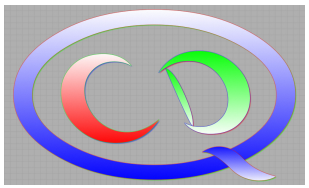
Deviation of Khuri-Treiman equations from Omnès functions

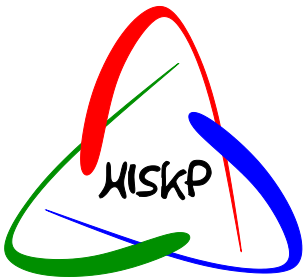
Dominik Stamen

MIAPP, TU München 2021



HISKP (Theorie)
Bonn University

20th December 2021

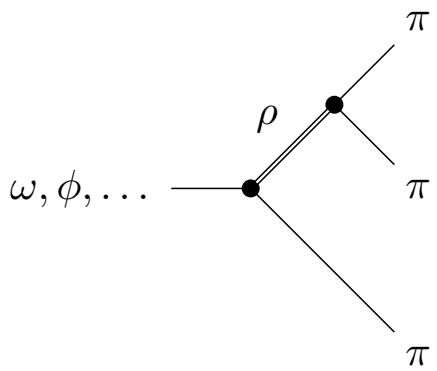


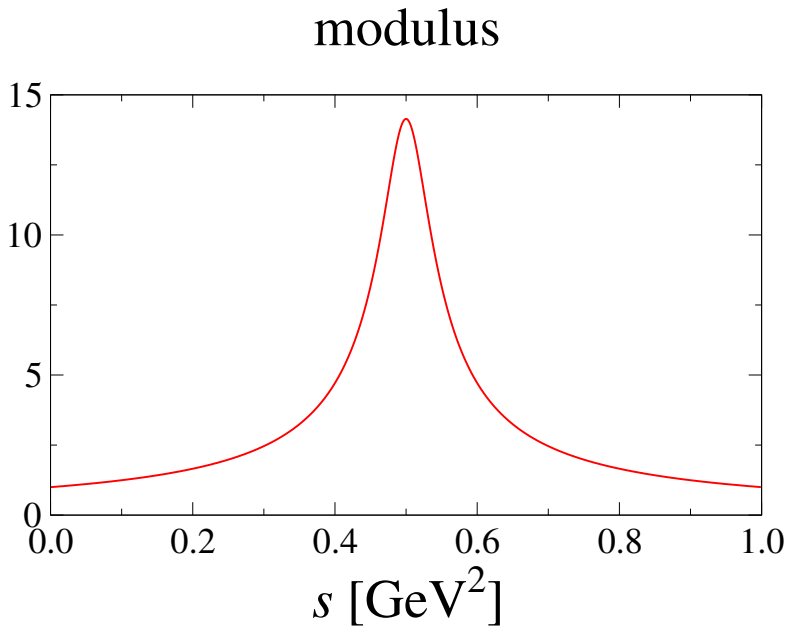
Tobias Isken, Bastian Kubis, and Malwin Niehus

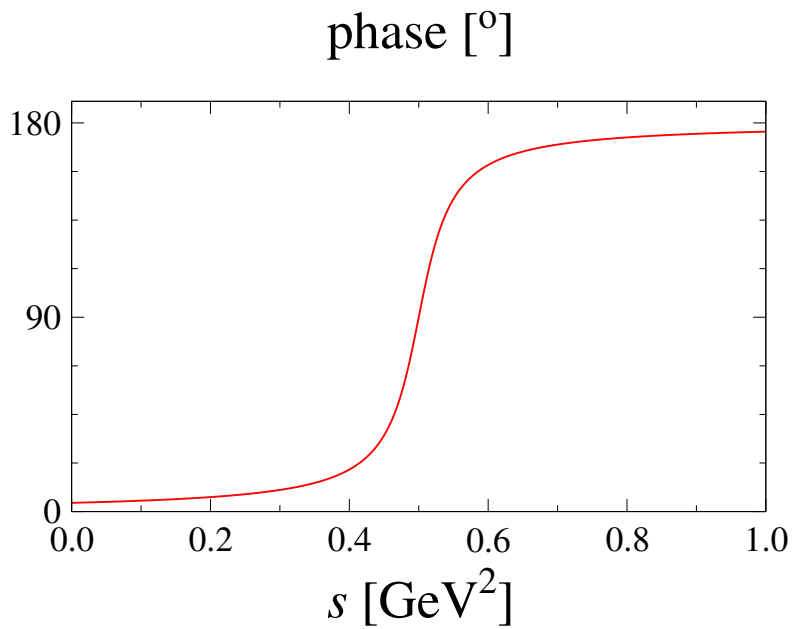
UNIVERSITÄT BONN

Motivation

The traditional picture: isobar model / Breit–Wigner resonances







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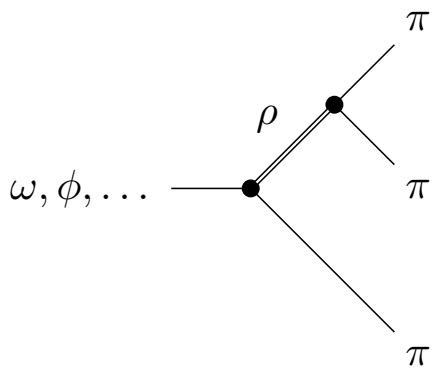
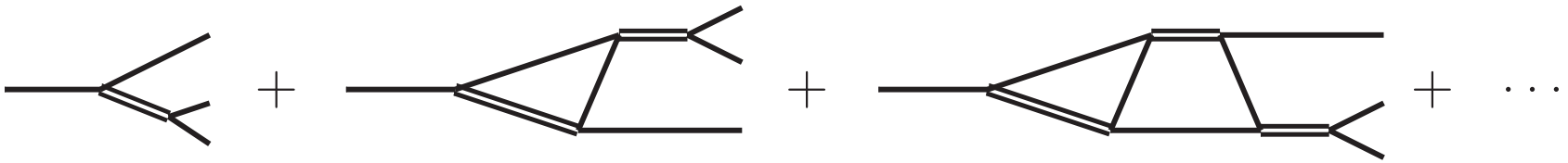


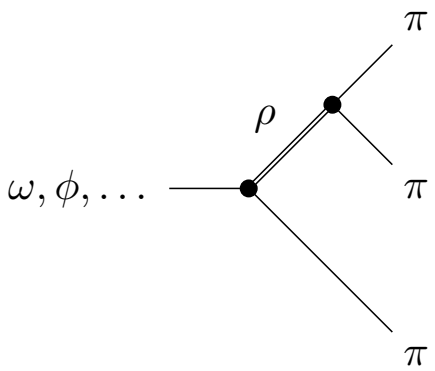
. . . so what's not to like?



- 3-particle rescattering

Motivation

The traditional picture: isobar model / Breit–Wigner resonances



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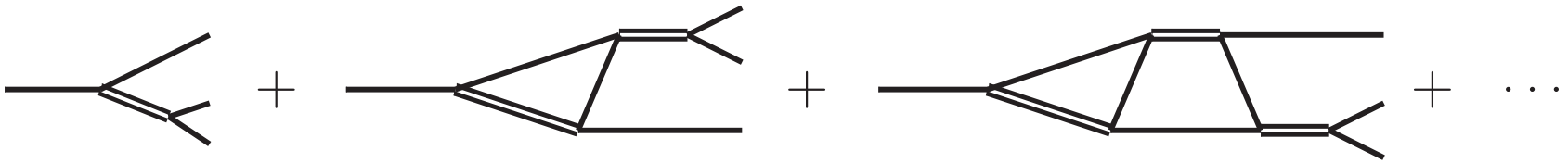
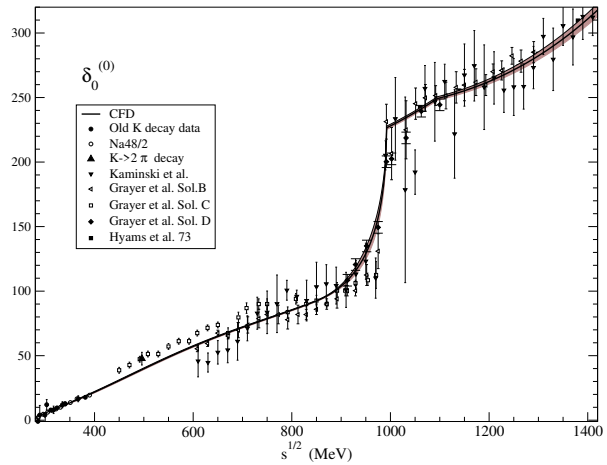


- 3-particle rescattering
- some resonances don't look like Breit–Wigners at all!

