Extracting the $\pi^-\pi^+$ Subsystem Amplitudes in diffractively produced $\pi^-\pi^-\pi^+$ at COMPASS

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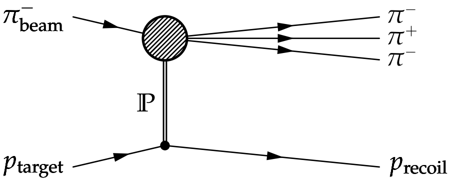
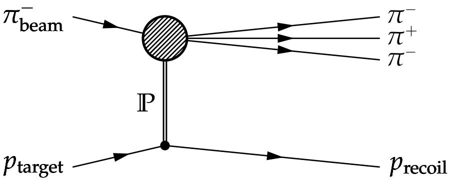


- 190 GeV/c pion beam
- Exclusive measurement
  - Clean data sample
- 46×10^6 $\pi^-\pi^-\pi^+$ events
- Squared four-momentum transfer $0.1 < t' < 1.0\,(\text{GeV}/c)^2$
- Well-known 3π resonances appear in $m_{3\pi}$ spectrum

Diffractive $\pi^-\pi^-\pi^+$ Production at COMPASS

COMPASS, PRD **95** (2017) 032004



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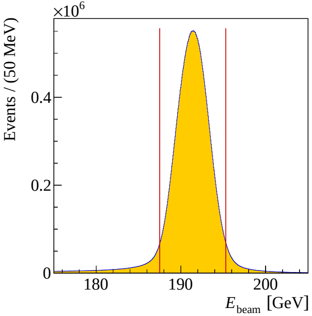

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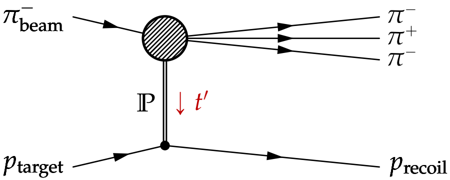

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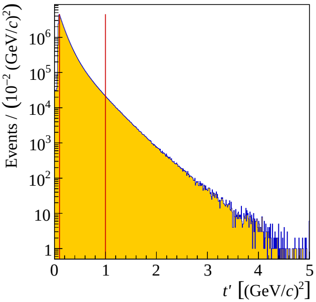

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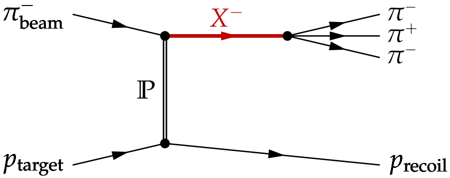

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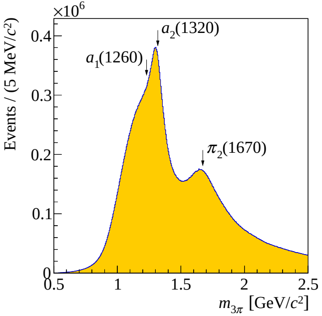

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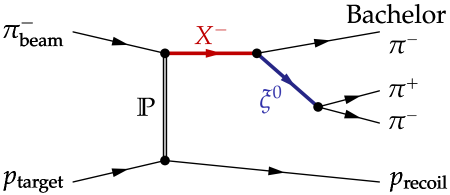

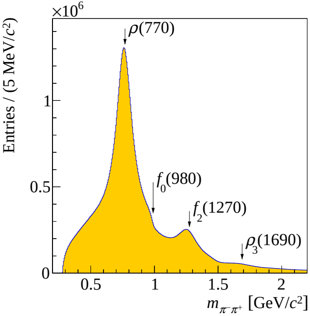

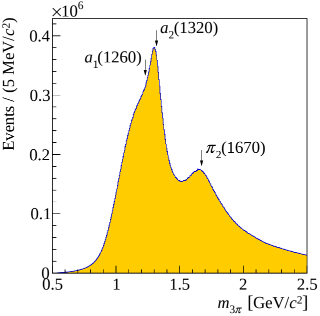

Decay of X via intermediate $\pi^-\pi^+$ resonances = “isobars”

