Precision event shapes with massive quarks

[2006.06383]+work in progess

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in colaboration with

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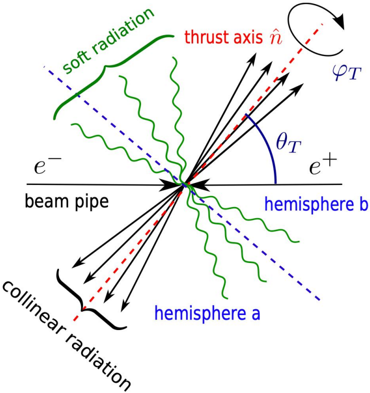
Outline

Outline

High energy $e^+e^- \to$ Hadrons



Scales:

- CM energy: Q
- Collinear: $p^2_{n,\bar{n}} \sim Q^2\lambda^2$
  Primary production $\Rightarrow \hat{m} \sim \lambda;\ \ \hat{m} \equiv \frac{m}{Q}$
- Ultrasoft: $p^2_s \sim Q^2\lambda^4$

Light-cone basis:

→ Usual convention: $n_1^\mu = n^\mu = (1,0,0,1),\ \ n_2^\mu = \bar{n}^\mu = (1,0,0,-1),\ \ n\cdot\bar{n} = 2$

→ Vector decomposition:
$p^\mu = \bar{n}\cdot p\,\frac{n^\mu}{2} + n\cdot p\,\frac{\bar{n}^\mu}{2} + p_\perp^\mu \equiv p^-\frac{n^\mu}{2} + p^+\frac{\bar{n}^\mu}{2} + p_\perp^\mu \equiv \left(p^+, p^-, p_\perp^\mu\right)$

→ Dijet momenta scaling: $p_n \sim Q\left(\lambda^2, 1, \lambda\right),\ \ p_{\bar{n}} \sim Q\left(1, \lambda^2, \lambda\right),\ \ p_s \sim Q\left(\lambda^2, \lambda^2, \lambda^2\right)$

Event Shapes

In this talk: defined in the context of $e^+e^- \to$ Hadrons

→ Observables which characterize the geometrical properties of final states.

→ They are a combination of the 4-momentum components of particles in the final state.

→ Their distributions are very sensitive to QCD dynamics ⇒ Measurements of QCD parameters

- 2-Jet Event Shapes:
  - → They reach their minimun value when the event is 2-Jet type (geometrically the final state consists on two back-to-back narrow jets).
  - → Distribution regions:
    - ∗ Peak: pure Dijet Configuration. Hadronization is $\mathcal{O}(1)$
    - ∗ Tail: transition between 2- and 3-jet events. Hadronization is a correction
    - ∗ Far Tail: multi-jet (isotropic) events. Hadronization through OPE
  - → Examples:

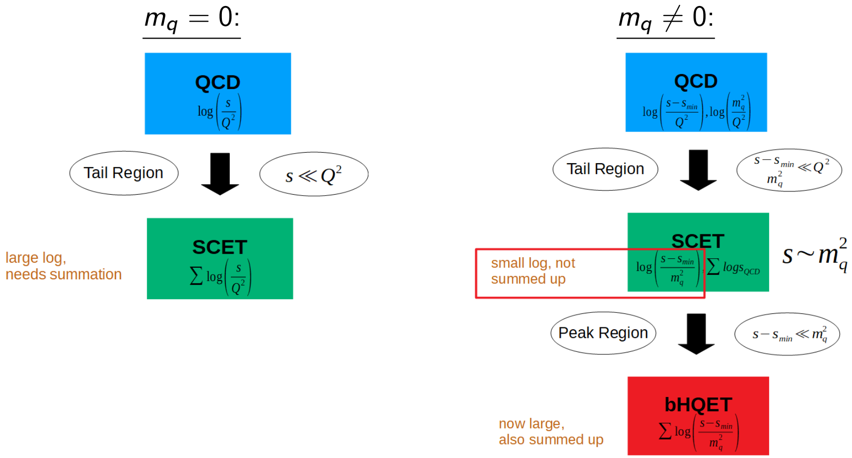
Thrust	C-parameter	Hemisphere Jet Mass
$\tau = \frac{1}{Q_P} \min_{\hat{t}} \sum_i (\lvert\vec{p}_i\rvert - \lvert\hat{t}\cdot\vec{p}_i\rvert)$	$C = \frac{3}{2}\left[1 - \frac{1}{Q_P^2}\sum_{i,j}\frac{(\vec{p}_i\cdot\vec{p}_j)^2}{\lvert\vec{p}_i\rvert\lvert\vec{p}_j\rvert}\right]$	$s = \left(\sum_{i\in h} p_i\right)^2$

h is one of the hemispheres delimited by the plane normal to the thrust axis $\hat{t}$

$$Q_P \equiv \sum_i \lvert\vec{p}_i\rvert$$

Dijets' primary quarks production

(s = jet invariant mass)



