SU(N)-natural inflation

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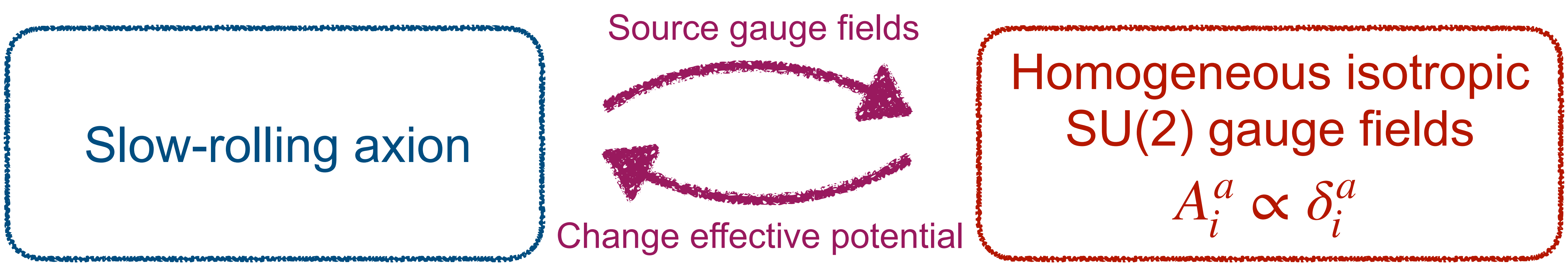
- Chromo-natural inflation (CNI) [P. Adshead and M. Wyman (2012)]

$$\mathscr{L} = \underbrace{\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)}_{\text{axion/inflaton}} \underbrace{- \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}}_{\text{SU(2) gauge fields}} + \underbrace{\frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}}_{\text{Axion-gauge fields coupling}}$$



With the interaction between axion and gauge fields,
we can reproduce the observed $\mathscr{P}_\zeta$ and n_s with $f < M_{\rm Pl}$.

Introduction

■ Chromo-natural inflation (CNI) [P. Adshead and M. Wyman (2012)]

$$\mathscr{L} = \underbrace{\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)}_{\text{axion/inflaton}} \underbrace{- \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}}_{\text{SU(2) gauge fields}} + \underbrace{\frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}}_{\text{Axion-gauge fields coupling}}$$

Due to the SU(2) gauge field background,
the gauge field perturbations are amplified.

→ The gauge field perturbations source the gravitational waves.
CNI suffers from the GW overproduction…
[P. Adshead, E. Martinec, and M. Wyman (2013)]

If the axion is a spectator, this model can predict observable GWs.
[E. Dimastrogiovanni, M. Fasiello, T. Fujita (2016)
K. Ishiwata, E. Komatsu, I. Obata (2021)]

How about SU(N) extension of CNI?

Chromo-natural inflation

- Review of CNI

$$\mathscr{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{\phi}{4f}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}$$

$$A^a_i(t) = \delta^a_i a(t)Q(t), \quad m_Q \equiv \frac{gQ}{H} \simeq \left(\frac{-g^2 f\partial_\phi V}{3H^4}\right)^{1/3},$$

$$\xi \equiv \frac{\dot{\phi}}{2fH} \simeq m_Q + m_Q^{-1}.$$

Spatial rotation of A^a_i can be represented by a gauge transf. :

$$^\forall R, ^\exists G : R_{ij}A^a_j = G^{ab}A^b_i$$

$A^a_i \propto \delta^a_i$ indicates SSB: spatial SO(3) × SU(2) → SO(3)

SU(N)-natural inflation

- **SU(N)-natural inflation** [T. Fujita, K. Mukaida, KM, and H. Nakatsuka (2021)]

Consider SU(2) ⊂ SU(N) and decompose A_i as

$$A_i = A_i^a \mathcal{T}_a = A_i^j \mathcal{T}_j + A_i^A \mathcal{T}_A$$

SU(N) generators
$a = 1, \ldots, N^2 - 1$
$\mathrm{Tr}\left[\mathcal{T}_a \mathcal{T}_b\right] = \frac{\delta_{ab}}{2}$

SU(2) generators
$i = 1,2,3$
$[\mathcal{T}_i, \mathcal{T}_j] = i\lambda\epsilon_{ijk}\mathcal{T}_k$

The other SU(N) generators
$A = 4, \ldots, N^2 - 1$

$$A_i^j(t) = \delta_i^j\, a(t) Q(t), \quad A_i^A(t) = 0,$$

$$m_Q \equiv \frac{g\lambda Q}{H} \simeq \left(\frac{-g^2\lambda^2 f \partial_\phi V}{3H^4}\right)^{1/3}, \qquad \xi \equiv \frac{\dot\phi}{2fH} \simeq m_Q + m_Q^{-1}.$$

λ depends on the choice of SU(2) or SSB pattern.
λ is degenerate with g.

