Revisiting Inclusive Decay Widths of Charmed Mesons

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“Charming clues for existence”

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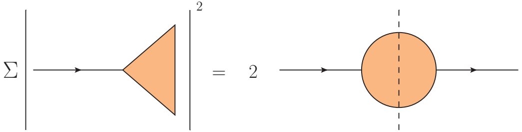
The theoretical framework

The total decay width of a heavy hadron

- $\Gamma(H_Q)$ can be computed as [Shifman, Voloshin '85]

$$\Gamma(H_Q) = \frac{1}{2m_{H_Q}} \mathrm{Im}\,\langle H_Q| i \int d^4x\, \mathrm{T}\Big\{\mathcal{H}_{eff}(x)\,, \mathcal{H}_{eff}(0)\Big\}|H_Q\rangle$$

  - $\mathcal{H}_{eff}$ - weak effective Hamiltonian describing Q quark decays
  - Use optical theorem



to relate Γ with imaginary part of forward scattering amplitude

The HQE

- Q carries most of the momentum of the hadron $p^{\mu}_{H_Q} = m_{H_Q} v^{\mu}$
- Introduce parametrisation

$$p^{\mu}_{Q} = m_Q v^{\mu} + k^{\mu} \qquad k \sim \Lambda_{QCD} \ll m_Q$$

- Rescale the heavy quark field

$$Q(x) = e^{-i m_Q v \cdot x} Q_v(x)$$

- The action of the covariant derivative becomes

$$i D_{\mu} Q(x) = e^{-i m_Q v \cdot x} \big(m_Q v_{\mu} + i D_{\mu} \big) Q_v(x)$$

$$D_{\mu} = \partial_{\mu} - i A^{a}_{\mu}(x) t^{a}$$

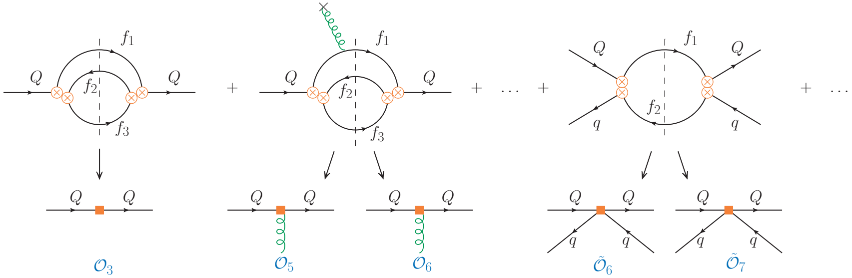
The HQE

- Systematic expansion

$$\Gamma(H_Q) = \underbrace{\Gamma_3}_{\Gamma(Q)} + \underbrace{\Gamma_5 \frac{\langle \mathcal{O}_5 \rangle}{m_Q^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle}{m_Q^3} + \ldots + 16\pi^2 \left[\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_Q^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_Q^4} + \ldots \right]}_{\delta\Gamma(H_Q)}$$

  - $\Gamma_d, \tilde{\Gamma}_d$ - short distance coefficients
  - $\mathcal{O}_d, \tilde{\mathcal{O}}_d$ - local operators bilinear in the heavy quark field
  - $\Gamma(Q)$ - total decay width of free quark Q
  - $\delta\Gamma(H_Q)$ - effects due to interaction with soft gluons and quarks

$$\Gamma(H_Q) = \Gamma_3 + \Gamma_5 \frac{\langle \mathcal{O}_5 \rangle}{m_Q^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle}{m_Q^3} + \ldots + 16\pi^2 \left[\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_Q^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_Q^4} + \ldots \right]$$



Very advanced framework thanks to huge effort of big community

Status of HQE: perturbative side

$$\Gamma_d = \Gamma_d^{(0)} + \left(\frac{\alpha_s(m_Q)}{4\pi}\right)\Gamma_d^{(1)} + \left(\frac{\alpha_s(m_Q)}{4\pi}\right)^2 \Gamma_d^{(2)} + \ldots$$