* Q center-of-mass energy

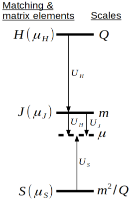
SCET (Soft-Collinear Effective Theory): [0801.4569], [0703207], [0711.2079]

($\hat{\sigma} \to$ partonic cross section)

$$\frac{1}{\sigma_0}\frac{d\hat{\sigma}_{\rm SCET}}{d\tau} = Q^2 \overbrace{H(Q,\mu)}^{\text{matching coefficient}} \int_0^{Q(\tau-\tau_{\rm min})} d\ell\, J_\tau(Q^2\tau - Q\ell,\mu)\, \overbrace{S_\tau(\ell,\mu)}^{\text{soft radiation}} + \text{p.c.}$$

$$J_\tau(s,\mu) \equiv \int_{s_{\rm min}}^{s-s_{\rm min}} ds'\, J_n(s-s',\mu) J_{\bar{n}}(s',\mu) \quad \text{collinear radiation}$$

each element evaluated at its natural scale. Running to a common scale sums up large logs

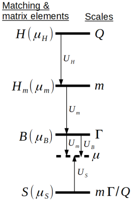


bHQET (Boosted Heavy Quark): [0801.4569], [0703207], [0711.2079]

1) Integrate out heavy quark mass
2) Boost back to c.o.m. frame
3) Match onto SCET in order to account for global soft radiation.

$$\frac{1}{\sigma_0}\frac{d\hat{\sigma}_{\rm bHQET}}{d\tau} = Q^2 H(Q,\mu_m)\, \overbrace{H_m\!\left(m, \frac{Q}{m}, \mu_m, \mu\right)}^{\text{new matching coefficient}} \int d\ell\, \overbrace{B_\tau\!\left(\frac{Q^2(\tau-\tau_{\rm min}) - Q\ell}{m}, \mu\right)}^{\text{new jet function}} S_\tau(\ell,\mu) + \text{p.c.}$$

$$B_\tau(\hat{s},\mu) = m\int_0^{\hat{s}} d\hat{s}'\, B_n(\hat{s}-\hat{s}',\mu) B_n(\hat{s}',\mu)$$



Outline

Massive schemes

Generalization of event shape definition for massive particles:

$$e = f(p_i)$$

E-scheme: $p_i = (p_i^0, \vec{p}_i) \to p_{i,E} = p_i^0\,(1, \vec{p}_i/|\vec{p}_i|)$
P-scheme: $p_i = (p_i^0, \vec{p}_i) \to p_{i,P} = |\vec{p}_i|\,(1, \vec{p}_i/|\vec{p}_i|)$
M-scheme: $p_i = (p_i^0, \vec{p}_i) = p_{i,M}$

* For massless particles: $p_E = p_P = p$
* $p_E^2 = p_P^2 = 0$ for massive and massless particles.
* Lorentz covariance lost $\Rightarrow$ defined in the c.o.m. frame.
* Change the cross section sensitivity to the quark mass:

	τ	C	s
M-scheme	$1-\beta$	$12\hat{m}^2(1-\hat{m}^2)$	m^2
P- and E- schemes	0	0	0

Threshold position for primary production of a stable quark-antiquark pair. $\hat{m} = \frac{m}{Q}$ and $\beta = \sqrt{1-4\hat{m}^2}$

Collinear limit

- Dijet limit:
  - → event shape can be decomposed in sectors
  - → event shape = sum of single-particle contributions

$$e_{\text{dijet}} = \boxed{e_n} + e_{\bar{n}} + e_s \qquad\qquad e_{n,\bar{n},s} = \sum_{i\in n,\bar{n},s} f_{n,\bar{n},s}(p_i)$$