  $\rightarrow$ use exact scattering phase shifts instead



Omnès function

- (elastic) unitarity

$$\text{disc} f(s) = 2i f(s) e^{-i\delta(s)} \sin\delta(s)$$

- analyticity
- crossing symmetry
- solution: Omnès function [Omnès, 1958]

$$\Omega(s) = \exp\left(\frac{s}{\pi}\int_{s_{\text{th}}}^{\infty} \text{d}s' \frac{\delta(s')}{s'(s'-s)}\right)$$

Omnès function

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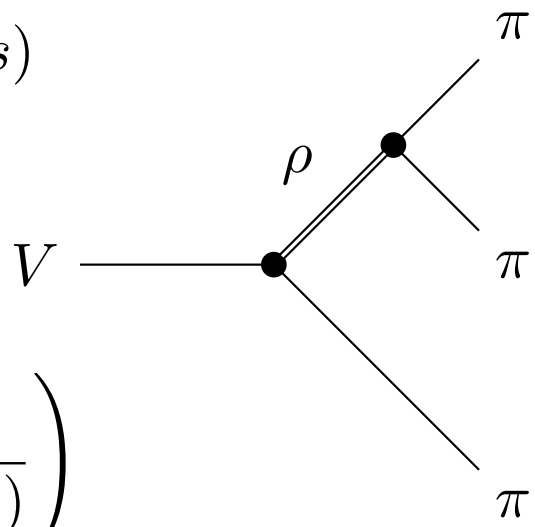


$$\Omega(s) = \exp\left(\frac{s}{\pi}\int_{s_{\text{th}}}^{\infty} \text{d}s' \frac{\delta(s')}{s'(s'-s)}\right)$$

- reconstruction theorem for 1^{--} decay

$$\mathcal{M} = i\varepsilon_{\mu\nu\alpha\beta}\epsilon^{\mu}p_1^{\nu}p_2^{\alpha}p_3^{\beta}\mathcal{F}(s,t,u)$$
$$\mathcal{F}(s,t,u) = \mathcal{F}(s) + \mathcal{F}(t) + \mathcal{F}(u)$$

- approximate by

$$\mathcal{F}(s,t,u) = \lambda\left(\Omega(s) + \Omega(t) + \Omega(u)\right)$$

KT equations

- take cross-channel rescattering into account

$$\text{disc}\mathcal{F}(s) = 2i(\mathcal{F}(s) + \hat{\mathcal{F}}(s))e^{-i\delta(s)}\sin\delta(s)$$

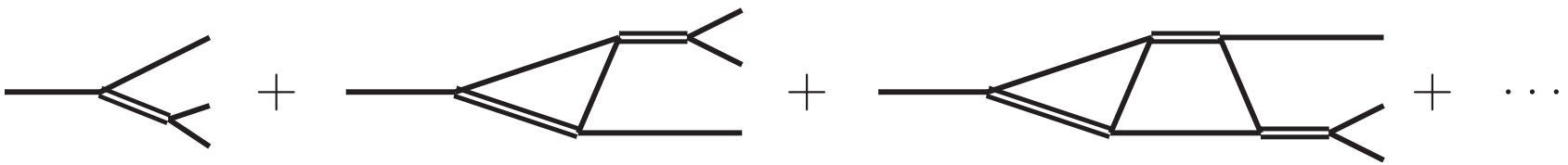
- solve with separation ansatz

$$\mathcal{F}(s) = \Omega(s)\left(P_{n-1}(s) + \frac{s^n}{\pi}\int_{s_{\text{th}}}^{\infty}\frac{\mathrm{d}s'}{s'^n}\frac{\hat{\mathcal{F}}(s)\sin\delta(s')}{|\Omega(s')|(s'-s)}\right)$$

- for 1^{--}

$$\hat{\mathcal{F}}(s) = \frac{3}{4}\int_{-1}^{1}\mathrm{d}z_s(1-z_s^2)\left(\mathcal{F}(t(s,z_s,M_V)) + \mathcal{F}(u(s,z_s,M_V))\right)$$

- solve iteratively or via matrix inversion

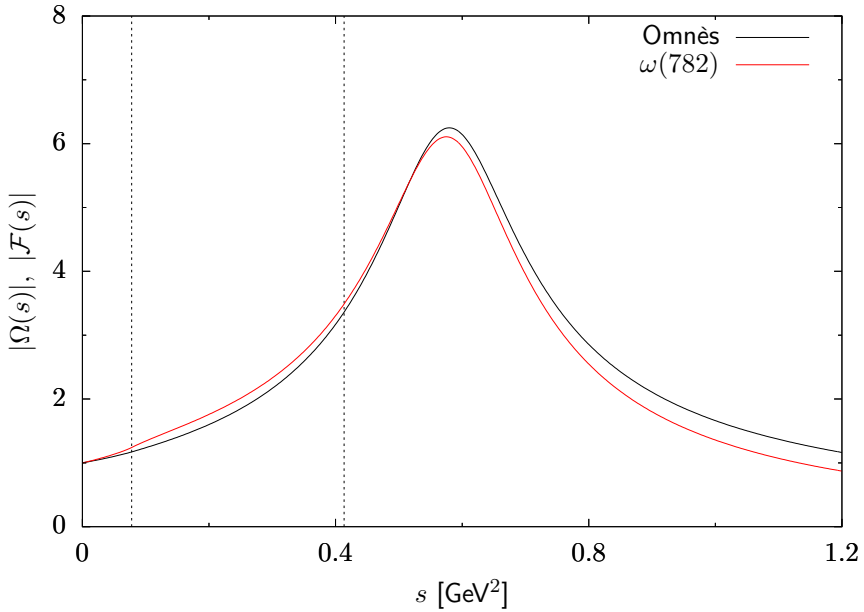


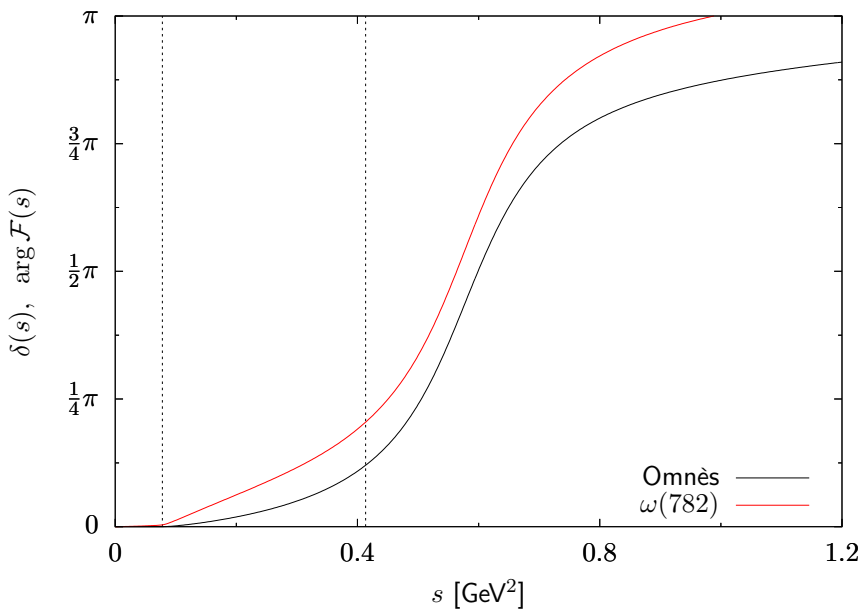
Crossed-channel rescattering effects: M_V

$$\mathcal{F}(s, M_V) = \Omega(s) \left(1 + \frac{s}{\pi} \int_{s_{\text{th}}}^{\infty} \frac{\mathrm{d}s'}{s'} \frac{\hat{\mathcal{F}}(s', M_V) \, \sin \delta(s')}{|\Omega(s')|(s' - s)} \right)$$

light mesons: $\omega \to 3\pi$ vs. $\phi \to 3\pi$

crossed-channel effects dependent on decay mass!





