Search for ρ and ρ' in the $\pi^-\ \pi^+$ Subsystem of the 1^{++} Wave in $\pi^- p \to \pi^-\pi^+\pi^- p$

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Hadronic three-body Christmas

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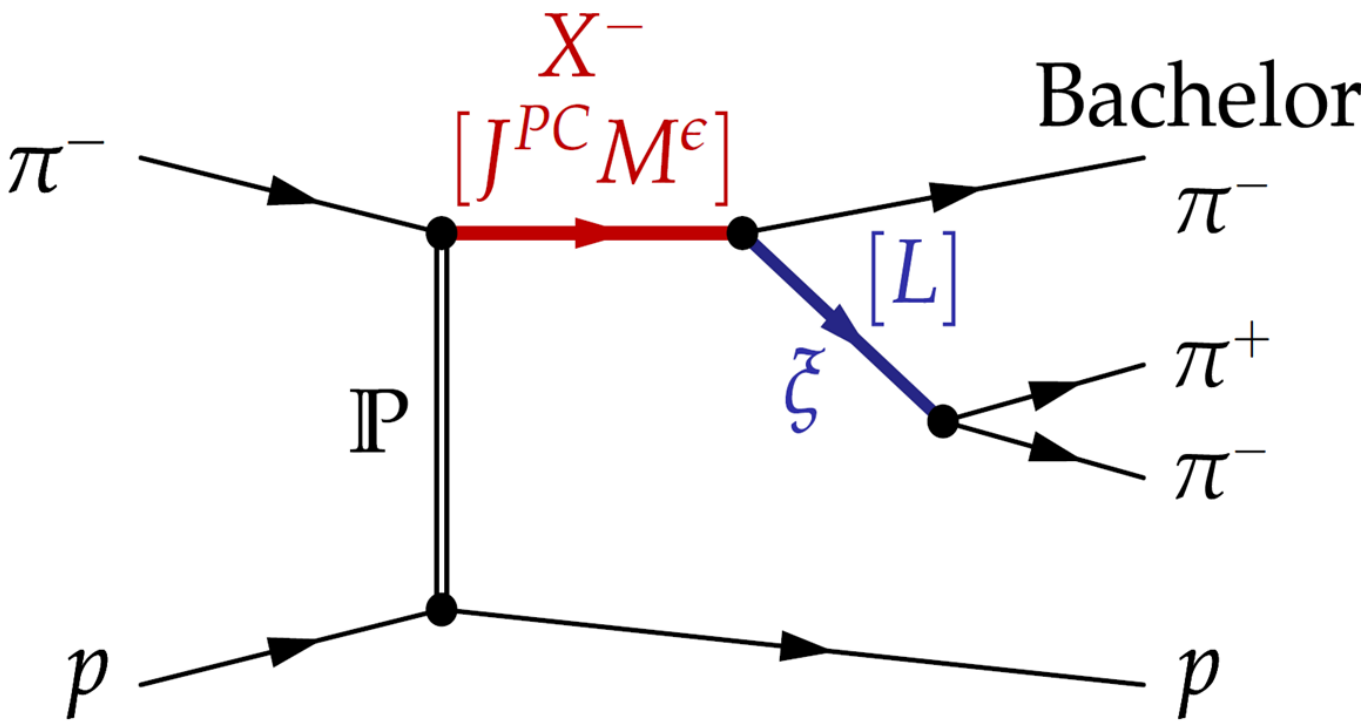
MIAPP, Garching

Analyzed Wave

- $J_X^{PC} = 1^{++}$, $M^{\epsilon} = 0^{+}$, $J_{\xi}^{PC} = 1^{--}$:
  largest wave (1/3 of total intensity)
- $46 \cdot 10^6$ events in our data sample
- analyzed in 50 $m_{3\pi}$-bins and 4 t'-bins
- for every $m_{3\pi}$-bin extracted amplitude
  of the $\pi^- \pi^+$ subsystem in $m_{2\pi}$-bins
- result of freed-isobar PWA are complex amplitudes
  that may be correlated among each other



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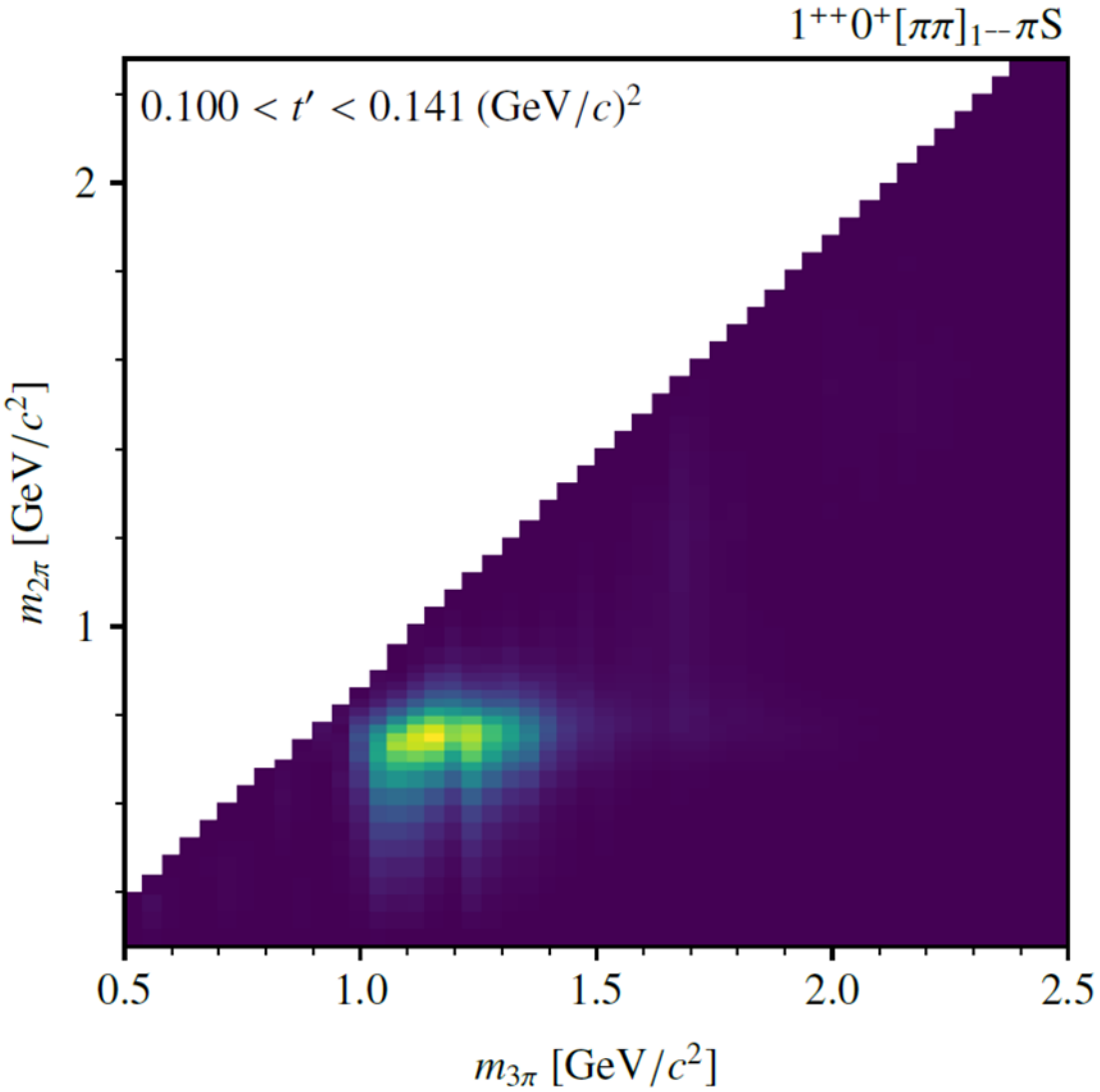
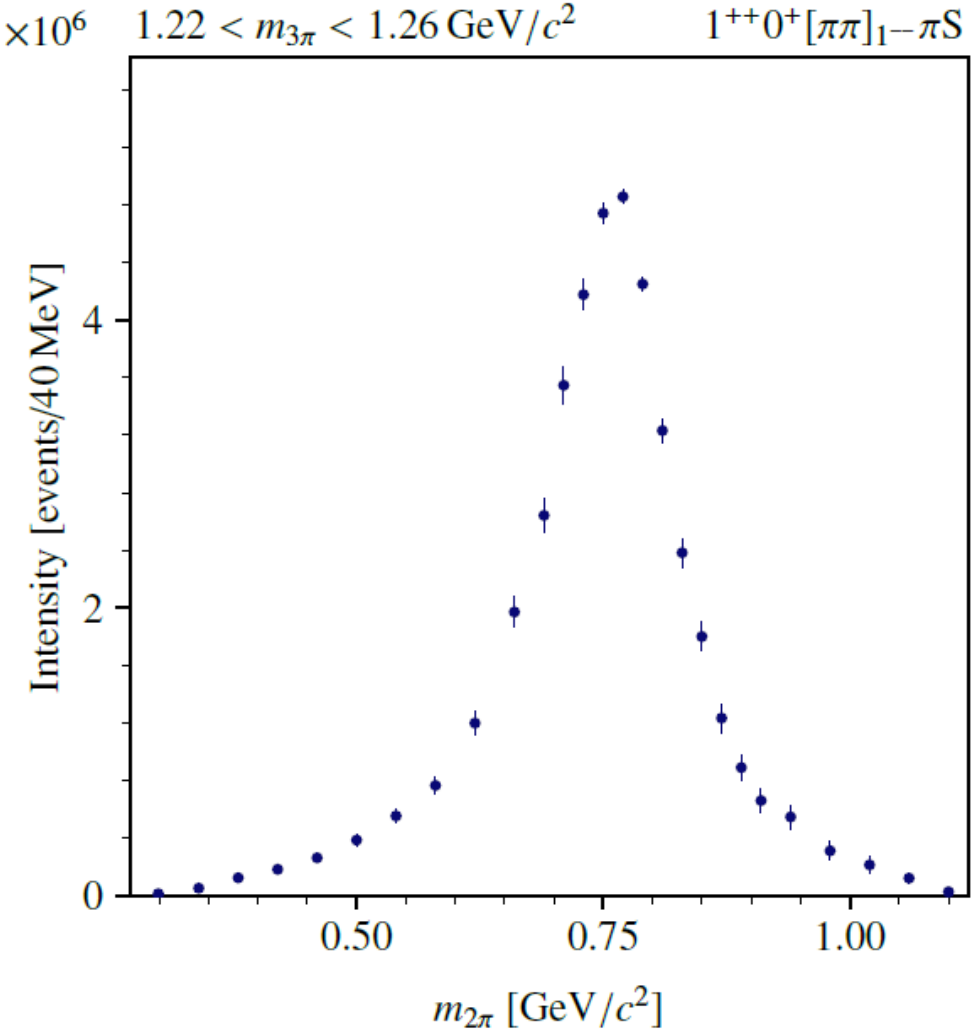
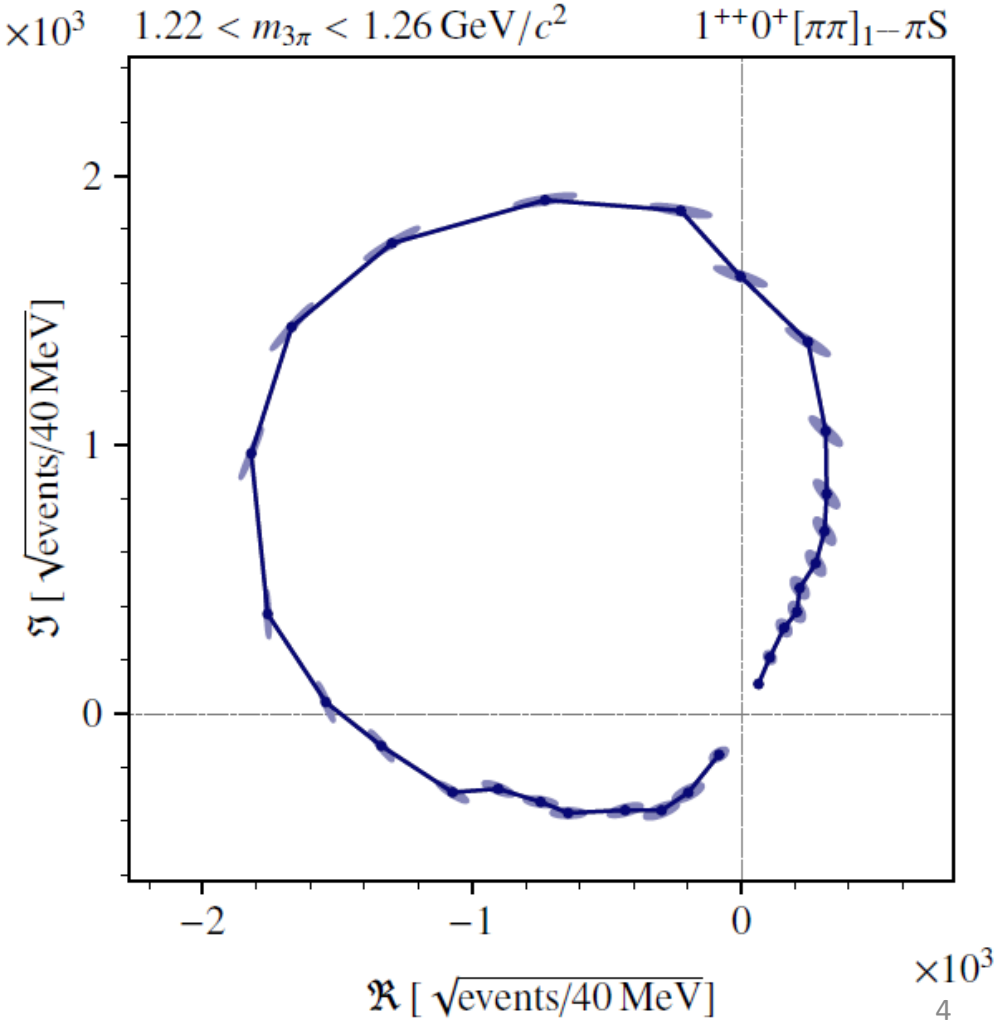
  that may be correlated among each other
- X^- dominated by $a_1(1260)$, ξ dominated by $\rho(770)$