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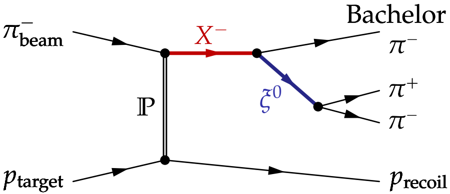

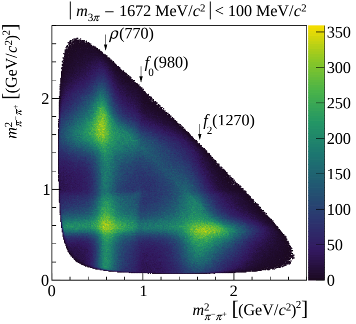

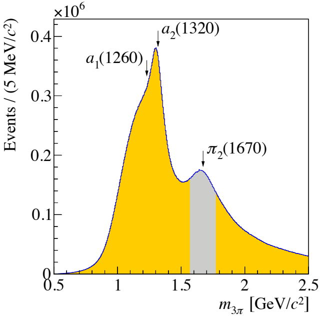

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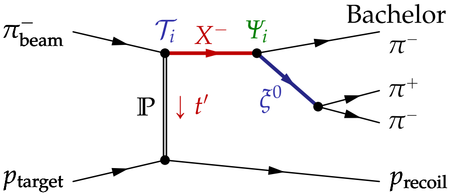

Ansatz: Factorization of production and decay

$$\mathrm{d}\sigma \propto \left| \sum_i^{\text{waves}} \mathcal{T}_i(m_{3\pi}, t')\, \Psi_i(m_{3\pi}, \tau) \right|^2 \mathrm{d}m^2_{3\pi}\ \mathrm{d}t'\ \mathrm{dLIPS}_3(m_{3\pi}, \tau)$$

- PWA model: coherent sum of partial-wave amplitudes
- Decay amplitudes $\Psi_i(m_{3\pi}, \tau)$
  - Describe 5-dimensional τ distribution of partial waves
  - Calculated using isobar model and helicity formalism
- Transition amplitudes $\mathcal{T}_i(m_{3\pi}, t') \Longrightarrow$ **interesting physics**
  - Dependence on $m_{3\pi}$ and t' unknown
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Partial-Wave Decomposition of $\pi^-\pi^-\pi^+$ Final State

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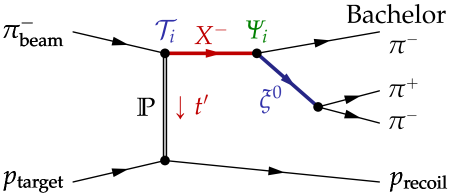

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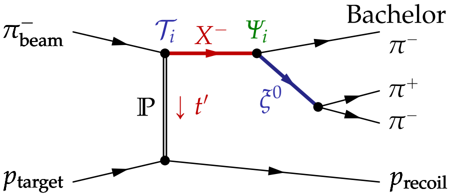

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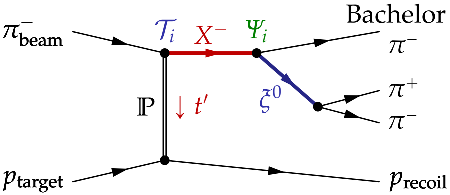

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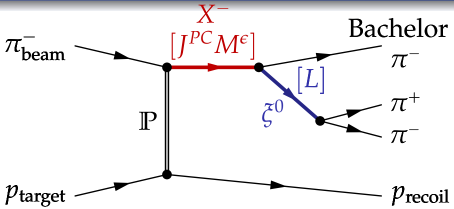