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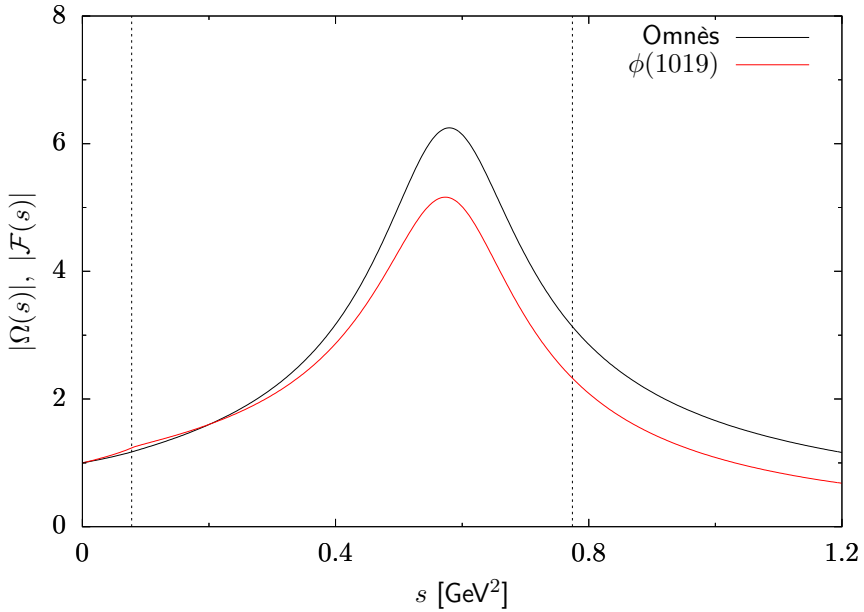
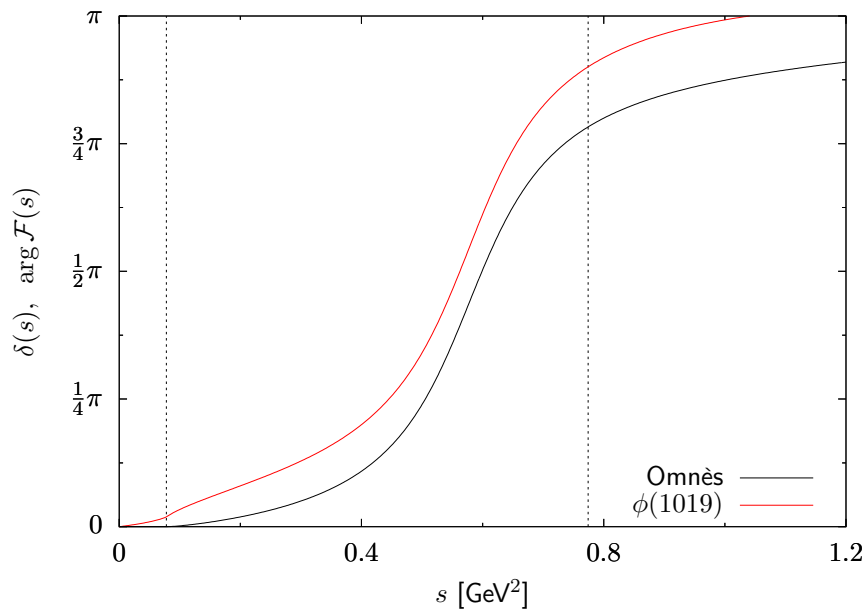




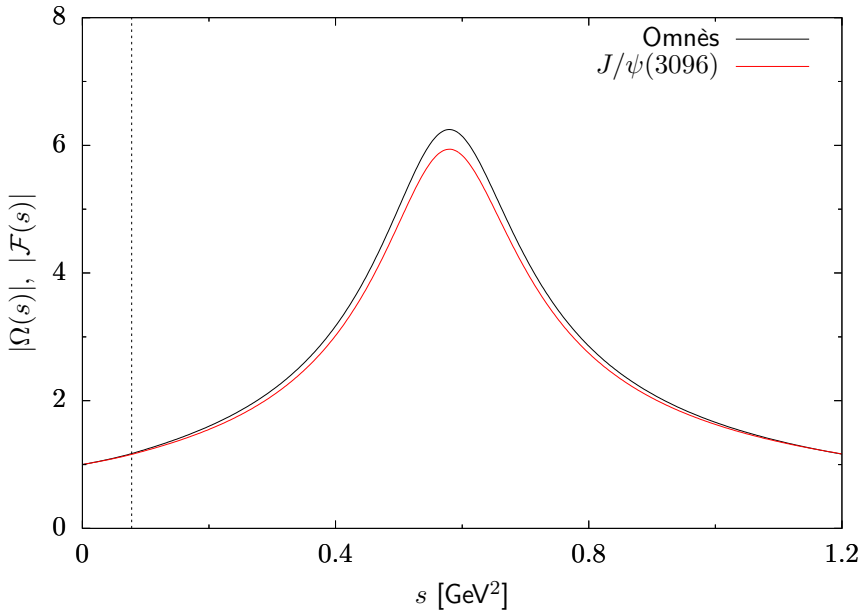
- ρ line shape modified by 3-body effects
- $\arg \mathcal{F}$ not universal: strong dependence on M_V

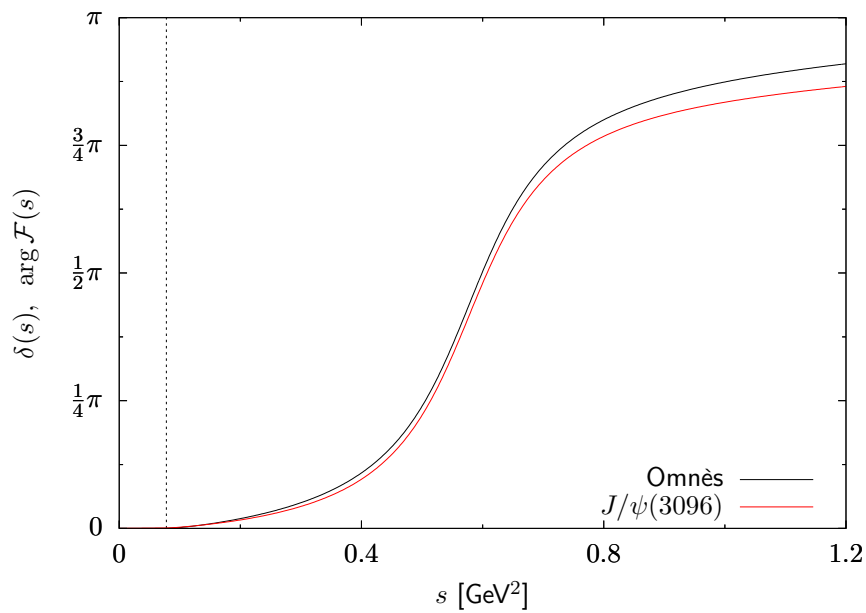
Crossed-channel rescattering effects: M_V

$$\mathcal{F}(s) = \Omega(s)\left\{1 + \frac{s}{\pi}\int_{s_{\text{th}}}^{\infty} \frac{\mathrm{d}s'}{s'} \frac{\hat{\mathcal{F}}(s')\,\sin\delta(s')}{|\Omega(s')|(s'-s)}\right\}$$

extension to higher energies: $J/\psi \to 3\pi$, $\Upsilon \to 3\pi$, etc.

