On-shell Pion Exchange in *X(3872)*

Eric Braaten
Ohio State University

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On-shell Pion Exchange in *X(3872)*

collaborators: Jun Jiang (Shandong University)

Li-Ping He (Bonn University)

based on BHJ [arXiv:2010.05801]

E. Braaten, L.-P. He & J. Jiang,
Galilean-invariant effective field theory for the *X(3872)* at next-to-leading order,
Phys. Rev. D 103, 036014 (2021)

On-shell Pion Exchange in *X(3872)*

- punchline: breakdown of effective range expansion

- Galilean invariant XEFT

- $D^{*0}\bar{D}^0$ scattering at NLO

- summary

$X(3872) \equiv \chi_{c1}(3872)$

discovery Belle 2003

$B^+ \to K^+ X$, $X \to J/\psi\, \pi^+\pi^-$

- <u>quantum numbers</u> $J^{PC} = 1^{++}$ LHCb 2013

- <u>mass</u>

  extremely close to $D^{*0}\bar{D}^0$ threshold

  $E_X \equiv M_X - (M_{D^{*o}} + M_{D^o}) = (-0.07 \pm 0.12)$ MeV LHCb 2020

  $\Longrightarrow E_X > -0.22$ MeV at 90% CL

- <u>width</u> LHCb 2020

  $\Gamma_X = (1.19 \pm 0.19)$ MeV (assuming Breit-Wigner line shape)

Unitarity

imposes strong constraints on few-body physics

2 → 2 scattering

phase shift: $\delta(k)$

T-matrix element: $$T(k) = \frac{1}{k \cot \delta(k) - i\, k}$$

short-range interactions only:

unitarity $\Longrightarrow$ effective range expansion

$k \cot \delta(k)$ has expansion in powers of k^2 with real coefficients

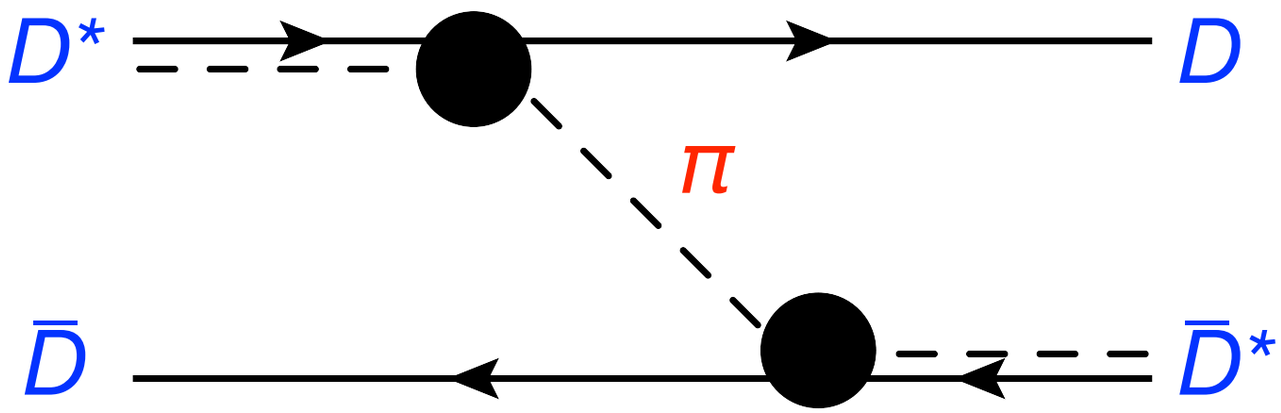
$$k \cot \delta(k) = -1/a + \tfrac{1}{2} r k^2 + \ldots$$

a = scattering length

r = effective range

Charm Mesons

exchange of pion between charm mesons *D*, *D**



pion can be on-shell if $M_{D^*} > M_D + m_\pi$

X(3872) channel: $D^{*0}\bar{D}^0 + D^0\bar{D}^{*0}$

$M_{D^{*o}} - (M_{D^o} + m_{\pi^o}) = 7.0$ MeV

$\Longrightarrow$ π^0 can be on shell

long-range interactions ?

