d*(2380) decays in hadronic molecule picture: in preparation

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Introduction

- Dibaryon has been long history in hadron spectroscopy.

- Dibaryon was predicted by Dyson and Xuong in 1964.

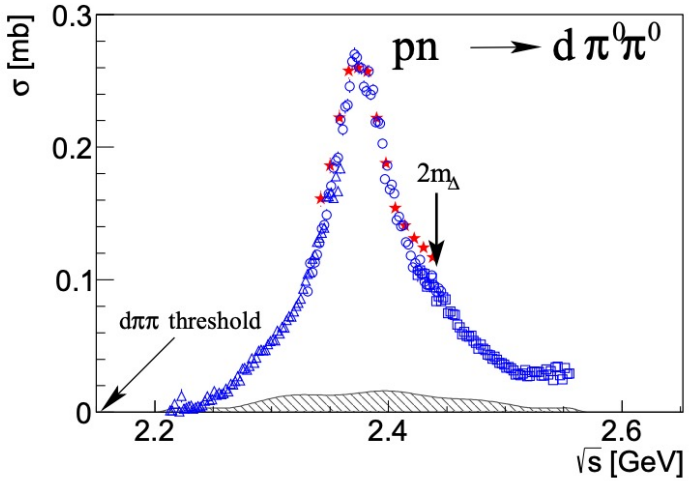
notation	I	J	asymptotic baryon-baryon configuration	mass (formula)	mass (value) (MeV)
D_{01}	0	1	deuteron	A	1876
D_{10}	1	0	${}^1S_0$ NN virtual state	A	1876
D_{12}	1	2	ΔN	A + 6B	2160
D_{21}	2	1	ΔN	A + 6B	2160
D_{03}	0	3	$\Delta\Delta$	A + 10B	2350
D_{30}	3	0	$\Delta\Delta$	A + 10B	2350

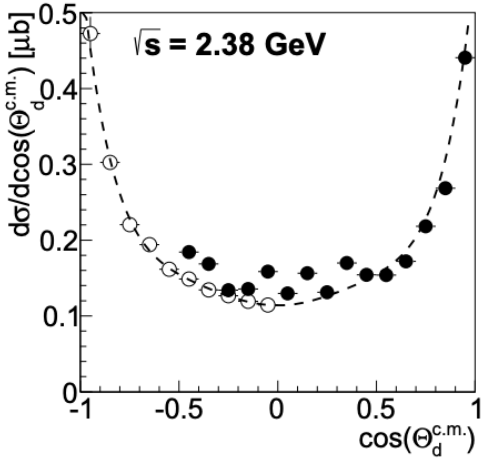
Clement (2016)

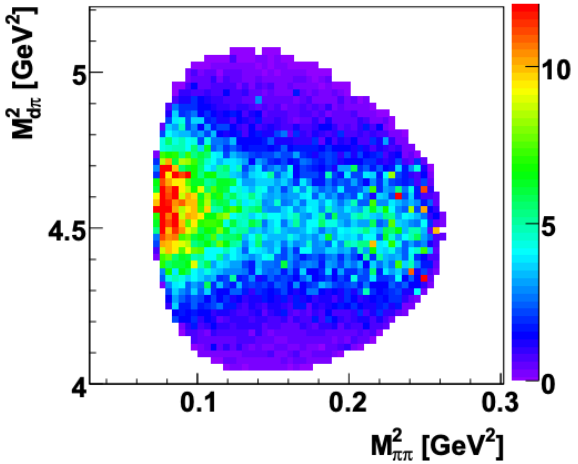
- The deuteron is the first example of the dibaryon.

The d*(2380): very brief introduction

- The discovery of the six-quark state called d*(2380) is confirmed in the WASA@COSY laboratory: Adlarson et al.(2011).







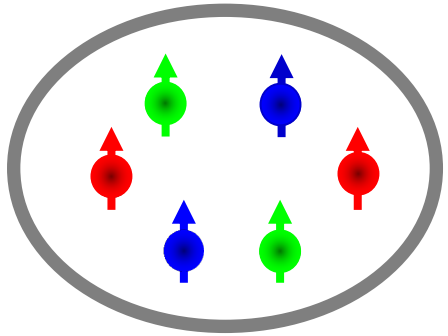
More details about d*(2380): Dan's talk (next two talks)

- Mass of d*(2389) ≈ 2380 MeV with binding energy ≈ 84 MeV.
- Total width ≈ 70 MeV: $\Gamma(d^* \to \Delta\Delta) \approx 64$ MeV, $\Gamma(d^* \to \mathrm{pn}) \approx 6$ MeV.
- The quantum number $(I)\, J^P = (0)\, 3^+$.