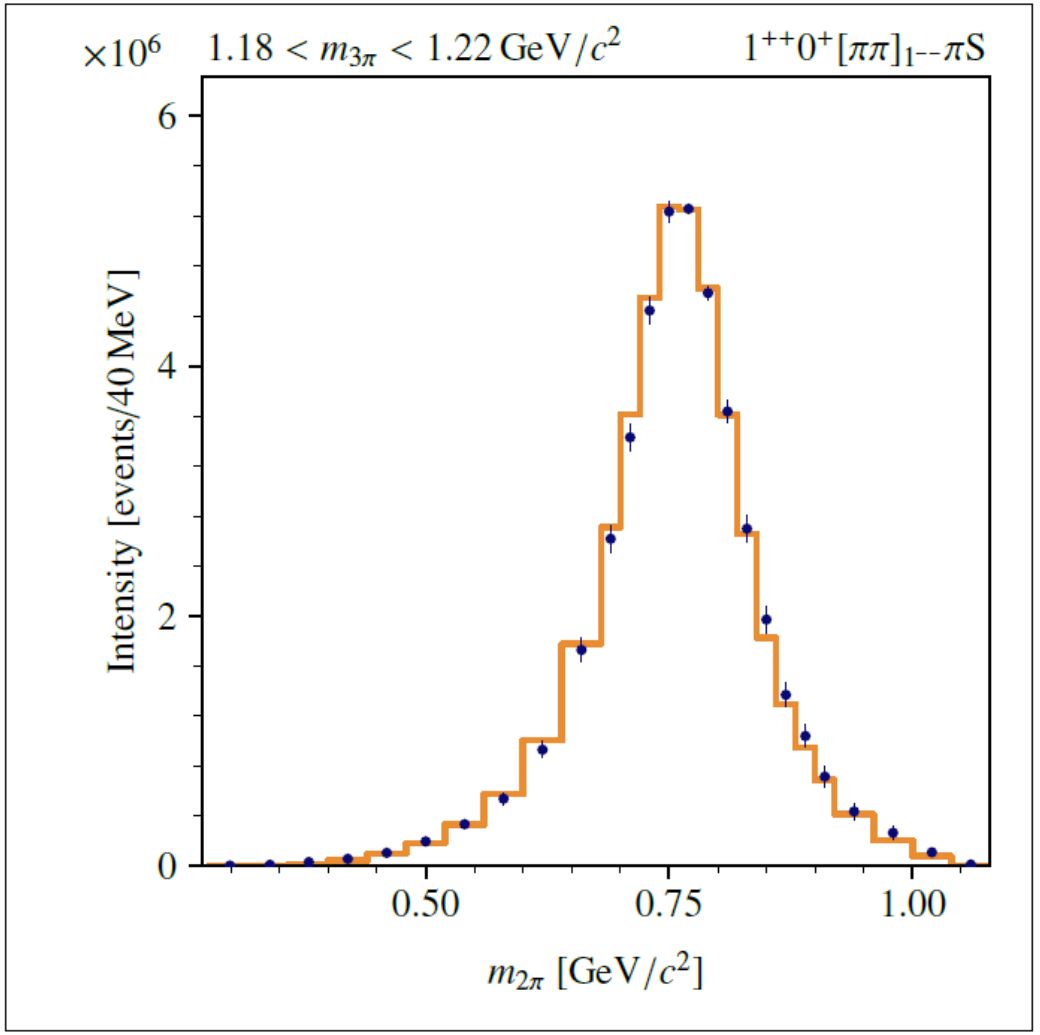
Analyzed Wave:
Amplitude of the $\pi^-\,\pi^+$ subsystem with $J^{PC}_{\xi} = 1^{--}$





Resonance Model for the $\pi^-\,\pi^+$ subsystem with $J^{PC}_{\xi} = 1^{--}$

- resonance-model fit of $m_{2\pi}$ dependence of extracted amplitudes
- unitarity resonance model[2] containing multiple resonances
- extraction of pole positions possible
- less process-dependent than Breit-Wigner parameters
- fit also fixes ambiguity (zero mode) in extracted amplitudes



2: Lukas Bayer. 'Consistent extraction of ρ-parameters from various reactions'. Bachelor's Thesis. Rheinischen Friedrich-Wilhelms-Universität Bonn, Mar. 2019.

Resonance Model in Detail

$$F(s) = \frac{M(s)}{1 - V(s)\Sigma(s)} = \left\{ 1 - {g_1}^2 \left[a_1 + \Sigma(0) \right] - a_1 \frac{{m_1}^2 {g_1}^2}{s - {m_1}^2} - \sum_{i=2}^{n} a_i \frac{s {g_i}^2}{s - {m_i}^2} \right\} \Bigg/$$

$$\left\{ 1 + \left(\frac{{m_1}^2 {g_1}^2}{s - {m_1}^2} + \sum_{i=2}^{n} \frac{s {g_i}^2}{s - {m_i}^2} \right) \frac{1}{2\pi} \left[k^2(s) \left\{ h({m_1}^2) - h(s) \right\} \right. \right.$$

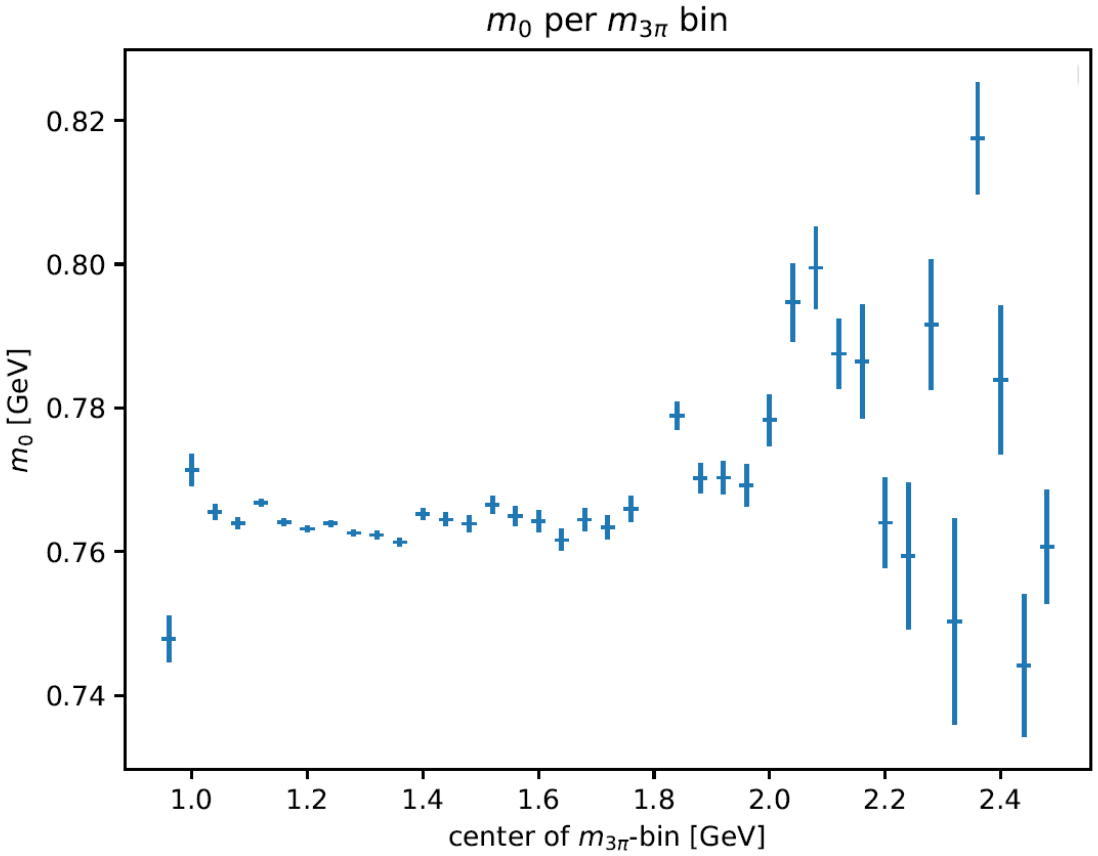
$$\left. \left. - k^2({m_1}^2)\, h'({m_1}^2)\, ({m_1}^2 - s) + \mathrm{i} \frac{k^3(s)}{\sqrt{s}} \right] \right\}.$$

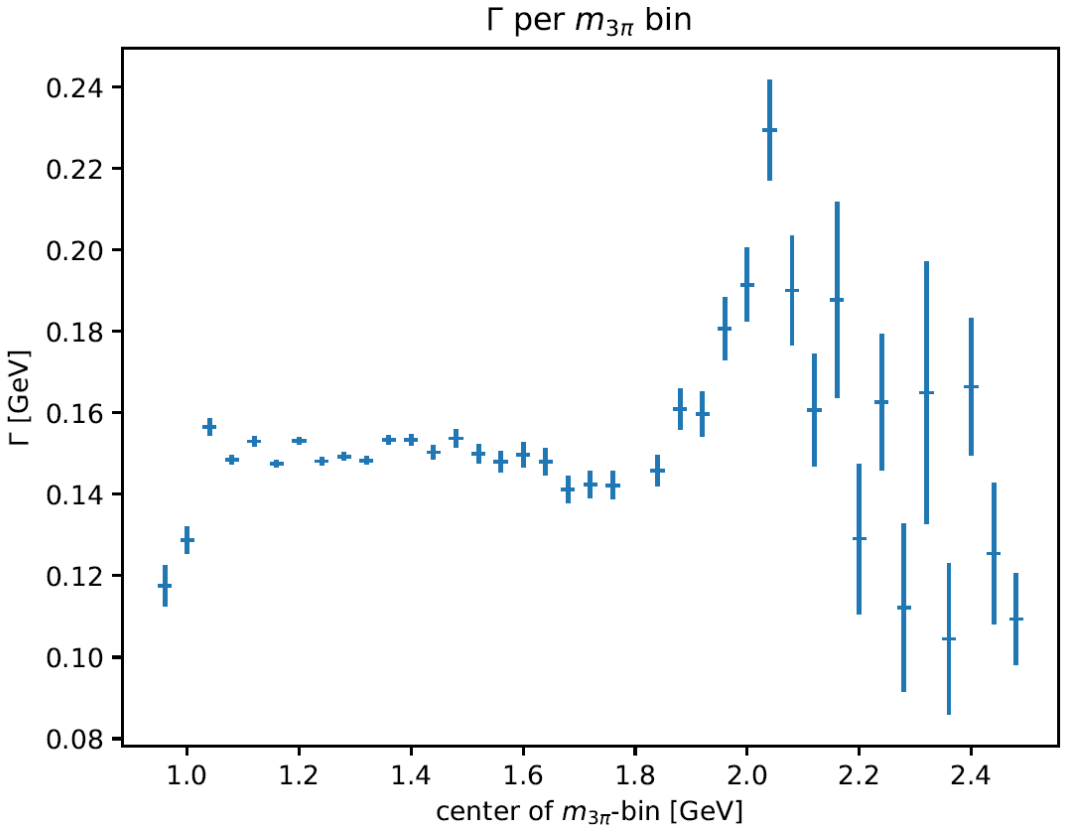
- $F(s)$ is the pion vector form factor consisting of:
  - the source term $M(s)$, the self energy term $\Sigma(s)$, the potential term $V(s)$ adjusted to multiple resonances
- free parameters:
  - couplings g_i and (complex) masses m_i, same for each t' and $m_{3\pi}$ bin
  - complex-valued a_i, individual for each t' and $m_{3\pi}$ bin

$$k(s) = \sqrt{\frac{s}{4} - m_\pi^2},$$

$$h(s) = \frac{2}{\pi} \frac{k(s)}{\sqrt{s}} \ln \left[\frac{\sqrt{s} + 2k(s)}{2m_\pi} \right]$$