On-shell Pion Exchange in *X(3872)*

punchline:

$D^{*0}\bar{D}^0$ scattering

In XEFT at NLO

exchange of on-shell pions between charm mesons

causes effective range expansion

for T-matrix element *T(k)* to break down

$\Longrightarrow$ effective range is not well defined

XEFT

S. Fleming, M. Kusunoki, T. Mehen & U. van Kolck,
Pion interactions in the X(3872),
Phys. Rev. D 76, 034006 (2007) [hep-ph/0703168]

XEFT

effective field theory for charm mesons and pions
with fine tuning that puts S-wave resonance *X(3872)*
extremely close to the threshold
in the 1^{++} channel $D^{*0}\bar{D}^{0} + D^{0}\bar{D}^{*0}$

- contact interactions in resonant channel
must be treated nonperturbatively

- other contact interactions and pion interactions
can be treated perturbatively

Galilean invariance

possible space-time symmetry of a nonrelativistic field theory

- requires conservation of kinetic mass
  (denominator in kinetic energy $p^2/2m$)

- simplifies analytic calculations
  (like Lorentz invariance)

- simplifies renormalization
  (by constraining ultraviolet divergences)

Galilean invariance is a good approximation
for charm mesons and pions !

$M_{D^{*o}} - (M_{D^o} + m_{\pi^o}) = 7.0$ MeV $\ll m_{\pi^o}$

Galilean invariant XEFT

E. Braaten,
Galilean-invariant effective field theory for the *X(3872)*,
Phys. Rev. D 91, 114007 (2015) [arXiv:1503.04791]

Galilean invariant effective field theory
for charm mesons and pions
with *X(3872)* resonance

- requires conservation of pion number
  = (# of π) + (# of D^*) + (# of $\bar{D}^*$)

  reduces number of diagrams

E. Braaten, L.-P. He & J. Jiang,
Galilean-invariant effective field theory for the *X(3872)*
at next-to-leading order,
Phys. Rev. D 103, 036014 (2021) [arXiv:2010.05801]

- simplified Feynman rules for NLO calculations
- simplified renormalization scheme

XEFT at LO

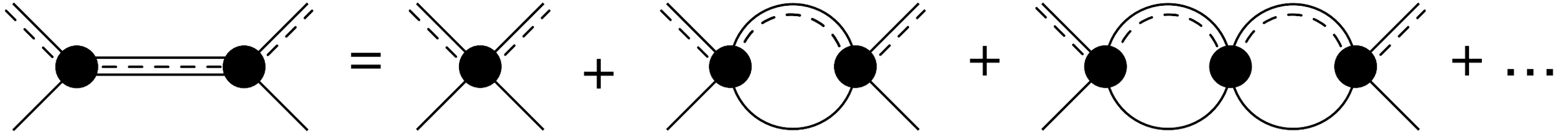
D^* propagator 

pole at complex energy:
$E_* = (M_{D^{*0}} - M_{D^0} - m_{\pi^0}) - i\,\Gamma_{D^{*0}}/2$