Candidates of the d*(2380)

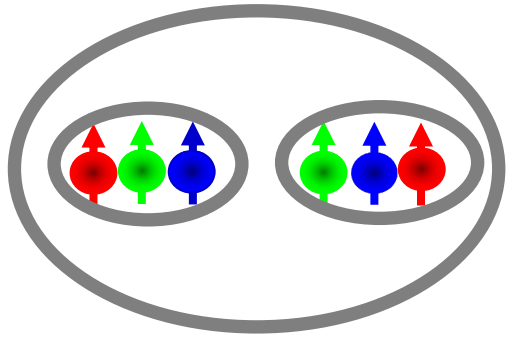
- Compact hexa-quark:

  Quark models by Thomas, Oka, Yamazaki, Goldman, Wang, Yuan, Zhang, Yu, Shen, Dong and others



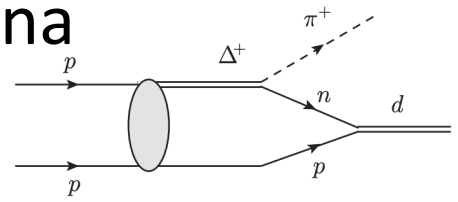
- Molecular state:

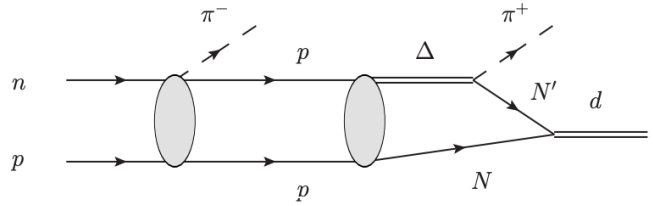
  Huang, Ping, Wang, Dong, Shen and others



- Kinematic triangle singularity effect:

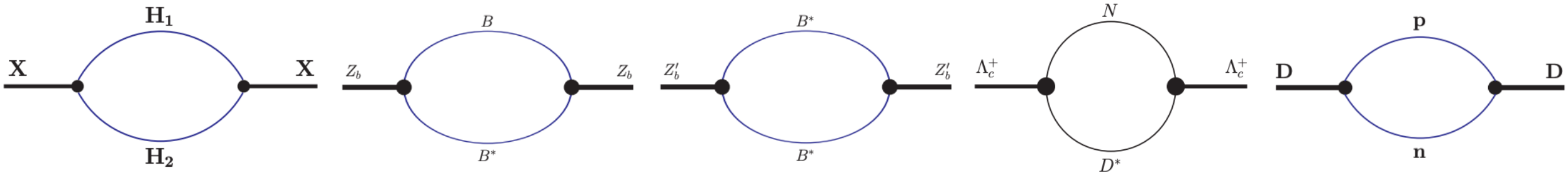
  Bar-nir, et.al and Oset, Ikeno and Molina





Phenomelogical Lagrangian approach to molecular states of exotic hadrons

- The (non-local) phenomenological Lagrangian is proposed by Tubingen-Beijing group to study the molecular-like exotic hadron states e.g., X(3872), $Z_b(10610)$, $Z_b(10650)$, $\Lambda_c(2940)$ and deuteron: see review: Dong, Faessler, Lyubovitskij (2017).



- We assume that the d*(2380) is a hadronic molecular state of ΔΔ pair.

- In this work, we will extract coupling constant of d*-ΔΔ and use to describe the d*→ d$\pi^0\pi^0$ decay.

Phenomenological Lagrangian of the d*ΔΔ coupling

- The phenomenological Lagrangian of the d*(2380) as ΔΔ molecular state is given by : Dong and Shen (2022)

$$\mathscr{L}_{d^*\Delta\Delta}(x) = g_{d^*\Delta\Delta}\, d^{*\mu\nu\rho}(x) \int d^4y\, \Phi_{\Delta\Delta}(y^2) \bar{\Delta}^c_\mu(x + y/2)\gamma_\nu \Delta_\rho(x - y/2) + \text{h.c.}$$