Semileptonic (SL) modes	
$\Gamma_3^{(3)}$	Fael, Schönwald, Steinhauser '20 Czakon, Czarnecki, Dowling '21
$\Gamma_3^{(2)}$	Czarnecki, Melnikov, v. Ritbergen, Pak, Dowling, Bonciani, Ferroglia, Biswas, Brucherseifer, Caola '97-'13
$\Gamma_5^{(1)}$	Alberti, Gambino, Nandi, Mannel, Pivovarov, Rosenthal '13-'15
$\Gamma_6^{(1)}$	Mannel, Pivovarov '19
$\Gamma_7^{(0)}$	Dassinger, Mannel, Turczyk '06
$\Gamma_8^{(0)}$	Mannel, Turczyk, Uraltsev '10

* Partial result

Non-leptonic (NL) modes	
$\Gamma_3^{(2)}$	Czarnecki, Slusarcyk, Tkachov '05 *
$\Gamma_3^{(1)}$	Ho-Kim, Pham, Altarelli, Petrarca, Voloshin, Bagan, Ball, Braun, Gosdzinsky, Fiol, Lenz, Nierste, Ostermaier, Krinner, Rauh '84-'13
$\Gamma_5^{(0)}$	Bigi, Uraltsev, Vainshtein, Blok, Shifman '92
$\Gamma_6^{(0)}$	Lenz, MLP, Rusov, Mannel, Moreno, Pivovarov '20-'21
$\tilde{\Gamma}_6^{(1)}$	Beneke, Buchalla, Greub, Lenz, Nierste, Franco, Lubicz, Mescia, Tarantino, Rauh '02-'13
$\tilde{\Gamma}_7^{(0)}$	Gabbiani, Onishchenko, Petrov '03-'04

Status of HQE: non-perturbative side

★ Fit to experimental data on semileptonic B decays ★ HQET sum rules ★ Lattice QCD

	B_d, B^+	B_s	$D^{+,0}$	D_s^+
$\langle \mathcal{O}_5 \rangle$	Gambino, Schwanda, Alberti Healey, Nandi, Bordone, Capdevila, '13, '14, '21 ★ Ball, Braun, Neubert '93-'95 ★ Kronfeld, Simone, Gambino, Melis, Simula '00 -'17 ★	$SU(3)_f$-breaking for μ_π^2 Bigi, Mannel, Uraltsev,'11 Spectroscopy relation for μ_G^2	HQE symmetry for μ_π^2; Spectroscopy relation for μ_G^2	HQE symmetry for μ_π^2; Spectroscopy relation for μ_G^2
$\langle \mathcal{O}_6 \rangle$	Gambino, Schwanda, Alberti Healey, Nandi '13 -'14 ★ EOM relation to $\langle \tilde{\mathcal{O}}_6 \rangle$	EOM relation to $\langle \tilde{\mathcal{O}}_6 \rangle$		
$\langle \tilde{\mathcal{O}}_6 \rangle$	Kirk, Lenz, Rauh '17 ★	King, Lenz, Rauh '21 ★		
$\langle \tilde{\mathcal{O}}_7 \rangle$	Vacuum insertion approximation (VIA)			

- Fit to experimental semileptonic D-decays highly desirable

  Belle II, BESIII, LHCb, super tau charm factory ?

- Even less input available for baryons

HQE: bottom vs. charm

- Two expansion parameters: $\alpha_s(m_Q)$ and $\frac{\Lambda_{QCD}}{m_Q}$
- If $m_Q \gg \Lambda_{QCD}$
  - Expect $\Gamma(H_Q) = \Gamma(Q) + \delta\Gamma(H_Q)$ with $\delta\Gamma(H_Q) \ll \Gamma(Q)$
- For $Q = b$: $\alpha_s \sim 0.22$ and $\frac{\Lambda_{QCD}}{m_b} \sim 0.10$
  - Lifetime ratios of bottom mesons differ from 1 up to 10 percent

    [Kirk, Lenz, Rauh '17]
  - Good agreement with experimental data
- For $Q = c$: $\alpha_s \sim 0.33$ and $\frac{\Lambda_{QCD}}{m_c} \sim 0.30$
  - Applicability of HQE to charm becomes questionable

Charmed mesons - Experimental status

- Charmed meson lifetimes are measured precisely

	D^0	D^+	D_s^+
$\tau\,[\mathrm{ps}]$	0.4101(15)	1.040(7)	0.504(4)
$\Gamma\,[\mathrm{ps}^{-1}]$	2.44(1)	0.96(1)	1.98(2)
$\tau(D_X)/\tau(D^0)$	1	2.54(2)	1.20(1)

[PDG 2021] *

- Spectator quark effects must give large contribution

$$\Gamma(D) = \Gamma(c) + \delta\Gamma(D) \quad \text{with} \quad \delta\Gamma(D) \sim \Gamma(c)$$

- Can we explain the observed pattern within the HQE?