- Any scheme: momentum scaling of collinear and soft particles remains the same.

n-collinear limit: $n = (1,0,0,1)$; $\bar{n} = (1,0,0,-1)$; $p_n = (p^+, p^-, p_\perp) \sim (\lambda^2, 1, \lambda)$

$$\left.\begin{aligned} p^0 &= (p^+ + p^-)/2 \simeq p^-/2 + \mathcal{O}(\lambda^2) \\ |\vec{p}\,| &= \sqrt{(p^0)^2 - m^2} \simeq p^-/2 + \mathcal{O}(\lambda^2) \end{aligned}\right\} \overset{LO}{\Longrightarrow} \boxed{\begin{aligned} p^- &= p_E^- = p_P^- \\ p^\perp &= p_E^\perp = p_P^\perp \end{aligned}}$$

"large" components identical at leading power in all schemes

$$\left.\begin{aligned} p^+ &= p^0 - p_z = p^0 - \sqrt{|\vec{p}\,|^2 - |\vec{p}_\perp|^2} \simeq \frac{p_\perp^2 + m^2}{2p^0} + \mathcal{O}(\lambda^4) \\ p_P^+ &= |\vec{p}\,| - p_z = |\vec{p}\,| - \sqrt{|\vec{p}\,|^2 - |\vec{p}_\perp|^2} \simeq \frac{|\vec{p}_\perp|^2}{2p^0} + \mathcal{O}(\lambda^4) \\ p_E^+ &= p^0 - \frac{p^0}{|\vec{p}\,|}\, p_z = \frac{p^0}{|\vec{p}\,|}\, p_P^+ \simeq p_P^+ + \mathcal{O}(\lambda^4) \end{aligned}\right\} \overset{LO}{\Longrightarrow} \boxed{p_P^+ = p_E^+ = p^+ - \frac{m^2}{p^-}}$$

plus component identical at leading power in E- and P-schemes

end up with 2 different jet functions

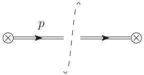
At Leading Order in λ: $p_{n,P} = p_{n,E} \neq p_n \quad \Longrightarrow \quad \boxed{e_n^P = e_n^E \neq e_n^M}$

SCET Jet function

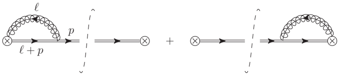
Feynman diagrams

Wave-function renormalization:

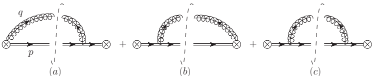
Tree-level:



One loop:



Virtual

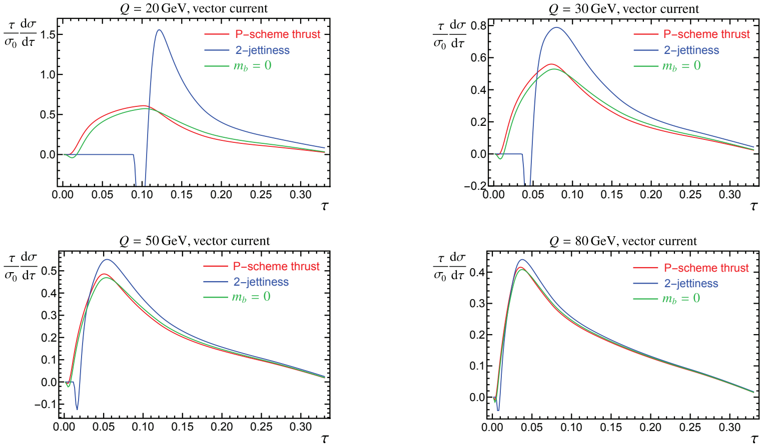


Real

SCET Results

Differential cross sections

[A. Bris, V. Mateu and M. Preisser, 2020]

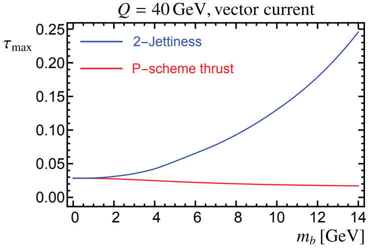


Differential cross section for massless quarks (green lines), 2-jettiness (blue lines) and P-scheme thrust (red lines) produced through vector current.