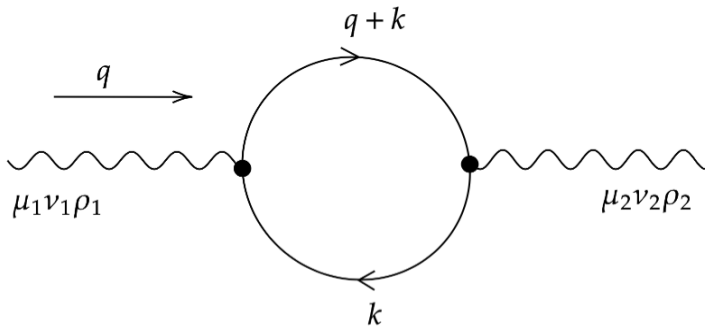
- The correlation function $\Phi(y^2)$ is described the finite length of the d$*$ size as a weakly bound state of ΔΔ baryons depending on the Jacobi coordinate y.

$$\Phi_{\Delta\Delta}(y^2) = \int \frac{d^4k}{(2\pi)^4}\, e^{-i\,k\cdot y}\, \Phi_{\Delta\Delta}(-k^2)\,, \qquad \Phi_{\Delta\Delta}(-k^2) = e^{k^2/\Lambda_{d^*}}\,,$$

- The free parameter cut-off Λ_{d^*} characterizes the distribution of the constituents in the d*.

Estimation of the d*ΔΔ coupling constant

- The amplitude of the self-energy diagram of spin-3 d* is given by



$$i\mathcal{M} = \varepsilon_{\mu_1\nu_1\rho_1}\,\varepsilon^*_{\mu_2\nu_2\rho_2}\int\frac{d^4k}{(2\pi)^4}\,\Pi^{\mu_1\nu_1\rho_1;\mu_2\nu_2\rho_2}(k,q)$$

$$\begin{aligned}\Pi^{\mu_1\nu_1\rho_1;\mu_2\nu_2\rho_2} \;=\; -\mathrm{tr}\Big[&\gamma^{\nu_1}(\not{k}+\not{q}+M)\left(g^{\mu_1\rho_2}-\frac{\gamma^{\mu_1}\gamma^{\rho_2}}{3}-\frac{2(k+q)^{\mu_1}(k+q)^{\rho_2}}{3M^2}-\frac{\gamma^{\mu_1}(k+q)^{\rho_2}-\gamma^{\rho_2}(k+q)^{\mu_1}}{3M}\right)\\ &\times\gamma^{\nu_2}(\not{k}+M)\left(g^{\mu_2\rho_1}-\frac{\gamma^{\mu_2}\gamma^{\rho_1}}{3}-\frac{2k^{\mu_2}k^{\rho_1}}{3M^2}-\frac{\gamma^{\mu_2}k^{\rho_1}-\gamma^{\rho_1}k^{\mu_2}}{3M}\right)\Big]\\ &\times\frac{g^2_{d^*\Delta\Delta}\,\Phi^2\big((k+w_\Delta\,q)^2\big)}{[k^2-M^2]\,[(k+q)^2-M^2]}\,,\end{aligned}$$

$$\Theta^{\mu_1\nu_1\rho_1;\mu_2\nu_2\rho_2}_{(1)} \;=\; \frac{1}{6}\sum_{P(\mu\nu\rho)_{1,2}}\mathcal{P}^{\mu_1\mu_2}\,\mathcal{P}^{\nu_1\nu_2}\,\mathcal{P}^{\rho_1\rho_2}-\frac{1}{15}\sum_{P(\mu\nu\rho)_{1,2}}\mathcal{P}^{\mu_1\nu_1}\,\mathcal{P}^{\rho_1\mu_2}\,\mathcal{P}^{\nu_2\rho_2}$$

Transverse spin-3 projection operator: Kawasaki and Kobayashi (1978)

$$\Theta_{(1)\mu_1\nu_1\rho_1;\mu_2\nu_2\rho_2}\,\Pi^{\mu_1\nu_1\rho_1;\mu_2\nu_2\rho_2}\left(q^2\right)=\Pi_{(\perp)}$$

Transverse spin-3 operator

The Weinberg-compositeness condition

- The deuteron has been proved theoretically by Weinberg that it is bound state.
- The renormalization constant, Z can also be interpreted as the matrix element between the physical and the corresponding bare state.

- For Z= 0, the physical state does not contain the bare one and is solely described as a bound state.

- The Weinberg compositeness condition (Z=0 for bound state) use to evaluate the coupling constant.