$\Rightarrow$ well defined high-energy limit of KT amplitudes





Crossed-channel rescattering effects: M_V

$$\mathcal{F}(s) = \Omega(s)\left\{1 + \frac{s}{\pi}\int_{s_{\text{th}}}^{\infty} \frac{\mathrm{d}s'}{s'} \frac{\hat{\mathcal{F}}(s')\,\sin\delta(s')}{|\Omega(s')|(s'-s)}\right\}$$

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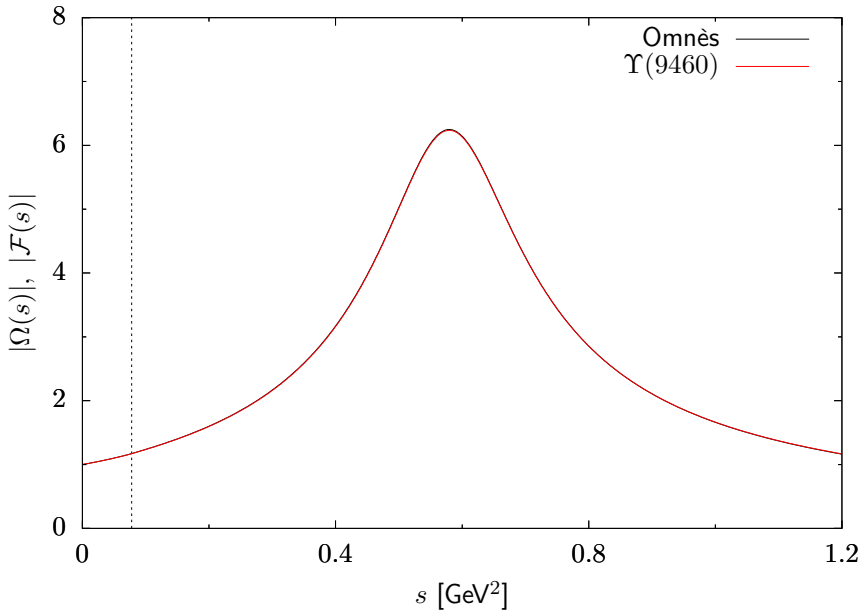
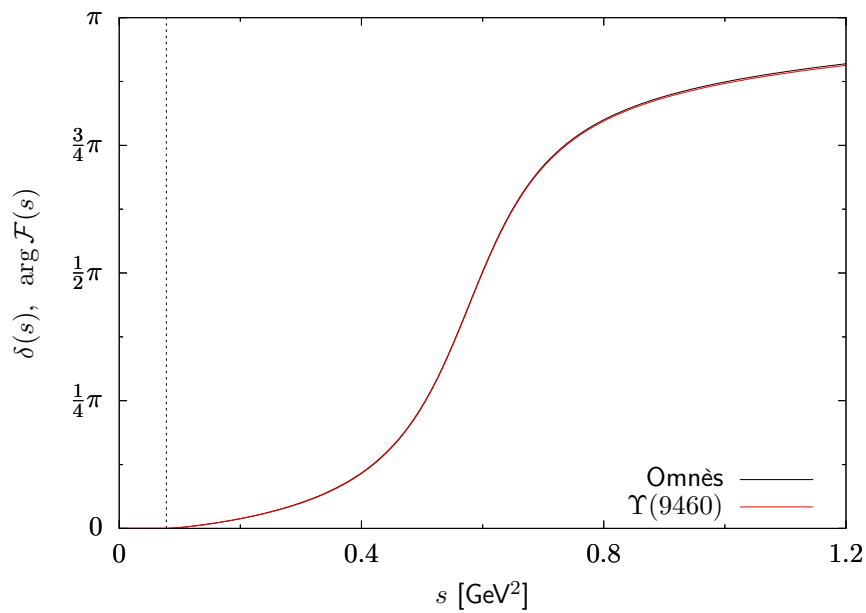




- 3-body effects vanish in the limit $M_V \to \infty$
- warning: naive continuation; inelastic effects neglected, ...

Crossed-channel rescattering effects: RT

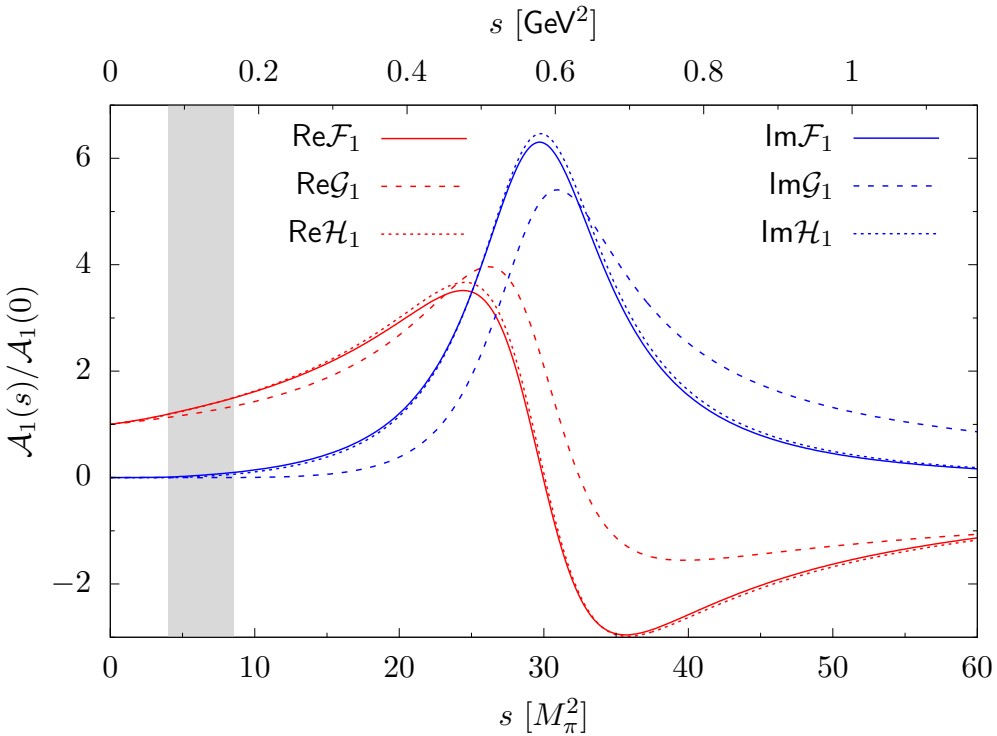
- $\eta \to 3\pi$ decay [Akdag, Isken, Kubis, 2021]
- reconstruction theorems

$$\mathcal{M}_1^C(s,t,u) = \mathcal{F}_0(s) + (s-u)\mathcal{F}_1(t) + (s-t)\mathcal{F}_1(u) + \mathcal{F}_2(t) + \mathcal{F}_2(u) - \frac{2}{3}\mathcal{F}_2(s)$$

$$\mathcal{M}_0^{\not C}(s,t,u) = (t-u)\mathcal{G}_1(s) + (u-s)\mathcal{G}_1(t) + (s-t)\mathcal{G}_1(u)$$