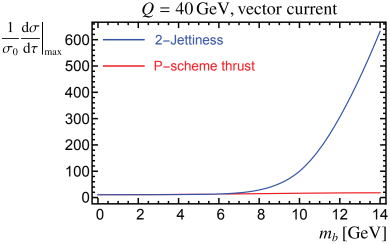
SCET Results

Mass dependence of the peak

[A. Bris, V. Mateu and M. Preisser, 2020]



(a) Peak position



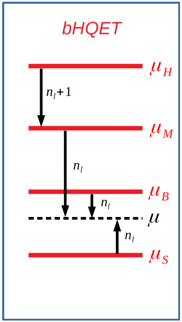
(b) Peak height

Peak position and peak height for 2-jettiness (blue) and P-scheme thrust (red) massive cross section.

Outline

Top quark mass extraction from peak cross section

* Hemisphere invariant mass sensitive to top quark mass ($s_{\mathrm{min}} = M^2$)
* Peak region (close to s_{min}) $\Rightarrow$ bHQET: ($\mu_M \sim$ primary mass)



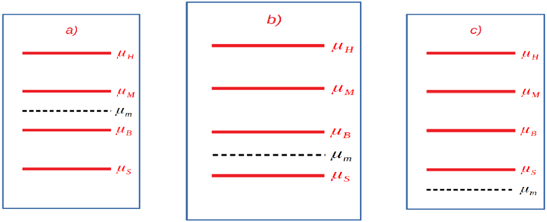
* Fluctuations around primary quark mass very small: $\frac{s-M^2}{M} \ll M \Rightarrow$ Non-vanishing secondary quark mass relevant for precise determination

bHQET

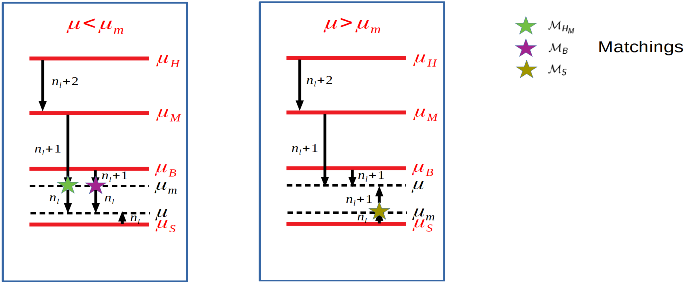
Massive secondary quark corrections

New scale (secondary quark mass) brings a richer structure of EFTs: (Based on [1405.4860] ↔ SCET)

→ Different scenarios: (b relevant in peak region)

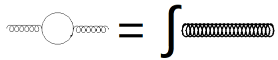


→ Consistency conditions: freedom to choose scale at which everyone runs to.

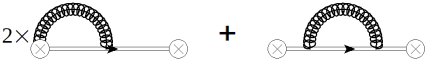


Feynman diagrams
(two loops with masses)

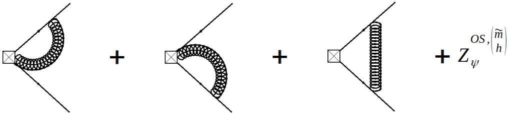
Dispersive Integral Method:



Jet function:



bHQET Hard Function:



Results. Pole-scheme cross section

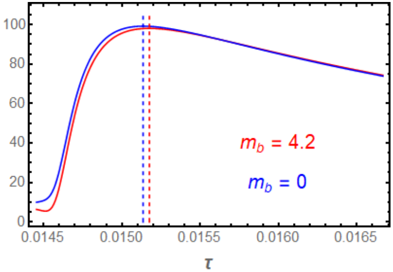
work in progress

Based on [B. Bachu, A. H. Hoang, V. Mateu, A. Pathak, I. W. Stewart]

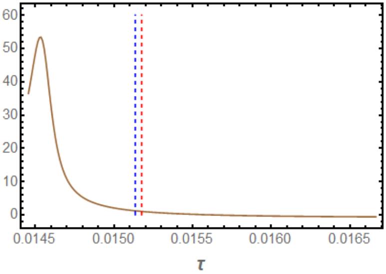
$$\tau \approx \frac{s_{h^+} + s_{h^-}}{Q^2} \quad \text{in the peak region}$$

$$M_t^{\text{pole}} = 170.034 \text{ GeV}$$
$$Q = 2000 \text{ GeV}$$
$$\Gamma_t = 1.32 \text{ GeV}$$

$$\left\{ \frac{1}{\sigma_0}\frac{d\sigma^{m_b=0}}{d\tau}, \; \frac{1}{\sigma_0}\frac{d\sigma^{m_b=4.2}}{d\tau} \right\}$$

