One-pion exchange process for Tcc+

[LHCb, arXiv:2109.01056]

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Hadronic three-body Christmas,
Garching, Germany
Thanks to all LHCb colleagues

Vanya Belyaev

Ivan Polyakov
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Vanya Belyaev

Ivan Polyakov

The oncoming paper is dedicated to the memory of Simon Eidelman
Selection of $D^0 D^0 \pi^+$

- Select $D^0 D^0 \pi^+$ candidates from primary vertex with detached $D^0 \to K^- \pi^+$
- Require detached $K^- \pi^+$ with high $p_T$
- Require good quality of tracks, vertexes, and particle ids.
- Ensure no $K/\pi$ candidates belong to one track (clones)
- Ensure no reflections via mis-ID
- Subtract fake-D background using 2d fit to $(m_{K\pi} \times m_{K\pi})$
Spectrum fit and significance
Breit-Wigner model

Naive model
BW signal \[((DD)_S \pi P\text{-wave}] + \text{ph.sp. background}
- significance > 10\sigma
- peak below (4.3\sigma)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>(N)</td>
<td>117 ± 16</td>
</tr>
<tr>
<td>(\delta m_{BW})</td>
<td>−273 ± 61 keV/c^2</td>
</tr>
<tr>
<td>(\Gamma_{BW})</td>
<td>410 ± 165 keV</td>
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Fundamental properties? Need better model \((D^* D \text{ threshold})\)
Three-body resonances

- Many resonances dominantly decay to three-particle final state
  (e.g. $a_1(1260) \to 3\pi$, $X(3872) \to D^0\bar{D}^0\pi^0$, $T_{cc}^{+}(3872) \to D^0\bar{D}^0\pi^+$, $\Lambda_{c/b}^* \to \Lambda_{c/b}\pi\pi$)
- Reaction amplitude need to include three-body effects
  (e.g. three-body threshold, dynamics of OPE, triangle singularity)
- Three-body unitarity [Fleming(1964)/Holman(1965)/Amado(1973)]:
  \[
  \mathcal{T} - \mathcal{T}^\dagger = \mathcal{T}^\dagger \tau \mathcal{T} + \mathcal{T}^\dagger \tau^\dagger \mathcal{D} \tau \mathcal{T} + \mathcal{D} \tau \mathcal{T} + \mathcal{T}^\dagger \tau^\dagger \mathcal{D} + \mathcal{D}
  \]
  - the $\mathcal{D}$ is an imaginary part of the OPE
    it is non-zero in the physical region: $(\sigma_1, \sigma_2) \in$ Dalitz
  - the $\tau$ indicates the phase space the particle-pair mass
  - the $\tau^\dagger \mathcal{D} \tau$ is an integral over the Dalitz

[Mai et al., EPJA 53, 177 (2017)]
[Jackura et al. (JPAC), EPJC 79, 56 (2019)]
[MM et al. (JPAC), JHEP 08 (2019) 080]
Ladders and resonances [MM et al. (JPAC), JHEP 08 (2019) 080]

\[ \mathcal{T} = \mathcal{T}_{\text{short range}} + \mathcal{T}_{\text{includes OPE}} \]

- \( \mathcal{T}_{\text{includes OPE}} \) has a short log cut due to real-pion exchange
- Splitting is not unique: \( \mathcal{T}_{\text{includes OPE}} \) must include contact term

**Is the \( \mathcal{T} \) what we are interested in?**

- OPE process is an integral part of the three-to-three scattering, however,
- OPE is a “background” for the resonance pole

\[ \mathcal{T} \xrightarrow{s \to s_{\text{pole}}} \frac{g^2}{s_{\text{pole}} - s} + \mathcal{B} + \ldots \]

- Any production amplitude \( 2 \to 3 \) neither include \( \mathcal{B} \). It is convoluted with the source.
Amplitude for resonance process

Two-body resonance

\[ \hat{T}_2(s) = \frac{g^2}{m^2 - s - ig^2 \Phi_2(s)} \rightarrow \frac{1}{(m^2 - s)/g^2 - \Sigma_2(s)} \]

