

Topological field theory in 3D

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Why topological field theories?

- appear in String theory and quantum field theory
- rigorous mathematical description
- interplay between String theory, QFT, and geometry
- fractional quantum Hall effect, topological quantum computing, knot theory, ...

What are TFTs?

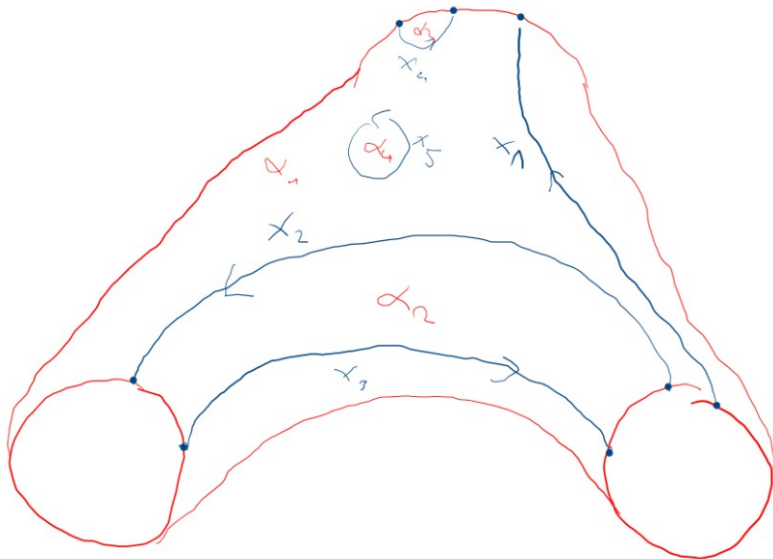
- Quantum field theory on a manifold (n-dim. surface)
- Observables are invariant under continuous deformations (depend only on topological properties)



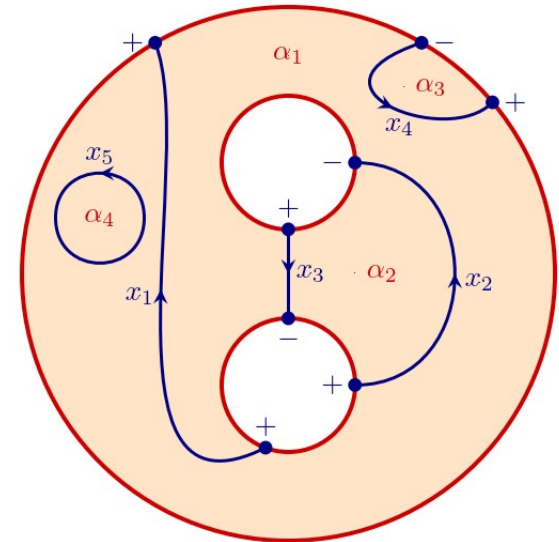
[<https://cems.riken.jp/en/laboratory/qmtrt>]

Defect TFT

- introduce lower-dimensional objects (lines, points)
- lines can separate different regions



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[Carqueville, 1607.05747]

Orbifolds

- In string theory: Take a symmetry, quotient it out, get a **new** string theory (with additional sectors)
- In TFT: similar procedure; related

$$\begin{array}{c} \phi \\ \bullet \end{array} \Big|_{gI} = \Big|_{gI} \begin{array}{c} g \cdot \phi \\ \bullet \end{array}$$

$$\begin{array}{c} \text{cap} \\ \text{cup} \end{array} : A \otimes A \longrightarrow A, \quad \begin{array}{c} \text{cap} \\ \text{cup} \end{array} : A \longrightarrow A \otimes A,$$

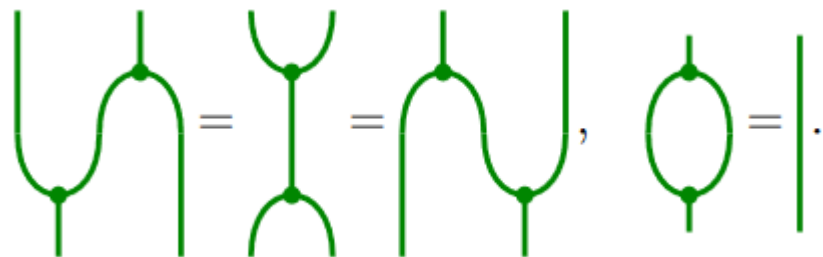
$$\begin{array}{c} \text{cap} \\ \text{cup} \end{array} : I_a \longrightarrow A, \quad \begin{array}{c} \text{cap} \\ \text{cup} \end{array} : A \longrightarrow I_a$$