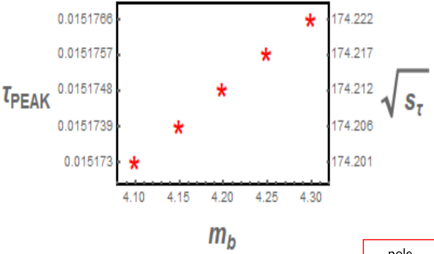


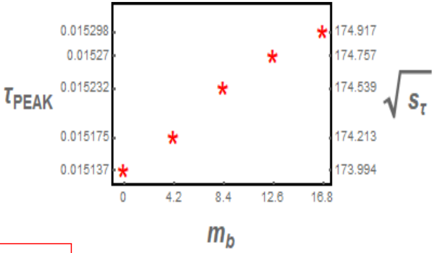
$$\left\{ \left[\frac{d\sigma^{m_b=0}}{d\tau} - \frac{d\sigma^{m_b=4.2}}{d\tau}\right] \Big/ \frac{d\sigma^{m_b=0}}{d\tau} \right\} \; \%$$



Results

work in progress





$$M_t^{\text{pole}} = 170.034 \ \text{GeV}$$
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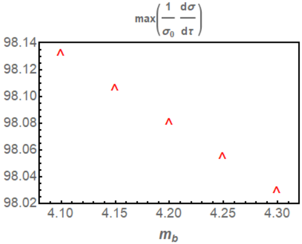
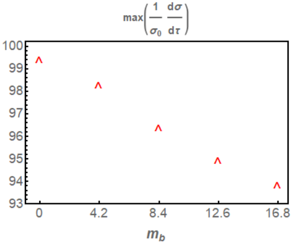
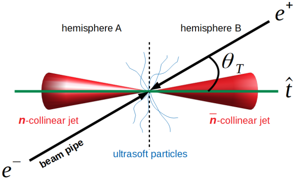




Outline

Orientation of dijet type events

- Use Thrust axis: θ_T angle with respect to beam



- Massless quarks [V. Mateu, G. Rodrigo, 2013]
  - → Only two angular structures:

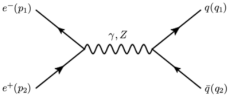
$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}e\,\mathrm{d}\cos\theta_T} = \frac{3}{8}\left(1+\cos^2\theta_T\right)\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}e} + \left(1-3\cos^2\theta_T\right)\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{\mathrm{ang}}}{\mathrm{d}e}$$

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta_T} = \frac{3}{8}\left(1+\cos^2\theta_T\right)R_{\mathrm{had}} + \left(1-3\cos^2\theta_T\right)R_{\mathrm{ang}}$$

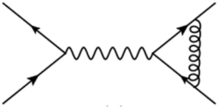
  - → $R_{\mathrm{ang}} \propto \alpha_s \Rightarrow \alpha_s$ determination
- Massive quarks??

Feynman diagrams

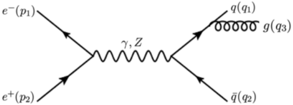
Leading Order:

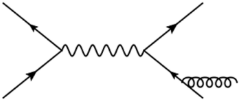


Virtual Radiation:



Real Radiation:



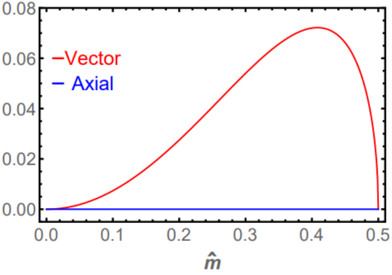


Results

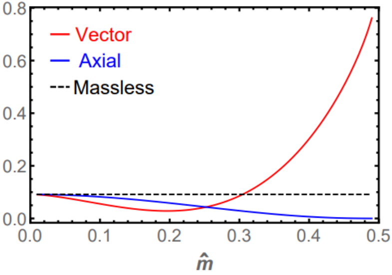
work in progress

* Oriented event shape diferential distributions:
  - **Vector current:** Contributions from real and virtual radiation. Singular and non-singular terms when $e \to e_{\text{min}}$
  - **Axial current:** Only real contributions. Non-singular terms
* Oriented total cross sections:

$$R_{\text{ang}}^{(0)}(\hat{m})$$



$$\frac{\pi}{\alpha_s\, C_F} R_{\text{ang}}^{(1)}(\hat{m})$$



Conclusions

- $\rightarrow$ Variations on the massive scheme of an event shape allow for different mass sensitivity in the corresponding cross section. This may be interesting to consider when fitting the distributions either for α_s or primary quark masses
- $\rightarrow$ P and E schemes are equivalent at leading order in collinear limit but they differ from M scheme in general.
- $\rightarrow$ We computed the missing pieces for the SCET and bHQET cross sections in P,E-scheme at N^{2}LL $+\mathcal{O}(\alpha_s)$ accuracy, the jet functions, and worked out resummation.
- $\rightarrow$ For a precise determination of the peak position, massive secondary quark corrections need to be taken into account.
- $\rightarrow$ Missing pieces of thrust and hemisphere mass distribution in the bHQET factorization theorem were computed.
- $\rightarrow$ We studied the oriented event shape distributions at $\mathcal{O}(\alpha_s)$ for massive quarks
- $\rightarrow$ Enhancement of oriented total cross section for vector current when $\hat{m} \rightarrow 1/2 \Rightarrow$ top mass determination

BACKUP SLIDES

Thrust, Hemisphere Jet Mass, C-parameter

	τ	C	s
E	$\frac{1}{Q}\min_{\hat{t}}\sum_i \frac{p_i^0}{\lvert\vec{p}_i\rvert}(\lvert\vec{p}_i\rvert - \lvert\hat{t}\cdot\vec{p}_i\rvert)$	$\frac{3}{2}\left[1-\frac{1}{Q^2}\sum_{i,j}\frac{p_i^0 p_j^0(\vec{p}_i\cdot\vec{p}_j)^2}{\lvert\vec{p}_i\rvert^2\lvert\vec{p}_j\rvert^2}\right]$	$\sum_{i,j\in \mathrm{h}}\frac{p_i^0 p_j^0(\lvert\vec{p}_i\rvert\lvert\vec{p}_j\rvert-\vec{p}_i\cdot\vec{p}_j)}{\lvert\vec{p}_i\rvert\lvert\vec{p}_j\rvert}$
P	$\frac{1}{Q_P}\min_{\hat{t}}\sum_i(\lvert\vec{p}_i\rvert - \lvert\hat{t}\cdot\vec{p}_i\rvert)$	$\frac{3}{2}\left[1-\frac{1}{Q_P^2}\sum_{i,j}\frac{(\vec{p}_i\cdot\vec{p}_j)^2}{\lvert\vec{p}_i\rvert\lvert\vec{p}_j\rvert}\right]$	$\sum_{i,j\in \mathrm{h}}(\lvert\vec{p}_i\rvert\lvert\vec{p}_j\rvert - \vec{p}_i\cdot\vec{p}_j)$
M	$\frac{1}{Q}\min_{\hat{t}}\sum_i(p_i^0 - \lvert\hat{t}\cdot\vec{p}_i\rvert)$	$\frac{3}{2}\left[2-\frac{1}{Q^2}\sum_{i\neq j}\frac{(p_i\cdot p_j)^2}{p_i^0 p_j^0}\right]$	$\left(\sum_{i\in h} p_i\right)^2$

Table: Thrust, C-parameter and hemisphere jet mass in the three massive schemes. In green, the original definitions.

	τ_n	C_n	s_n
E, P	$\tau_n^{E,P} = \frac{1}{Q}\sum_{i\in +}\left(p_i^+ - \frac{m_i^2}{p_i^-}\right)$	$C_n^{E,P} = \frac{6}{Q}\sum_{i\in +}\left(p_i^+ - \frac{m_i^2}{p_i^-}\right)$	$s_n^{E,P} = Q\sum_{i\in +}\left(p_i^+ - \frac{m_i^2}{p_i^-}\right)$
M	$\tau_n^J = \frac{1}{Q}\sum_{i\in +} p_i^+$	$C_n^J = \frac{6}{Q}\sum_{i\in +} p_i^+$	$s_n = Q\sum_{i\in +} p_i^+$

Table: Thrust, C-parameter and hemisphere jet mass collinear limits.