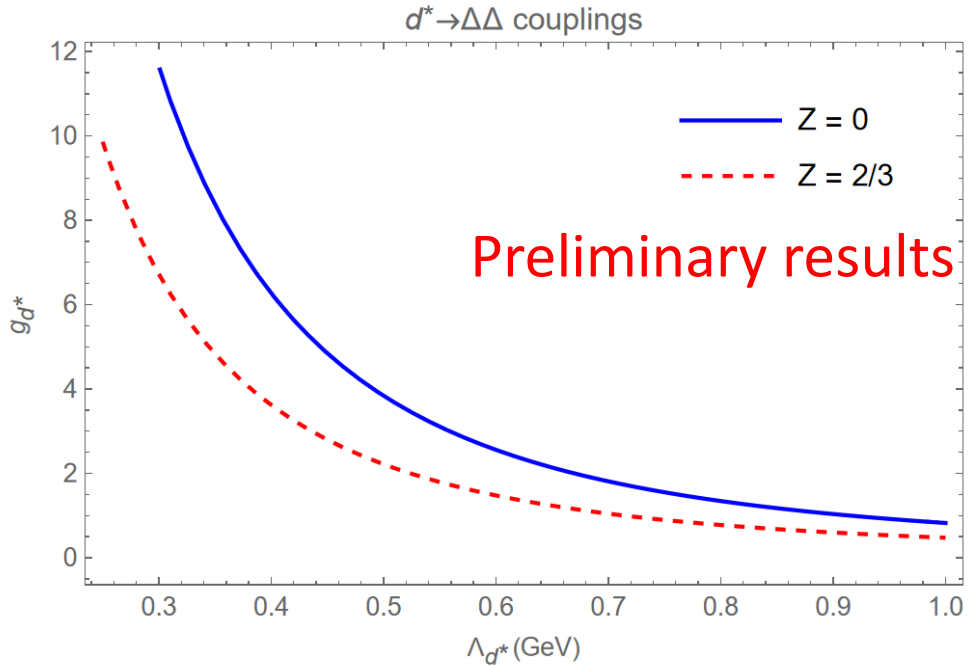
Weinberg compositeness condition

$$Z = 1 - \left. \frac{d\,\Pi_{(\perp)}\left(q^2\right)}{d\,q^2} \right|_{q^2 = m_{d*}^2}$$

Numerical (preliminary) results

- According to Huang et.al.(2015) have shown that the d* is composed of ΔΔ bound state and Compact-six quark (CC) about 1/3 and 2/3 respectively i.e., $|d^*\rangle = \sqrt{1/3}|\Delta\Delta\rangle + \sqrt{2/3}|\mathrm{CC}\rangle$.

- We consider in two cases that Z=0 (pure bound state) and Z=2/3 (1/3 component of d*).



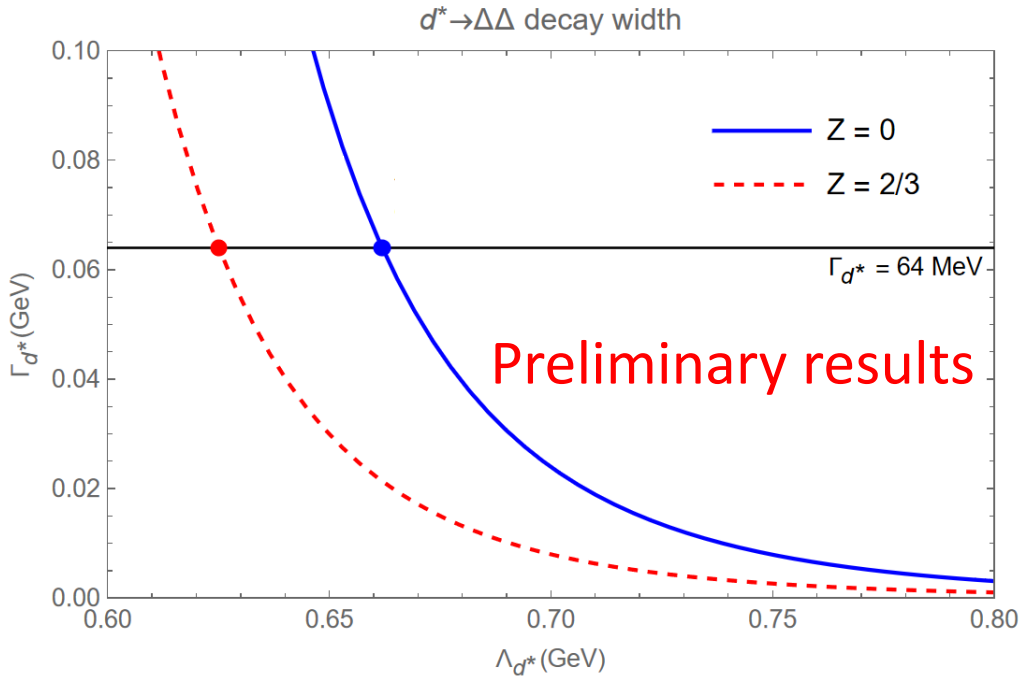
Fixing the d* coupling with d*→ ΔΔ decay