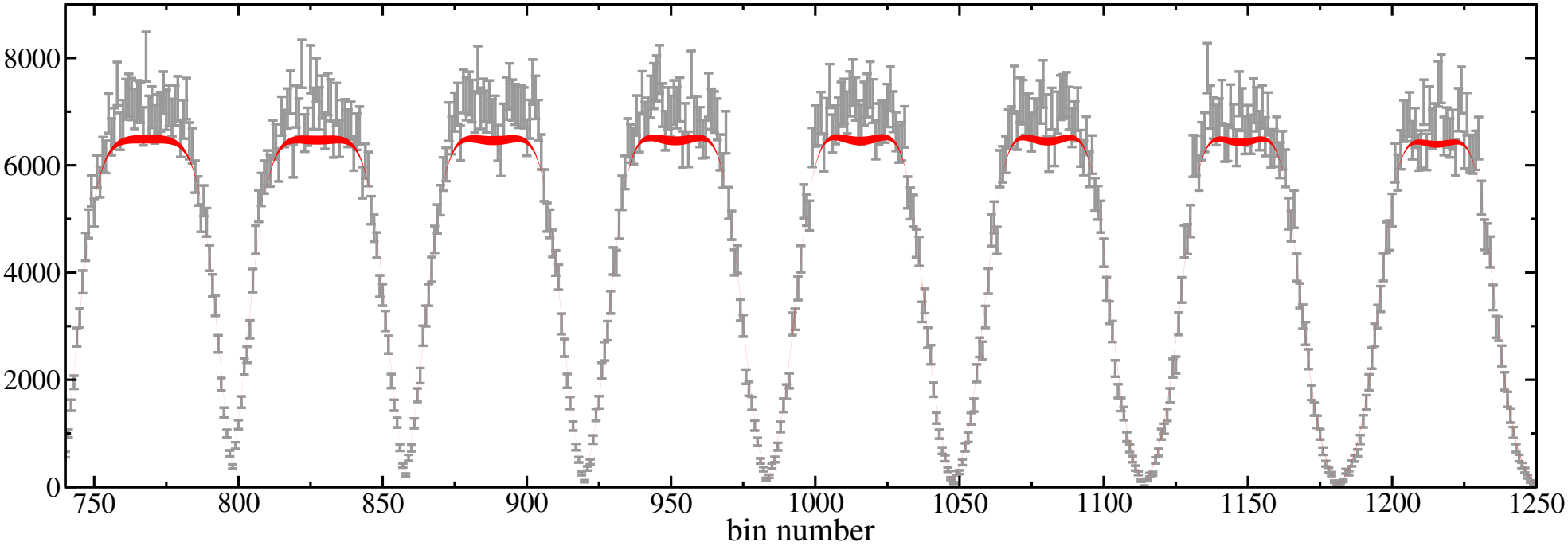
$$\mathcal{M}_2^{\not C}(s,t,u) = 2(u-t)\mathcal{H}_1(s) + (u-s)\mathcal{H}_1(t) + (s-t)\mathcal{H}_1(u) - \mathcal{H}_2(t) + \mathcal{H}_2(u)$$

- 3 different P-wave SVAs
- due to different reconstruction theorems



Example for $\phi \to 3\pi$ Dalitz plot

KLOE Dalitz plot: $2 \cdot 10^6$ events, 1834 bins [Niecknig, Kubis, Schneider, 2012]



	$\hat{\mathcal{F}} = 0$
χ^2/dof	$1.71 \ldots 2.06$

$$\mathcal{F}(s) = a\Omega(s) = a \exp\left(\frac{s}{\pi}\int_{s_{\text{th}}}^{\infty} \mathrm{d}s' \frac{\delta(s')}{s'(s'-s)}\right)$$

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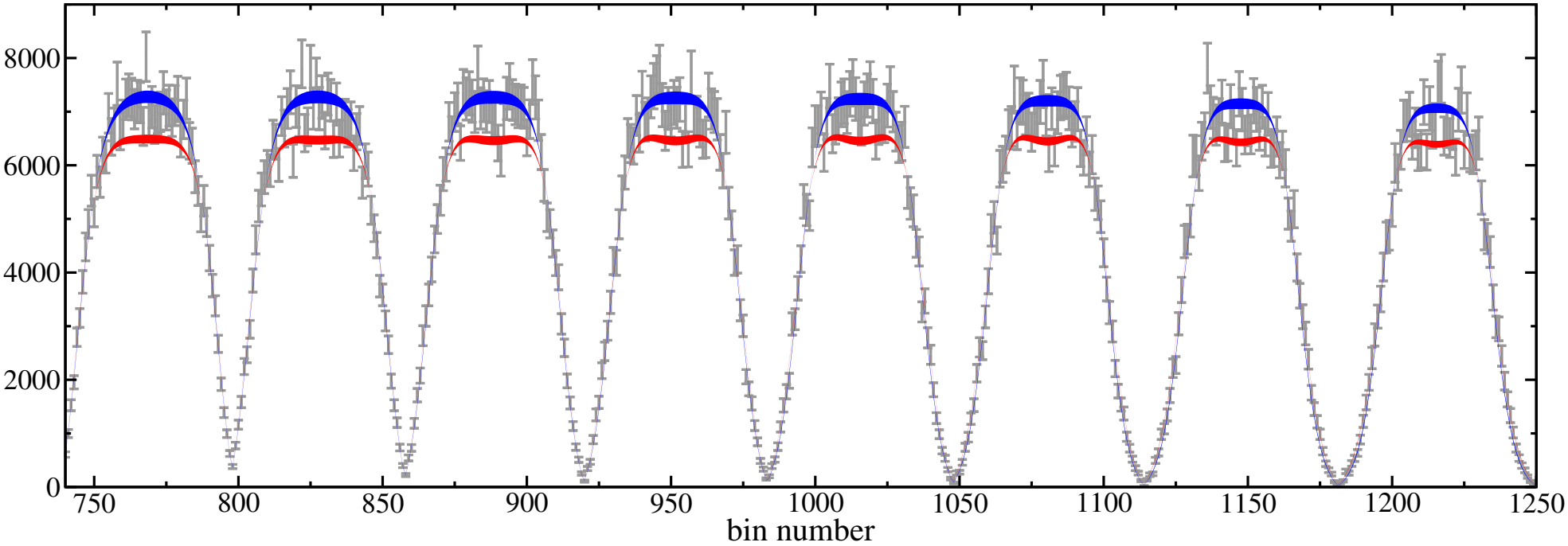

	$\hat{\mathcal{F}} = 0$	once-subtracted
χ^2/dof	$1.71 \ldots 2.06$	$1.17 \ldots 1.50$

$$\mathcal{F}(s) = a\Omega(s) \left(1 + \frac{s}{\pi} \int_{s_{\text{th}}}^{\infty} \frac{\mathrm{d}s'}{s'} \frac{\hat{\mathcal{F}}(s') \sin \delta(s')}{|\Omega(s')|(s' - s)} \right)$$

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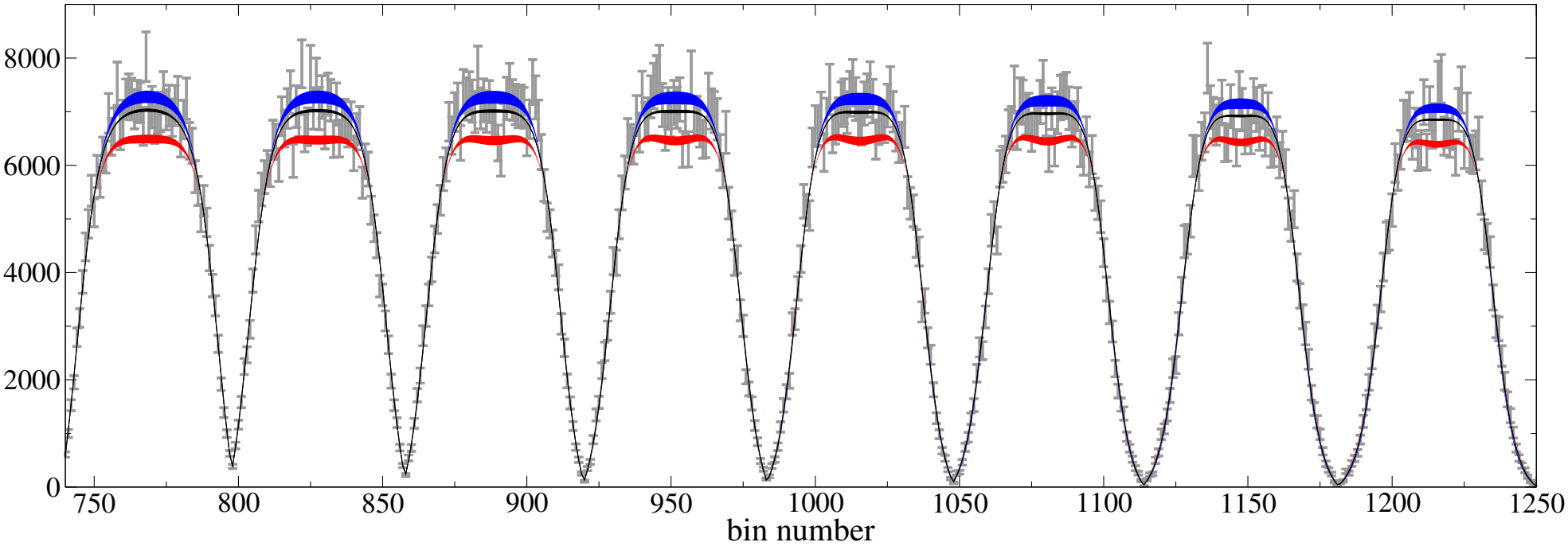


