Hadronic Form Factors for the b Anomalies

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Introduction

b decays are multi purpose laboratories

- extract Standard Model (SM) parameters
  - direct determination of CKM matrix elements $|V_{cb}|$ and $|V_{ub}$
  - determination of CKM angles (from non-leptonic decays)
- indirectly search for Beyond the Standard Model (BSM) effects in e.g.
  - $b \to s\mu^+\mu^-$ and $b \to se^+e^-$, including lepton-flavour universality (LFU) checks
  - $b \to c\tau^-\overline{\nu}$ and $b \to c\{e^-, \mu^-\}\overline{\nu}$ as LFU checks
- test our theoretical understanding of QCD

Experimental Landscape

theoretical predictions for the b decays are being challenged experimentally:

BaBar at the Stanford Linear Accelerator Center (California, USA)
e^+e^- collisions — ended 2008

Belle (II) at KEK (Tsukuba, Japan)
e^+e^- collisions — Belle ended 2010
Belle II data taking started in 2019

ATLAS,CMS,LHCb at the Large Hadron Collider (CERN)
pp collisions — LHC “run 2” ended in 2018
LHC “run 3” planned for 2022
⋮ ⋮ ⋮
LHC “run 5” planned until 2035

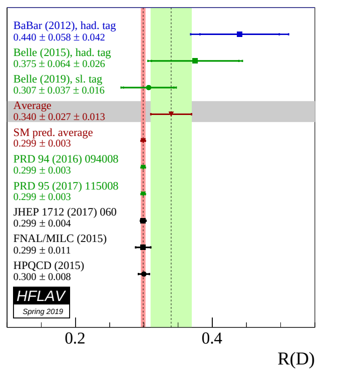
long-standing and significant tensions between measurements and theory predictions of more than $2\,\sigma$ but less than $5\,\sigma$ individually have been dubbed “b anomalies”

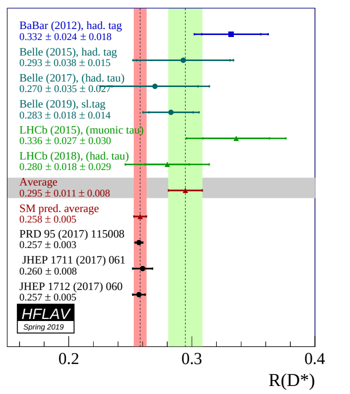
The anomalies in plots (1)

test of lepton flavour universality in $b \to c\ell\overline{\nu}$

[HFLAV Spring 2019]

$$\frac{b \to c\tau\overline{\nu}}{b \to c\mu\overline{\nu}} \quad \to \quad R(X) \equiv \frac{\Gamma\left(\overline{B} \to X\tau\overline{\nu}\right)}{\Gamma\left(\overline{B} \to X\mu\overline{\nu}\right)} \qquad X = D, D^*$$





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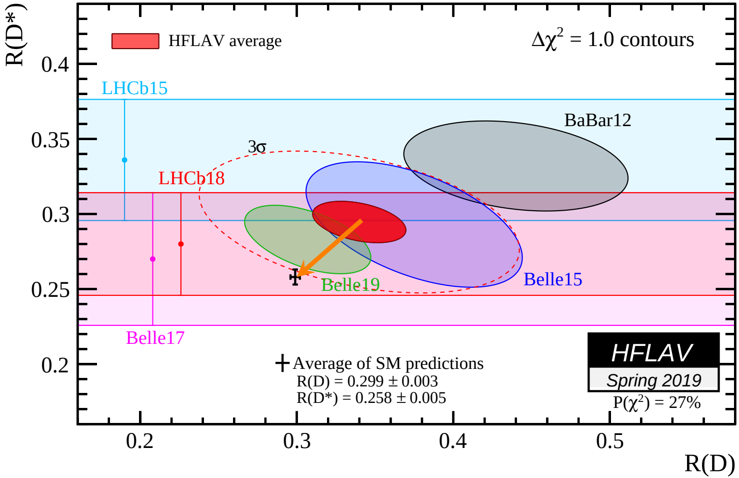
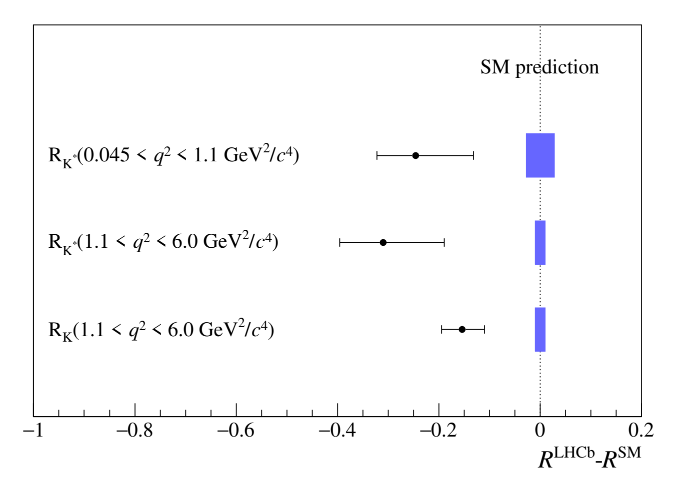