- We will fix the d* coupling constant with the d* → ΔΔ decay.

$$i\mathcal{M}_{d^*\to\Delta\Delta} = i\, g_{d^*\Delta\Delta}\, \varepsilon_{\mu\nu\rho}(k,\lambda)\, \bar{u}^{\mu}(k_1,\lambda_1)\, \gamma^{\nu}\, \bar{u}^{\rho,T}(k_2,\lambda_2)\, \Phi_{\Delta\Delta}\big(-(k_1-k/2)^2\big)$$

$$\Gamma_{d^*\to\Delta\Delta} = \frac{1}{\pi^2}\int_0^{m^2} \frac{dk_1^2\, k_1\, \Gamma_\Delta}{(k_1^2-M^2)^2+(M\,\Gamma_\Delta)^2} \int_0^{(m-k_1)^2} \frac{dk_2^2\, k_2\, \Gamma_\Delta}{(k_2^2-M^2)^2+(M\,\Gamma_\Delta)^2}\, \Gamma^{(0)}_{d^*\to\Delta\Delta}(k_1,k_2) \;;\; \Gamma^{(0)}_{d^*\to\Delta\Delta} = \frac{1}{8\pi}\frac{|\vec{k}_1^*|}{m^2}\left|\mathcal{M}_{d^*\to\Delta\Delta}\right|^2$$

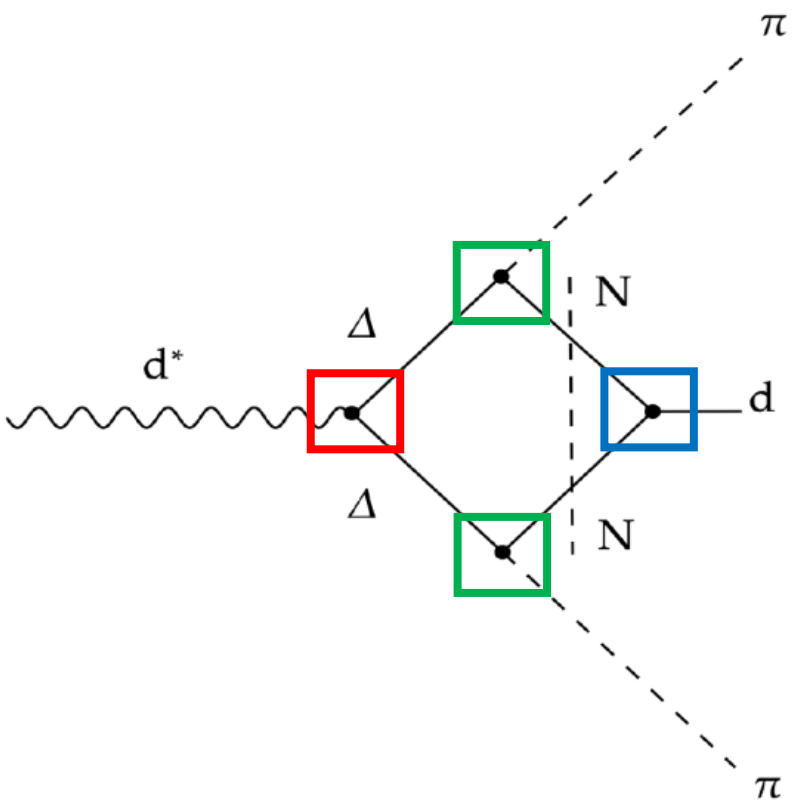
- $g_{d^*\Delta\Delta} = 2.05$, $\Lambda_{d^*} = 0.660$ GeV (Z=0)
- $g_{d^*\Delta\Delta} = 1.35$, $\Lambda_{d^*} = 0.625$ GeV (Z=2/3)



Experiment : $\Gamma_{d^*\to\Delta\Delta}$= 64 MeV

The d*→ $d\pi^0\pi^0$ in molecular picture with phenomenological Lagrangian

- Next, we consider the d*→ $d\pi^0\pi^0$ being two step decay as d*→ ΔΔ → $p\pi^0 n\pi^0 \rightarrow d\pi^0\pi^0$



$$\mathscr{L}_{d^*\Delta\Delta}(x) = g_{d^*\Delta\Delta}\, d^{*\mu\nu\rho}(x) \int d^4y\, \Phi_{\Delta\Delta}(y^2)\bar{\Delta}^c_\mu(x+y/2)\gamma_\nu\Delta_\rho(x-y/2) + \text{h.c.}$$