* Does not include new measurement of $\tau(D^{0,+})$ by Belle II arXiv:2108.03216

HQE for charmed mesons

The effective Hamiltonian

- Describe NL and SL decays of c-quark

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}}\left[\sum_{q_{1,2}=d,s}\lambda_{q_1 q_2}\left(C_1 Q_1^{q_1 q_2} + C_2 Q_2^{q_1 q_2}\right) + \sum_{q=d,s}\sum_{\ell=e,\mu} V_{cq}^* Q^{q\ell}\right] + \text{h.c.}$$

$C_{1,2}(\mu_1)$ – Wilson coefficients at scale $\mu_1 \sim m_c$

$\lambda_{q_1 q_2} = V_{cq_1}^* V_{uq_2}$

- $\Delta C = 1$ operator basis

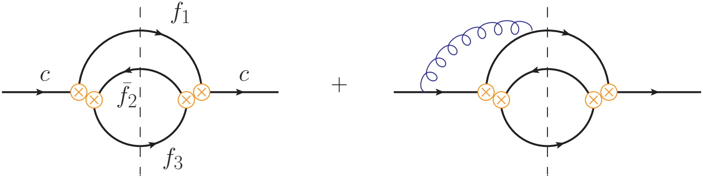
$$Q_1^{q_1 q_2} = (\bar{q}_1^i \gamma_\rho (1-\gamma_5) c^i)(\bar{u}^j \gamma^\rho (1-\gamma_5) q_2^j)$$

$$Q_2^{q_1 q_2} = (\bar{q}_1^i \gamma_\rho (1-\gamma_5) c^j)(\bar{u}^j \gamma^\rho (1-\gamma_5) q_2^i)$$

$$Q^{q\ell} = (\bar{q}^i \gamma_\rho (1-\gamma_5) c^i)(\bar{\nu}_\ell \gamma^\rho (1-\gamma_5) \ell)$$

- Neglect contribution of penguin operators $Q_{j\geq 3}$

Dimension-three contribution



- Total decay width of free c-quark

$$\Gamma_3 = \underbrace{\frac{G_F^2 m_c^5}{192\pi^2}}_{\Gamma_0} \left(c_3^{(0)} + \frac{\alpha_s}{4\pi} c_3^{(1)} + \ldots \right)$$

- Large uncertainty from mass-scheme dependence of Γ_0

Dimension-three contribution

- At NLO observe suppression between SL and NL corrections
  - In the Pole scheme*

$$\Gamma_3 = \Gamma_3^{\rm LO}\left[1 + \left(\overbrace{\underbrace{1.84}_{\rm oper.} - \underbrace{0.74}_{\rm WC}}^{\rm NL} - \overbrace{0.67}^{\rm SL}\right)\frac{\alpha_s}{\pi} + \mathcal{O}\left(\frac{\alpha_s}{\pi}\right)^2\right]$$

  - In the $\overline{\rm MS}$-scheme**

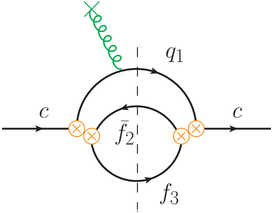
$$\Gamma_3 = \Gamma_3^{\rm LO}\left[1 + \left(\overbrace{\underbrace{2.10}_{\rm oper.} - \underbrace{0.70}_{\rm WC}}^{\rm NL} - \overbrace{0.71}^{\rm SL} + \overbrace{6.66}^{\rm conv.fac.}\right)\frac{\alpha_s}{\pi} + \mathcal{O}\left(\frac{\alpha_s}{\pi}\right)^2\right]$$

- N^{2}LO corrections for NL modes important

[Egner, Fael, Schönwald, Steinhauser (in progress)]

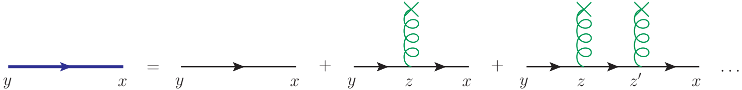
$^*m_c^{\rm Pole} = 1.48{\rm GeV}$ $\quad$ $^{**}\overline{m}_c(\overline{m}_c) = 1.27{\rm GeV}$