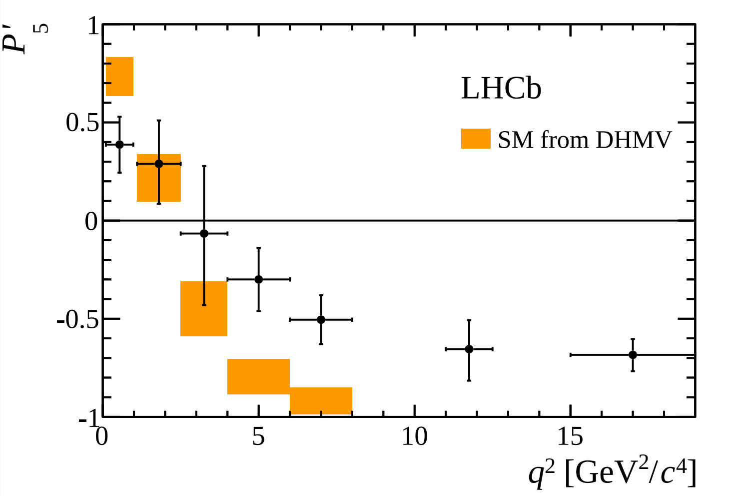
test of lepton flavour universality ($R(X)$), and angular distributions (P_5') in $b \to s\ell^+\ell^-$

[Albrecht,Langenbruch Physik-Journal '18 & update]



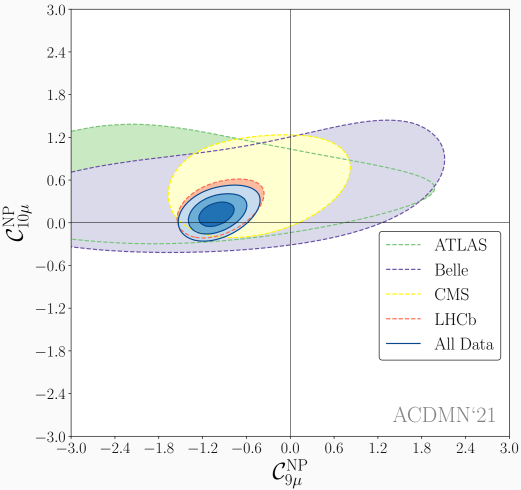
test of lepton flavour universality ($R(X)$), and angular distributions (P'_5) in $b \to s\ell^+\ell^-$

[LHCb]

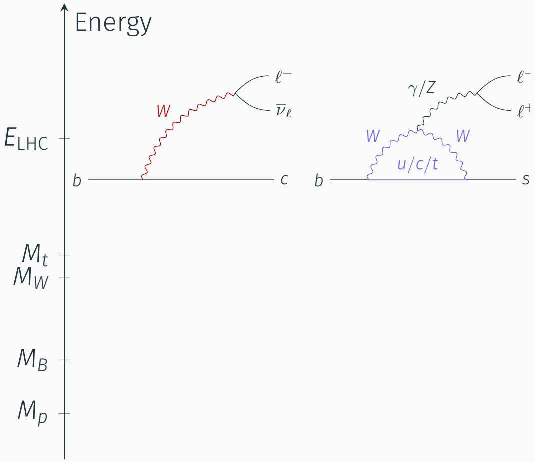


test of lepton flavour universality ($R(X)$), and angular distributions (P_5') in $b \to s\ell^+\ell^-$

[Alguero et al. '21]



- EFTs widely used tools of theoretical physics
- used to interpret the anomalies w/o assuming a concrete model beyond SM



Weak Effective Theory

- EFTs widely used tools of theoretical physics
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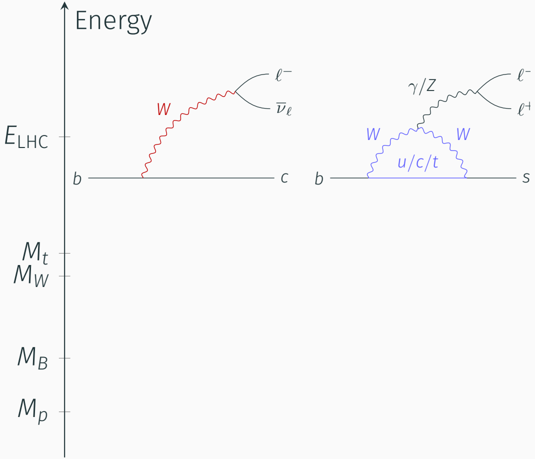
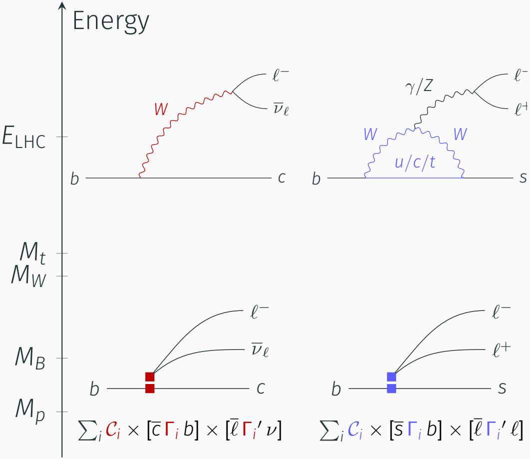
- replaces dynamical degrees of freedom (here: t, W, Z) with coefficients $\mathcal{C}_i$ and static structures Γ in local operators