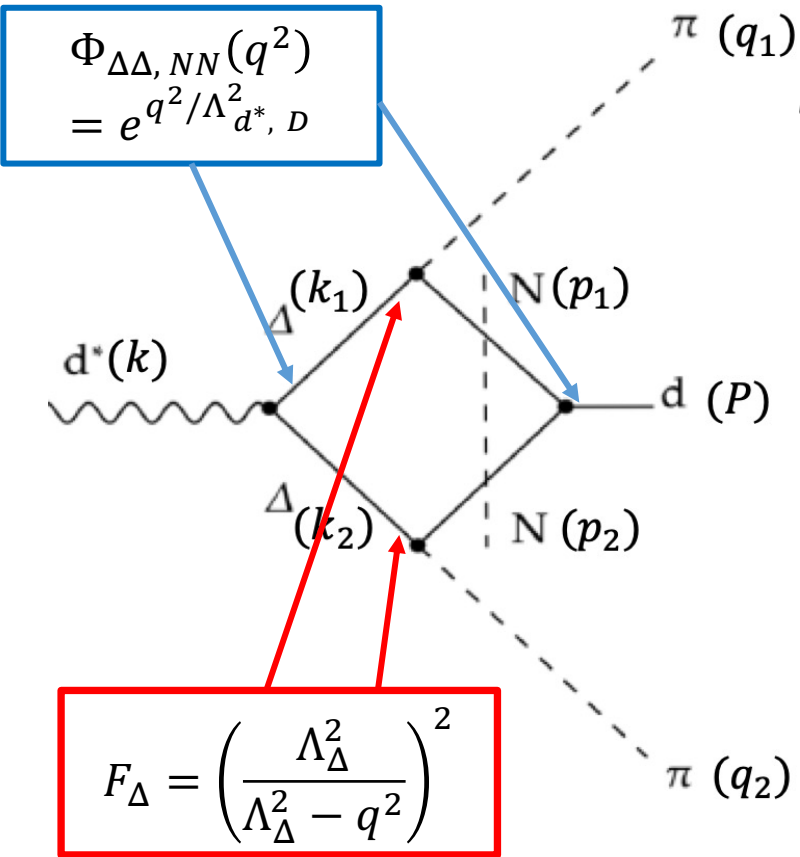
$$\mathscr{L}_{\Delta N\pi} = h_{\Delta N\pi}\bar{\Delta}^\mu\vec{T}\cdot\partial_\mu\vec{\pi}\, N + \text{h.c.}$$

$$\mathscr{L}_{dpn}(x) \;=\; g_{dpn}\, d_\mu(x) \int d^4y\, \Phi_{NN}(y)\bar{p}^c\,(x+y/2)\,\gamma^\mu n\,(x-y/2) + \text{h.c.}$$

Dong, Gutsche, Faessler, Lyubovitskij (2008)

Amplitude and on-shell nucleon approximation

$$
\begin{aligned}
i\mathcal{M}_{d^*\to d\pi^0\pi^0} &= -\int \frac{d^4l}{(2\pi)^4}\,\mathcal{G}\mathcal{F}\,\varepsilon_{\mu'\nu'\rho'}G^{\mu'\nu'\rho'}_{\mu\nu\rho}\,\varepsilon^*_\sigma \\
&\quad\times \mathrm{Tr}\Big[S^{\rho\rho_1}(l+L-q_1)iq_{1,\rho_1}S(l+L)\gamma^\sigma iq_{2,\rho_2}S(l-L)S^{\rho_2\mu}(l-L+q_2)\gamma^\nu\Big] \\
S(l\mp L) &= \frac{\not{l}\mp\not{L}}{(l\mp L)^2-m_N^2}\,, \quad l=\frac{1}{2}(p_1-p_2)\,, \quad L=\frac{1}{2}(p_1+p_2) \\
S_{\mu\nu}(k) &= \frac{\not{k}+M}{k^2-M^2-i\,M\,\Gamma_\Delta}\left(-g_{\mu\nu}+\frac{1}{3}\gamma_\mu\gamma_\nu+\frac{1}{3M}(\gamma_\mu k_\nu-\gamma_\nu k_\mu)+\frac{2}{3M^2}k_\mu k_\nu\right) \\
\mathcal{F} &= \Phi_{\Delta\Delta}\,\Phi_{NN}\,F_\Delta^2\,,\ \mathcal{G}=i\tfrac{2}{\sqrt{3}}g_{d^*\Delta\Delta}\,g_{dpn}\,h^2_{\Delta N\pi}
\end{aligned}
$$