Power corrections due to two-quark operators



- Parametrise interaction with soft gluons
- Expand up to order $\mathcal{O}(D^3)$
- Use background field method and Fock-Schwinger gauge

e.g. [Novikov et al. '84]



Dimension-five contribution

- Two possible operators

$$2m_D\mu_\pi^2(D) = -\langle D|\bar{c}_v(iD_\mu)(iD^\mu)c_v|D\rangle$$

$$2m_D\mu_G^2(D) = \langle D|\bar{c}_v(iD_\mu)(iD_\nu)(-i\sigma^{\mu\nu})c_v|D\rangle$$

- Result at order $1/m_c^2$

$$\Gamma_5\frac{\langle\mathcal{O}_5\rangle}{m_c^2} = \Gamma_0\left[c_{\mu_\pi}\frac{\mu_\pi^2}{m_c^2} + c_G\frac{\mu_G^2}{m_c^2}\right]$$

- The coefficient of the kinetic operator: $c_{\mu_\pi} = -c_3^{(0)}/2$

- The coefficient of the chromo-magnetic operator:

$$c_G = 3C_1^2\mathcal{C}_{G,11} + 2C_1C_2\mathcal{C}_{G,12} + 3C_2^2\mathcal{C}_{G,22} + \mathcal{C}_{G,SL}$$

Dimension-six contribution

- The Darwin operator

$$2m_D\rho_D^3(D) = \langle D|\bar{c}_v(iD_\mu)(iv\cdot D)(iD^\mu)c_v|D\rangle$$

- IR divergences from expansion of quark-propagator

$$\Gamma_6\frac{\langle\mathcal{O}_6\rangle}{m_c^3} \stackrel{!}{=} \Gamma_0\left[3C_1^2\mathcal{C}_{D,11} + 2C_1C_2\mathcal{C}_{D,12} + 3C_2^2\mathcal{C}_{D,22} + \mathcal{C}_{D,SL}\right]\frac{\rho_D^3}{m_c^3}$$

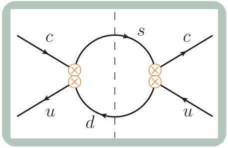
with

$$\mathcal{C}_{D,X} \underset{m_q\to 0}{\sim} \log\left(\frac{m_q^2}{m_c^2}\right) \qquad q = \{u,d,s\}$$

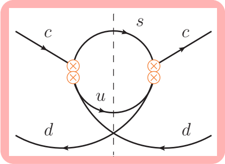
- Sensitivity to long-distance dynamics $\Lambda_{QCD} \sim m_q \ll m_c$
- Signal of mixing with four-quark operators at order $1/m_c^3$

Power corrections due to four-quark operators

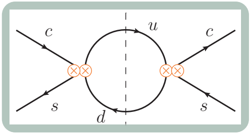
Weak Exchange



Pauli inteference



Weak Annihilation



- Appear first at order $1/m_c^3$
- Loop-enhanced with respect to two-quark operators
- Very different size of corresponding diagrams
  - Dominant contribution of Pauli interference to D^+ [Guberina et al. ’79]
  - Small contribution due to helicity suppression for D^0 and D_s^+

Dimension-six contribution

- Choice of operator basis

$$\tilde{\mathcal{O}}_1^q = (\bar{h}_v \gamma_\mu (1-\gamma_5) q)(\bar{q}\gamma^\mu (1-\gamma_5) h_v)$$

$$\tilde{\mathcal{O}}_2^q = (\bar{h}_v (1-\gamma_5) q)(\bar{q}(1+\gamma_5) h_v)$$

$$\tilde{\mathcal{O}}_3^q = (\bar{h}_v \gamma_\mu (1-\gamma_5) t^a q)(\bar{q}\gamma^\mu (1-\gamma_5) t^a h_v)$$

$$\tilde{\mathcal{O}}_4^q = (\bar{h}_v (1-\gamma_5) t^a q)(\bar{q}(1+\gamma_5) t^a h_v)$$