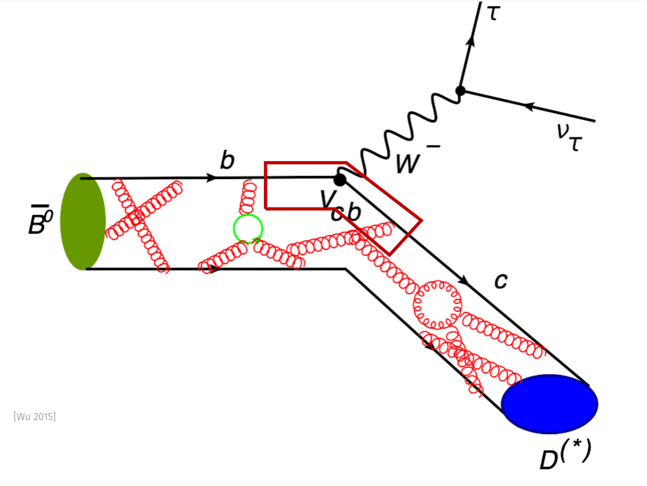


- EFTs widely used tools of theoretical physics
- used to interpret the anomalies w/o assuming a concrete model beyond SM
- replaces dynamical degrees of freedom (here: t, W, Z) with coefficients $\mathcal{C}_i$ and static structures Γ in local operators
- for $b \to c\ell\nu$
  - 5 semileptonic $[\bar{s}\Gamma b]\,[\bar{\ell}\Gamma'\ell]$ ops
- for $b \to s\ell\ell$
  - 10 semileptonic $[\bar{s}\Gamma b]\,[\bar{\ell}\Gamma'\ell]$ ops
  - large number of four-quark $[\bar{s}\Gamma b]\,[\bar{c}\Gamma' c]$ ops



$\overline{B}_{(s)} \to \{D_{(s)}, D^*_{(s)}\}\ell^-\overline{\nu}$

Matrix elements of local operators $\bar{c}\,\Gamma b$ parametrized through form factors



Hadronic form factors

Matrix elements of local operators $\bar{c}\,\Gamma b$ parametrized through form factors

- scalar functions of momentum transfer (typical notation: $q^2 = m^2_{\ell-\bar{\nu}}$)
- 3 independent functions for $0^- \to 0^-$ transitions (e.g. $B_q \to D_q$)
- 7 independent funktions for $0^- \to 1^-$ transitions (e.g. $B_q \to D_q^*$)
- total of 10 non-perturbative objects per spectator q
- use isospin symmetry to relate $q = u$ and $q = d$

QCD based approaches to control the form factors

- numerical simulation in discretized space time: lattice QCD (comp. expensive)
- QCD (light-cone) sum rules (large systematic uncertainties)
- heavy-quark expansion (relates D and D^*; only provides relations between form factors)

heavy-quark expansion of any of the 10+10 ($q = u, d$ and $q = s$) form factors:

$$F = \left(A(F) + \frac{\alpha_s}{\pi} B(F)\right)\xi + \sum_{i=1}^{6}\left[\frac{\Lambda}{2m_b} C_b^{(i)}(F) L_i + \frac{\Lambda}{2m_c} C_c^{(i)}(F) L_i\right] + \frac{\Lambda^2}{4m_c^2} D^{(i)}(F)\ell_i$$

$$+ \text{ higher order terms}$$

global fit of the form factors

- coefficients $A(F)$ to $D(F)$ can be computed
- non-perturbative functions ξ, L_i, and ℓ_i can be expanded in momentum transfer variable
  - fit their expansion coefficients
  - good fit w/ $\chi^2/\text{ndf} = 36.27/81$

theory inputs

- lattice QCD [FNAL/MILC; HPQCD]
- QCD light-cone sum rules [Gubernari,Kokulu,DvD '18]
- QCD sum rules [Ligeti,Neubert,Nir '92 & '93]

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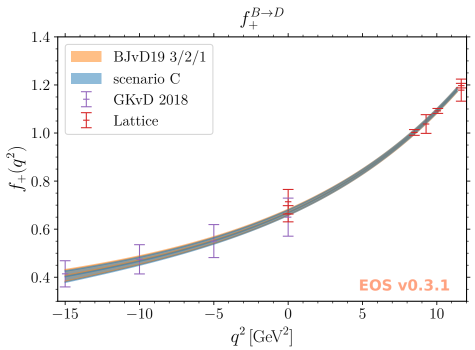


- determination of $|V_{cb}|$
- theory prediction of R_D and R_{D^*}

long-standing tension between $|V_{cb}|$ values extracted from exclusive $\overline{B} \to \{D, D^*\}\ell^-\overline{\nu}$ decay and inclusive $\overline{B} \to X_c\ell^-\overline{\nu}$ decay