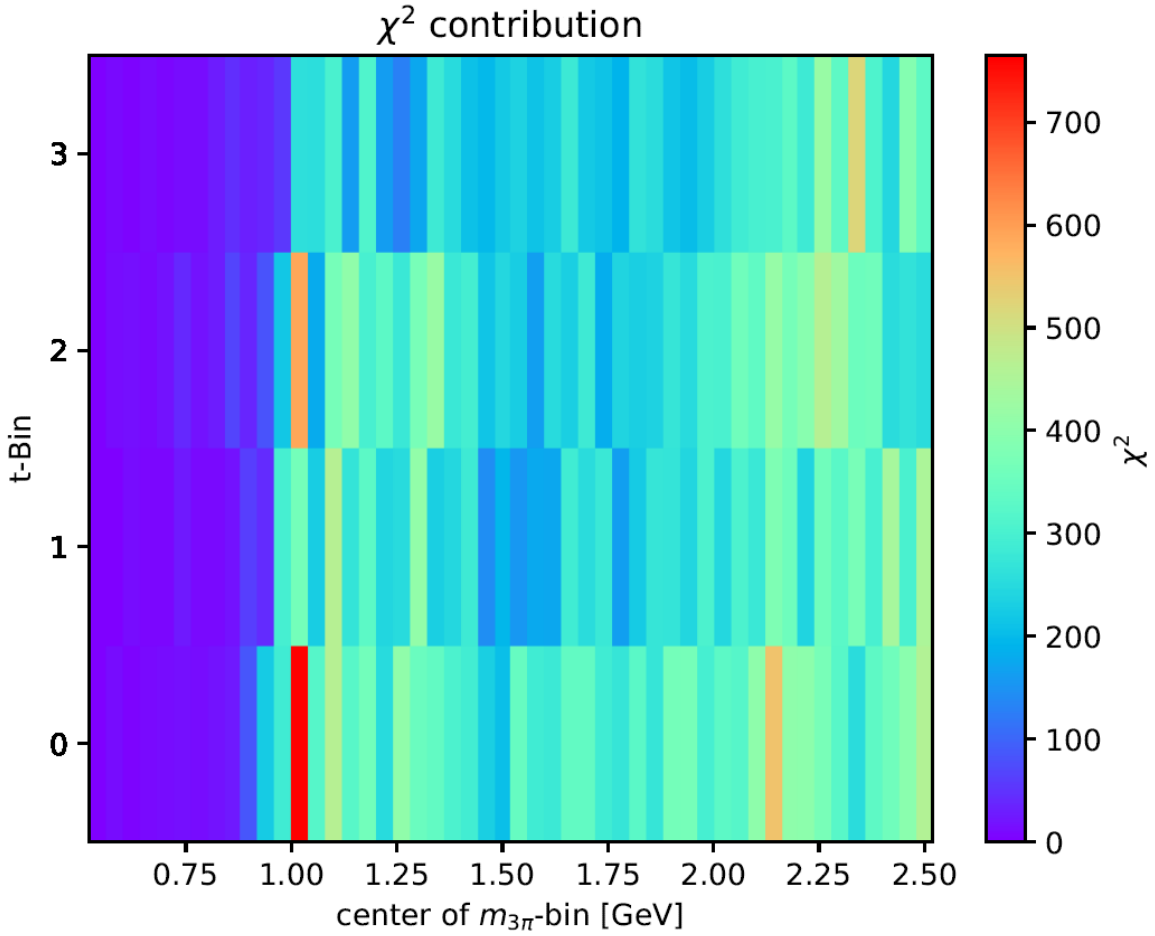
Results for Single $m_{3\pi}$ Bins

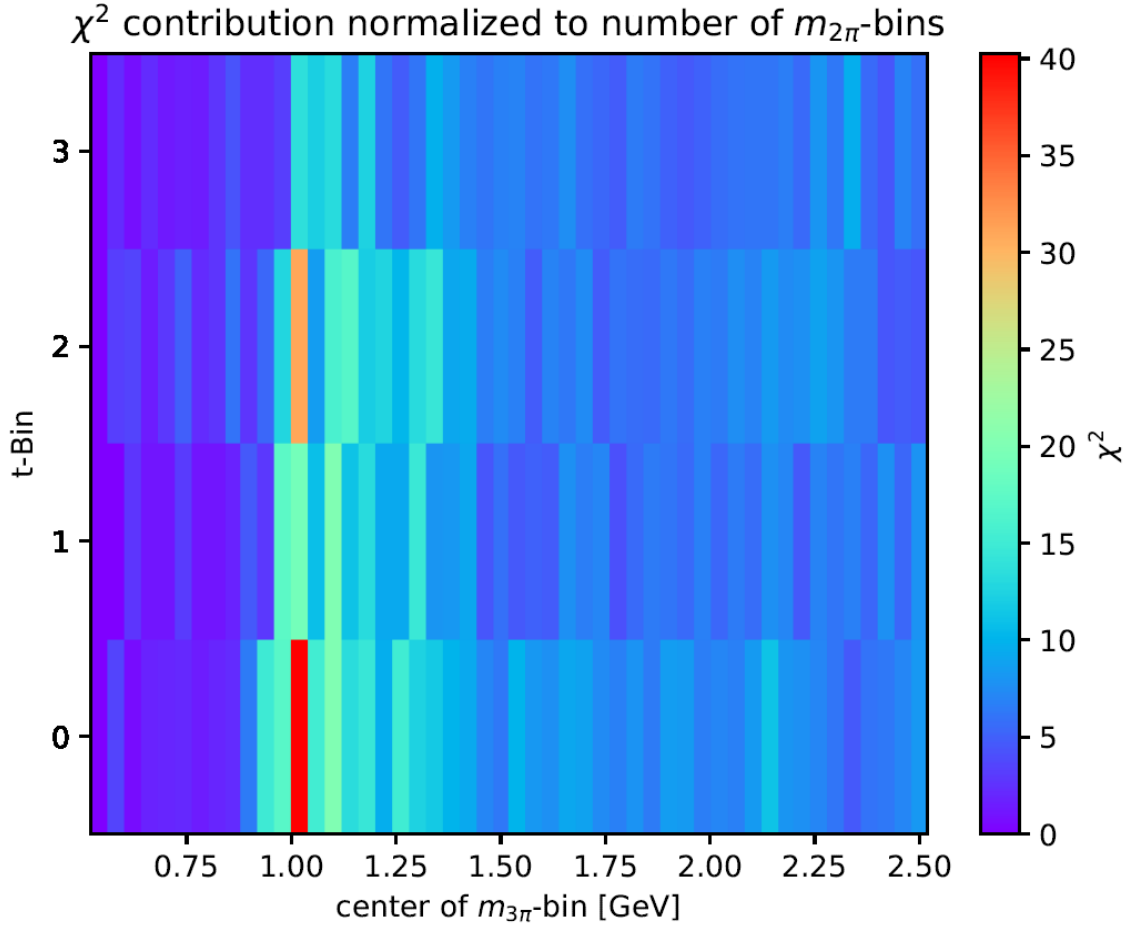




mass and width are consistent between 1.1 and 1.8 GeV; similar picture in all 4 t' bins
→ fit of all $(m_{3\pi}, t')$ cells should be possible

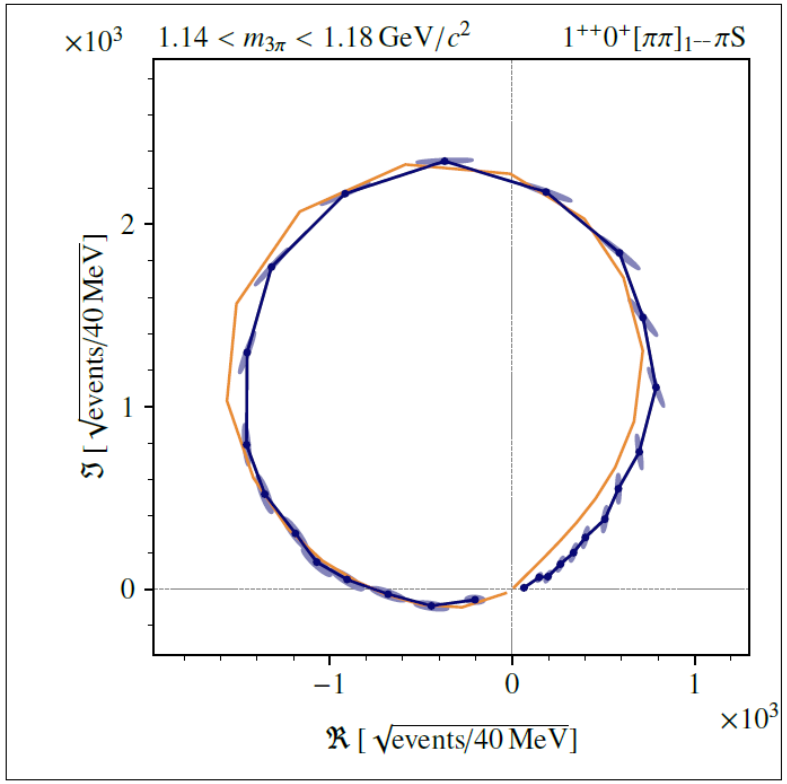
χ^2-contribution

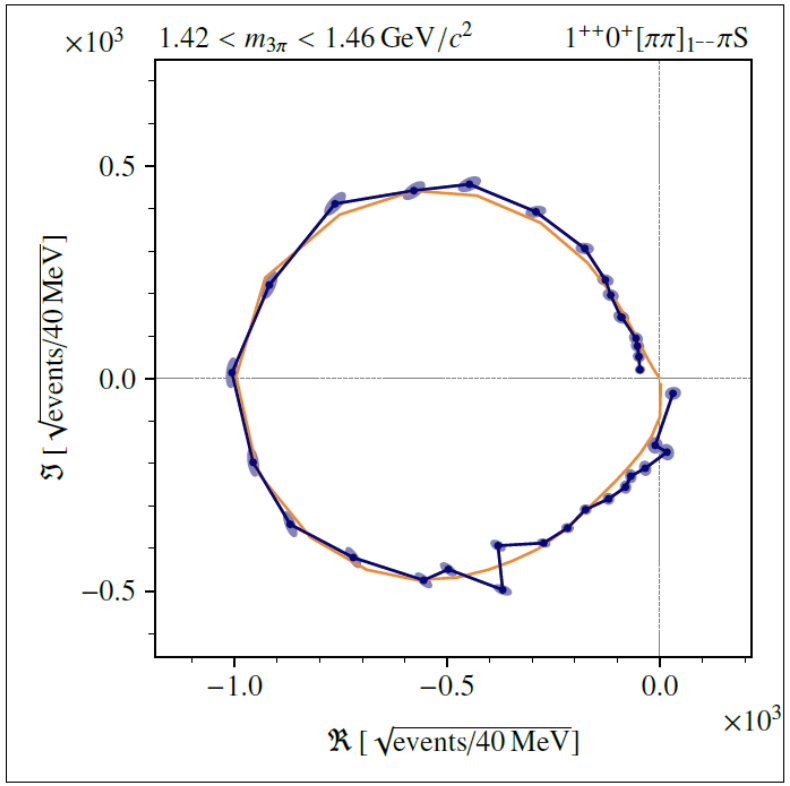


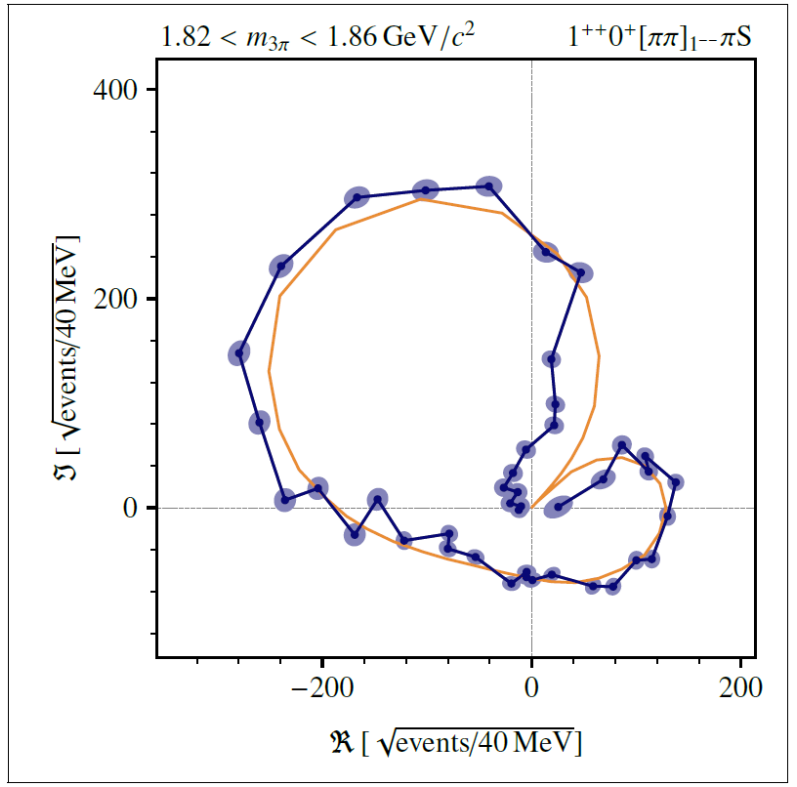


- study contributions from ($m_{3\pi}$,t') cells to χ^2
- largest deviations observed in $a_1(1260)$ region, where most events are concentrated

Argand Diagrams of Extracted Amplitudes in selected $m_{3\pi}$-Bins







amplitude resonance model

overall fair description of the data over complete $m_{3\pi}$ and t' range

Uncertainty for the Poles

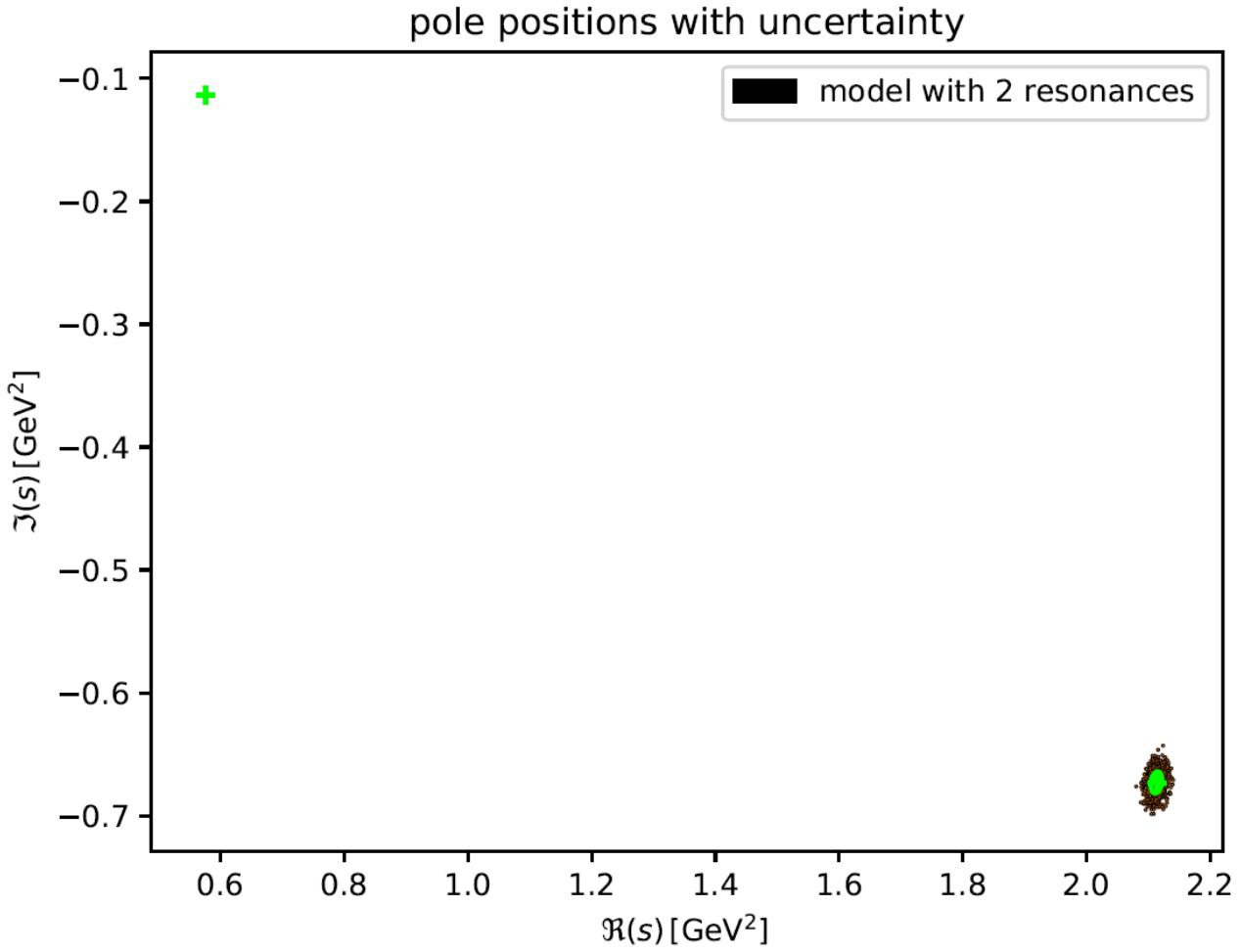
- poles s are found by minimizing $\frac{1}{|A(s)|^2}$

  → analytic uncertainty propagation is difficult

  → Monte Carlo approach

  → repetition of pole finding for fit-parameters randomly drawn from the Gaussian covariance matrix obtained from the fit