D propagator 

pair propagator in resonant channel $D^{*0}\bar{D}^0 + D^0\bar{D}^{*0}$

sum geometric series of bubble diagrams from contact interactions to all orders



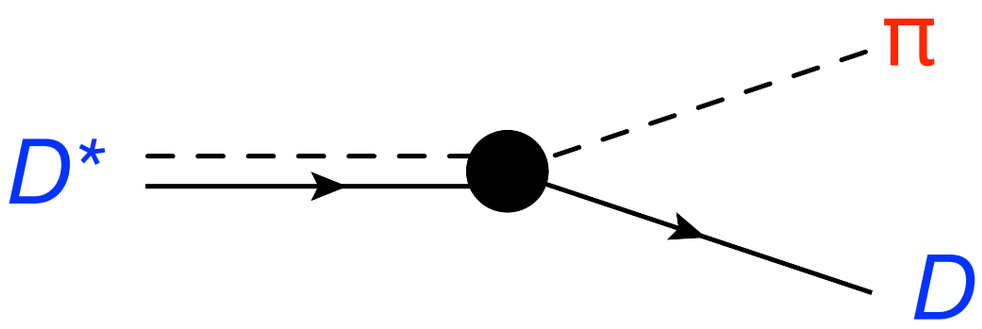
$$= \frac{-i}{-\gamma + \sqrt{-2\mu(E_{\rm cm} - E_*)}}$$

$E_{\rm cm} = E - P^2/[2(M_{D^{*0}}+M_{D^0})]$

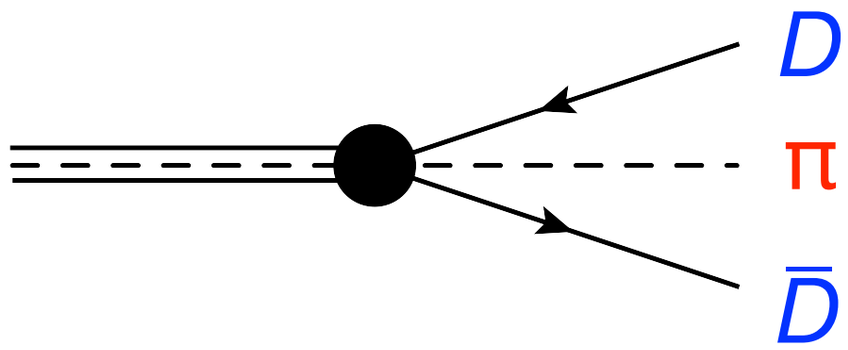
single interaction parameter: binding momentum γ

XEFT at NLO

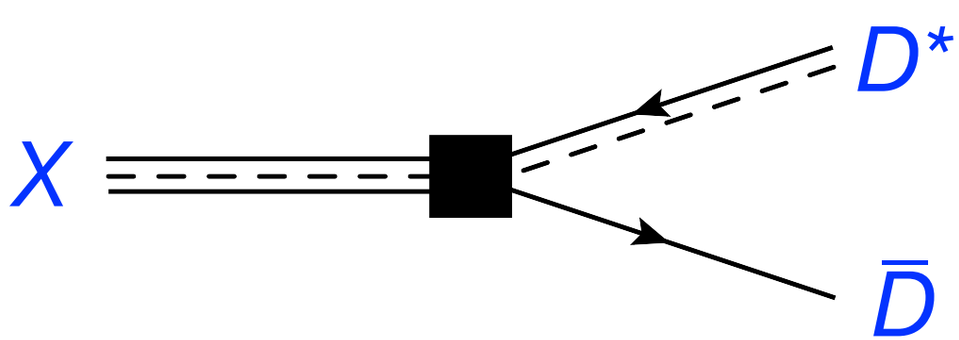
pion interaction vertex



DDpi interaction vertex



∇^2 interaction vertex



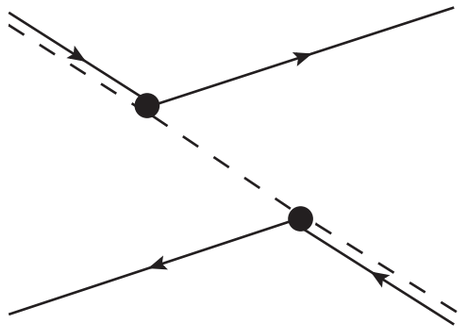
no other interactions required by renormalization