- inclusive determination @ 3 loops, $1/m_b^2$ [Bordone,Capdevilla,Gambino '21]

$$|V_{cb}|^{\text{incl}} = (42.2 \pm 0.5) \times 10^{-3}$$

- average of exclusive determinations from global fit [Bordone,Gubernari,Jung,DvD '19]

$$|V_{cb}|^{\text{excl}} = (40.0 \pm 0.9) \times 10^{-3}$$

- mutually compatible at the 1.8σ level, with average

$$|V_{cb}|^{\text{avg}} = (41.1 \pm 0.5) \times 10^{-3}$$

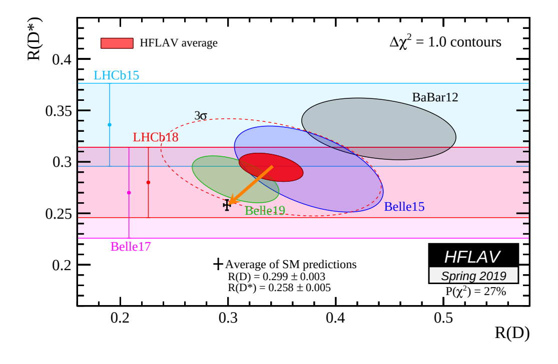
- determination of $|V_{cb}|$
- theory prediction of R_D and R_{D*}

- global fit makes precise predictions of LFU ratios possible

$$R(D) = 0.2989 \pm 0.0032$$
$$R(D^*) = 0.2472 \pm 0.0050$$

- $R(D)$ fixed due to precise lattice inputs
- $R(D^*)$ shifts down w.r.t. HFLAV point

- in good shape for upcoming precise Belle II & LHCb data



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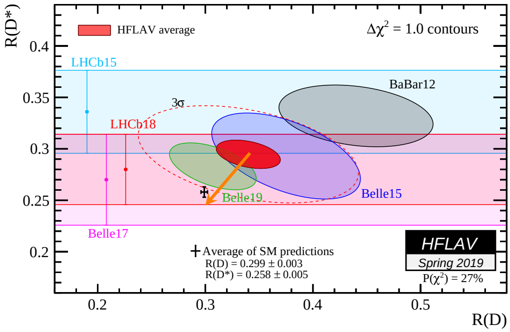
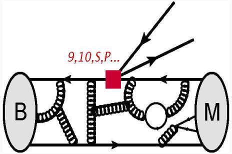
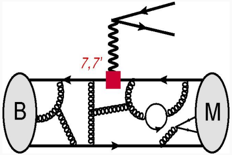


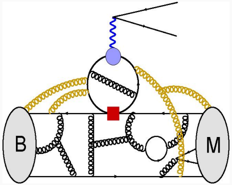
- significance increases to $> 4\sigma$

$B \to \{K, K^*\}\ell^+\ell^-$

Anatomy of exclusive $b \to s\ell^+\ell^-$ decay amplitudes







$$\mathcal{A}_\lambda^\chi = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2 m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

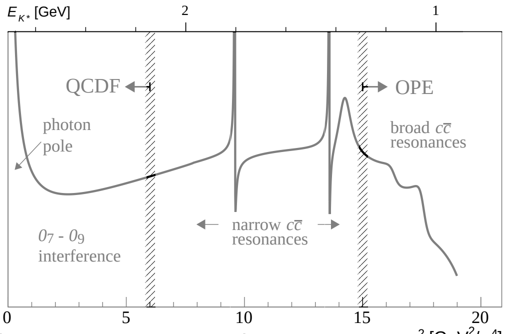
nomenclature of the essential hadronic matrix elements $\quad q^2 = m_{\ell\ell}^2$

- $\mathcal{F}_\lambda$ local form factors of dimension-three $\bar{s}\gamma^\mu b$ & $\bar{s}\gamma^\mu\gamma_5 b$ currents
- $\mathcal{F}_\lambda^T$ local dipole form factors of dimension-three $\bar{s}\sigma^{\mu\nu} b$ currents
- $\mathcal{H}_\lambda$ nonlocal form factors of dimension-five nonlocal operators

all three needed for consistent description to leading-order in α_e

Spectrum

$$\mathcal{H}_\lambda(q^2) = P(\lambda)_\mu \langle H_s| \int d^4x\, e^{iq\cdot x}\, \mathcal{T}\{ J^\mu_{\text{em}}(x), [C_1 O_1^c + C_2 O_2^c](0) \} |H_b\rangle$$



[sketch from Blake, Gershon, Hiller 1501.03309]

- $O^c_{1,2} \sim [\bar{s}\Gamma b]\ [\bar{c}\Gamma' c]$

source of dominant systematic uncertainties in theoretical predictions!

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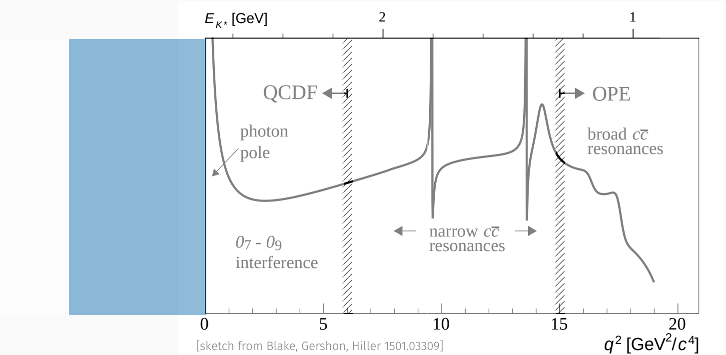
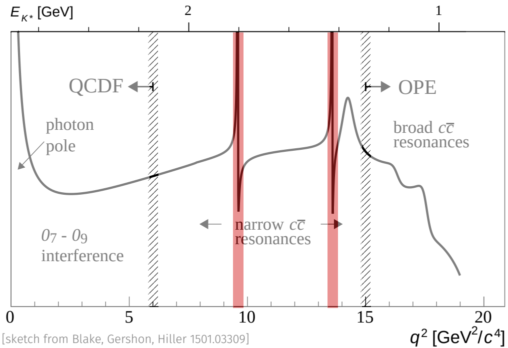