For $m_{3\pi} =$ const, 3π kinematic distribution is completely defined by

- Quantum numbers of X
- Orbital angular momentum L between ξ and bachelor π
- Isobar resonance $\xi \Longrightarrow$ model for $m_{\pi^-\pi^+}$ dependence of amplitude; e.g. Breit-Wigner amplitude for $\rho(770) \to \pi^-\pi^+$

Partial wave: short-hand notation

$$J^{PC}\, M^\varepsilon\, \xi\, \pi\, L$$

- Represents specific 5-dimensional kinematic distribution

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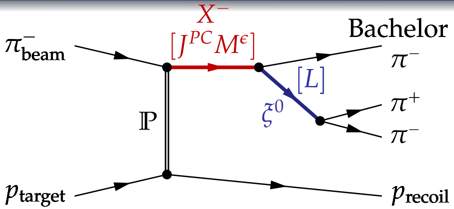

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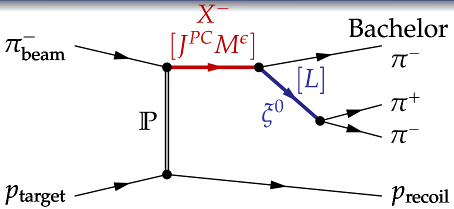

Fit model

- Included isobar resonances:
  - $[\pi\pi]_S$ — $J^{PC} = 0^{++}$
  - $\rho(770)$ — 1^{--}
  - $f_0(980)$ — 0^{++}
  - $f_2(1270)$ — 2^{++}
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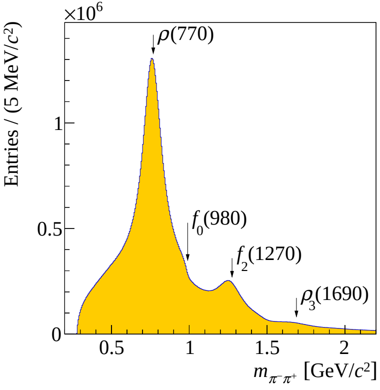
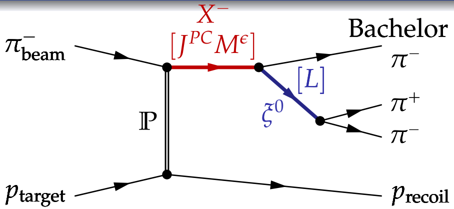
  - $\rho_3(1690)$ — 3^{--}
- Requires precise knowledge of isobar $\to \pi^-\pi^+$ amplitudes





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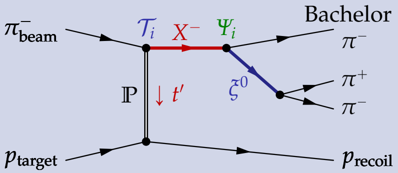
- J and L up to 6
- 87 partial waves
- Additional incoherent isotropic background wave

The Freed-Isobar Partial-Wave Analysis Method

Krinner *et al.*, PRD **97** (2018) 114008



- Calculation of decay amplitudes $\Psi_i(\tau)$ needs exact, parameter-free knowledge of amplitude for isobar $\to \pi^-\pi^+$
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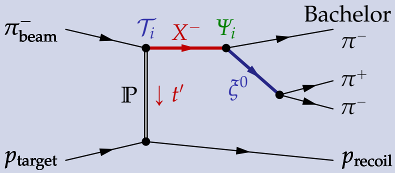

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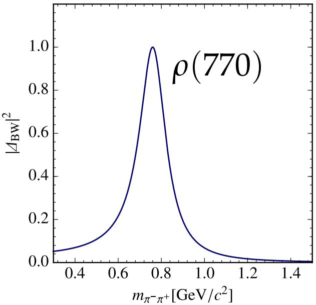