- Corresponding parametrisation

$$\langle D_q | \tilde{\mathcal{O}}_i^q | D_q \rangle = f_{D_q}^2 m_{D_q}^2 \left(1 + \frac{4}{3}\frac{\alpha_s(m_c)}{\pi}\right) \tilde{B}_i^q$$

$$\langle D_q | \tilde{\mathcal{O}}_i^{q'} | D_q \rangle = f_{D_q}^2 m_{D_q}^2 \tilde{\delta}_i^{q' q} \qquad q \neq q'$$

$\tilde{B}_{1,2}^q = 1 + \delta\tilde{B}_{1,2}^q$ $\quad$ $\tilde{B}_{3,4}^q = 0 + \tilde{\epsilon}_{1,2}^q$ $\quad$ $\tilde{\delta}_i^{q' q} \ll 1$ (eye-contractions)

Dimension-six contribution

- Total result: $$16\pi^2 \, \tilde{\Gamma}_6^{D_q} \frac{\langle \tilde{\mathcal{O}}_6 \rangle^{D_q}}{m_c^3}$$

Mass scheme	D^0	D^+	D_s^+
VIA			
Kinetic	$\underbrace{-0.012}_{\text{NLO}} = \underbrace{0.000}_{\text{LO}} \underbrace{-0.012}_{\Delta\text{NLO}}$	$\underbrace{-2.53}_{\text{NLO}} = \underbrace{-1.42}_{\text{LO}} \underbrace{-1.11}_{\Delta\text{NLO}}$	$\underbrace{-0.19}_{\text{NLO}} = \underbrace{-0.10}_{\text{LO}} \underbrace{-0.09}_{\Delta\text{NLO}}$
HQET SR			
Kinetic	$\underbrace{0.014}_{\text{NLO}} = \underbrace{0.016}_{\text{LO}} \underbrace{-0.002}_{\Delta\text{NLO}}$	$\underbrace{-2.76}_{\text{NLO}} = \underbrace{-1.58}_{\text{LO}} \underbrace{-1.18}_{\Delta\text{NLO}}$	$\underbrace{-0.20}_{\text{NLO}} = \underbrace{-0.13}_{\text{LO}} \underbrace{-0.07}_{\Delta\text{NLO}}$

- NLO corrections are very large!

Dimension-seven contribution

- Expand in p_q/m_c and k/m_c
- Generate both local and non-local operators (in total 18 oper.)
- Conversion between QCD and HQET decay constant

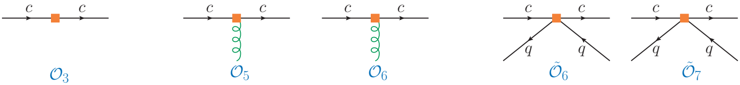
e.g. [Neubert '92]

$$f_{D_q} = \frac{F(m_c)}{\sqrt{m_{D_q}}}\left(1 - \frac{2}{3}\frac{\alpha_s(m_c)}{\pi} + \frac{G_1(m_c)}{m_c} + 6\frac{G_2(m_c)}{m_c} - \frac{1}{2}\frac{m_{D_q} - m_c - m_q}{m_c}\right)$$

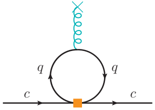
- In VIA, the effect of 12 operators accounted by parametrisation of dimension-six operators

Operator mixing at dimension-six

- Recall



- $\tilde{\mathcal{O}}_6$ and $\mathcal{O}_6$ mix under renormalisation

$\langle$  $\rangle$

$$\sim \left[\frac{1}{\epsilon} + \log\left(\frac{\mu^2}{m_q^2}\right) + c\right] \langle \mathcal{O}_{\rho_D} \rangle + \mathcal{O}\left(\frac{1}{m_c}\right)$$

- All dependence on $\log(m_q^2)$ in $\mathcal{C}_{D,X}$ cancels!