[sketch from Blake, Gershon, Hiller 1501.03309]

- $O_{1,2}^c \sim [\bar{s}\Gamma b]\ [\bar{c}\Gamma' c]$

- for $q^2 - 4m_c^2 \ll \Lambda_{\text{had}} m_b$, expand T-product in light-cone operators
- leading contributions expressed through local form factors $\mathcal{F}_\lambda$
- correction suppressed by $1/(q^2 - 4m_c^2)$ can by systematically obtained

$$\mathcal{H}_\lambda(q^2) = P(\lambda)_\mu \langle H_s| \int d^4x\, e^{iq\cdot x}\, \mathcal{T}\{ J^\mu_{em}(x), [C_1 O_1^c + C_2 O_2^c](0) \} |H_b\rangle$$



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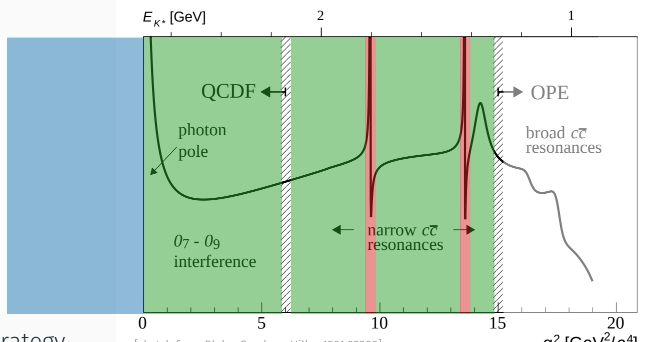
- $O^c_{1,2} \sim [\bar{s}\Gamma b]\ [\bar{c}\Gamma' c]$

- for $q^2 = M^2_{J/\psi}$ and $q^2 = M^2_{\psi(2S)}$, spectrum dominated by non-leptonic decays
- experimental measurements provide additional information about $\mathcal{H}_\lambda$

$$\mathcal{H}_\lambda(q^2) = P(\lambda)_\mu \langle H_s| \int d^4x\, e^{iq\cdot x}\, \mathcal{T}\{ J^\mu_{em}(x), [C_1 O_1^c + C_2 O_2^c](0) \} |H_b\rangle$$



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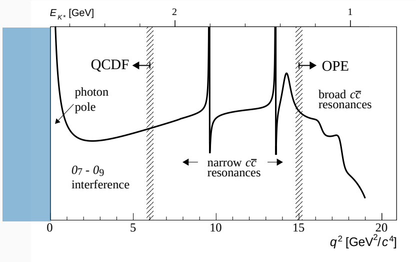
- $O^c_{1,2} \sim [\bar{s}\Gamma b]\ [\bar{c}\Gamma' c]$

[Bobeth,Chrzaszcz,DvD,Virto '17]

new strategy

- compute $\mathcal{H}_\lambda$ at spacelike q^2
- extrapolate to timelike $q^2 \leq 4M_D^2$ using suitable parametrization
- include information from hadronic decays to narrow charmonia J/ψ and $\psi(2S)$

$$4m_c^2 - q^2 \gg \Lambda_{\text{hadr.}}^2$$

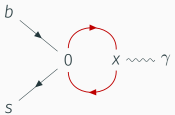


- expansion in ops w/ light-like sep. $x^2 \simeq 0$

[Khodjamirian, Mannel, Pivovarov, Wang 2010]

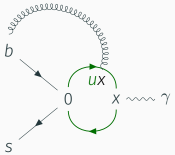
- employing light-cone expansion of charm propagator

[Balitsky, Braun 1989]



$$\xrightarrow{q^2 \ll 4m_c^2} \underbrace{\left(\frac{C_1}{3} + C_2\right) g(m_c^2, q^2)}_{\text{coeff \#1}} [\bar{s}\,\Gamma\, b] + \cdots$$

$$+ (\text{coeff \#2}) \times [\bar{s}_L \gamma^\alpha (in_+ \cdot \mathcal{D})^n \tilde{G}_{\beta\gamma} b_L]$$



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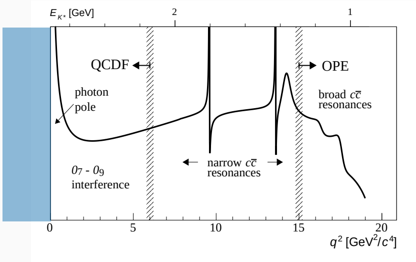

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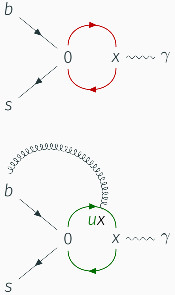

$$\Rightarrow \mathcal{H}_\lambda = \text{coeff \#1} \times \mathcal{F}_\lambda + \mathcal{H}_\lambda^{\text{spect.}} + \text{coeff \#2} \times \tilde{\mathcal{V}}_\lambda$$

- ▶ leading part identical to QCD Fact. results

[Beneke, Feldmann, Seidel ’01&’04]