- from the pole-distribution calculate covariance matrices for pole positions



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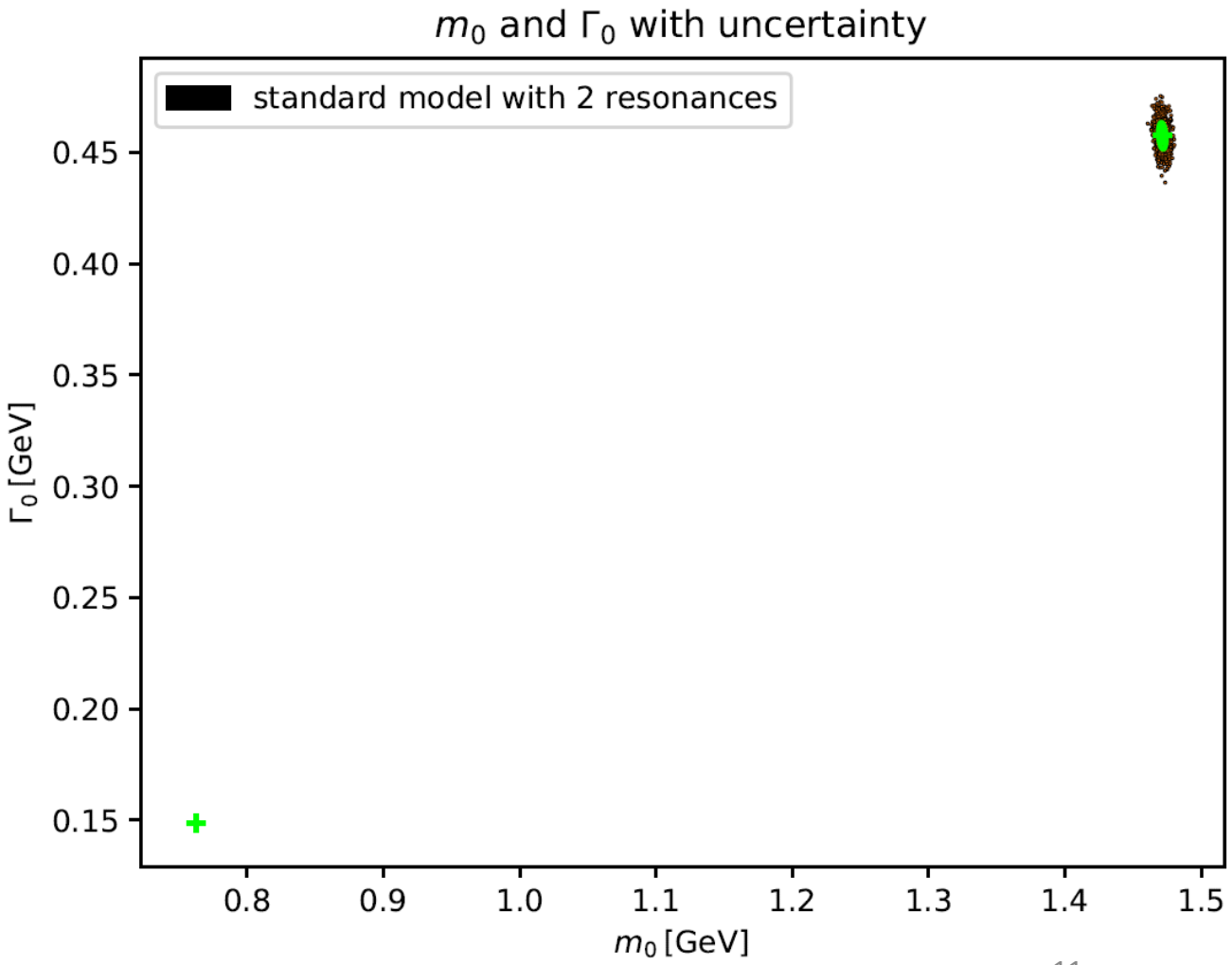
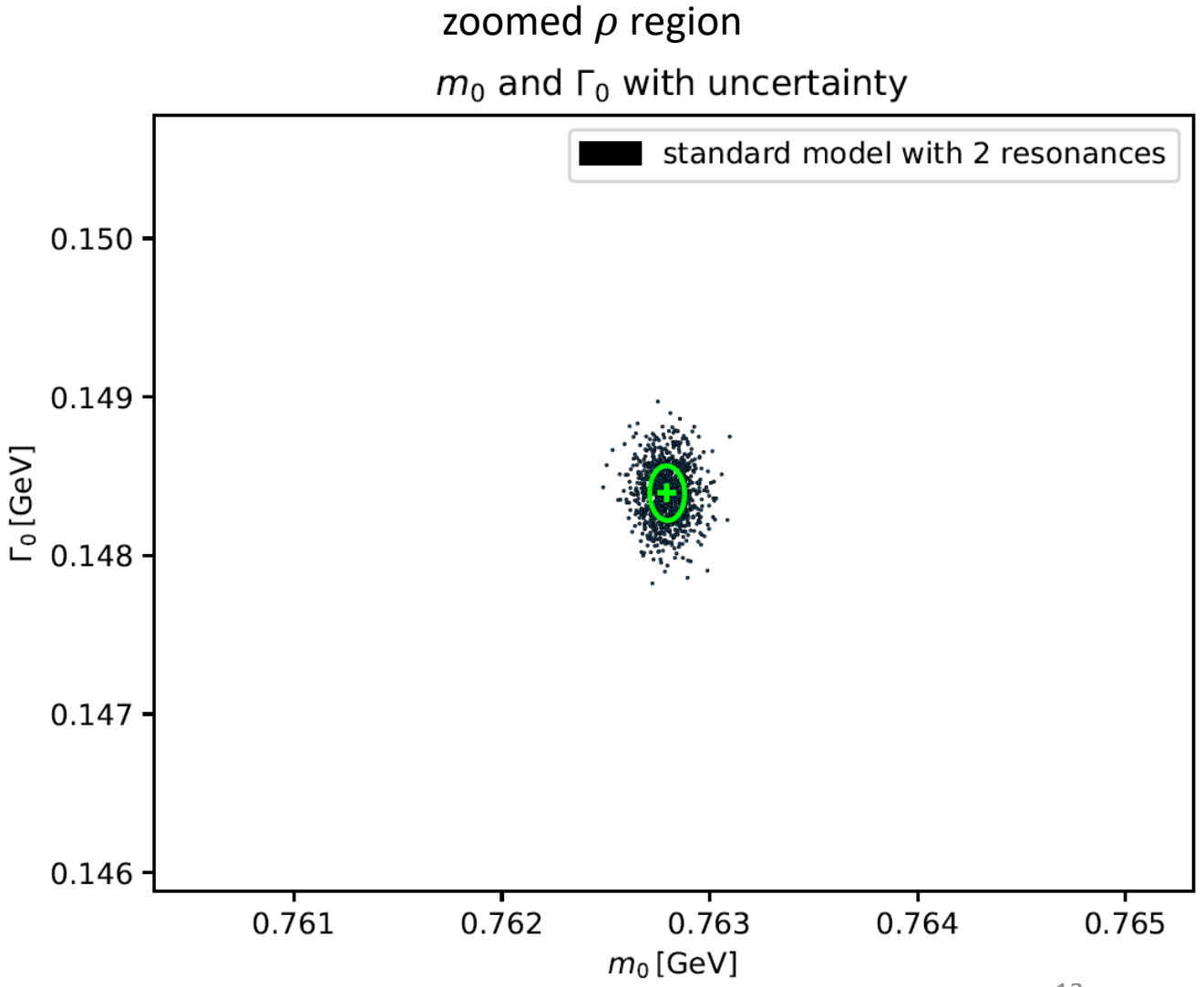
- from the pole-distribution calculate covariance matrices for pole positions

  → calculation of uncertainties for mass and width possible



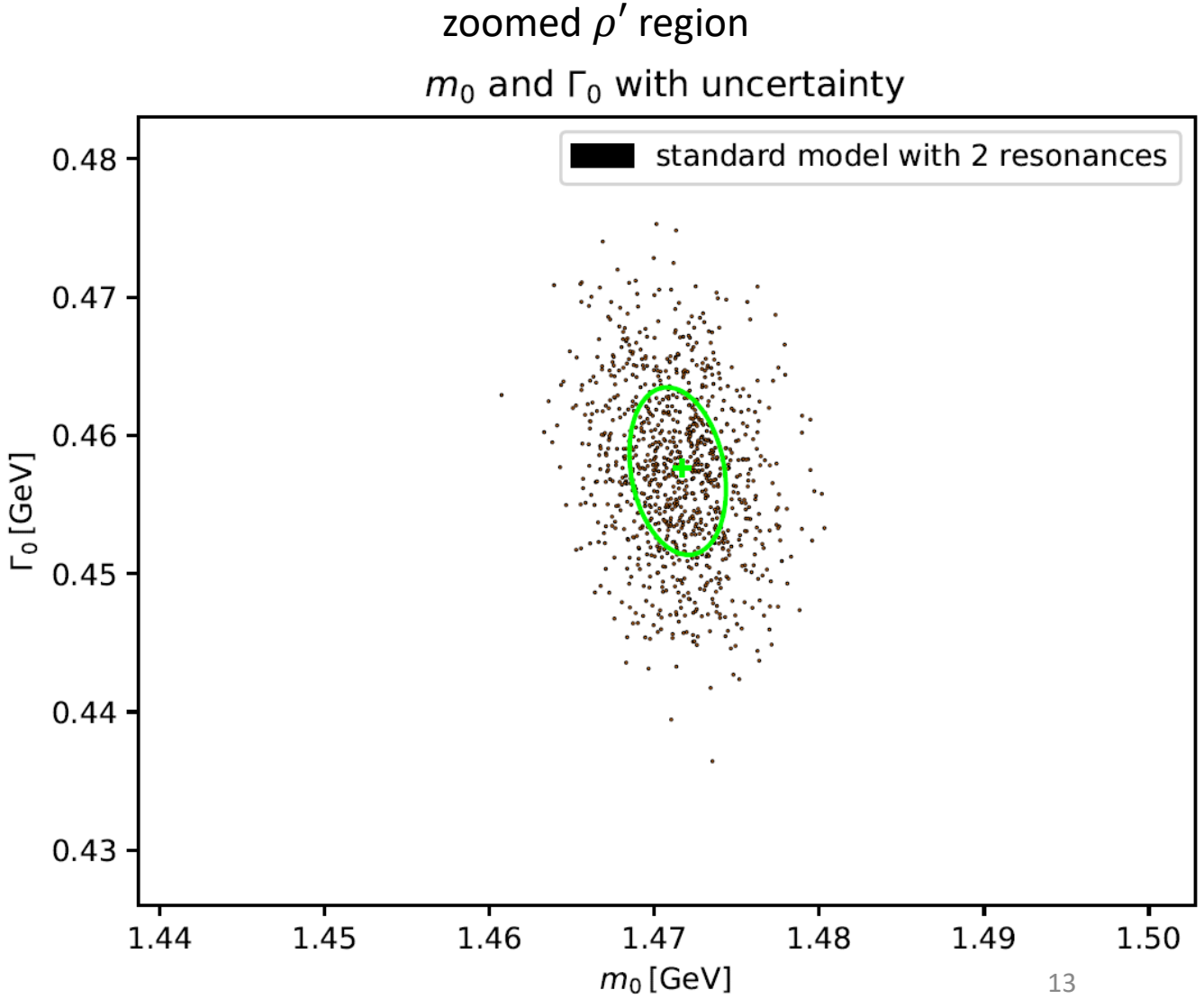
Resonance Parameters

- extracted parameters:
- $m_0 = 762.8 \pm 0.1$ MeV
- $\Gamma_0 = 148.4 \pm 0.2$ MeV
- other measurements in PDG (T-Matrix Pole):
- $m_0 = 761^{+4}_{-3}$ MeV, $\Gamma_0 = 142^{+4}_{-5}$ MeV
- $m_0 = 763.7^{+1.7}_{-1.5}$ MeV, $\Gamma_0 = 146.4^{+2.0}_{-2.2}$ MeV
- $m_0 = 754 \pm 18$ MeV, $\Gamma_0 = 148 \pm 10$ MeV
- our values are in good agreement

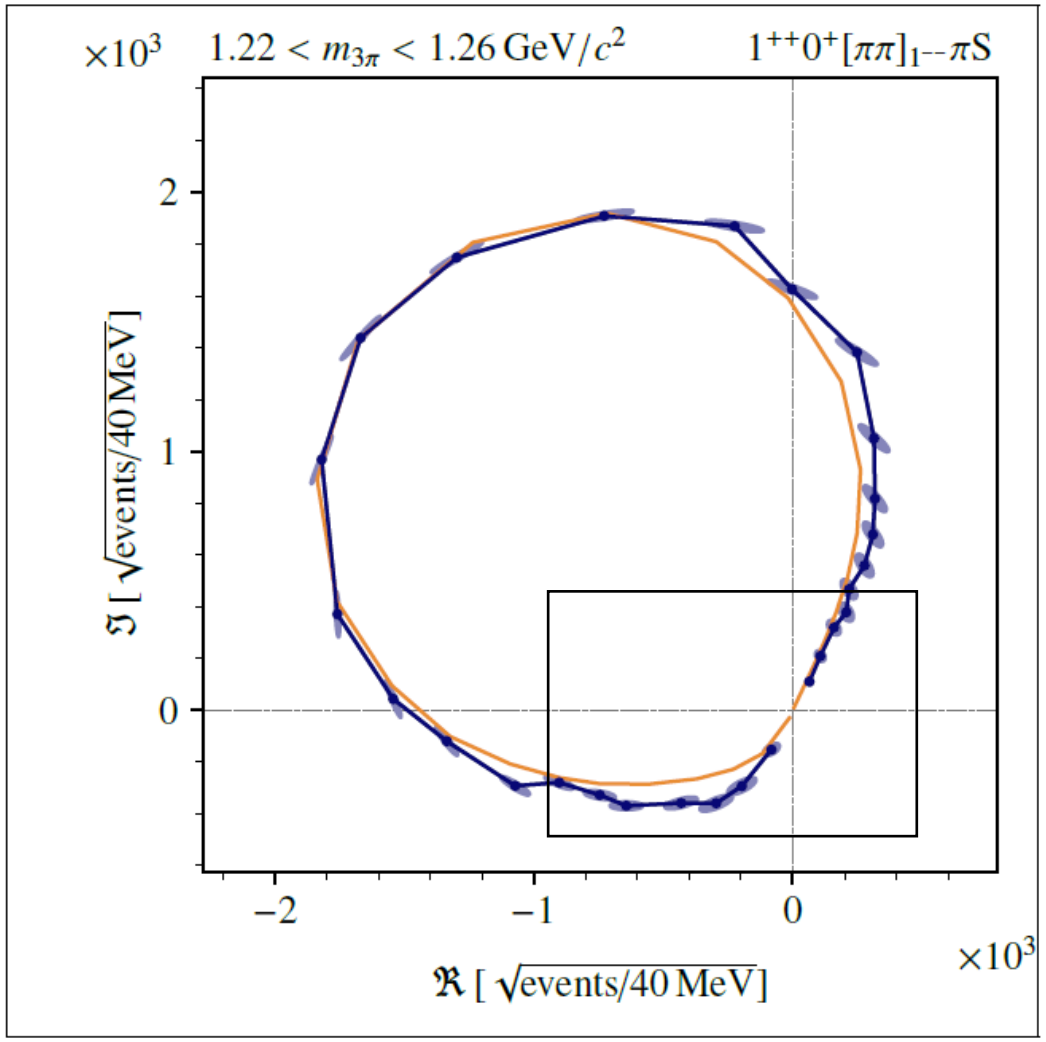


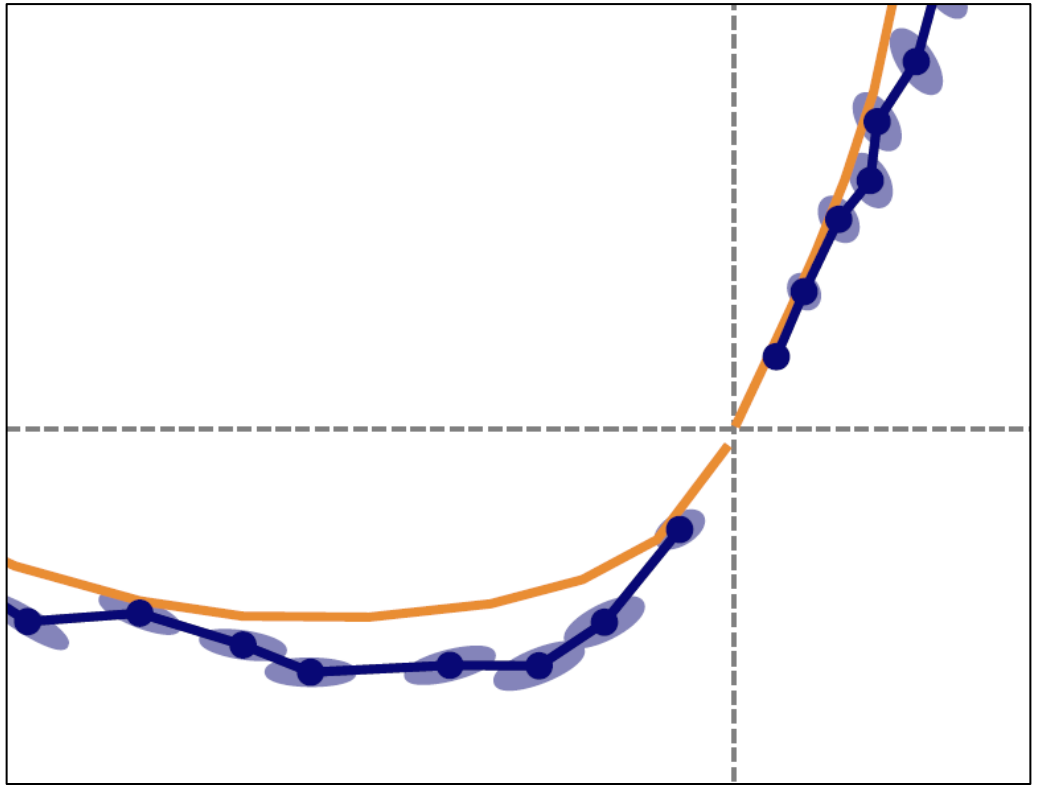
Resonance Parameters

- mass and width of ρ':
- $m_0 = 1471 \pm 3$ MeV
- $\Gamma_0 = 457 \pm 6$ MeV
- PDG average (BW-parameters):
- $m_0 = 1465 \pm 25$ MeV
- $\Gamma_0 = 400 \pm 60$ MeV