- Constant c depends on choice of operator basis

Our setup

- Most up-to-date analysis

$$\Gamma(D) = \Gamma_3 + \Gamma_5 \frac{\langle \mathcal{O}_5 \rangle}{m_c^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle}{m_c^3} + \ldots + 16\pi^2 \left[\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_c^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_c^4} + \ldots \right]$$

$$\tau(D^+_{(s)})/\tau(D^0) = 1 + \left[\Gamma(D^0)^{\mathrm{HQE}} - \Gamma(D^+_{(s)})^{\mathrm{HQE}} \right] \tau(D^+_{(s)})^{\mathrm{exp}}$$

- What we have included

	SL	NL
Γ_3	NLO	NLO
Γ_5	LO	LO
Γ_6	LO	LO New *
$\tilde{\Gamma}_6$	NLO	NLO
$\tilde{\Gamma}_7$	LO	LO

	Source
$\langle \mathcal{O}_5 \rangle$	Heavy quark symmetry;
	Spectroscopy relations
$\langle \mathcal{O}_6 \rangle$	EOM relation to $\langle \tilde{\mathcal{O}}_6 \rangle$ New
$\langle \tilde{\mathcal{O}}_6 \rangle$	HQET sum rules New **
$\langle \tilde{\mathcal{O}}_7 \rangle$	VIA

* Based on [Lenz, MLP, Rusov '20] ** [King, Lenz, Rauh '21]

Results

The total width of D^+

$$\Gamma(D^+) = \Gamma_0\Big[\underbrace{6.15}_{c_3^{\rm LO}} + \underbrace{2.95}_{\Delta c_3^{\rm NLO}} - 1.66\,\frac{\mu_\pi^2(D)}{\rm GeV^2} + 0.13\,\frac{\mu_G^2(D)}{\rm GeV^2} + 23.6\,\frac{\rho_D^3(D)}{\rm GeV^3}$$

$$-\ 16.9\,\tilde{B}_1^q + 0.56\,\tilde{B}_2^q + 84.0\,\tilde{\epsilon}_1^q - 1.34\,\tilde{\epsilon}_2^q + \underbrace{6.76}_{\rm dim-7}$$

$$-\ 0.06\,\tilde{\delta}_1^{qq} + 0.06\,\tilde{\delta}_2^{qq} - 16.8\,\tilde{\delta}_3^{qq} + 16.9\,\tilde{\delta}_4^{qq} - 29.3\,\tilde{\delta}_1^{sq} + 28.8\,\tilde{\delta}_2^{sq} + 0.56\,\tilde{\delta}_3^{sq} + 2.36\,\tilde{\delta}_4^{sq}\Big]$$

$$= 6.15\,\Gamma_0\Big[1 + 0.48 - 0.13\,\frac{\mu_\pi^2(D)}{0.48\,\rm GeV^2} + 0.01\,\frac{\mu_G^2(D)}{0.34\,\rm GeV^2} + 0.31\,\frac{\rho_D^3(D)}{0.082\,\rm GeV^3}$$

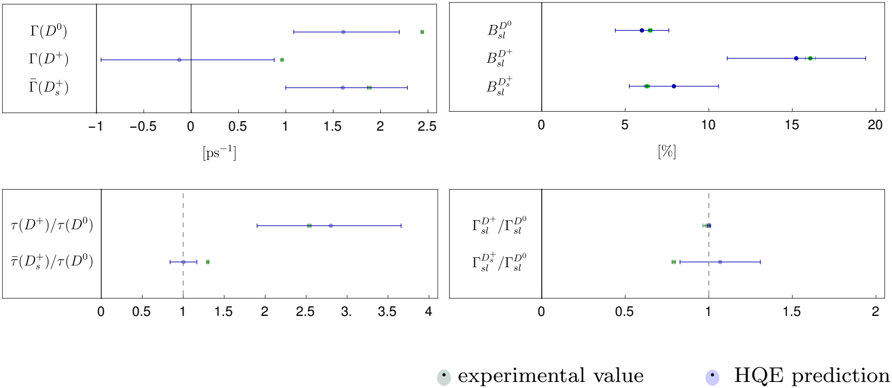
$$-\ \underbrace{2.66}_{\rm dim-6,VIA} - 0.055\,\frac{\delta\tilde{B}_1^q}{0.02} + 0.002\,\frac{\delta\tilde{B}_2^q}{0.02} - 0.546\,\frac{\tilde{\epsilon}_1^q}{-0.04} + 0.009\,\frac{\tilde{\epsilon}_2^q}{-0.04} + \underbrace{1.10}_{\rm dim-7,VIA}$$

$$-\ 0.0000\,r_1^{qq} - 0.0000\,r_2^{qq} + 0.0011\,r_3^{qq} + 0.0008\,r_4^{qq}$$

$$-\ 0.0109\,r_1^{sq} - 0.0080\,r_2^{sq} - 0.0000\,r_3^{sq} + 0.0001\,r_4^{sq}\Big]$$

Obtained in the kinetic scheme with $\mu^{\rm cut} = 0.5$ GeV

Result



Obtained in the kinetic scheme with $\mu^{\mathrm{cut}} = 0.5$ GeV

Conclusion and outlook

- HQE consistent with experimental data albeit huge uncertainties

- Large room for theoretical improvement

  - For $\langle \mathcal{O}_5 \rangle$, $\langle \mathcal{O}_6 \rangle$ exp. data highly desirable → control of $SU(3)_F$

    BESIII, Belle II, LHCb, super tau charm factory?