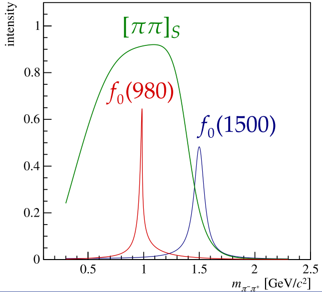

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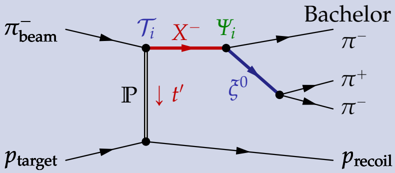
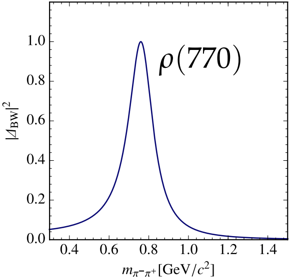
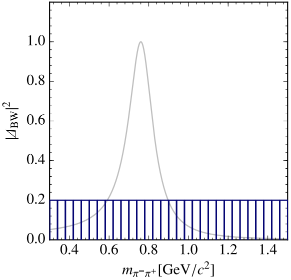


- *Novel technique:* **"freed-isobar" PWA**
  - Replace fixed isobar parametrizations by step-like functions
  - Extract isobar amplitude from data
  - No assumptions about isobar resonances
    - Reduced model dependence
    - Study $m_{3\pi}$ and t' dependence

Freed-isobar PWA of COMPASS data

- Replace 16 of the 87 waves by 12 freed-isobar waves





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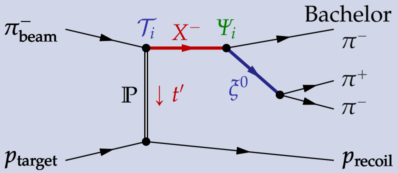

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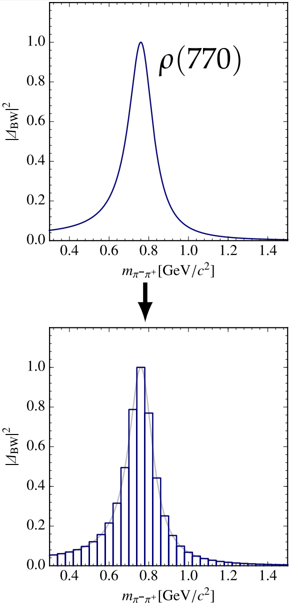

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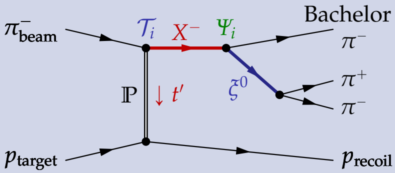

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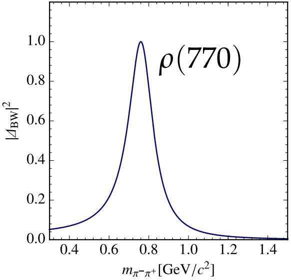

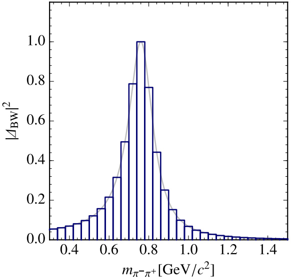

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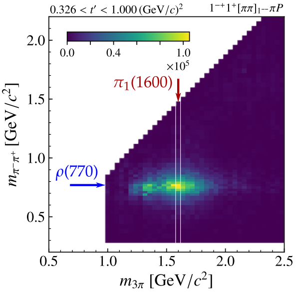
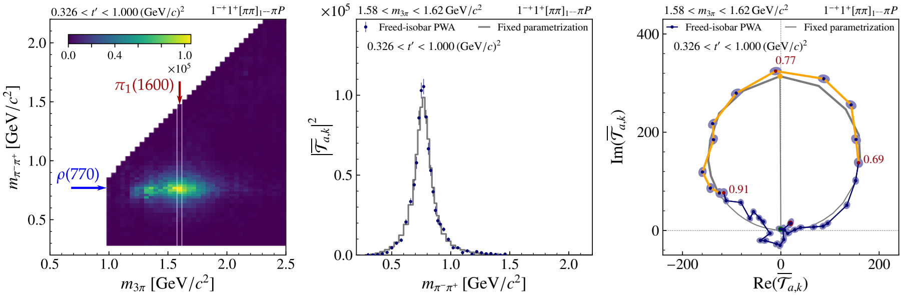