$D^{*0}\bar{D}^0$ Scattering at NLO

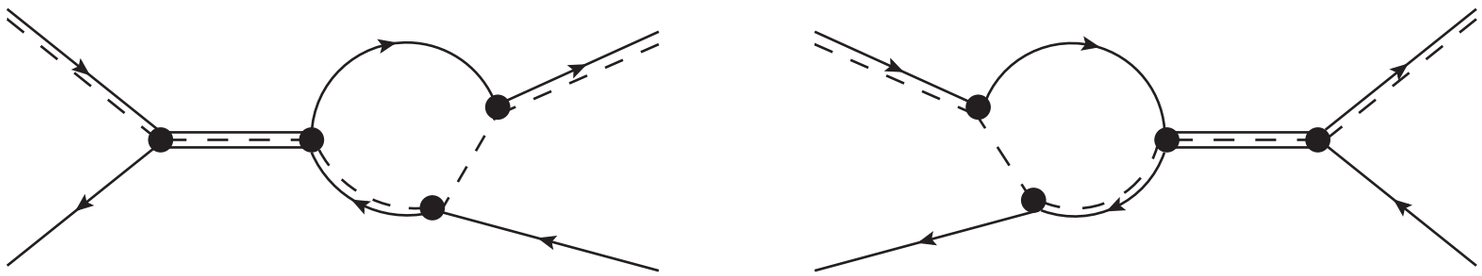
BHJ [arXiv:2010.05801]