	$\hat{\mathcal{F}} = 0$	once-subtracted	twice-subtracted
χ^2/dof	$1.71 \ldots 2.06$	$1.17 \ldots 1.50$	$1.02 \ldots 1.03$

$$\mathcal{F}(s) = a\Omega(s)\left(1 + bs + \frac{s^2}{\pi}\int_{s_{\text{th}}}^{\infty} \frac{\mathrm{d}s'}{s'^2} \frac{\hat{\mathcal{F}}(s')\sin\delta(s')}{|\Omega(s')|(s'-s)}\right)$$

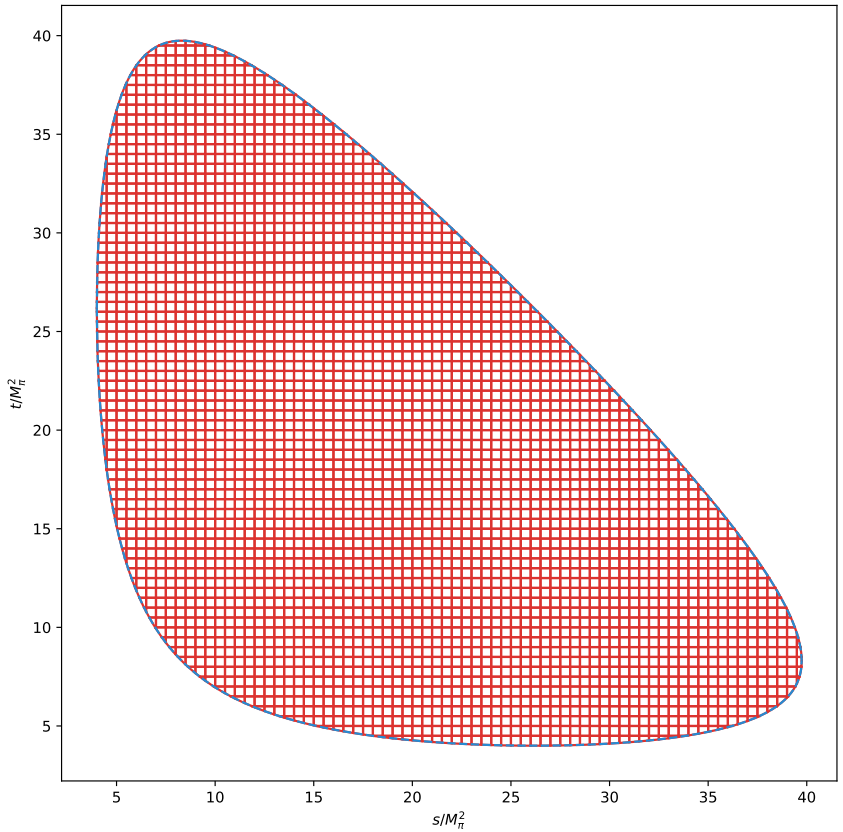
How to quantify KT corrections on Dalitz plot level?

- Dalitz plot for kinematically allowed decay region for $\phi \to 3\pi$
- KT amplitude $\kappa(s,t)$, Omnès amplitude $\omega(s,t)$
- define probability distribution

$$p(s,t) = \frac{|\kappa(s,t)|^2}{\int_D |\kappa(s,t)|^2 \mathrm{d}s\mathrm{d}t}$$

- and the probability that the drawn sample hits bin D_k

$$p_k = \int_{D_k} p(s,t)\mathrm{d}s\mathrm{d}t$$



Definition of χ^2

- function for Omnès solution

$$\omega_k(\lambda) = |\lambda|^2 \int_{D_k} \rho_{3\pi}(s,t)|\Omega(s) + \Omega(t) + \Omega(u)|^2 \mathrm{d}s\mathrm{d}t$$

- χ^2 for N events

$$\begin{aligned}\chi^2 &= \sum_{k=1}^{|\mathcal{D}|} \frac{(Np_k - \omega_k(\lambda))^2}{Np_k(1-p_k)} \\ &= N \sum_{k=1}^{|\mathcal{D}|} \frac{\left(p_k - \omega_k\left(\frac{\lambda}{\sqrt{N}}\right)\right)^2}{p_k(1-p_k)}\end{aligned}$$

