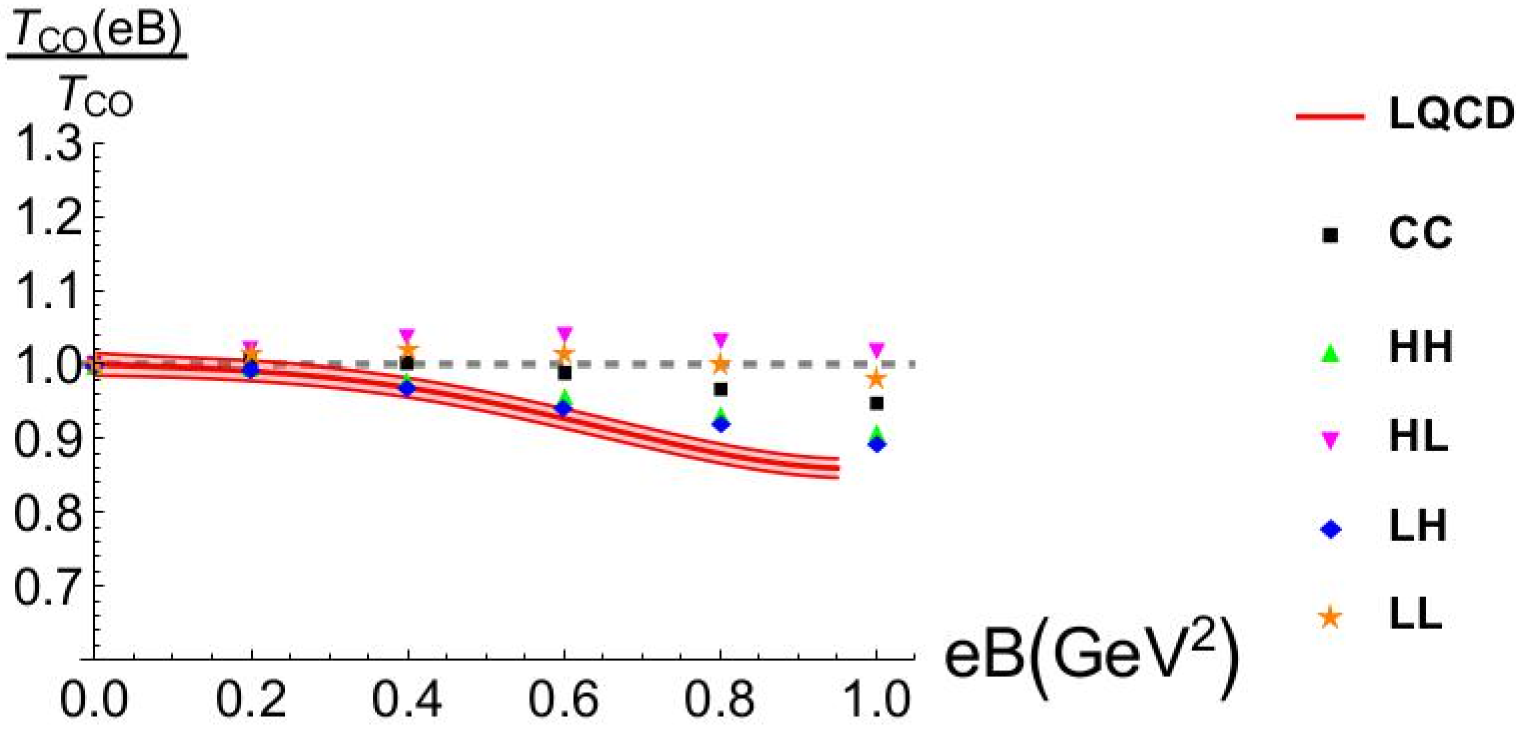




	c	χ^2 per DoF
Parameter Set CC	0.276 ± 0.068	0.211
Parameter Set HH	0.044 ± 0.079	0.149
Parameter Set HL	0.374 ± 0.051	0.290
Parameter Set LH	0.149 ± 0.103	0.634
Parameter Set LL	0.465 ± 0.062	0.551