Visible Problems of the Resonance Model





Visible Problems of the Resonance Model

visible problems of the model in some $(m_{3\pi},t')$ cells:

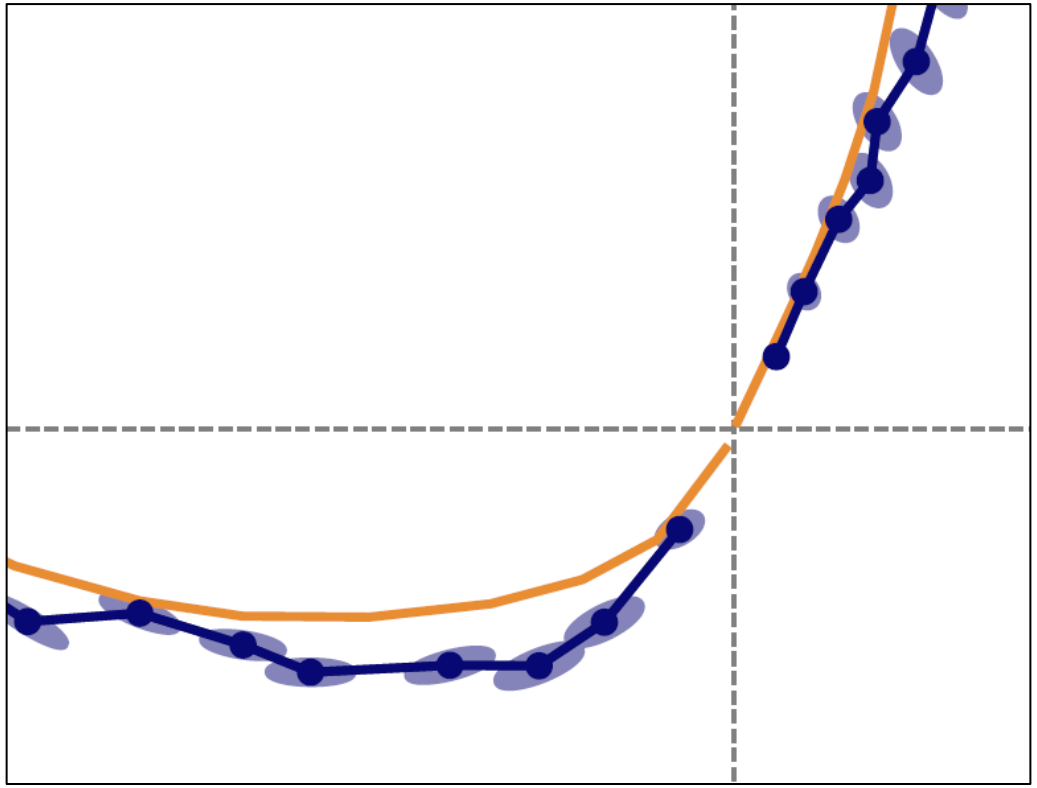
- model by construction starts in origin, the data don't
- either imaginary or real part of the model amplitude tends to be "too slow" in certain $m_{3\pi}$-bins

possible solution:

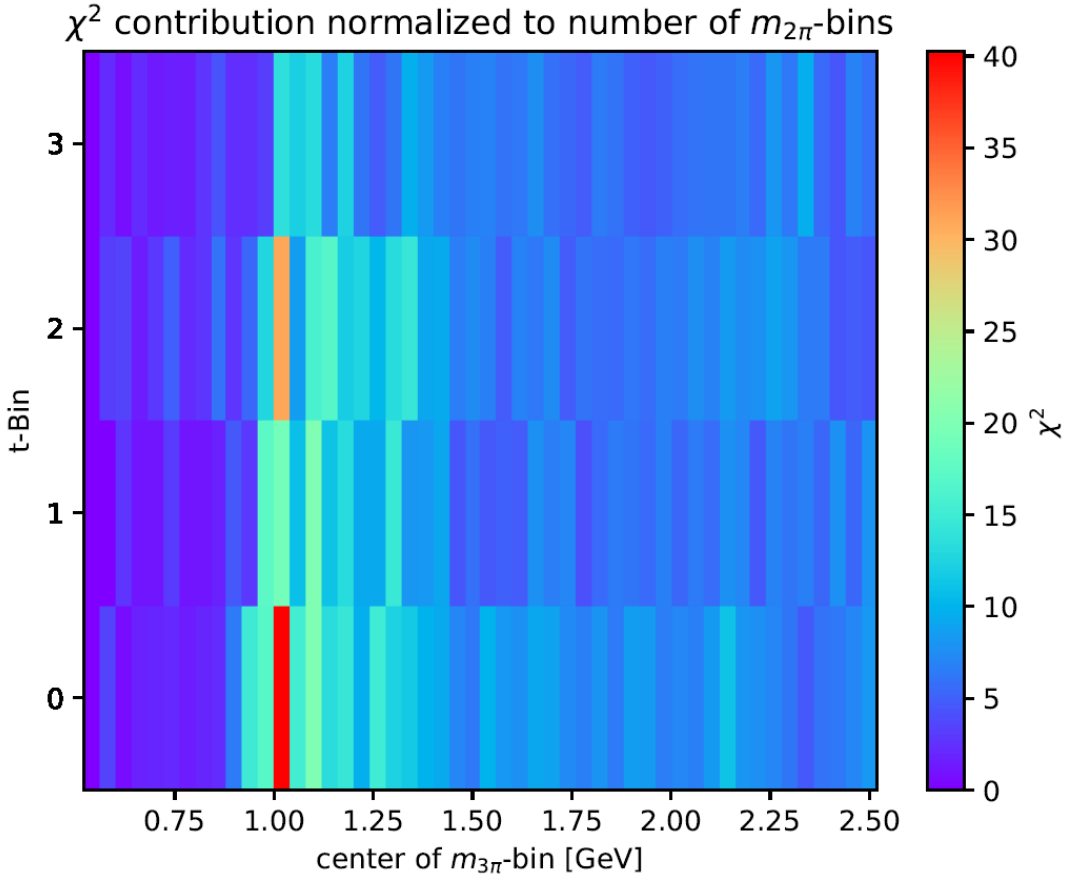
- add background-like terms

→ first simple approach: addition of a complex-valued offset C for each $(m_{3\pi}, t')$ cell (not physically motivated)