pion exchange diagrams

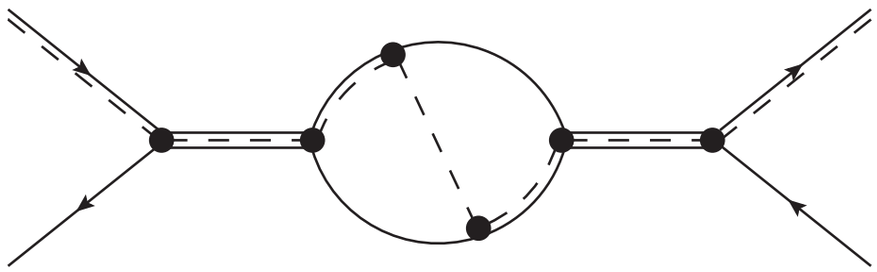
tree level



one loop

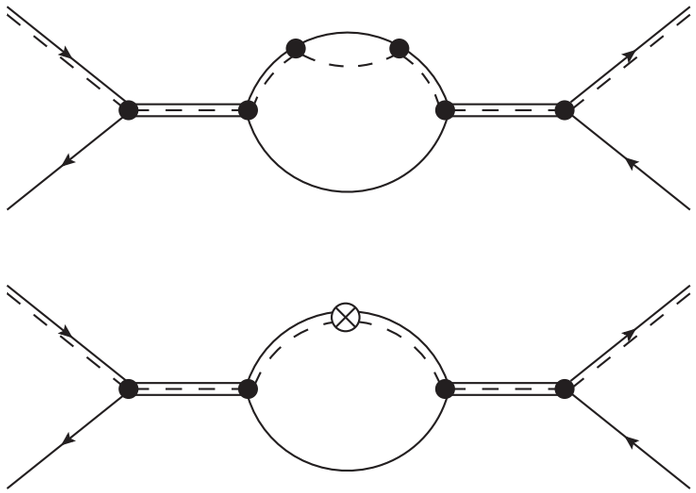


two loop

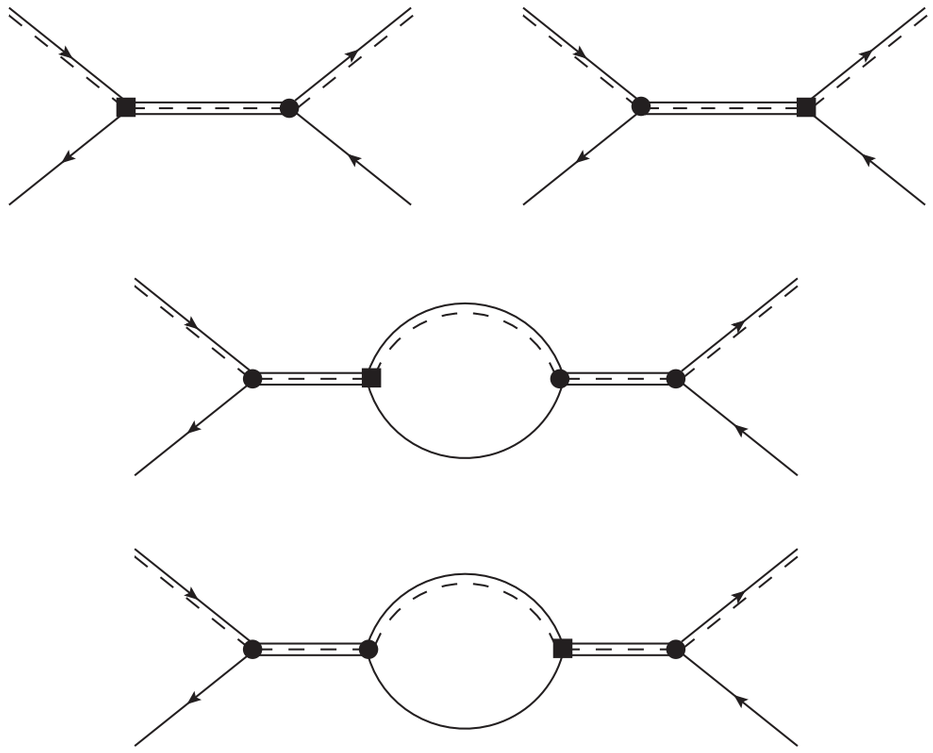


$D^{*0}\bar{D}^0$ Scattering at NLO

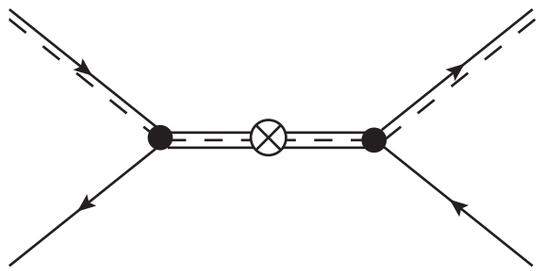
*D** propagator corrections



∇^2 interaction vertices



pair propagator counterterm



$D^{*0}\bar{D}^0$ Scattering at NLO

BHJ [arXiv:2010.05801]

- dimensional regularization

  linear, logarithmic UV divergences $\Longrightarrow$ poles in *d-2*, *d-3*

- renormalization prescription

  BHJ: pole in pair propagator exact at LO

- T-matrix element in limit d → 3:

$$T_{s+}(p) = A_\pi(p^2) + \frac{W_\pi(p)^2}{(-\gamma - ip) - \Pi(p)}$$

$A_\pi(p^2)$, $W_\pi(p)$: analytic in terms of logarithms

$\Pi(p)$: analytic in terms of logarithms
and one integral: $\int_0^1 dw \frac{w}{\sqrt{1-w}\,(1-w+rw^2/4)} \log \frac{(1-w)p_*^2 + p^2}{(1-w)p_*^2}$

$D^{*0}\bar{D}^0$ Scattering at NLO

BHJ [arXiv:2010.05801]

reciprocal of S-wave C=+ T-matrix element: $\dfrac{1}{T_{s+}(p)}$

expand in powers of *p*:

$$= \left[1 - i\left(\frac{r^{5/2}}{3\pi\sqrt{1-r}} + \frac{2\sqrt{1-r}\,r^{3/2}}{3\pi}\right) g_\pi^2 \kappa_*\right](-\gamma) + \left[1 - i\frac{(2-r)r^{3/2}}{3\pi\sqrt{1-r}} g_\pi^2 \kappa_*\right](-i\,p) + \mathcal{O}(p^2)$$

compare with effective range expansion

$$\left(-1/a + \tfrac{1}{2} r p^2 + \dots\right) - i\,p$$

- (-γ) has complex correction factor
- (- *i p*) has complex correction factor

  $\Longrightarrow$ effective range is not defined
- p^3 term with imaginary coefficient

breakdown of effective range expansion !