- Intensity peak at $m_{3\pi} \approx 1.6\,\text{GeV}/c^2$ and $m_{\pi^-\pi^+} \approx 0.8\,\text{GeV}/c^2$
- Clear $\rho(770)$ signal: peak in intensity + circular structure in Argand diagram

- $\rho(770)$ parametrization used in conventional PWA (gray line) agrees fairly well with measured $\pi^-\pi^+$ amplitude

Example: $\pi^-\pi^+$ Amplitude in $1^{-+} \to [\pi\pi]_{1^{--}} + \pi^-$

$0.326 < t' < 1.000\,(\mathrm{GeV}/c)^2$ *[arXiv:2108.01744]*



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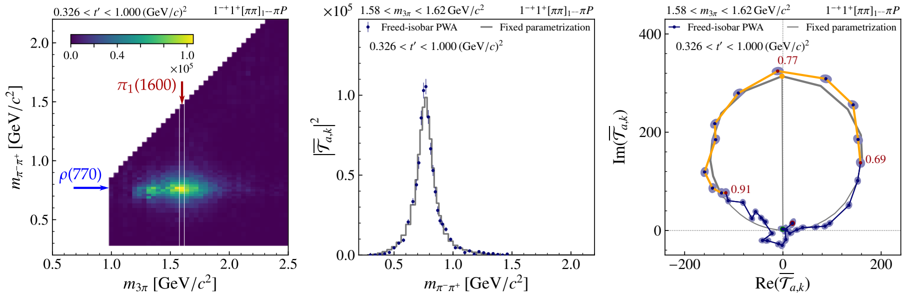

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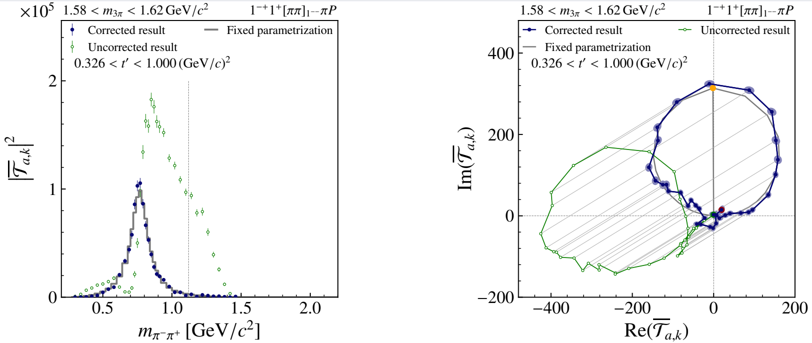
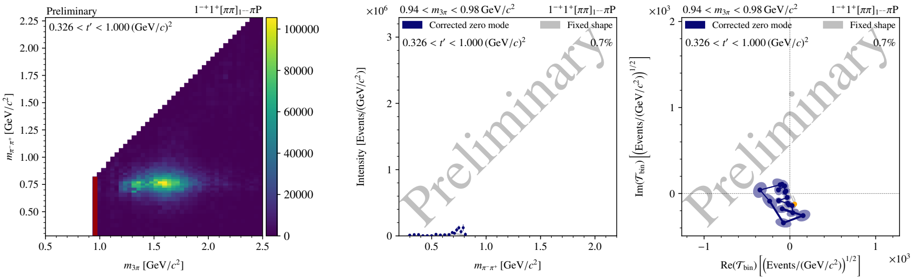
Zero-Mode Ambiguity

[arXiv:2108.01744]