- ▶ subleading matrix element $\tilde{\mathcal{V}}_\lambda$ can be inferred from B-LCSRs

[Khodjamirian, Mannel, Pivovarov, Wang ’10; Gubernari, DvD, Virto ’21] matrix elements of a single operator appearing at subleading power in the LCOPE

$$\tilde{\mathcal{V}}_\lambda \sim \langle M|\,\bar{s}(0)\gamma^\rho P_L G^{\alpha\beta}(-un^\mu)b(0)\,|B\rangle$$

for $B \to K^{(*)}$ and $B_s \to \phi$ transitions

- matrix element has been prev. calculated in light-cone sum rules (KMPW2010)

  [Khodjamirian et al, 1006.4945]

- physical picture provides that the soft gluon field originates from the B meson
  - analytical results independent of two-particle $b\bar{q}$ Fock state inside the B
  - expressions start with three-particle $b\bar{q}G$ Fock state, and their light-cone distribution amplitudes (LCDAs)

$$\Phi(t,u) \sim \langle 0|\,\bar{q}(x)G^{\mu\nu}(ux)\Gamma h_v^b(0)\,|\bar{B}(vM_B)\rangle \qquad x^\mu = tn^\mu$$

  - original results missing out on four out of eight three-particle LCDAs

Soft gluon matrix elements

- we calculate the soft-gluon contributions $\tilde{V}_\lambda$ to the full set of $B \to V$ and $B \to P$ nonlocal form factors using light-cone sum rules
  - analytic results for restricted set of LCDAs in full agreement with KMPW2010

    [Khodjamirian, Mannel, Pivovarov, Wang 2010]
  - result of restricted set fails to reproduce duality thresholds obtained from local form factor sum rules [Gubernari, Kokulu, DvD '18]
  - using the full set of LCDAs, our results reproduce the (local) duality thresholds!
  - our numerical results differ significantly from KMPW2010, but differences are well understood!
  - conclusion: soft-gluon contributions are not numerically relevant for $q^2 < 0$

- Taylor expand $\mathcal{H}_\lambda$ in q^2/M_B^2 around 0 [Ciuichini et al. '15]
  - \+ simple to use in a fit
  - \- incompatible with analyticity properties, does not reproduce resonances
  - \- expansion coefficients unbounded! $\Rightarrow$ impossible to estimate truncation error

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- use information from hadronic intermediate states in a dispersion relation [Khodjamirian et al. '10]

$$\mathcal{H}_\lambda(q^2) - \mathcal{H}_\lambda(q^2) = \int ds \frac{\text{Im}\,\mathcal{H}_\lambda(s)}{(s-s_0)(s-q^2)} + \dots$$

  - \+ reproduces resonances
  - \- hadronic information above the threshold must be modelled
  - \- complicated to use in a fit, relies on theory input in single point s_0

Extrapolate Parametrisations

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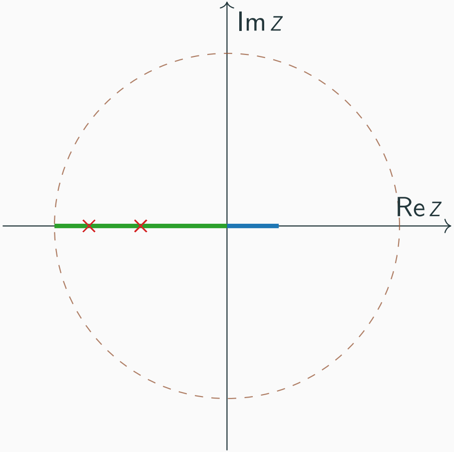
- expand the matrix elements in variable $z(q^2)$ with branch cut at $q^2 = 4M_D^2$ [Bobeth/Chrzaszcz/DvD/Virto '17]

Parametrisation of the nonlocal form factors

- map q^2 to new variable z that develops branch cut at $q^2 = 4M_D^2$ [Bobeth/Chrzaszcz/DvD/Virto '17]
  - branch cut is mapped onto unit circle in z
- data and theory live insides the unit circle
  - real-valued $q^2 \leq 4M_D^2$ is mapped to real-valued z
- expand in z
  - \+ resonances J/ψ, $\psi(2S)$ can be included (poles/Blaschke factors)
  - \+ easy to use in a fit to theory and data
  - \+ compatible with analyticity
  - \- expansion coefficients unbounded!



matrix elements $\mathcal{H}$ arise from nonlocal operator [Gubernari,DvD,Virto '20]

$$O^\mu(Q;x) \sim \int d^4y\, e^{iQ\cdot y}\, T\{J^\mu_{\text{em}}(x+y), [C_1 O_1 + C_2 O_2](x)\}$$

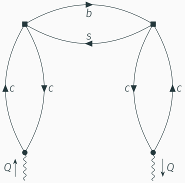
construct four-point operator to derive a dispersive bound

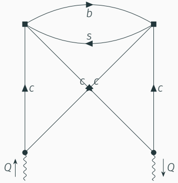
- define matrix element of “square“ operator

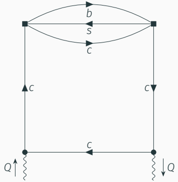
$$\left[\frac{Q^\mu Q^\nu}{Q^2} - g^{\mu\nu}\right] \Pi(Q^2) \equiv \int d^4x\, e^{iQ\cdot x} \langle 0|\, T\{O^\mu(Q;x) O^{\dagger,\nu}(Q;0)\}\, |0\rangle$$

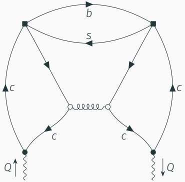
- $\Pi(Q^2)$ has two types of discontinuities
    - from intermediate unflavoured states ($c\bar{c}$, $c\bar{c}c\bar{c}$, ...)
    - from intermediate $b\bar{s}$-flavoured states ($b\bar{s}$, $b\bar{s}g$, $b\bar{s}c\bar{c}$, ...)