- Self-energy: \( ig^2 \Phi_2(s) \rightarrow \Sigma_2(s) \), Chew-Mandelstam to ensure analyticity.
- \( \mathcal{K} \)-matrix: \( g^2/(m^2 - s) \) is uncontrolled real part

Three-body resonance

\[ \hat{T}_R(s) = \frac{1}{(m^2 - s)/g^2 - \Sigma(s)} = \frac{1}{\mathcal{K}_1^{-1}(s) - \Sigma(s)} \]

- Dispersion relation for the self-energy:

\[ \Sigma(s) = \frac{s}{2\pi} \int_{s_{th}}^{\infty} \frac{ds'}{s'(s' - s)} \int_{\text{Dalitz}(s')} |\hat{A}_{R \rightarrow 1,2,3}(s', \sigma_1', \sigma_2')|^2 d\Phi'_3 \]

- The \( |\hat{A}_{R \rightarrow 1,2,3}(s, \sigma_1, \sigma_2)|^2 \) is observable (+FSI) Dalitz-plot distribution
Iterative solution

Bethe-Salpeter / Blankenbeckler-Sugar / Lippmann-Schwinger / B-matrix (Mai/JPAC) approaches

\[ \mathcal{T} = \mathcal{V} + \mathcal{V}_t \mathcal{T}, \quad \mathcal{V} = \mathcal{V}_0 + \mathcal{V}_1 \]

- Integral equation (system of eqs) that yields the unitary solution if \( \mathcal{V} \) includes OPE, \( \mathcal{V}_0 \).
- The real part of \( \mathcal{V} \) is unconstrained by unitarity (\( \mathcal{K} \)-matrix).

- Incorporates all three-body effects:
  - Three-body unitarity cut
  - Dynamics of the OPE
  - Triangle singularities

- Analyticity of \( \mathcal{T} \) is not ensured
  - Spurious left-hand singularities
  - First-sheet wooly cut (©Aitchison)
Relation to BS equations


1. Potential has two terms ($V_0$ includes OPE):
   \[ V = V_1 + V_0. \]

2. Obtain $T_0$:
   \[ T_0 = V_0 + V_0 \tau T_0, \]
   (the ladder: long-range)

3. Find a solution in the form ($T_0 +$ something)
   \[ T = T_0 + (1 + T_0 \tau) \hat{T} (1 + \tau T_0), \]

4. by solving,
   \[ \hat{T} = V_0 + V_0 (\tau + \tau T_0 \tau) \hat{T}, \]
   (resonance: short-range)

5*. Algebraic relation for decay amplitude
   \[ i \left| \hat{A}_{R \rightarrow 1,2,3} \right|^2 = i \tau^\dagger \frac{1}{\rho_3} \tau + \tau T_0 \tau - \tau^\dagger T_0^\dagger \tau^\dagger \]
   \[ = (1 + \tau^\dagger T_0^\dagger) \left[ i \tau^\dagger \frac{1}{\rho_3} \tau + \tau^\dagger D \tau \right] (1 + T_0 \tau). \]
Physical meaning of the terms [MM et al. (JPAC), JHEP 08 (2019) 080]

\[ |\hat{A}_{R\rightarrow1,2,3}|^2 = \frac{1}{\rho_3} \left( 1 + \tau^\dagger \hat{T}_0 \right) \left( 1 + \tau^\dagger \hat{D}\tau \right) (1 + \hat{T}_0 \tau). \]

Relation to KT
- Khuri-Trieman framework: two-body unitarity continued to the decay domain
- Gives the rescattering effect, systematically accounts for triangle diagrams

\[ F_{\text{decay}} = (1 + \hat{T}_{\text{KT}} \tau) \hat{C} \]

- Originally, \( s \) is a fixed parameter, however,
- \( \hat{T}_{\text{KT}} \) is a valid construct for \( \hat{T}_0 \) [Aitchison(1986), Pasquier(1968)]

The \( |\hat{A}_{R\rightarrow1,2,3}(s, \sigma_1, \sigma_2)|^2 \) is observable (+FSI) Dalitz-plot distribution
$T_{cc}^+$ decay amplitude