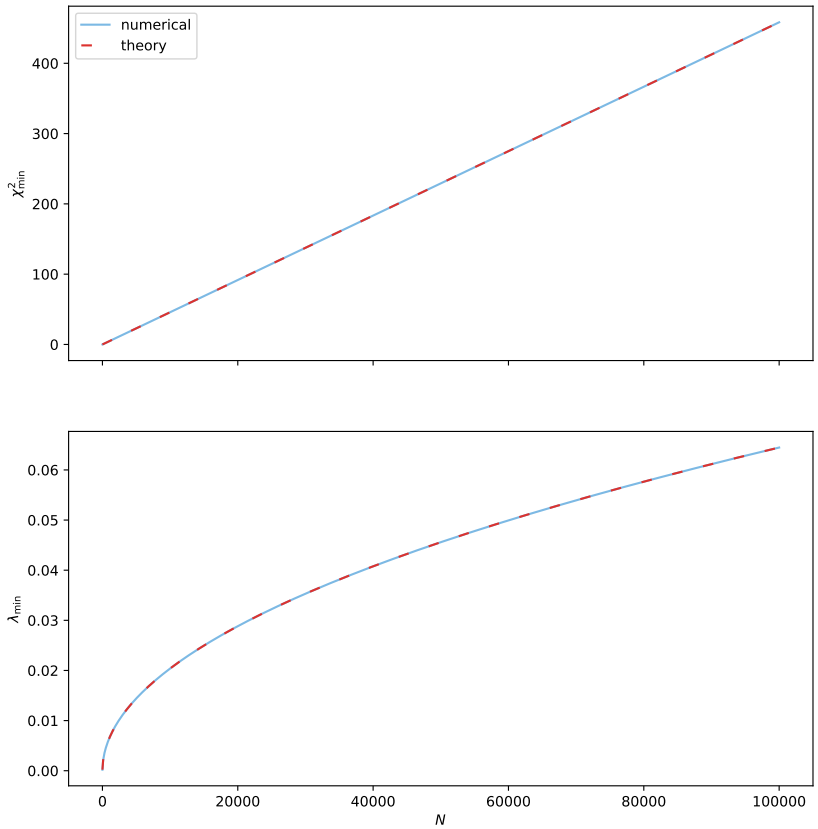
Minimizing χ^2

- use scaling to find

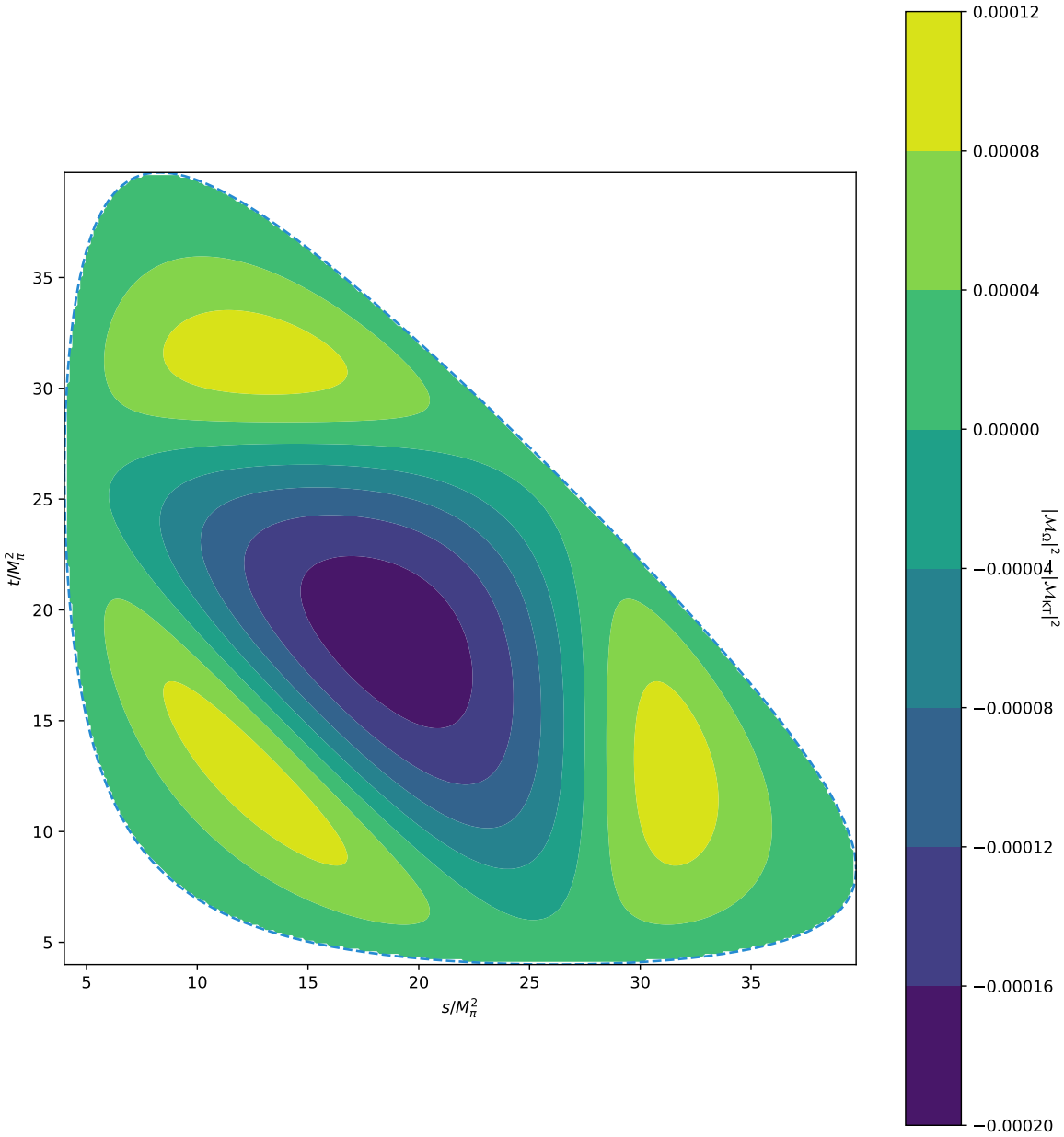
$$\chi^2_{\text{min}}(N, \mathcal{D})/\text{dof} = \frac{N}{|\mathcal{D}| - 1}\chi^2_{\text{min}}(1, \mathcal{D})$$

$$\lambda_{\text{min}}(N, \mathcal{D}) = \sqrt{N}\lambda_{\text{min}}(1, \mathcal{D})$$

- numerically checked



Fitting Omnès to KT



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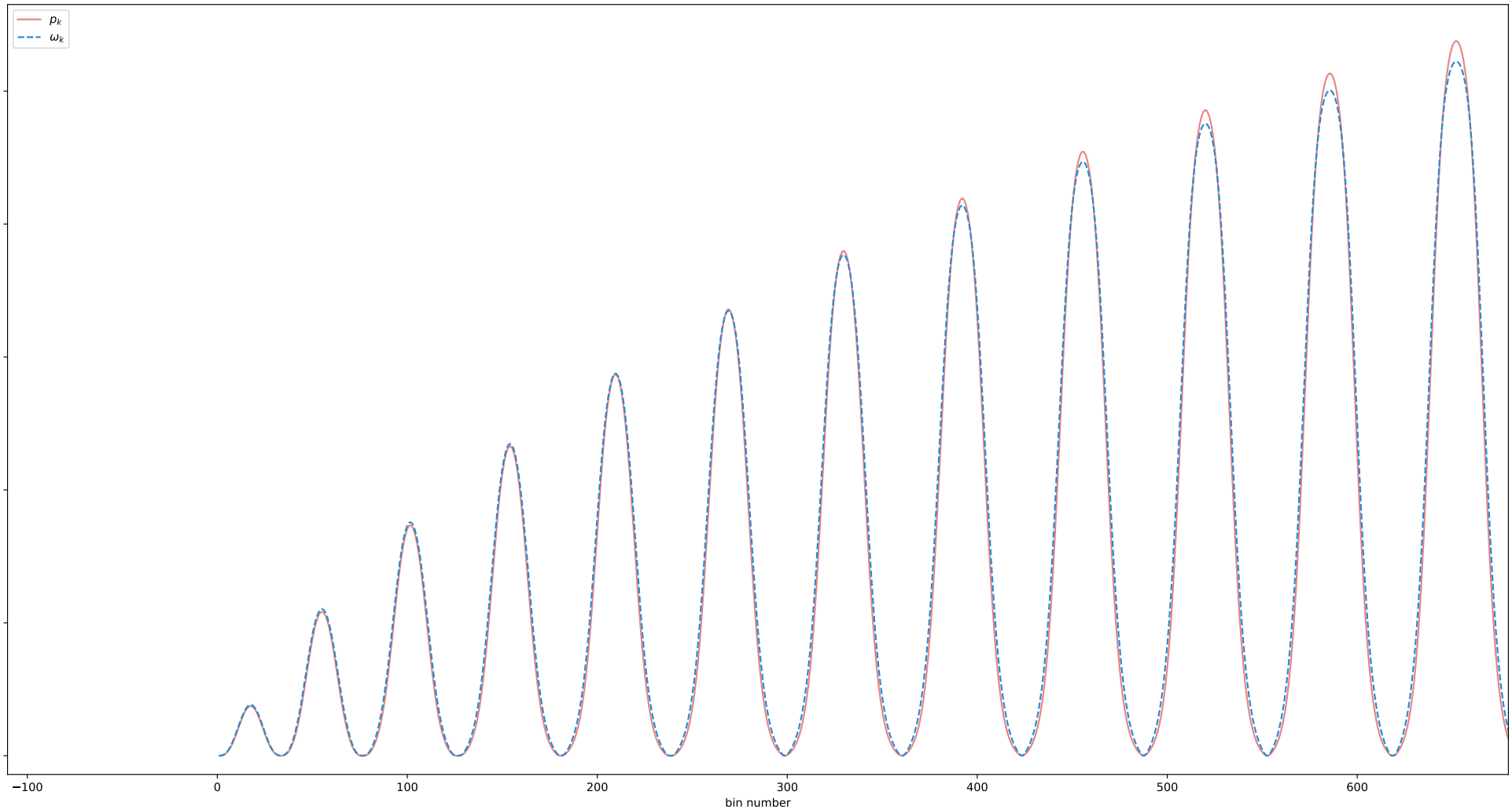
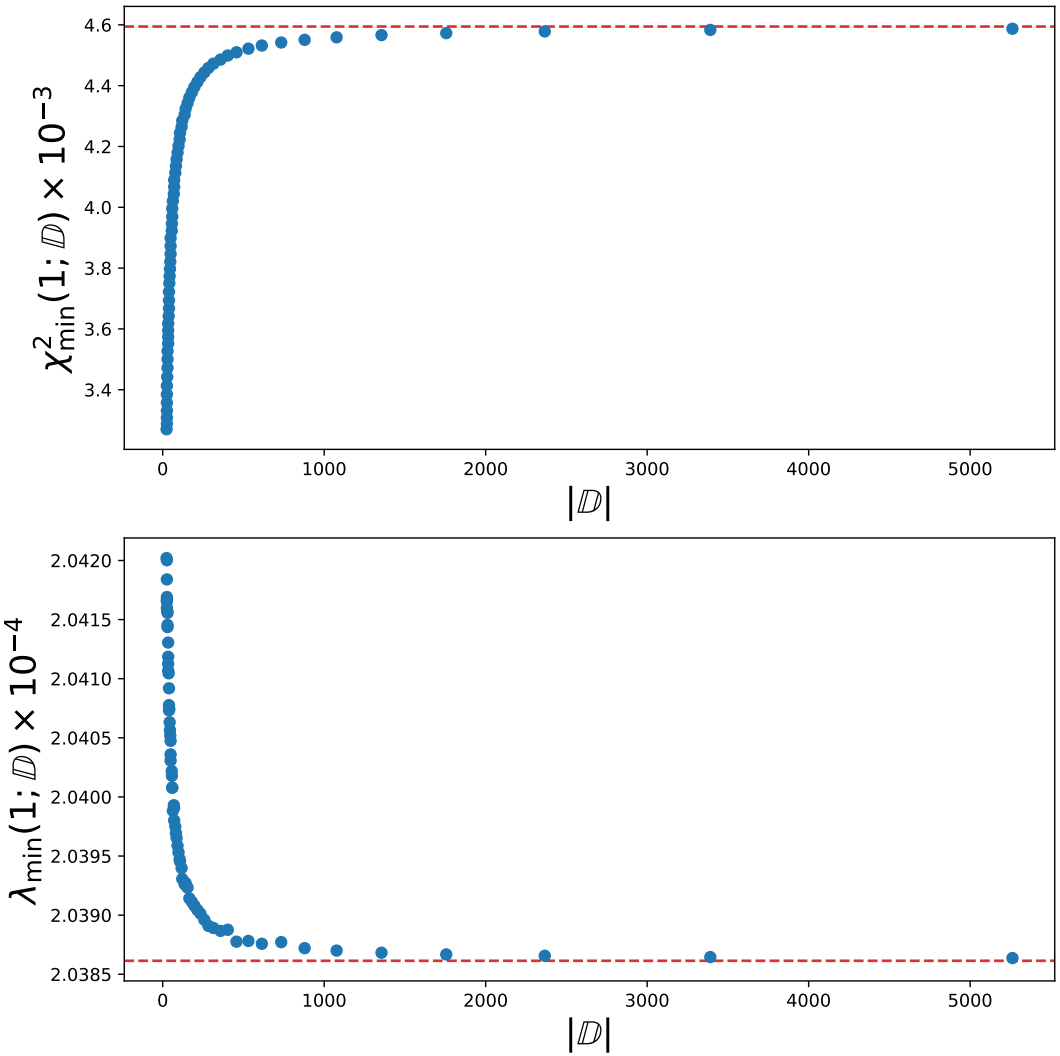