Consider the momenta of two nucleons in the loop as on-shell

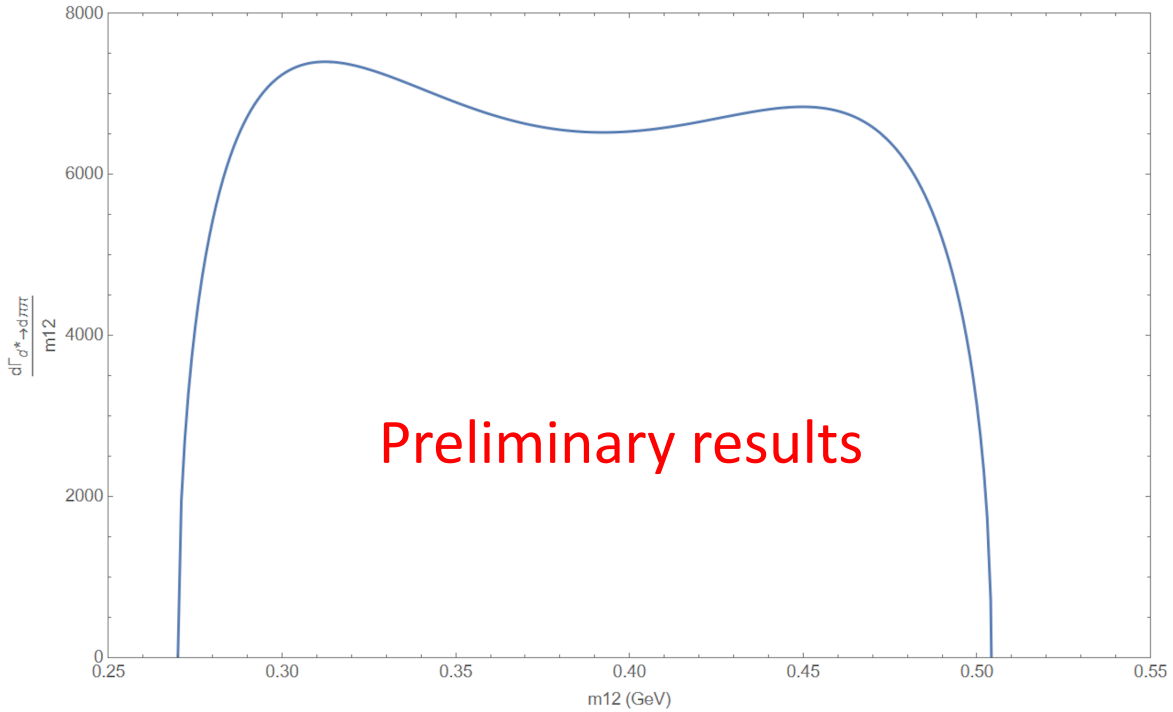
Risser and Shuster (1973)

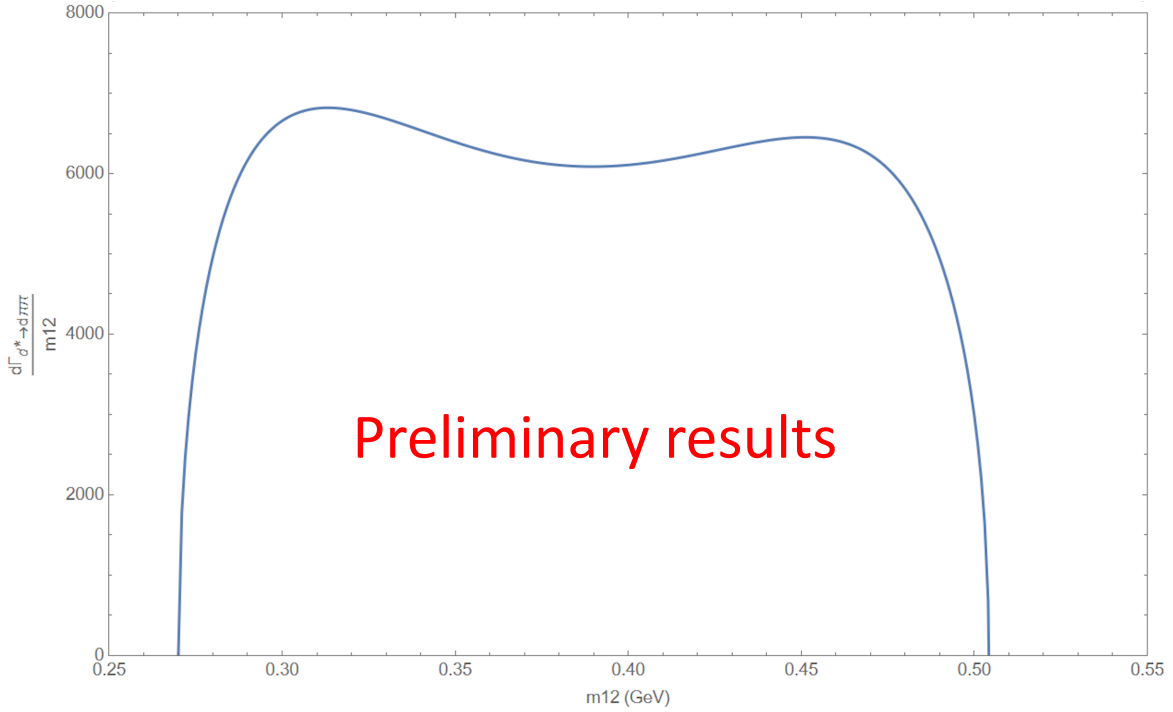
$$
\begin{aligned}
S(l\mp L) &= \frac{\not{l}\mp\not{L}}{(l\mp L)^2-m_N^2} \overset{\text{on-shell}}{\Longrightarrow} (\not{l}\mp\not{L})(-2\pi i)\delta^{(4)}\left((l\mp L)^2-m_N^2\right) \\
(l\mp L)^2 &= m_N^2 \quad\Longrightarrow\quad p_1=p_2
\end{aligned}
$$