Orbifold data

Identities need to be fulfilled:



$$\mu_{gh,i} \circ (\mu_{g,h} \otimes 1_i) = \mu_{g,hi} \circ (1_g \otimes \mu_{h,i})$$

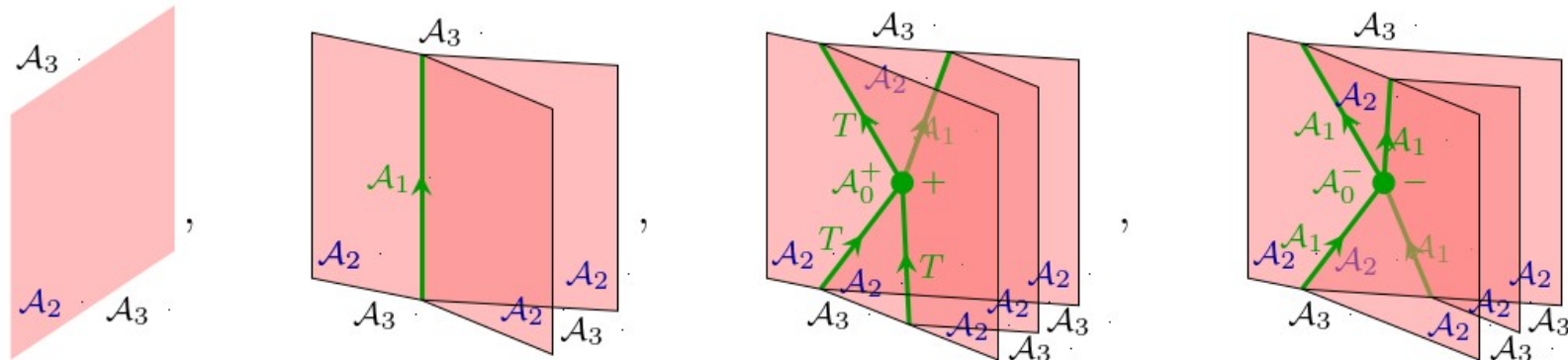


2D and 3D

- Many well-understood examples known in 2D (e.g. Landau-Ginzburg models)
- Far fewer examples in 3D
- Our focus: Rozansky-Witten theory
- Can we do orbifolds in RW-theory?
What kind of theories do we find?