- In freed-isobar PWA mathematical ambiguities may arise on amplitude level
  - $\mathcal{T}^{\text{fit}}(m_{\pi^-\pi^+}) = \mathcal{T}^{\text{phys}}(m_{\pi^-\pi^+}) + \mathfrak{C}\,\Delta(m_{\pi^-\pi^+})$
  - Any value of $\mathfrak{C}$ leads to identical total amplitude $\Longrightarrow$ continuous mathematical ambiguity $\Longrightarrow$ "zero mode"
  - "Zero-mode shape" $\Delta(m_{\pi^-\pi^+})$ can be calculated
  - $\mathfrak{C}$ determined by additional constraints



Challenges

- Modelling of $m_{\pi^-\pi^+}$ dependence of amplitudes (see next talk by Martin Bartl)
  - Presence of excited isobar states?
  - Rescattering effects due to bachelor pion?
  - How to feed back information into conventional PWA?

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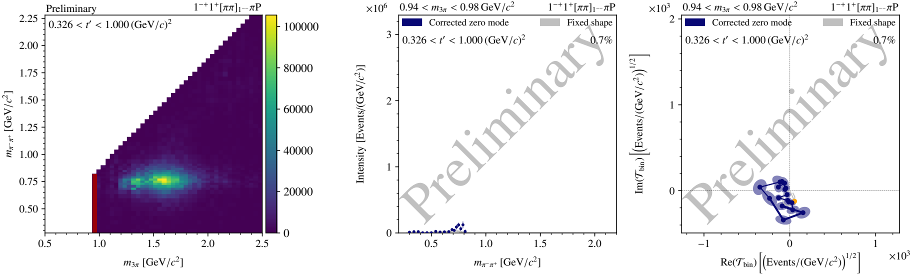

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Summary

Freed-Isobar Partial-Wave Analysis Method

- Greatly reduces model bias of $\pi^-\pi^-\pi^+$ PWA
- Detailed insight into 2π vs. 3π dynamics
  - Verify/learn isobar parametrizations from data
  - Search for higher excited isobar states
  - Study of final-state-interaction effects
- Ambiguities on amplitude level appear when many waves are freed
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Challenges

- Requires large data sets
- Modeling of extracted isobar amplitudes
  - See next talk by Martin Bartl
- *Ultimate goal:* 2D fit of $m_{3\pi}$ and $m_{\pi^-\pi^+}$ dependence of amplitudes

Summary

Freed-Isobar Partial-Wave Analysis Method

- Greatly reduces model bias of $\pi^-\pi^-\pi^+$ PWA
- Detailed insight into 2π vs. 3π dynamics
  - Verify/learn isobar parametrizations from data
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Backup Slides

Partial-Wave Decomposition of $\pi^-\pi^-\pi^+$ Final State

COMPASS, PRD **95** (2017) 032004

PWA fit in given $(m_{3\pi}, t')$ bin

- Neglect $m_{3\pi}$ and t' dependence within bin

$$\mathrm{d}\sigma \propto \underbrace{\left|\sum_{i}^{\text{waves}} \mathcal{T}_i \Psi_i(\tau)\right|^2}_{\equiv \mathcal{I}(\tau;\{\mathcal{T}_i\})} \mathrm{dLIPS}_3(\tau)$$

- Model for measured τ distribution: intensity $\mathcal{I}(\tau;\{\mathcal{T}_i\})$
- Extended likelihood function

$$\mathcal{L}_{\text{ext}}(\{\mathcal{T}_i\}) = \underbrace{\frac{\overline{N}^N e^{-\overline{N}}}{N!}}_{\substack{\text{Poisson} \\ \text{distribution}}} \prod_{k=1}^{N} \underbrace{\frac{\mathcal{I}(\tau_k;\{\mathcal{T}_i\})}{\int \mathrm{dLIPS}_3(\tau)\,\eta(\tau)\,\mathcal{I}(\tau;\{\mathcal{T}_i\})}}_{\substack{\text{Probability density} \\ \text{for event } k}}$$