$\Phi_{NN}(q^2)$ → s-wave deuteron wave function: Dong, Gutsche, Faessler, Lyubovitskij (2008)

Numerical (preliminary) results

- The width is given by $d\Gamma = \frac{1}{(2\pi)^3 32 m_{d^*}^3} |M_{d^* \to d\pi^0\pi^0}|^2 dm_{12}^2 dm_{23}^2$.
- The $d\Gamma/dm_{12}^2$ vs m_{12} with $\Lambda_\Delta = 1$ GeV and $\Lambda_D = 0.45$ GeV.





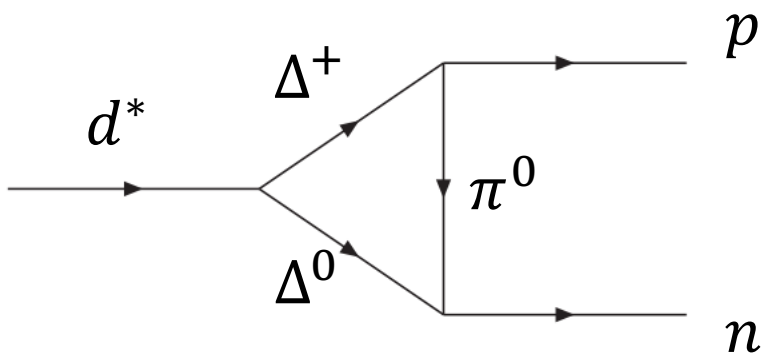
Z= 0 case: $\Gamma_{d^* \to d\pi^0\pi^0}$ = 13.5 MeV

Z= 2/3 case: $\Gamma_{d^* \to d\pi^0\pi^0}$ = 10 MeV

Experiment : $\Gamma_{d^* \to d\pi^0\pi^0}$ = 10.2 MeV

Forthcoming next results

- We are doing calculation of the d* $\rightarrow pn$ with the phenomenological Lagrangian and same values of the parameters.



- We hope that the theoretical calculation migth reproduces the experimental data (decay width and Branching ratio and, etc.).

- Caution: the results of this approach strongly depend on the cut-off parameters!!!

Summary and conclusions

- We assume the d* as molecular bound state of ΔΔ. The non-local phenomenological Lagrangian approch is used with one free-parameter, Λ_{d^*}.
- We fixed the coupling $g_{d^*\Delta\Delta}(\Lambda_{d^*})$ with the off-shell decay of the $d^* \to \Delta\Delta$.
- The decay width of the $d^* \to d\pi^0\pi^0$ is reproduced with $\Lambda_\Delta = 1$ GeV and $\Lambda_\mathrm{D} = 0.45$ GeV.
- The d*(2380) likely composes of ΔΔ molecule (1/3) and compact six quark (2/3).
- The $d^* \to pn$ decay width will be reproduced (hopefully) with the same parameters of $d^* \to d\pi^0\pi^0$ decay.
- The compositeness condition might be useful to estimate the d* coupling to other hadrons and can be applied to the EOS of the neutron stars.

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THANK YOU

for your attention