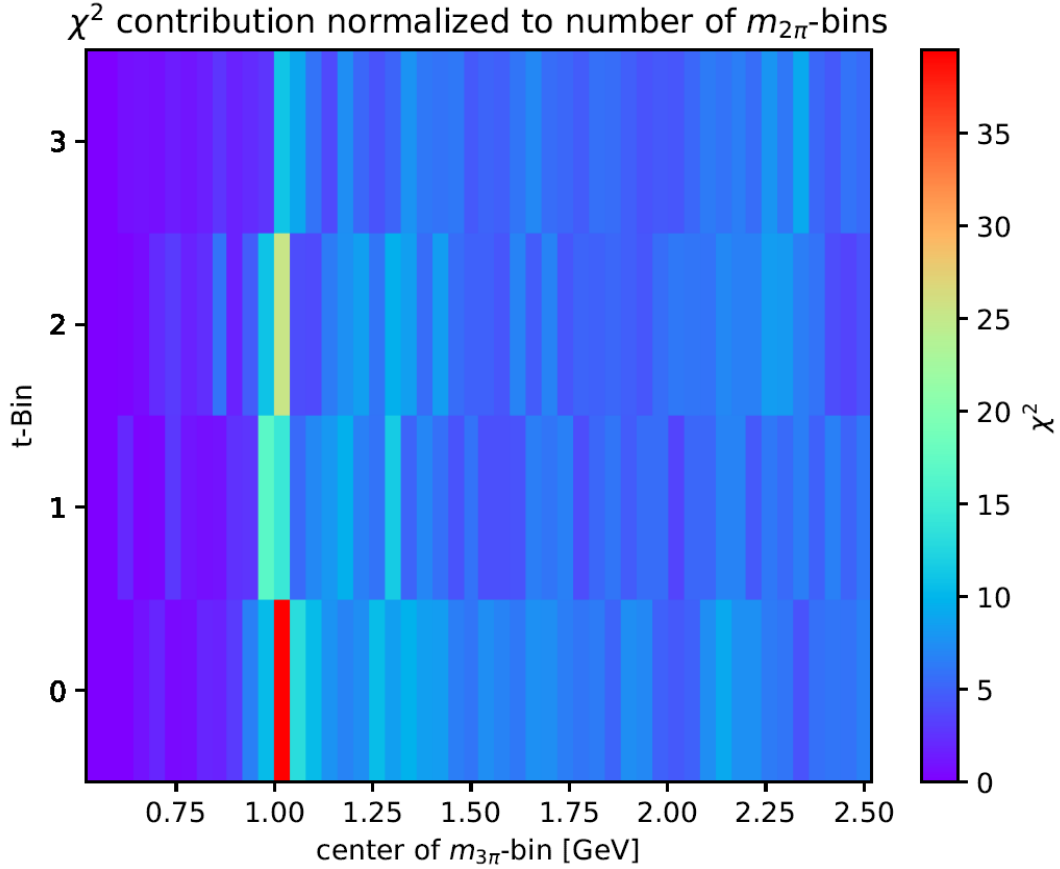
→ $\tilde{F} = F + C$



Change in χ^2



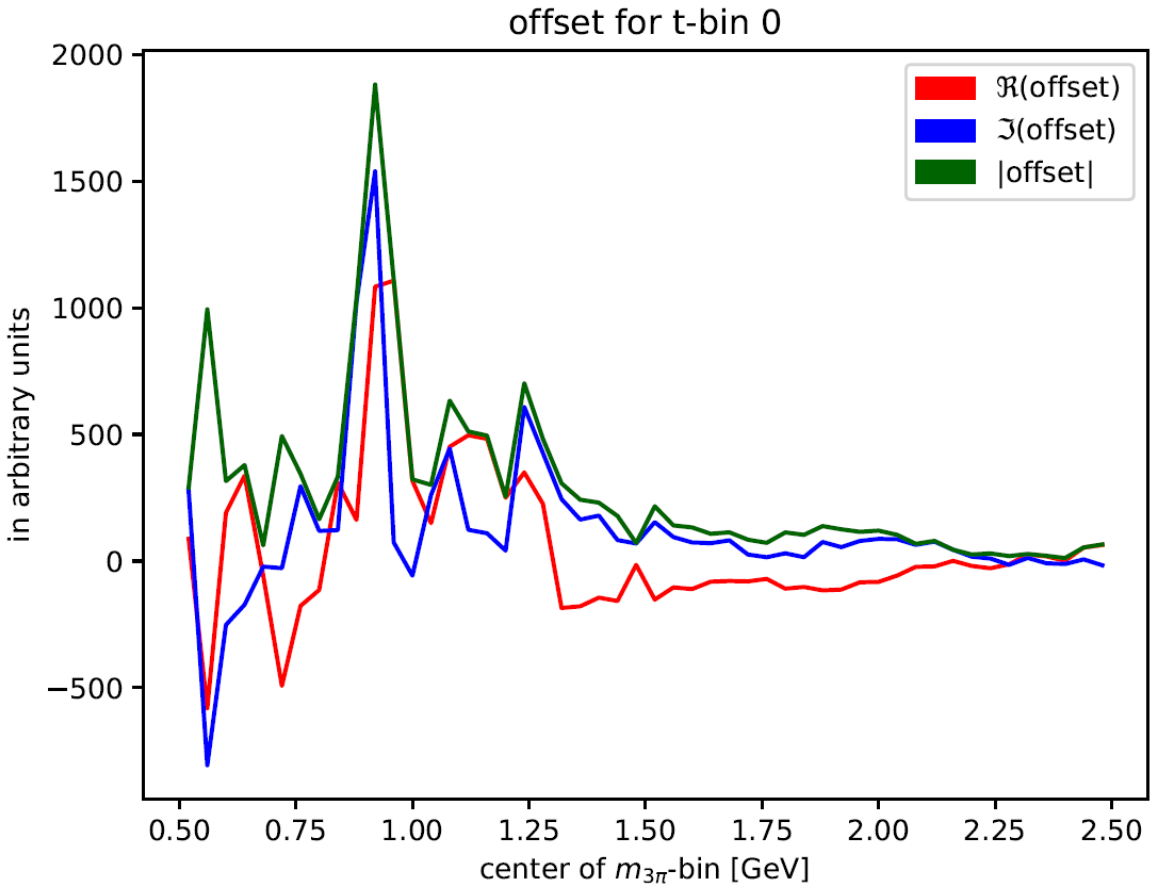
no offset

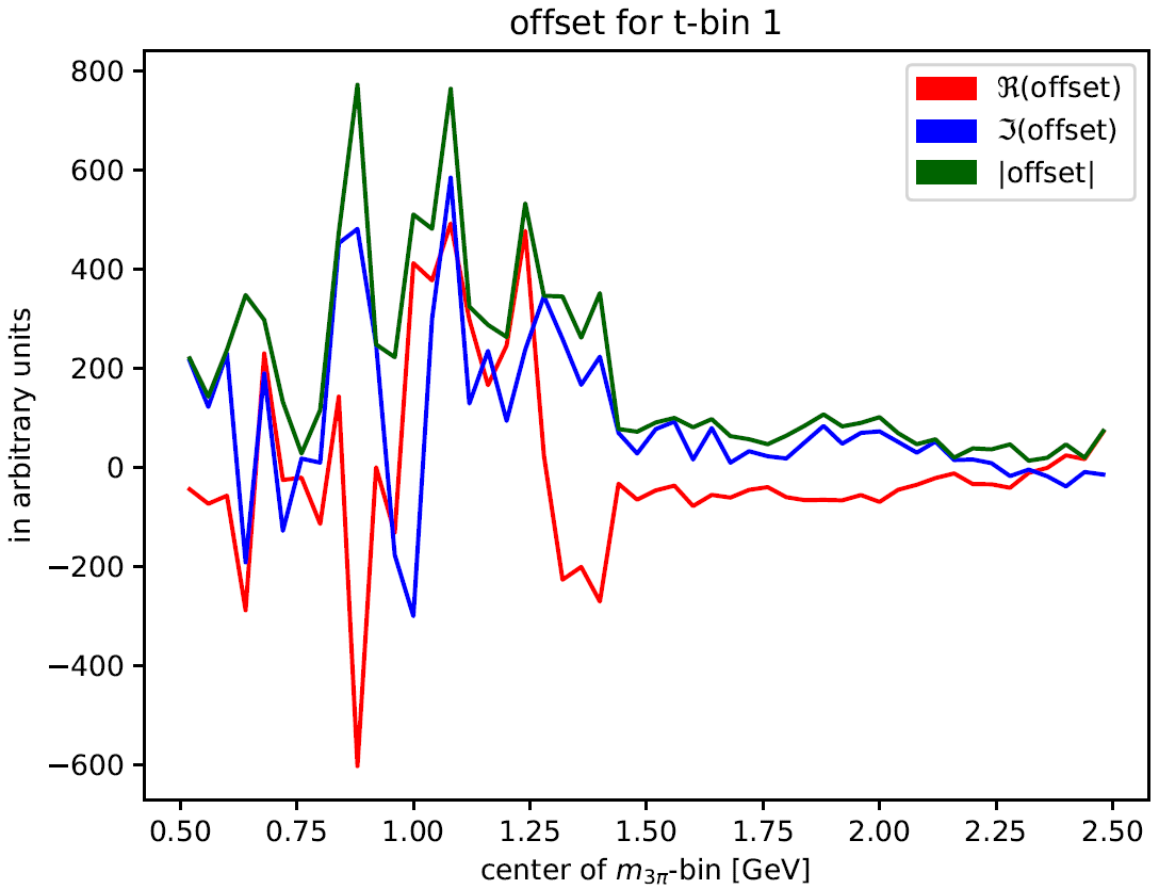


with offset

- significant improvement of description of data across entire kinematic range

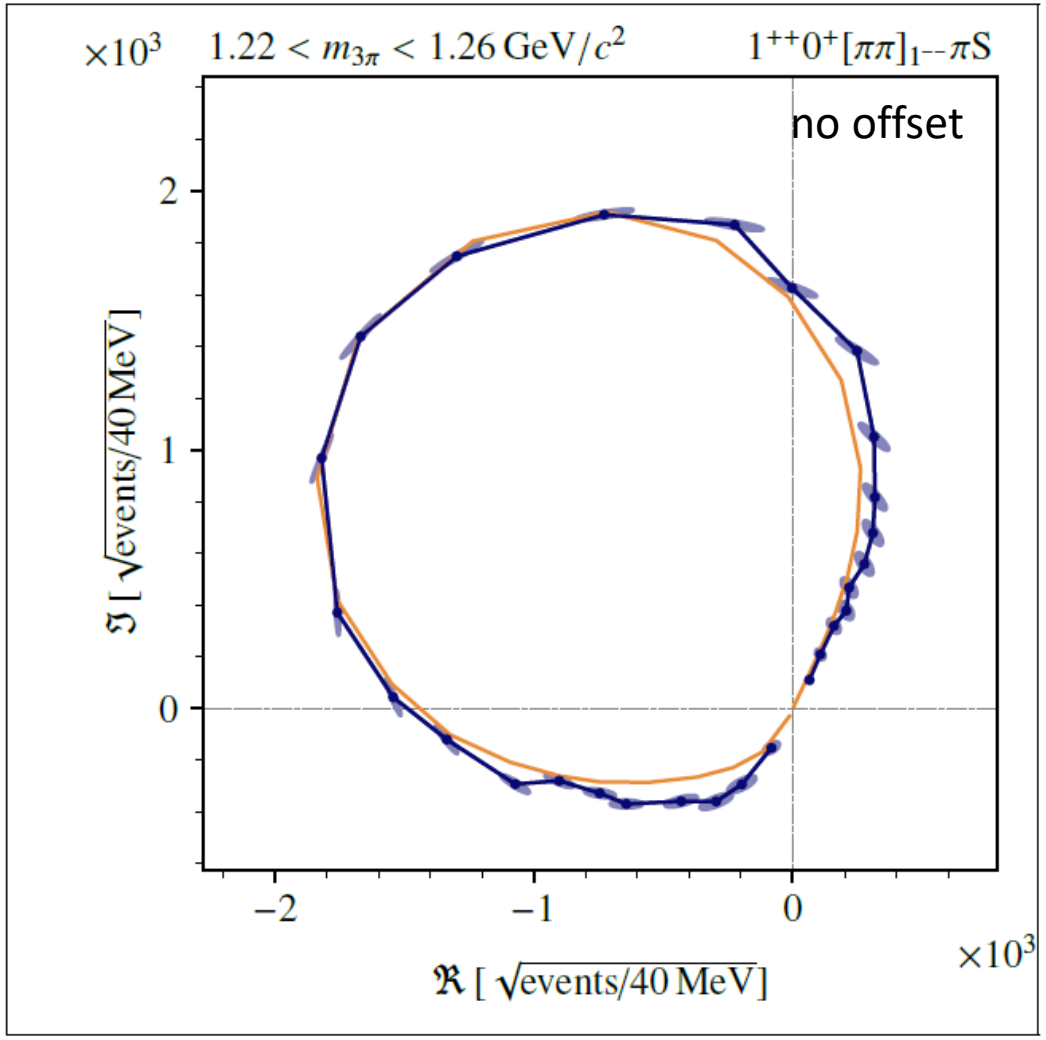
Offset Parameter Distribution



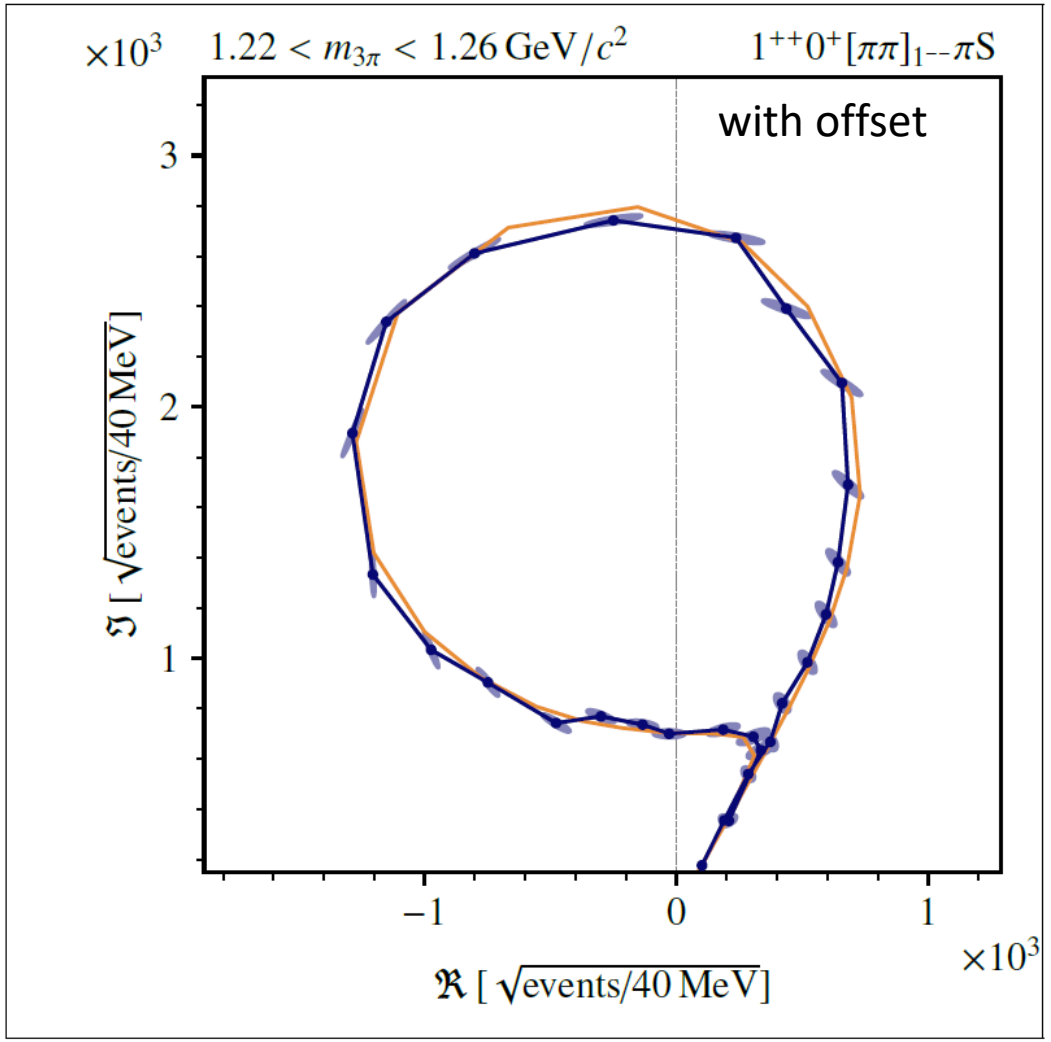


- qualitatively similar behavior in all 4 t′ bins
- C sizeable in region $m_{3\pi}$ < 1.5 GeV

Argand Diagram Examples in $m_{3\pi}$-Bins



better agreement



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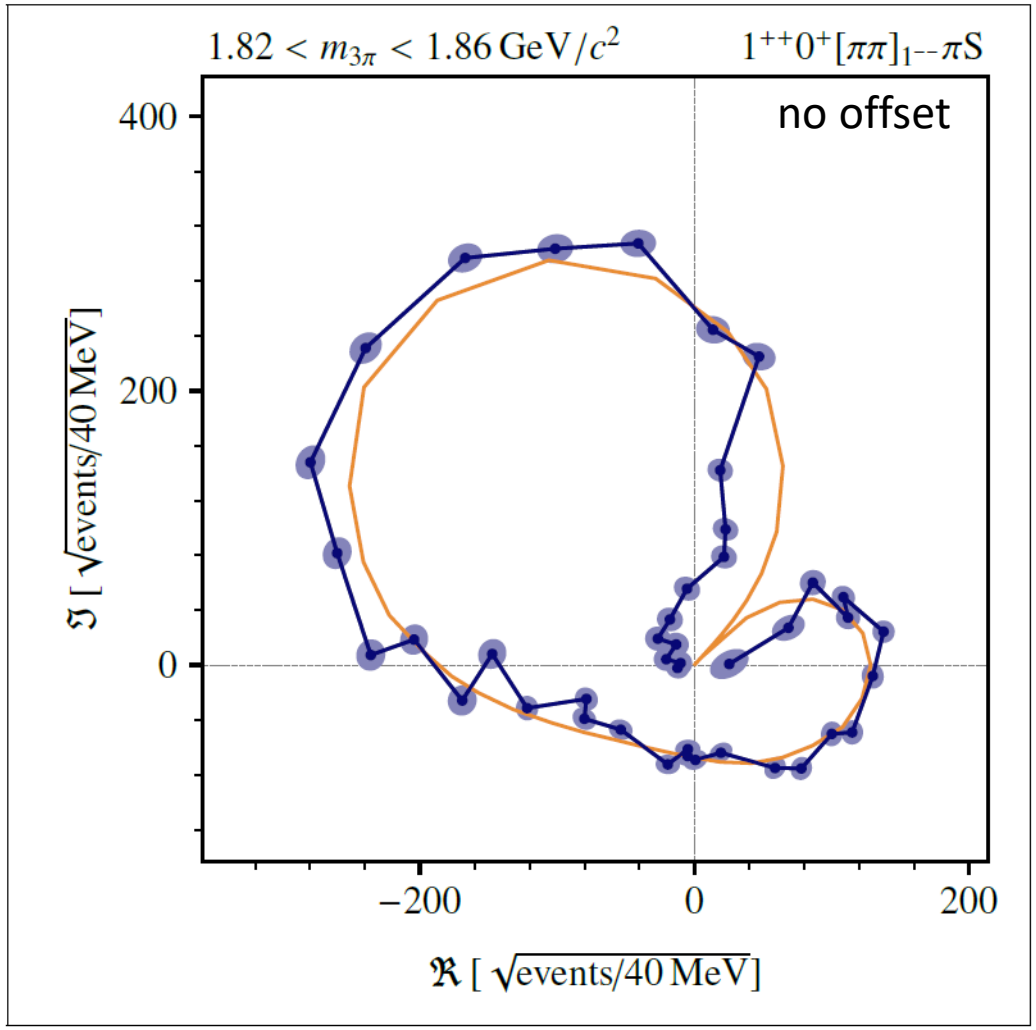
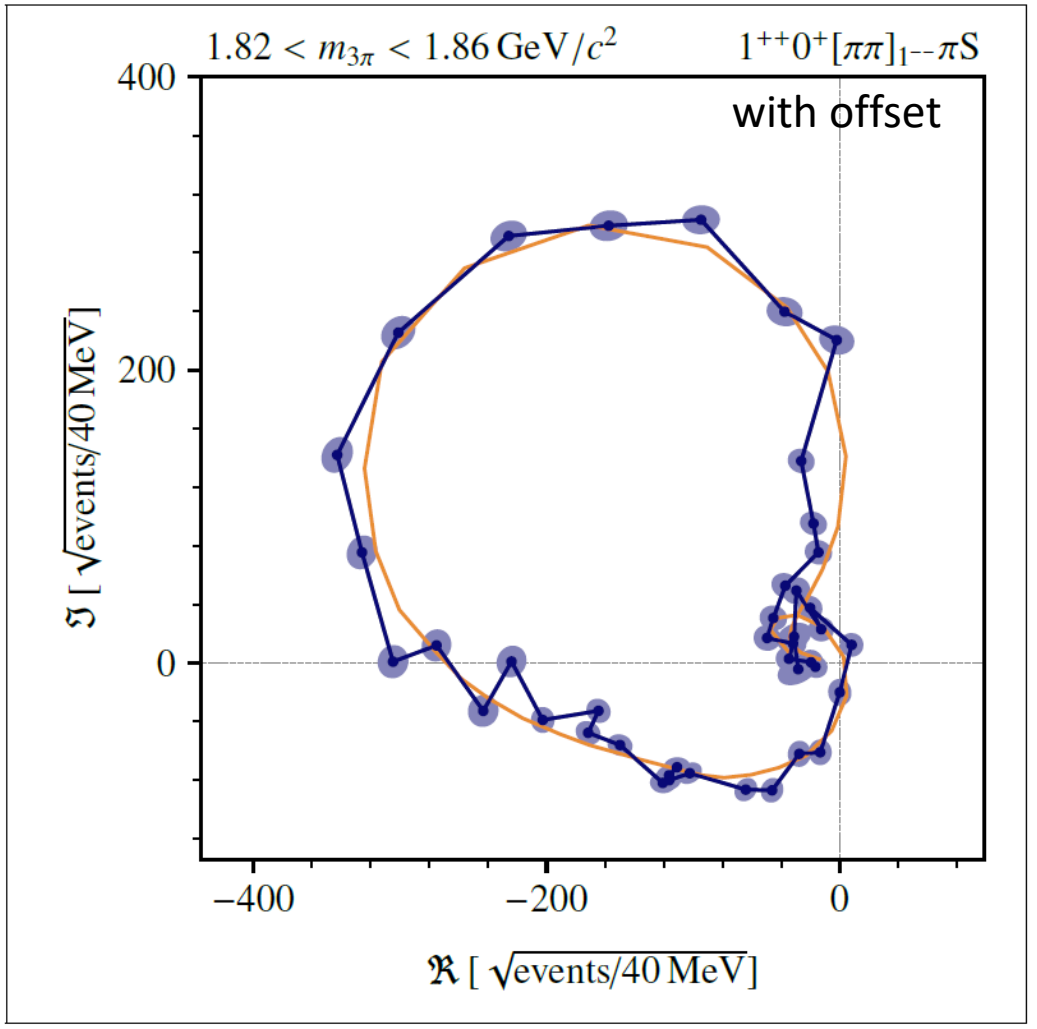
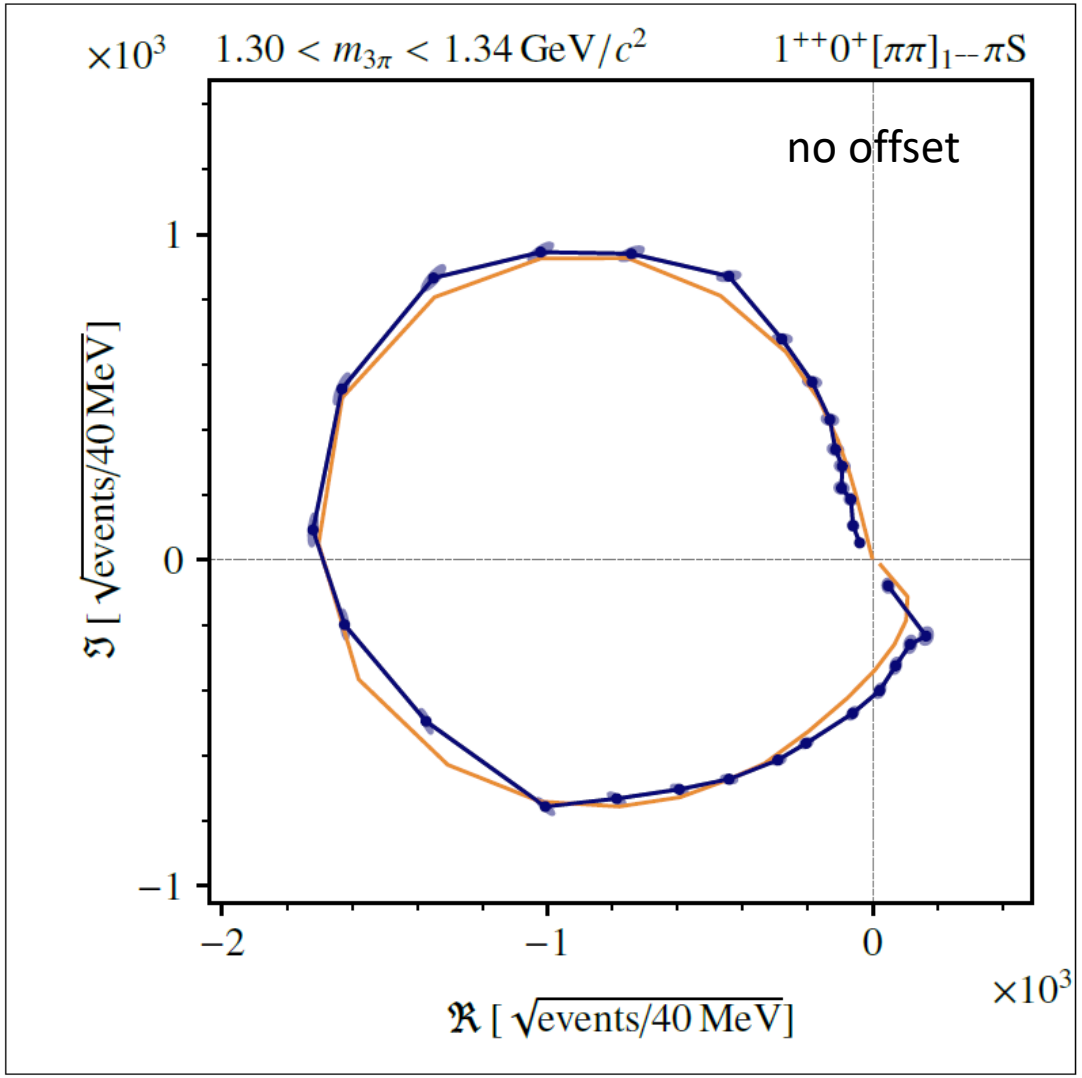