$D^{*0}\bar{D}^0$ Scattering at NLO

BHJ [arXiv:2010.05801]

breakdown of effective range expansion for $T_{s+}(p)$
from exchange of on-shell π^0

BUT $T_{s+}(p)$ at NLO seems to be compatible with the form

$$T_{s+}(p) = A_\pi(p^2) + W(p^2)\, T_{\rm SD}(p)\, W(p^2)$$

$A_\pi(p^2)$: pion exchange term
$W(p^2)$: complex phase factor
$T_{SD}(p)$: short-distance T-matrix element
with an effective range expansion

$$1/T_{\rm SD}(p) = \left(-\gamma_{s+} + \tfrac{1}{2} r_{s+} p^2 + \dots \right) - i\, p$$

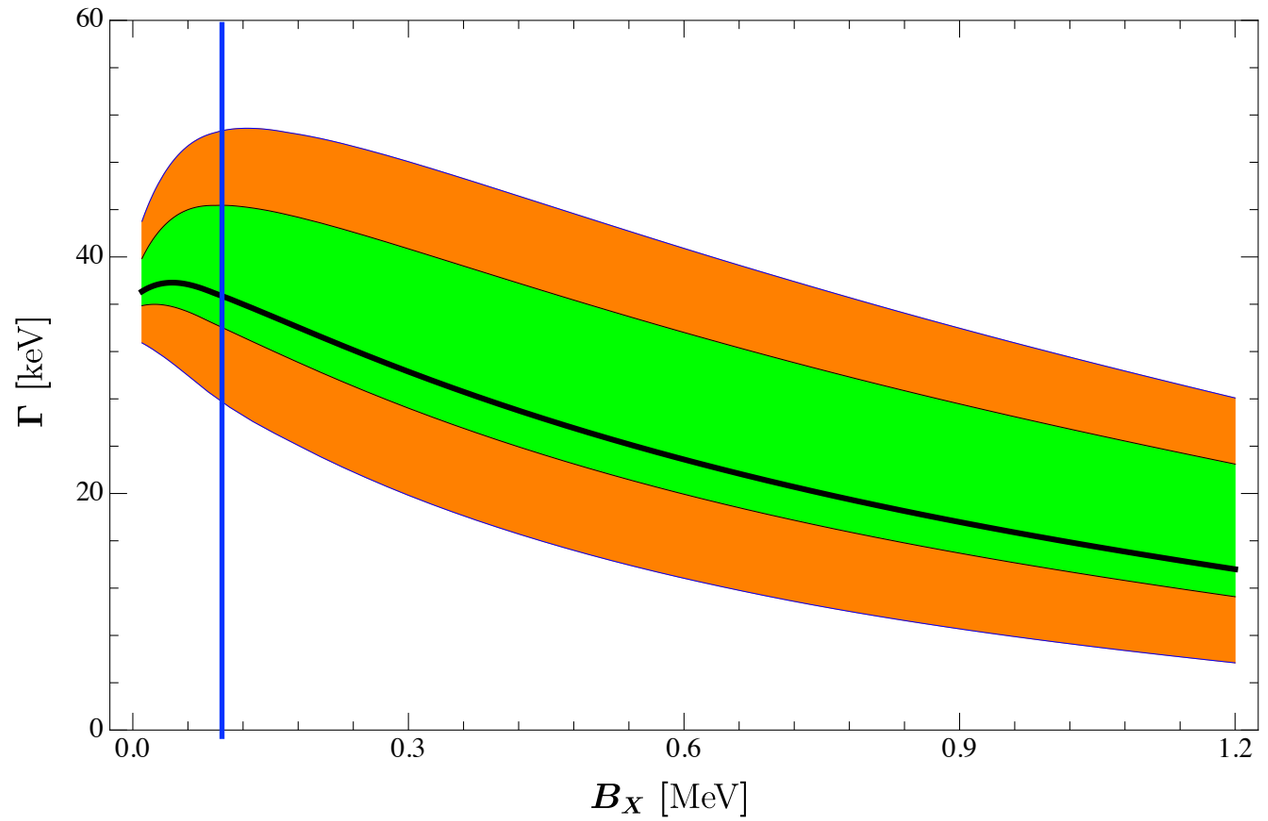
complex coefficients γ_{s+}, r_{s+}, ...