Extrapolate Cuts of Π









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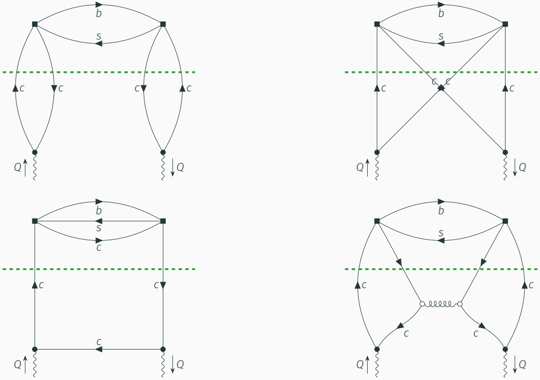

- from intermediate unflavoured states ($c\bar{c}$, $c\bar{c}c\bar{c}$, ...)

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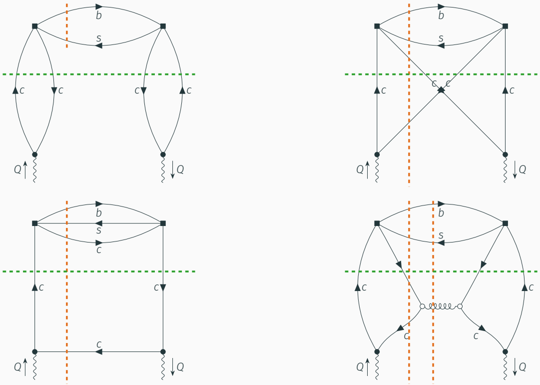


- from intermediate unflavoured states ($c\bar{c}$, $c\bar{c}c\bar{c}$, ...)
- from intermediate $b\bar{s}$-flavoured states ($b\bar{s}$, $b\bar{s}g$, $b\bar{s}c\bar{c}$, ...)

Dispersion relation for Π

dispersive representation of the $b\bar{s}$ contribution to derivative of Π

$$\chi(Q^2) \equiv \frac{1}{2!}\left[\frac{d}{dQ^2}\right]^2 \frac{1}{2i\pi} \int_{(m_b+m_s)^2}^{\infty} ds\, \frac{\mathrm{Disc}_{b\bar{s}}\,\Pi(s)}{s-Q^2} > 0 \qquad \text{if } Q^2 < 0$$

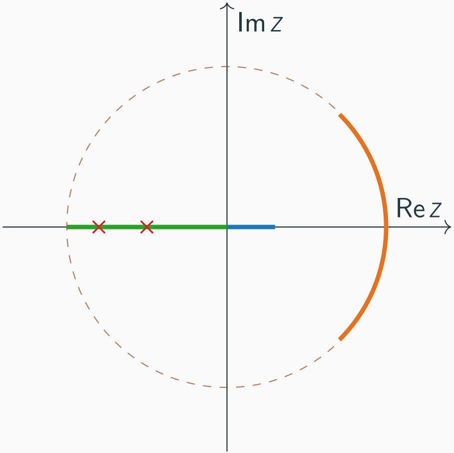
- $\mathrm{Disc}_{b\bar{s}}\,\Pi$ can be computed in the local OPE
  $\rightarrow \chi^{\mathrm{OPE}}(Q^2)$
- $\mathrm{Disc}_{b\bar{s}}\,\Pi$ can be expressed in terms of the nonlocal form factors $|\mathcal{H}_\lambda|^2$
  $\rightarrow \chi^{\mathrm{had}}(Q^2)$

- global quark hadron duality suggests that $\chi^{\mathrm{OPE}}(Q^2) = \chi^{\mathrm{had}}(Q^2)$
- parametrize $\mathcal{H}_\lambda \propto \sum_n a_{\lambda,n} f_n$ with orthonormal functions f_n
  $\Rightarrow$ dispersive bound: $\quad \chi^{\mathrm{OPE}} \geq \sum_n |a_{\lambda,n}|^2$
- *first application* of such a bound to nonlocal form factors
- technically more challenging than for local form factors

- expand in z
  - $f_n(z)$ orthogonal on arc
  - + accounting for behaviour on arc produces dispersive bound on each parameter ✓

[Gubernari/DvD/Virto '20]

- turns (so far!) hardly quantifiable systematic theory uncertainties into parametric uncertainties
- currently being implemented in EOS
  - open source software at *github.com/eos/eos*
  - available from PyPI for easy dissemination to both theory + experimental colleagues



New Puzzles

Non-leptonic decay $\overline{B}_s^0 \to D_s^{(*)+}\pi^-$ and $\overline{B}^0 \to D^{(*)+}K^-$