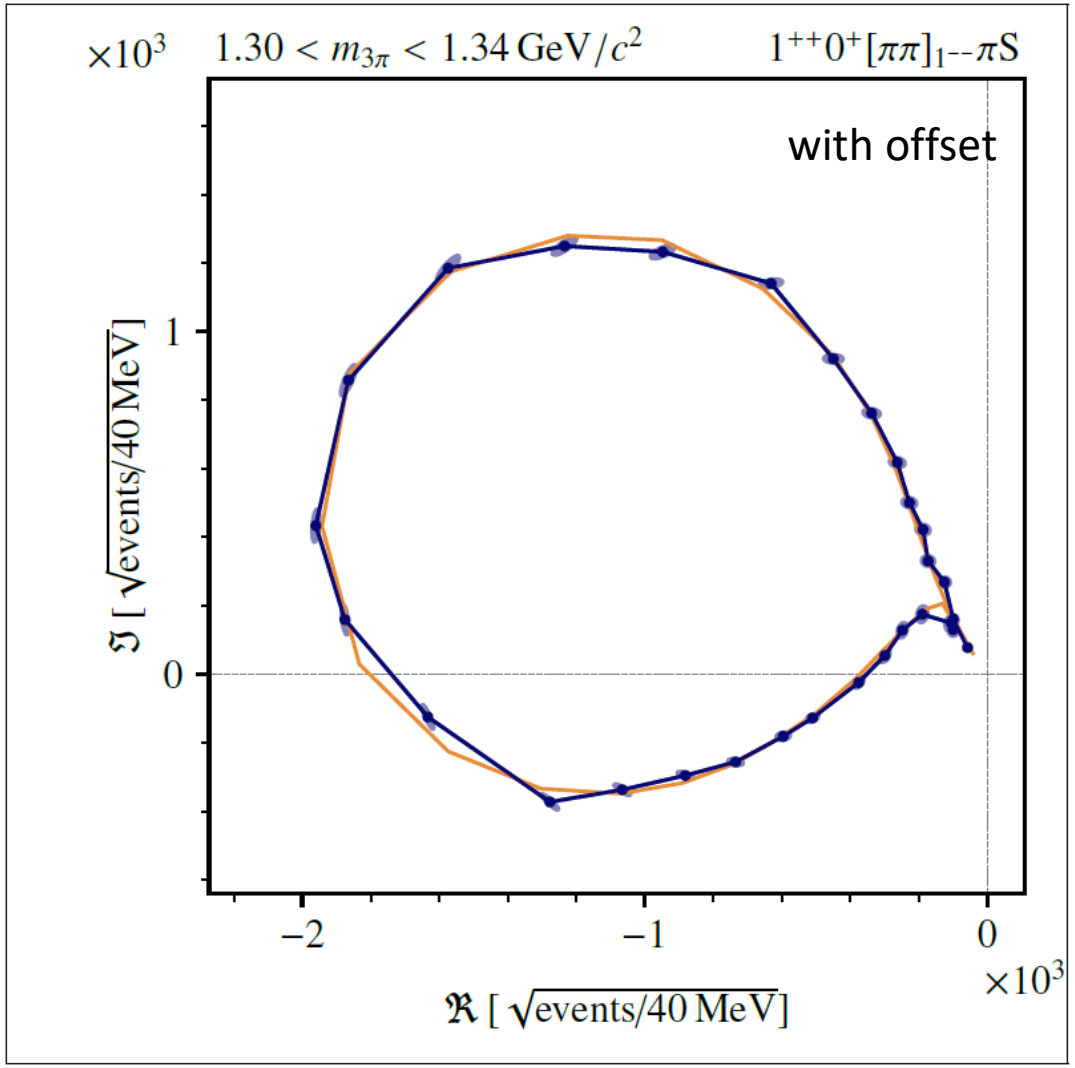
slightly better agreement



Argand Diagram Examples in $m_{3\pi}$-Bins



similar agreement



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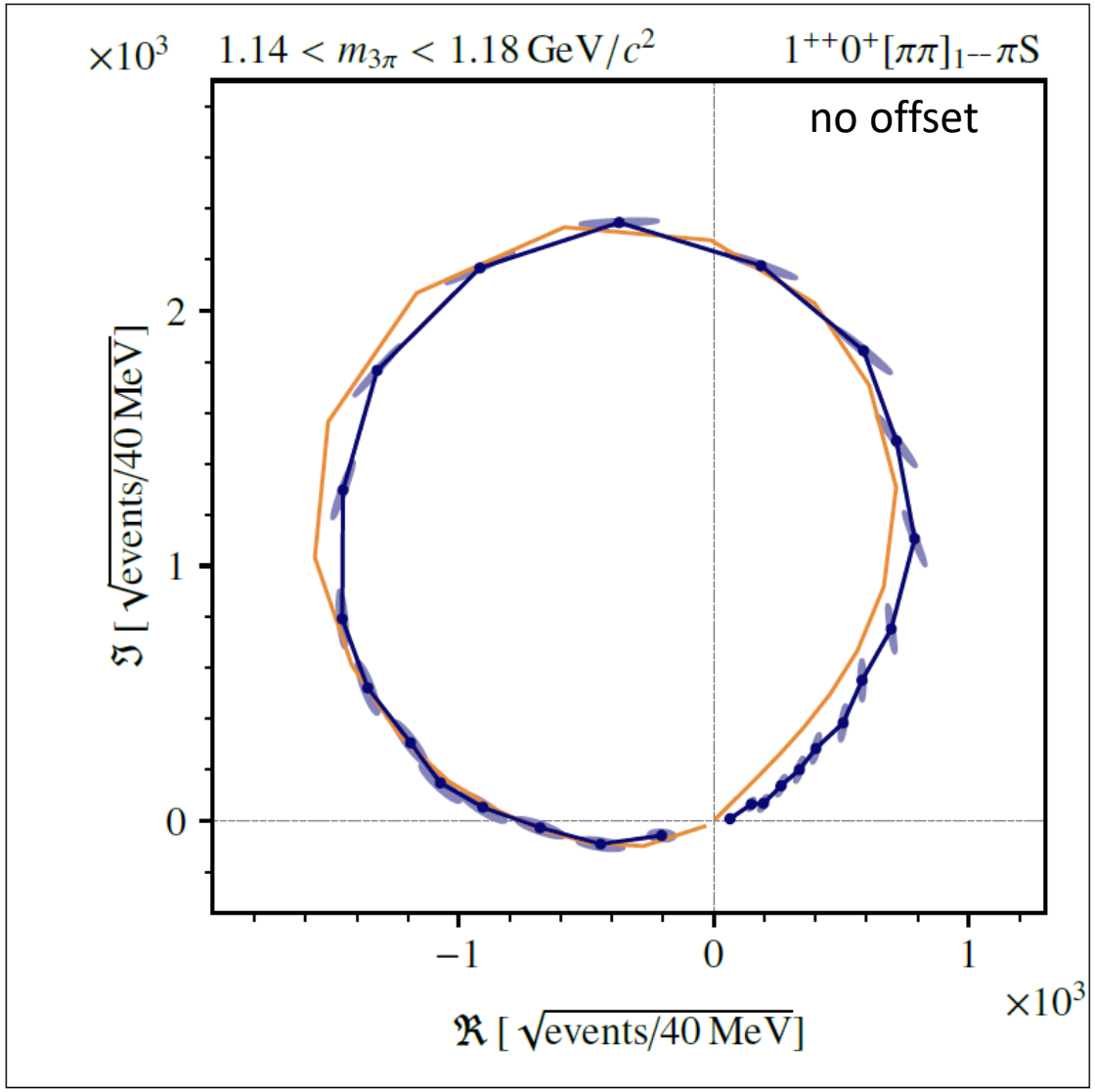
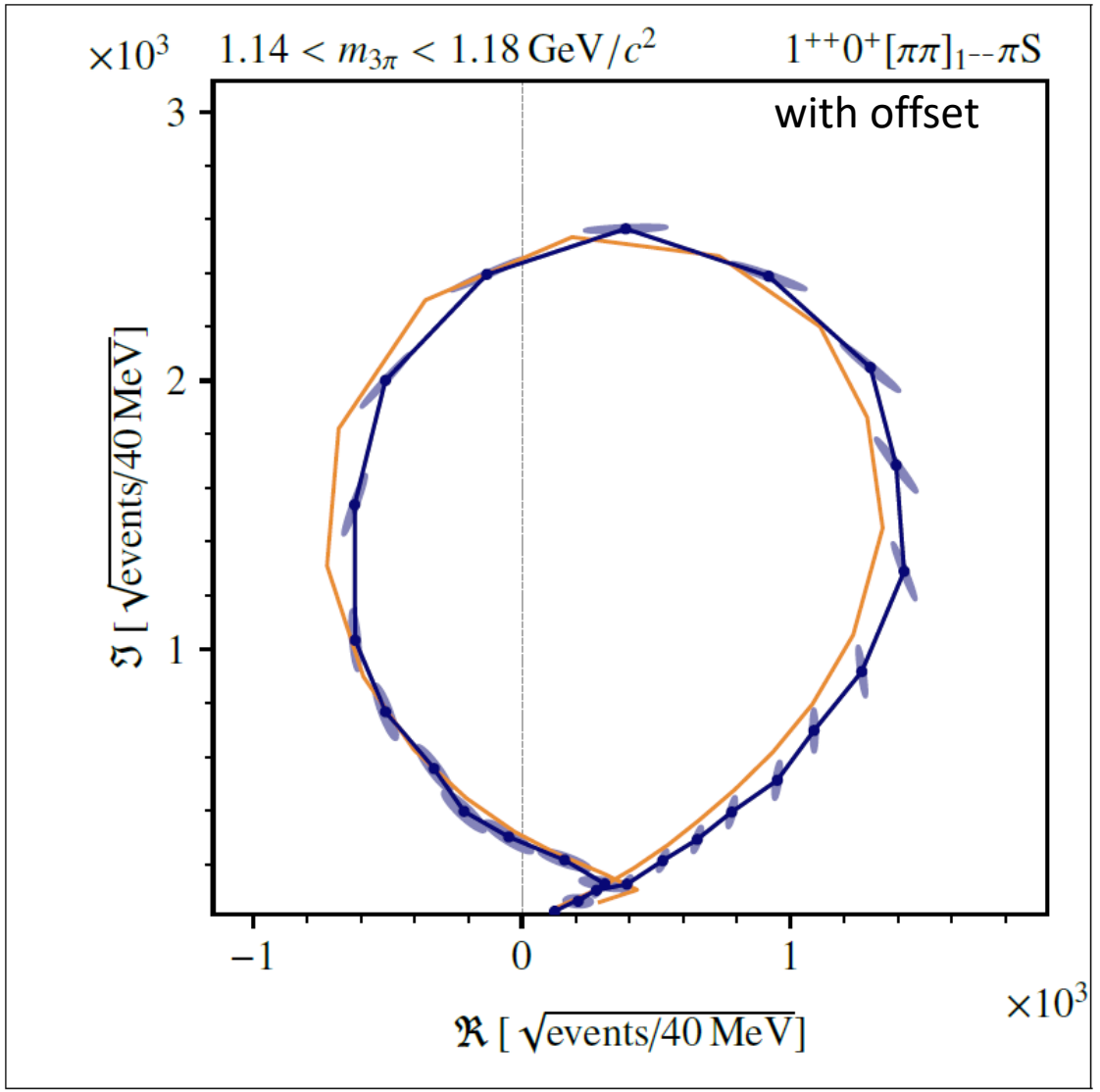


slightly worse agreement



Change in Resonance Parameters

improvement of fit quality
- standard: χ^2/n.d.f. = 4.277
- with offset: χ^2/n.d.f. = 3.646