SU(N)-natural inflation

- Examples: $N = 3$

SU(3) ⊃ SU(2) × U(1) $\mathbf{3} = \mathbf{2} + \mathbf{1}, \quad \mathbf{8} = \mathbf{3} + \mathbf{2} + \mathbf{2} + \mathbf{1}$

$$\mathcal{T}_i = \frac{1}{2}\begin{pmatrix} \sigma_i & 0 \\ 0 & 0 \end{pmatrix} \qquad \lambda = 1$$

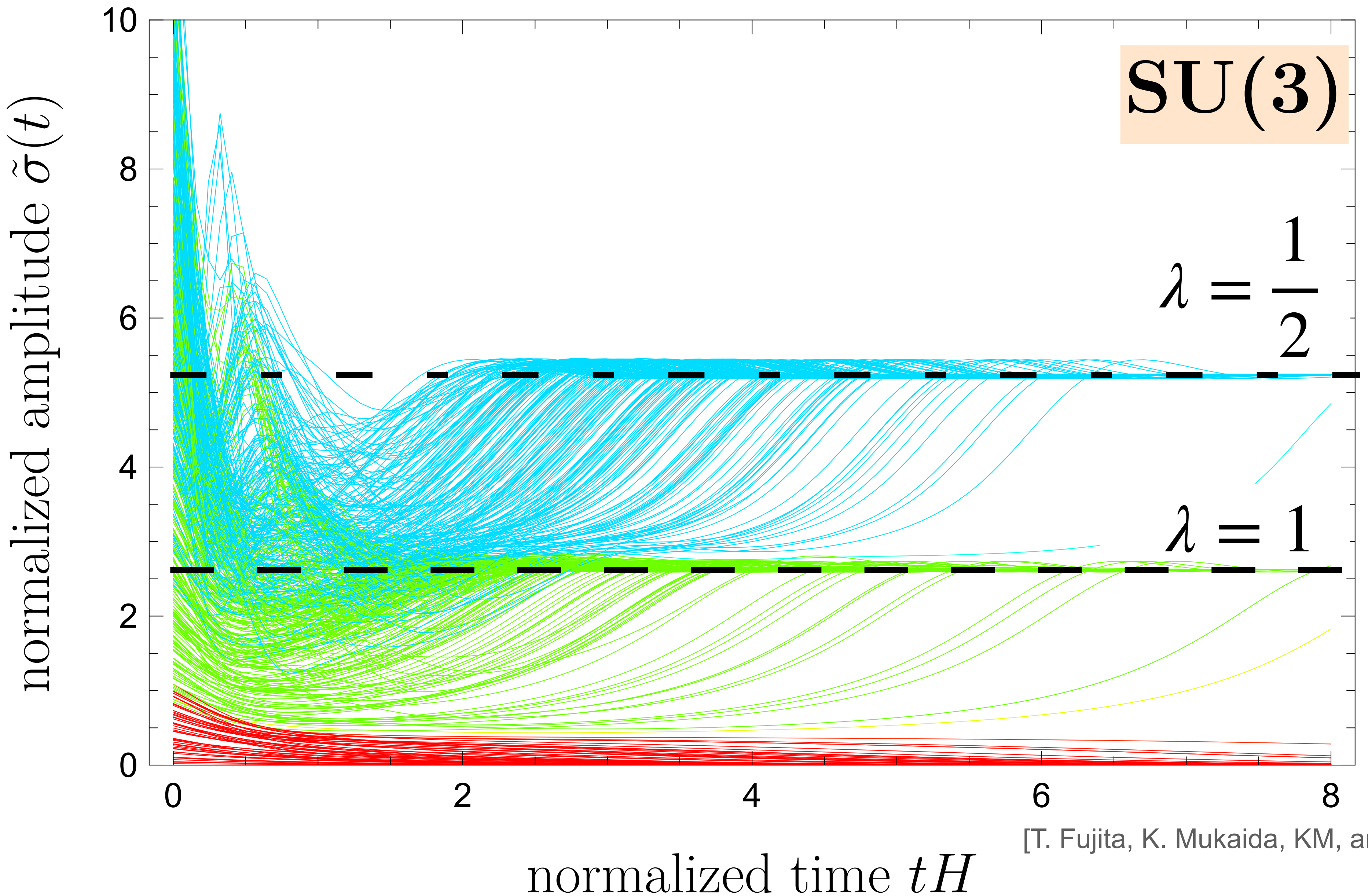
SU(3) ⊃ SU(2) $\mathbf{3} = \mathbf{3}, \quad \mathbf{8} = \mathbf{3} + \mathbf{5}$

$$\mathcal{T}_i = \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\lambda = \frac{1}{2}$$

- Time evolution of A_i^a

Assumptions: $a \propto e^{Ht}, \quad \xi \equiv \frac{\dot{\phi}}{2fH} = \mathrm{const}\,.$



[T. Fujita, K. Mukaida, KM, and H. Nakatsuka (2021)]

The solutions labeled by λ explain the numerical solutions.

Summary

- We study the SU(N) version of CNI
  and construct homogeneous and isotropic solutions
  by determining an SU(2) subgroup.

- We numerically check that
  the analytic solutions explain all the numerical solutions.

- The difference of the solutions or λ is degenerated with g.
- Transitions of solutions can break the degeneracy?