Model assumptions:
- $J^P = 1^+$: S-wave decay to $D^* D$
- $T_{cc}^+$ is an isoscalar: $|T_{cc}^+\rangle_{I=0} = \{|D^*0 D^0\rangle - |D^{*+} D^0\rangle\} / \sqrt{2}$
- No isospin violation in couplings to $D^{*+} D^0$ and $D^{*0} D^+$

Effective range of $T_{cc}^+$

$D^0 D^0 \pi^+$ ↔ 5.8 MeV ↔ $D^{*+} D^0$ ↔ 1.4 MeV ↔ $D^{*0} D^+$
$T_{cc}^+$ self-energy and hadronic reaction amplitude

Three-body unitarity [MM et al. (JPAC), JHEP 08 (2019) 080]

Dynamic amplitude of $D^*D \rightarrow D^*D$ scattering:

$$T_{2\times2}(s) = \frac{K}{1 - \Sigma K} = \frac{K(m^2 - s)}{m^2 - s - ig^2(\rho_{\text{tot}}(s) + i\xi(s))}$$

where $K$ is the isoscalar potential:

$$K = \frac{1}{m^2 - s} \begin{pmatrix} g \cdot g & -g \cdot g \\ -g \cdot g & g \cdot g \end{pmatrix},$$

and $\Sigma$ is the loop function:

$$\Sigma(s) = [D^*D \rightarrow DD\pi(\gamma) \rightarrow D^*D]$$

$$= \left[ \text{contributions} \right].$$

The construction is guided by Unitarity and Analyticity.

Model parameters: $|g|^2$ and $m^2$ – bare mass and coupling
Fit to the spectrum

Unitarized model

- The signal shape does not depend on $|g|$ for $|g| \to \infty$.
- The lower limit: $|g| > 7.7(6.2) \text{ GeV}$ at 90(95)% CL
- $\delta m_U$ is the only parameter

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<td>$N$</td>
<td>$186 \pm 24$</td>
</tr>
<tr>
<td>$\delta m_U$</td>
<td>$-359 \pm 40 \text{ keV}/c^2$</td>
</tr>
<tr>
<td>$</td>
<td>g</td>
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No direct sensitivity to the width, the value is driven by the model.
Predicted mass spectrum resolution removed

Visible characteristics:

- Peak position: $-359 \pm 40$ keV
  (The most precise ever wrt to the threshold)

- FWHM: $47.8 \pm 1.9$ keV,

- Lifetime: $\tau \approx 10^{-20}$ s.
  (Unprecedented for exotic hadrons)

A bound state below $D^{*+}D^0$ threshold with a narrow width due to $D^*$

Still, the NLL scan suggest the low limit to the width, $\Gamma > 10$ keV at 95 CL.
Analytic continuation is non-trivial due to three-body decays [MM et al. (JPAC), PRD 98 (2018) 096021]

The pole parameters:

\[ \delta m_{\text{pole}} = -360 \pm 40 \text{ keV} , \]
\[ \Gamma_{\text{pole}} = 48 \pm 2 \text{ keV} . \]
Details on analytic continuation \cite{MM et al. (JPAC), PRD 98 (2018) 096021}

\[ \hat{F}^{-1} = \frac{m^2 - s}{g^2} - \Sigma(s) \]

- $\Sigma(s)$ is a dispersion integral of three-body integral

**Function $\Sigma(s)$ on the unphysical sheet**

\[
\Sigma_{II}(s_{\text{complex}}) = \underbrace{\Sigma_{I}(s_{\text{complex}})}_{\text{dispersion integral (easy)}} + \underbrace{2i\rho(s_{\text{complex}})}_{\text{Dalitz integral (tough)}}
\]
The $D^*$ width gives the limit to $T_{cc}^+$ width, $< \Gamma_{T_{cc}^+}^{(\text{max})}$

Parameter $|g|$ sets the value in the range $[0, \Gamma_{T_{cc}^+}^{(\text{max})}]$

The fit prefers the limit value
Weinberg Compositeness and the Width [MM, preliminary]

\[ 1 - Z = \sqrt{\frac{1}{1 + 2r/\mathcal{R}a}} \approx 1 - \frac{r}{a}, \quad \Rightarrow \quad Z \propto \frac{1}{|g|^2} \text{ when } |g| \to \infty \]

And, finite $|g|$ gives deviation of the width from the asymptotic value

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**“Molecule” configuration:**
- two mesons are well separated,
- bound by forces similarly to el.mag. van der Waals,
- entirely coupled to $D^* + D^0$,
- $T_{cc}^+$ lives until $D^*0$ decays,
- spatially-extended object.