Fits for multiple bin configurations



Pearson's test

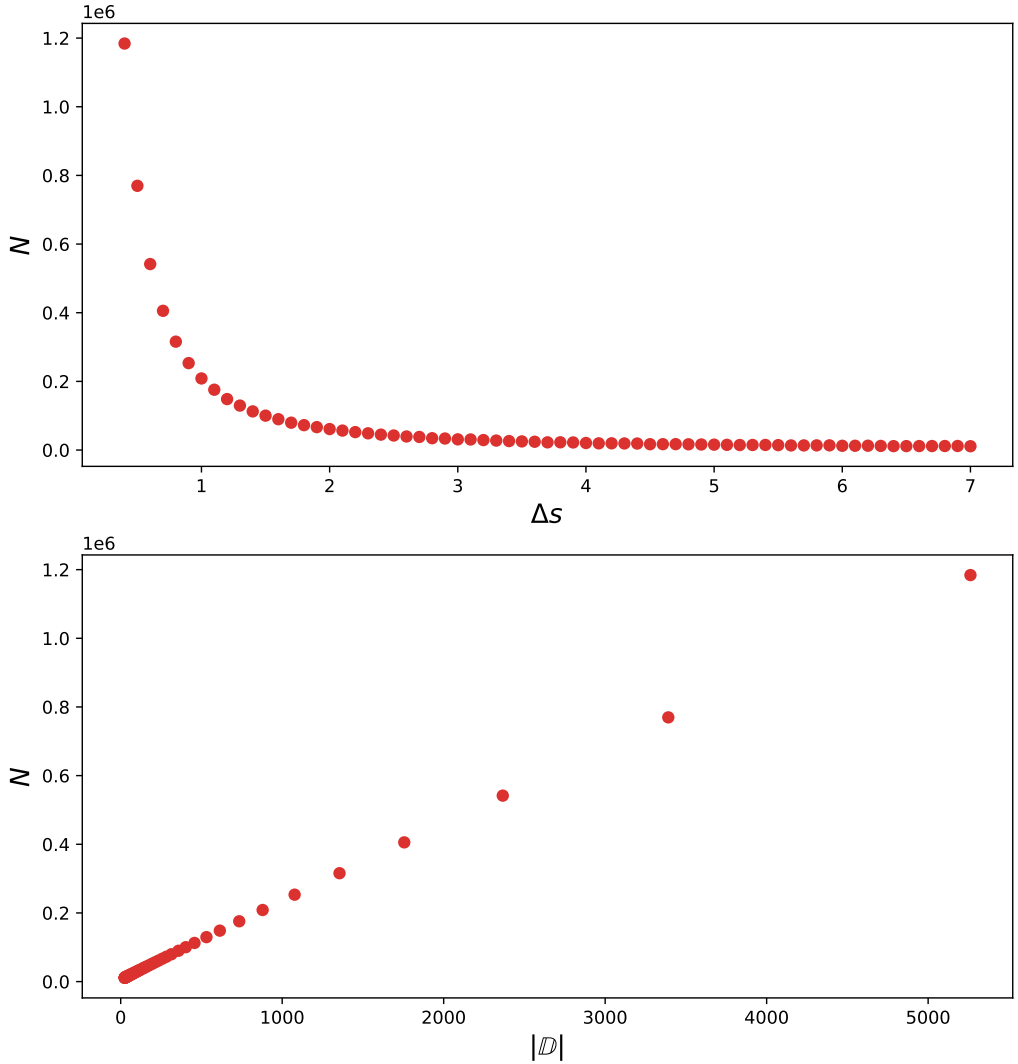
- null hypothesis: Omnès solution is able to describe KT solution
- test this hypothesis with Pearson's test
- reject the hypothesis if p-value below p_{thr}

$$1 - F(\chi^2_{\text{min}}(N, \mathcal{D}), |\mathcal{D}| - 1) < p_{\text{thr}}$$

- F is cumulative distribution function of χ^2
- recast condition

$$N > \frac{\chi^2_{\text{thr}}(|\mathcal{D}|)}{\chi^2_{\text{min}}(1, \mathcal{D})}$$

Pearson's test



Outlook

- same analysis for 1^{-+}
- reconstruction theorem known

$$\mathcal{F}(s,t,u) = \mathcal{F}(t) - \mathcal{F}(u)$$

- possible to constrain $\tau \to 3\pi\nu_\tau$ reconstruction theorem to one constant?
- derive reconstruction theorem for $a_2 \to 3\pi$
- compare the different processes