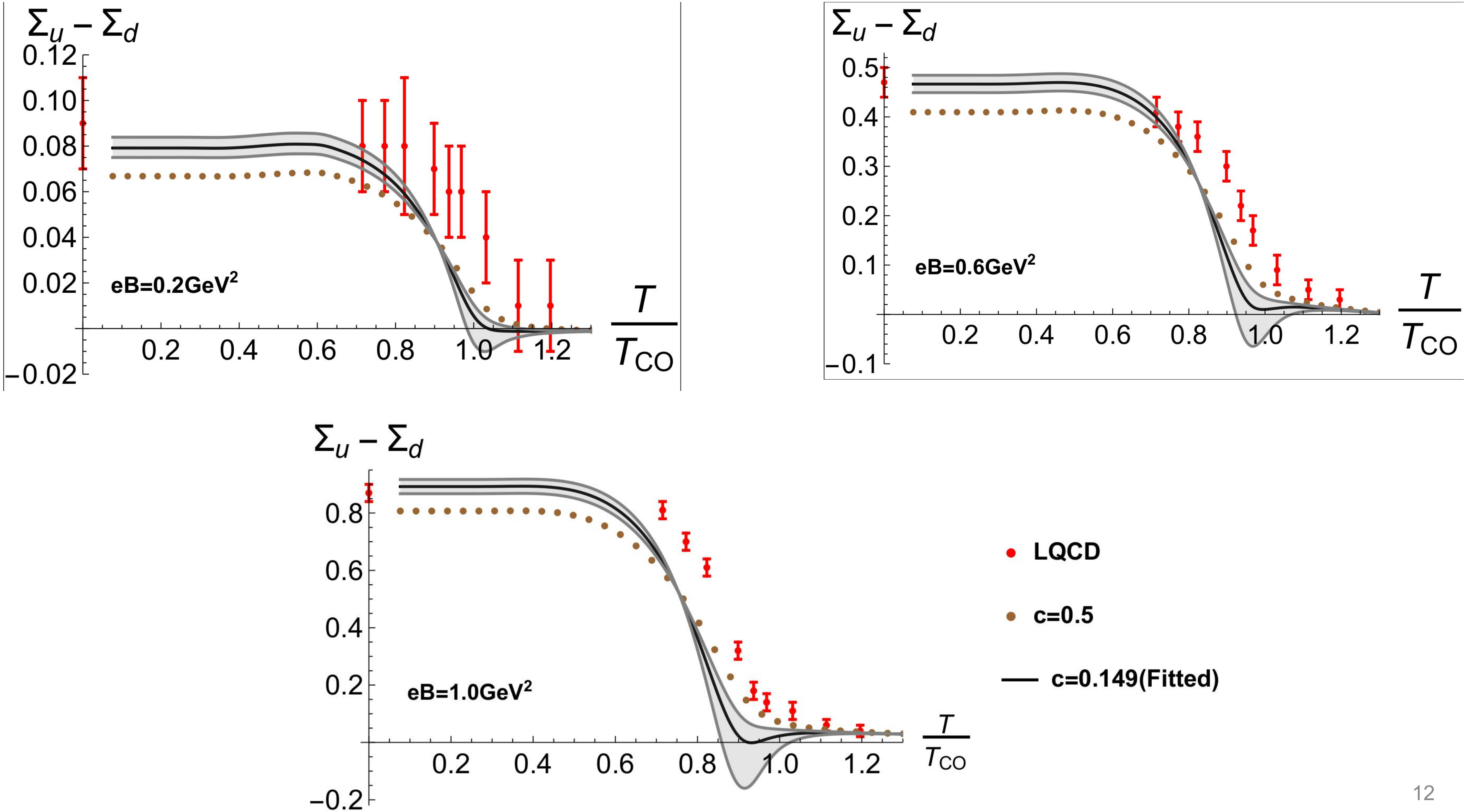
QCD phase diagram in $T - eB$ plane

- Transition temperature does not depend on c as the condensate is almost independent of c.



- Low condensate value and high F_π produce a stronger IMC effect around the crossover temperature.

Condensate difference at finite temperature



Condensate difference at finite temperature with constraint from M_{η^*} >400 MeV