- decay modes can be accurately predicted in collinear factorization [Beneke et al. '01]
  - no penguin contributions or weak annihilation
  - expansion in Λ/m_b, Λ/m_c, and α_s
  - at leading-power $\overline{B}_q \to D_q^{(*)}$ form factors contribute
- theory predictions recently updated using precise form factor input

  [Bordone,Huber,Gubernari,Jung,DvD '20]
- measured branching ratios are systematically lower than measurements
  - correspond to downward shift in the weak coupling by 17%
  - tension at the $4.4\,\sigma$ level

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possible explanation

- large power corrections
- underestimated exp. uncertainties
- large e.m. corrections [Beneke '21]
- BSM physics

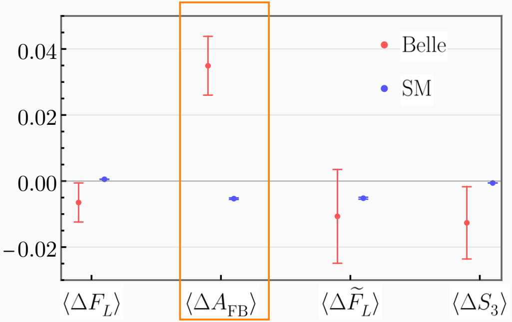
none of them is satisfactory

LFNU in $\overline{B} \to D^*\{e^-, \mu^-\}\overline{\nu}$

- existing Belle data can be used to extract LF specific angular observables

  [Bobeth et al. '21]

  - A_{FB}: leptonic forward-backward asymmetry
  - current presentation hides non-zero value for $\Delta A_{\mathrm{FB}} = A_{\mathrm{FB}}^{(\mu)} - A_{\mathrm{FB}}^{(e)}$
- difference of e and μ observables very accurately predicted in SM
  - form factor uncertainties cancel
  - fit to all measurements sees $4\,\sigma$ tension with theory predictions



requires careful re-assessment by Belle (II)

Conclusion

- b-quark phenomenology hinges on accurate and precise information of a number of hadronic form factors
  - local form factors affect extraction of $|V_{cb}|$ and prediction of $R_{D^{(*)}}$
  - nonlocal form factors contribute the single-largest systematic uncertainty in exclusive $b \to s\ell\ell$ decays
- clear road toward controlling these objects, but much work still needs to be done
- key is a combined theory + data driven approach
- new puzzles appear to due increase in theory precision

Backup Slides

Extrapolate Dispersion relation for Π

the hadronic representation reads schematically:

$$1 \geq \frac{1}{\chi^{\mathrm{OPE}}(Q^2)\, 2!} \left[\frac{d}{dQ^2}\right]^2 \int_{(m_b+m_s)^2}^{\infty} ds \sum_\lambda \frac{\omega_\lambda(s)\, |\mathcal{H}_\lambda(s)|^2}{s - Q^2}$$

- aim: diagonalize this expression

Ansatz:

$$\hat{\mathcal{H}}_\lambda(q^2) \equiv P(q^2) \times \phi_\lambda(q^2) \times \mathcal{H}_\lambda(q^2) \equiv \sum_n a_{\lambda,n} f_n(q^2)$$

- Blaschke factor $P(q^2)$ removes poles of narrow charmonia
- outer function ϕ_λ accounts for weight function ω_λ and Cauchy integration kernel
- orthonormal polynomials $f_n(q^2)$ diagonalize remainder of the expression

normalisation to χ^{OPE} leads to a diagonal bound

$$1 \geq \sum_\lambda \sum_n |a_{\lambda,n}|^2$$

Compute Soft gluon matrix elements

Transition	$\tilde{\mathcal{V}}(q^2 = 1\,\text{GeV}^2)$	GvDV2020	KMPW2010
$B \to K$	$\tilde{\mathcal{A}}$	$(+4.9 \pm 2.8) \cdot 10^{-7}$	$(-1.3^{+1.0}_{-0.7}) \cdot 10^{-4}$
$B \to K^*$	$\tilde{\mathcal{V}}_1$	$(-4.4 \pm 3.6) \cdot 10^{-7}\,\text{GeV}$	$(-1.5^{+1.5}_{-2.5}) \cdot 10^{-4}\,\text{GeV}$
	$\tilde{\mathcal{V}}_2$	$(+3.3 \pm 2.0) \cdot 10^{-7}\,\text{GeV}$	$(+7.3^{+14}_{-7.9}) \cdot 10^{-5}\,\text{GeV}$
	$\tilde{\mathcal{V}}_3$	$(+1.1 \pm 1.0) \cdot 10^{-6}\,\text{GeV}$	$(+2.4^{+5.6}_{-2.7}) \cdot 10^{-4}\,\text{GeV}$
$B_s \to \phi$	$\tilde{\mathcal{V}}_1$	$(-4.4 \pm 5.6) \cdot 10^{-7}\,\text{GeV}$	—
	$\tilde{\mathcal{V}}_2$	$(+4.3 \pm 3.1) \cdot 10^{-7}\,\text{GeV}$	—
	$\tilde{\mathcal{V}}_3$	$(+1.7 \pm 2.0) \cdot 10^{-6}\,\text{GeV}$	—

reduction by a factor of ~ 200

- new structures in three-particle LCDAs account for factor 10 (due to cancellations!)
- updated inputs that enter the sum rules (mostly) linearly account for further factor 10
- similar relative uncertainties, but absolute uncertainties reduced by $\mathcal{O}(100)$