Large Corrections in XEFT at NLO ?

L. Dai, F.K. Guo & T. Mehen
Revisiting $X(3872) \to D^0\bar{D}^0\pi^0$ in an effective field theory for the *X(3872)*,
Phys. Rev. D 101, 054024 (2020) [arXiv:1912.04317]

numerical calculation
of decay rate for $X(3872) \to D^0\bar{D}^0\pi^0$ in XEFT to NLO

decay rate Γ
as function of
binding energy B_X



for B_X = 0.1 MeV, NLO error band is ±30% !

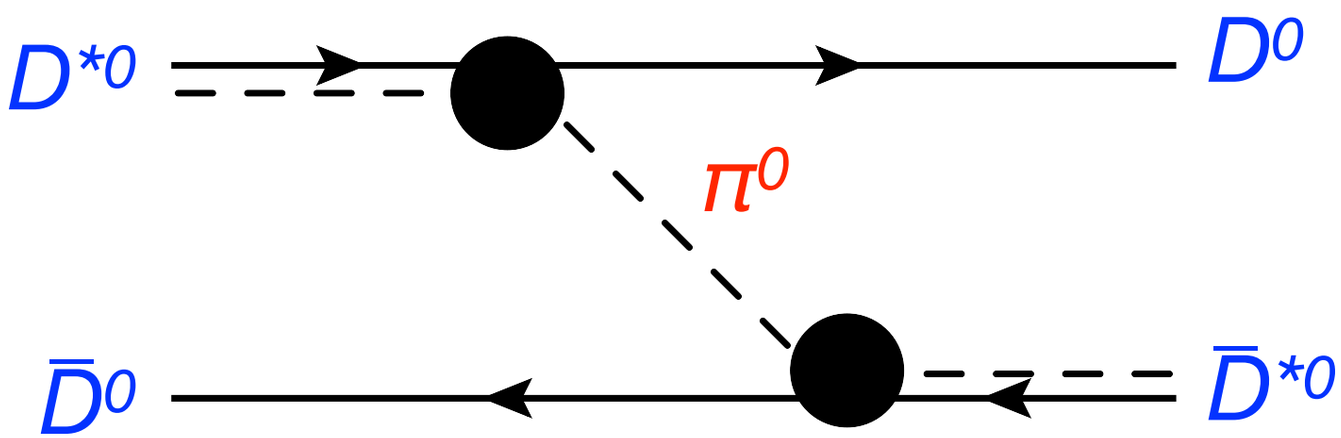
problem: inappropriate renormalization prescription?
calculate analytically using Galilean invariant XEFT ?

Summary

X(3872) is loosely bound neutral charm-meson molecule

$$X(3872) = \left(D^{*0}\bar{D}^0 + D^0\bar{D}^{*0}\right)/\sqrt{2}$$

π^0 exchanged between charm mesons can be on-shell



XEFT gives systematic treatment of $D^0\bar{D}^0\pi^0$ component

$D^{*0}\bar{D}^0$ scattering amplitude at NLO

$\Longrightarrow$ breakdown of effective range expansion for $T_{s+}(p)$

but there may be a short-distance T-matrix element $T_{SD}(p)$

with an effective range expansion