**“Atomic” (©ancient Greek) configuration:**
- genuine QCD state,
- bound by direct color forces
- $\sim T_{cc}^+$ cannot live shorter than $D^*0$,
- $\tau_{T_{cc}^+}$ can be arbitrary large (uncoupled from continuum)
- typical hadronic size of 1 fm.
How the width is made [MM, preliminary]

Components of the model:
- Coupled $D^*+D^0/D^*0D^+$ channels
- One-particle exchange
- $D^* \to D\pi$ decay

Analytic expression for the width:

$$\frac{1}{\Gamma} = \frac{1}{2\pi} \mathcal{P} \int_\text{th}^{\infty} \frac{\rho'(e)/\rho(\delta m_0)}{e - \delta m_0} \, de$$

- 48 keV: default model
- $-12$ keV if not consider the $D^*0D^+$
- $\sim 15$ keV is controlled by the tail

Two-body approx. [Albaladejo, M. (2021)]

48 keV $\stackrel{\text{remove}}{\longrightarrow}$ OPE $\rightarrow$ 30 keV $\stackrel{\text{remove}}{\longrightarrow}$ 75 keV

$\frac{\rho'(\delta m_0)/\rho(\delta m_0)}{e - \delta m_0}$

Meng-Lin Du et al. (2021) gets 56 keV:
- supposedly includes FSI, analitycity is questionable

$\frac{\delta m_{D^*0D^+}}{[\text{MeV}]}$

25% of integral

$\rho'(\delta m_0)/\rho(\delta m_0)$
Summary

**Crutial role of OPE for understanding \( T_{cc}^+ \) nature**
- The width is a physical observable sensitive tightly related to the Weinberg compositeness
- The width saturation limit is highly impacted by the OPE

An ambitious program to address further important effects:
- Final-state interaction (\( D^*D \) triangles)
- Validation of quantum numbers
- Isospin violation in couplings (potential)
- Production ratios
Backup
Effective range extraction

Effective range and Weinberg compositeness

[LHCb, arXiv:2109.01056]

\[ \mathcal{A}_{NR}^{-1} = \frac{1}{a} + r \frac{k^2}{2} - i k + O(k^4), \]

\[ \frac{2}{|g|^2} \mathcal{A}_U^{-1} = -\left[ \xi(s) - \xi(m_U^2) \right] + 2 \frac{m_U^2 - s}{|g|^2} - i \phi_{tot}. \]

Matching:

- \( r = 16w/|g|^2 \),
  where \( w \) is a normalization factor between \( \rho \) and \( k \)
- \( w \) excludes the contribution of the second threshold
- does not have the \( 1/\sqrt{\delta} \) term

- \( T_{cc}^+ \): \( a = (-7.16 \pm 0.51) + i(1.85 \pm 0.28) \) fm
- \( T_{cc}^+ \): \( r \) is negative in the model: \( 0 < -r < 11.9(16.9) \) fm at 90(95) % CL
- \( T_{cc}^- \): \( 1 - Z > 0.48(0.42) \). \( T_{cc}^+ \) is consistent with the molecule
Comparison to the deuteron

**Deuteron** [Garcon, Van Orden(2001)]
- Presumably molecule
- $1 - Z \approx 1$
- $R_{\text{charge}} = 2.1$ fm
- $R_{\text{matter}} = 1.9$ fm
- $a = -5.42$ fm
- $r = 1.75$ fm

**Tetraquark** $T_{cc}^+$ [LHCb, arXiv:2109.01056]
- [compact $cc$ core]
- [{$\bar{u}\bar{d}$ cloud}]
- Expected to be atomic
- $1 - Z \geq 0.48$ at 90% CL
- $R_{\text{charge}} = ??$
- $R_{\text{matter}} = ??$
- $a = -7.16$ fm
- $r > -11.9$ fm at 90% CL