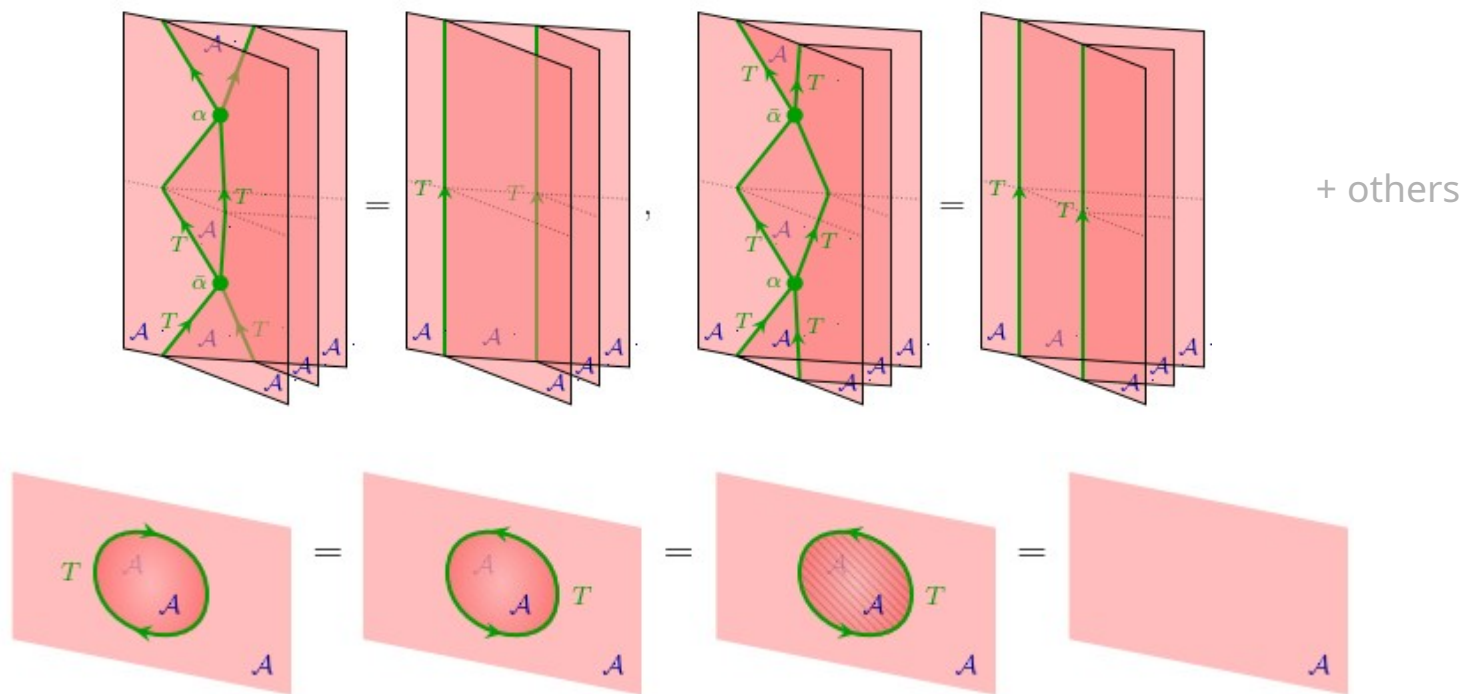
Orbifold data in 3D

- group action surface instead of group action line
- special surface, line, two points



Orbifold data in 3D

- identities:

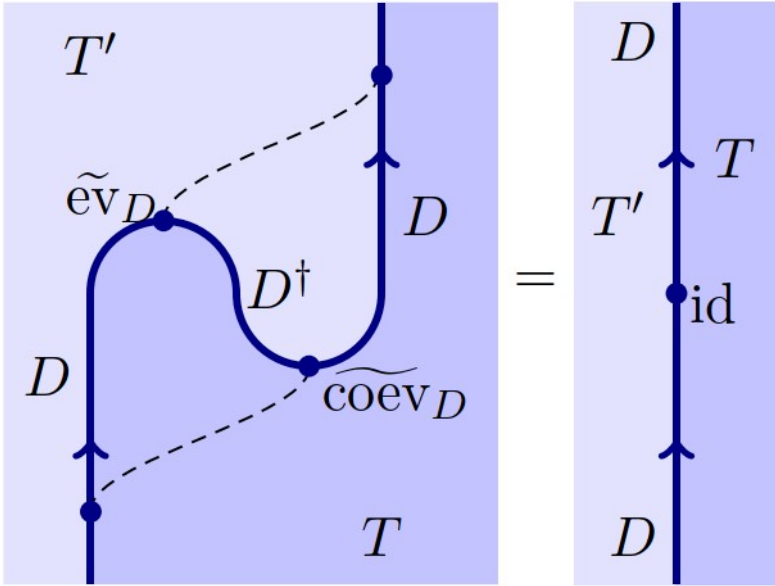


[Carqueville, Runkel, Schaumann, 1705.06085]

Our research

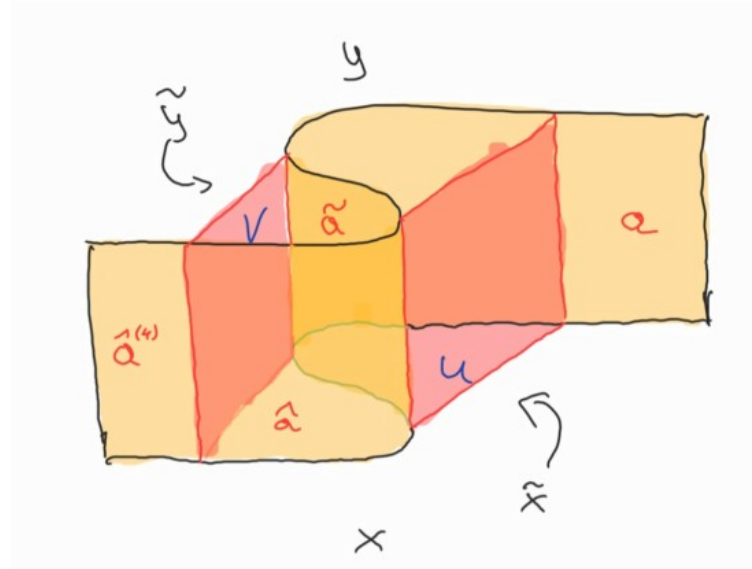
- Fill in details in Rozansky-Witten theory required for orbifold procedure (done)
- Construct orbifold data (surface, line, points) in RW theory (done)
- Prove the identities (WIP)
- Use the orbifold procedure to find new TFTs

Zorro moves and movies



[Klos, Roggenkamp; 1907.12339]

$$\lambda_X \circ (\tilde{ev}_X \otimes 1) \circ (1 \otimes \widetilde{coev}_X) \circ \rho_X^{-1} = 1_X$$



[Fragkos; unpublished]