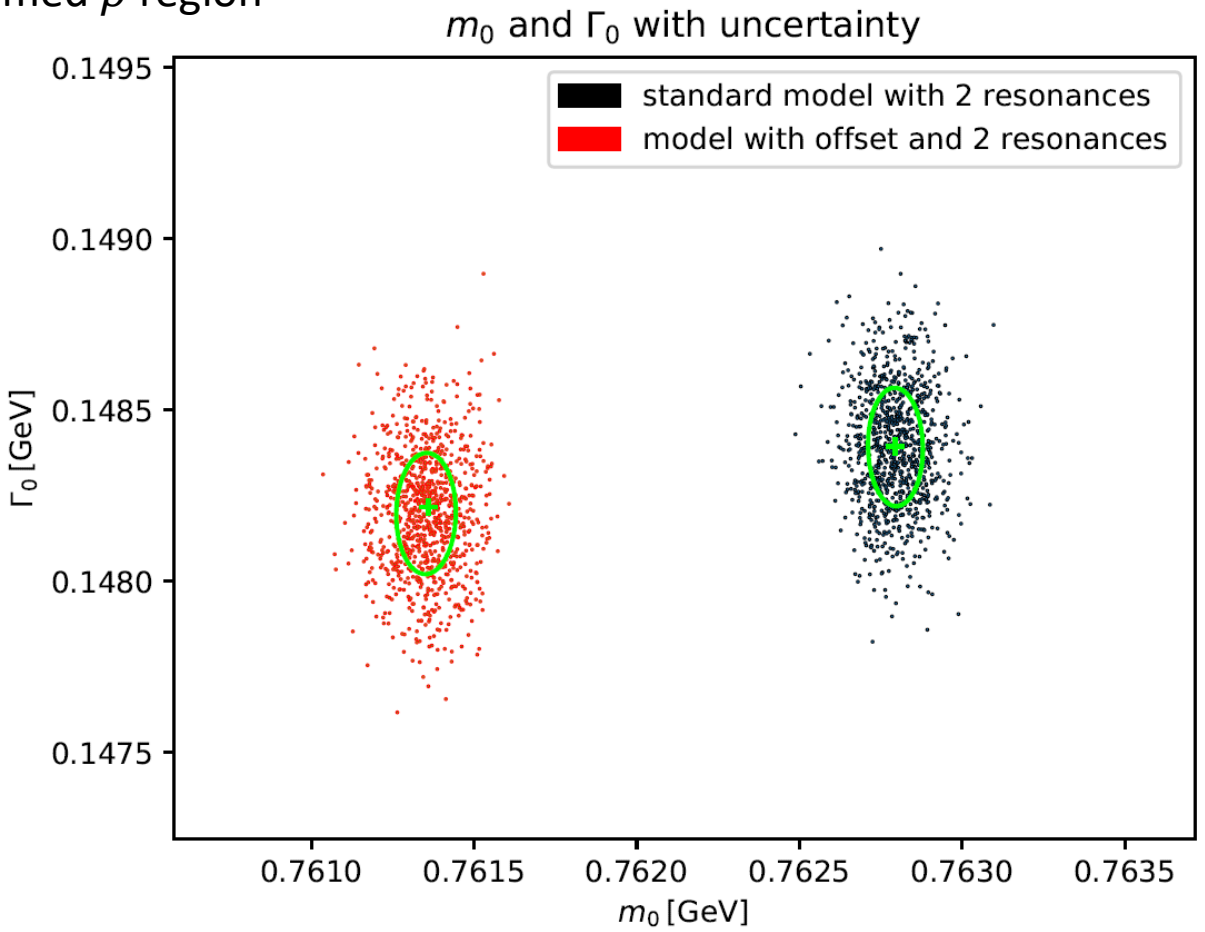
change of mass and width of ρ :
- standard:

$m_0 = 762.8 \pm 0.1$ MeV
$\Gamma_0 = 148.4 \pm 0.2$ MeV
- with offset:

$m_0 = 761.4 \pm 0.1$ MeV
$\Gamma_0 = 148.2 \pm 0.2$ MeV

compared to statistical uncertainties
offsets modify resonance parameters significantly
→ need to study systematic uncertainties

zoomed ρ region



Change in Resonance Parameters

improvement of fit quality
- standard: χ^2/n.d.f. = 4.277
- with offset: χ^2/n.d.f. = 3.646

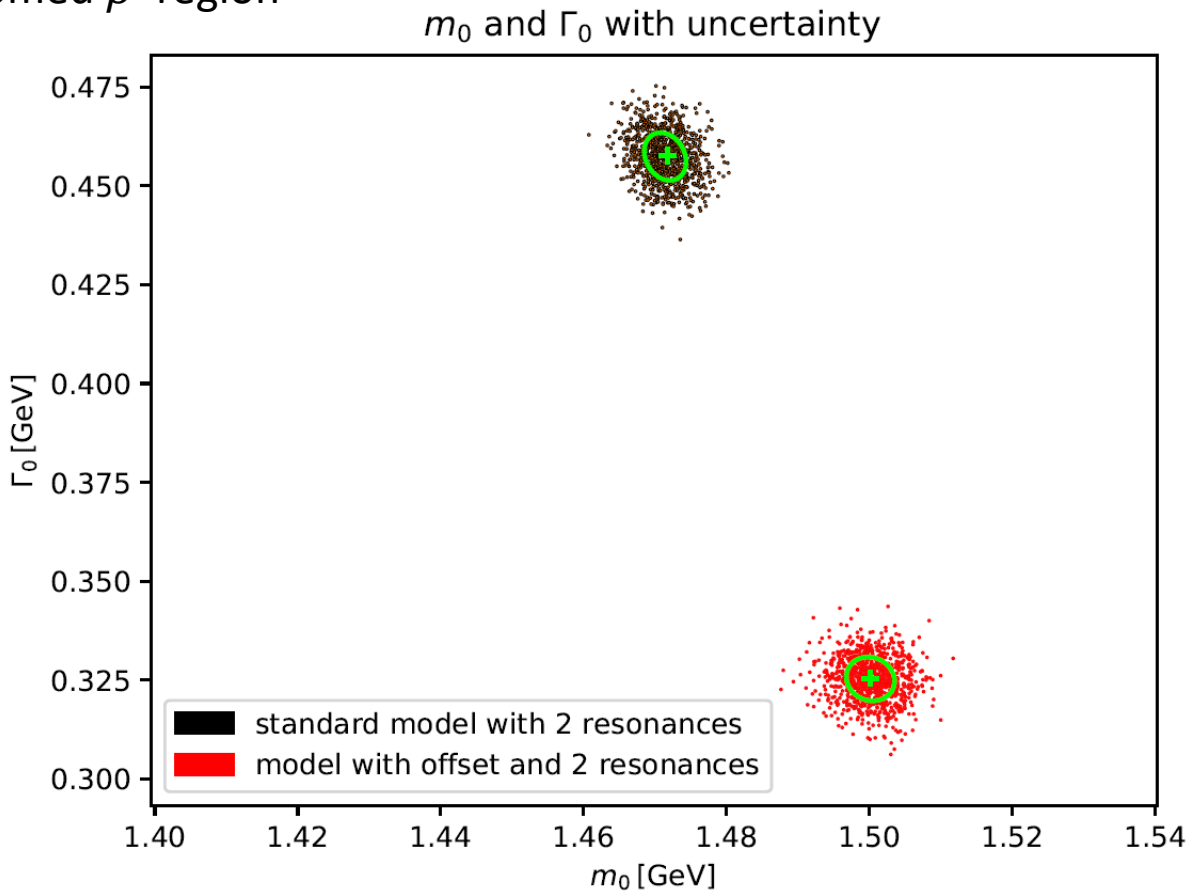
change of mass and width of ρ' :
- standard:

$m_0 = 1471 \pm 3$ MeV
$\Gamma_0 = 457 \pm 6$ MeV
- with offset:

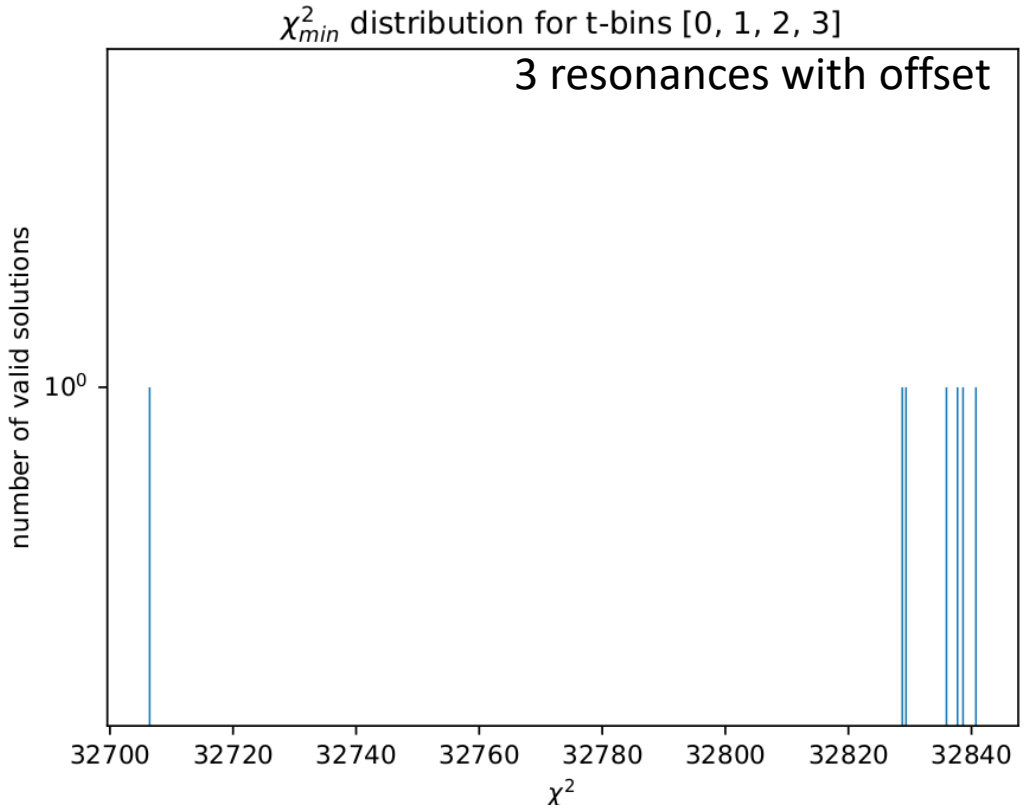
$m_0 = 1500 \pm 3$ MeV
$\Gamma_0 = 325 \pm 6$ MeV

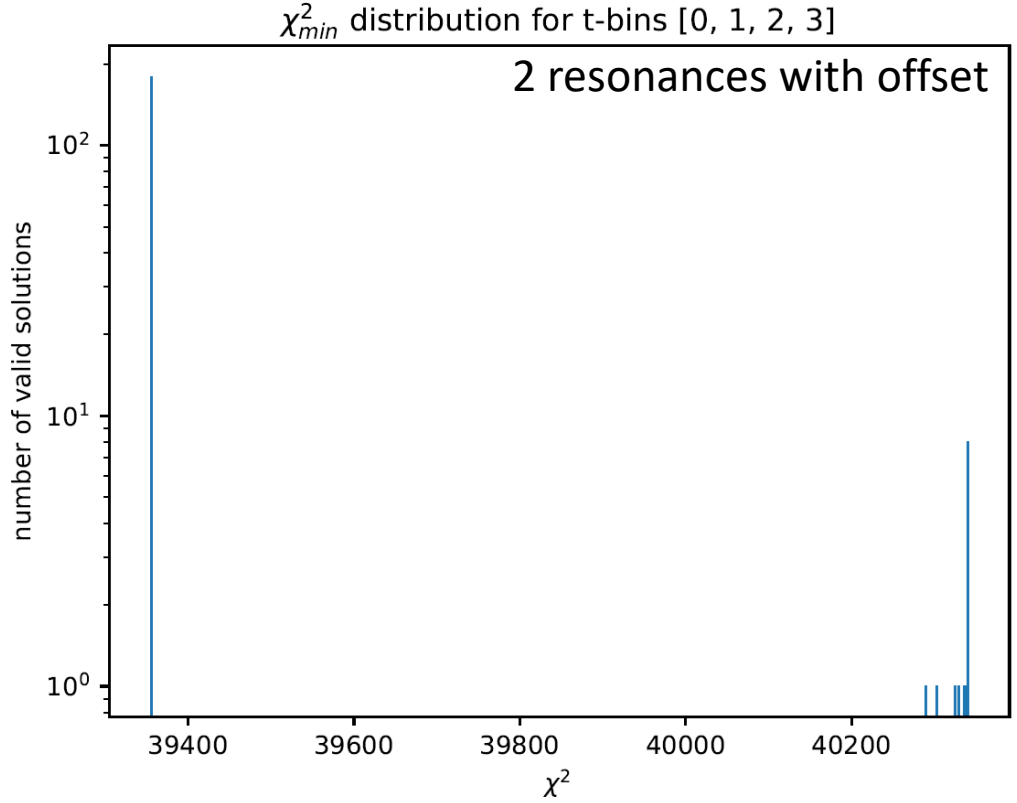
compared to statistical uncertainties
offsets modify resonance parameters significantly
→ need to study systematic uncertainties

zoomed ρ' region



Resonance-Model Fit with 3 ρ Resonances





- for each resonance-model fit, we perform 1000 fit attempts with random start values
- we check the validity of each fit attempt and take the valid attempt with the lowest χ^2 value
- for resonance-model with 3 resonances and offset only 7 of 1000 fit attempts valid

Summary

- Fitted amplitude of $\pi^- \pi^+$ subsystem with $J^{PC}_{\xi} = 1^{--}$ in $\pi^- \pi^+ \pi^-$ system with $J^{PC}_{X} = 1^{++}$
- proof of principle: COMPASS data can be described over wide $m_{3\pi}$ and t' range
- below $m_{3\pi}$ = 1.8 GeV $\rho(770)$ pole position independent of $m_{3\pi}$ and t'
- model not able to describe some features of the data
- seems that background-like component is needed
- fit with 3 ρ poles not (yet) reliable
  - data may be insufficient to constrain the model parameters

- Can the model be improved further?
- Machinery to test and compare different models on COMPASS data is working

Outlook

- study other 3π waves decaying into $\rho\ \pi$
- study combined fits of several $\rho\ \pi$ waves to better constrain ρ' mass region
- extend analysis to full COMPASS data set ($46 * 10^6$ events → $115 * 10^6$ events)
  - improved PWA wave set
  - PWA with more freed-isobar waves possible
  - finer binning in t' possible