- Estimate $\{\mathcal{T}_i\}$ by maximizing $\ln \mathcal{L}_{\text{ext}}$

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COMPASS, PRD **95** (2017) 032004

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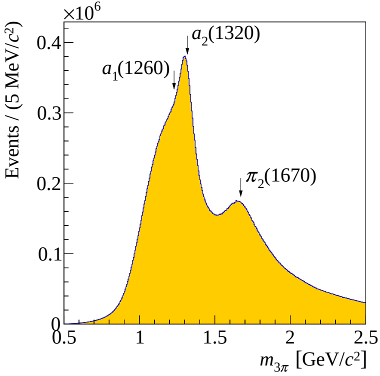
- PWA makes no assumptions about contributing 3π resonances

Partial-wave notation: $J^{PC}\ M^{\varepsilon}\ \xi\ \pi\ L$



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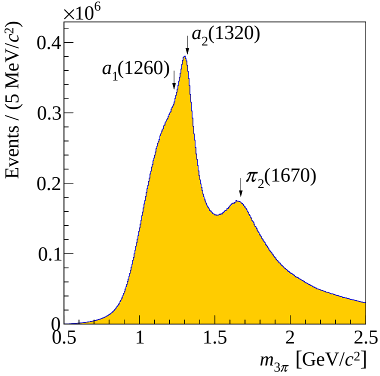

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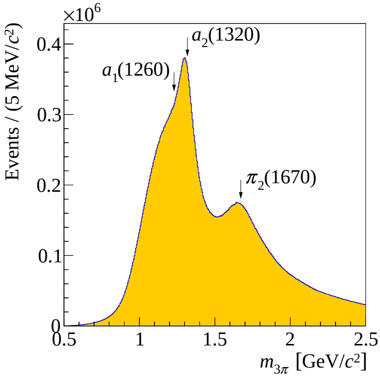
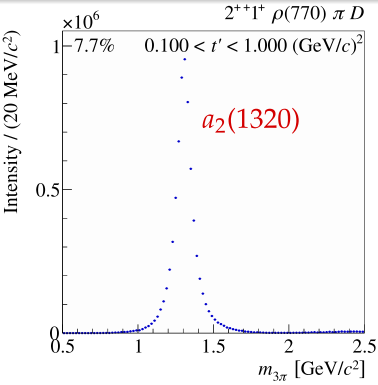
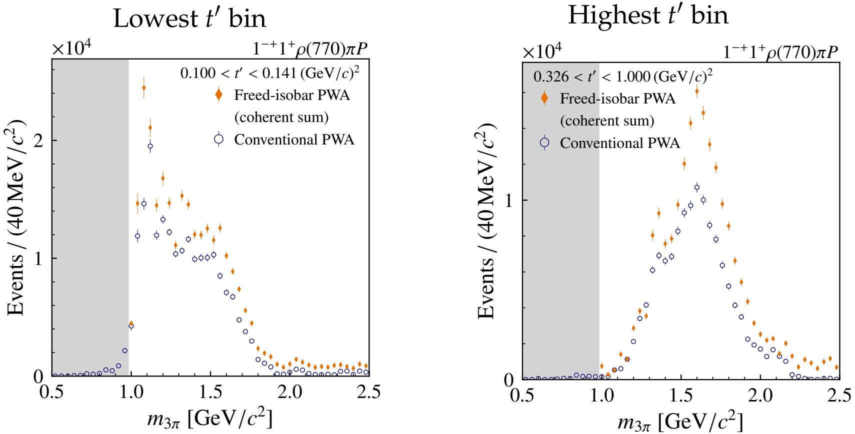




Comparison with Conventional PWA

Coherent Sum over full $m_{\pi^-\pi^+}$ Range

[arXiv:2108.01744]



- Freed-isobar PWA confirms existence of $\pi_1(1600) \to \rho(770)\pi$