  - Lattice determination for $\langle \tilde{\mathcal{O}}_6 \rangle$

    Planned by M. Black, O. Witzel (RBC-UKQCD, Siegen)

  - Complete dimension-seven contribution at LO-QCD: $\Gamma_{7,\mathrm{NL}}^{(0)}$

  - Higher order QCD corrections, particularly $\Gamma_{3,\mathrm{NL}}^{(2)}$, $\tilde{\Gamma}_6^{(2)}$, and $\tilde{\Gamma}_7^{(1)}$

    M. Egner, M. Fael, K. Schönwald, M. Steinhauser (in progress)

    Planned by U. Nierste, M. Steinhauser (Karlsruhe)

  - Study dependence of charm quark mass definition

Going now to the B-system

	HFLAV	Lenz, MLP, Rusov (in progress)
$\frac{\tau(B^+)}{\tau(B_d)}$	1.076(4)	1.08(2)
$\frac{\tau(B_s^+)}{\tau(B_d)}$	0.998(5)	$1.004(5) + 0.0456 \times \underbrace{\left[\frac{\rho_D^3(B_s)}{\rho_D^3(B_d)} - 1\right]}_{\approx 46\%} = 1.025(15)$

- The coefficient of the Darwin operator is found to be large
- Estimate of ρ_D^3 using EOM for gluon field e.g. [Bigi, Mannel, Uraltsev '11]

$$D_\mu G^{\mu\rho,a} = -g_s \sum_q (\bar{q}\gamma^\rho t^a q) \qquad [iD_\mu, [iD^\rho, iD^\mu]] = g_s D_\mu G^{\mu\rho}$$

The Darwin operator

- The Darwin operator can be rewritten as penguin operator

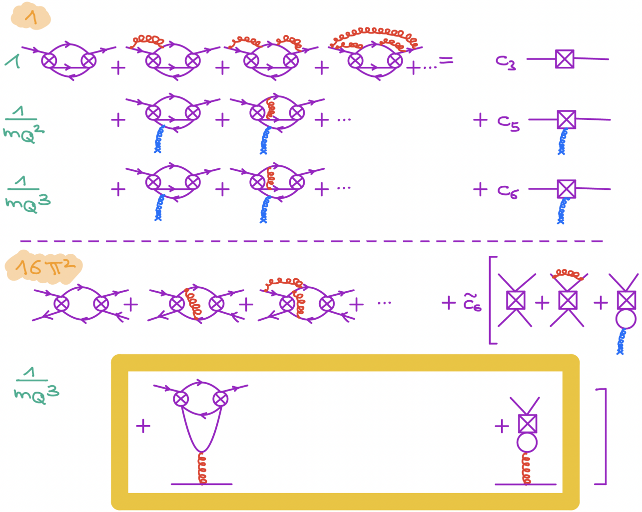
$$\mathcal{O}_{\rho_D^3} = \frac{1}{4m_B}\bar{b}_v[iD_\mu,[iD^\rho,iD^\mu]]v_\rho b_v = -\frac{g_s^2}{4m_B}(\bar{b}_v\gamma^\mu t^a b_v)\sum_q(\bar{q}\gamma_\mu t^a q) + \mathcal{O}\left(\frac{1}{m_b}\right)$$

- Using Fierz-transformation

$$\rho_D^3(B) = \frac{g_s^2}{18}f_B^2\, m_B\left[2\,\tilde{B}_2^{q'} - \tilde{B}_1^{q'} + \frac{3}{4}\tilde{\epsilon}_1^{q'} - \frac{3}{2}\tilde{\epsilon}_2^{q'} + \sum_{q=u,d,s}\left(2\tilde{\delta}_2^{qq'} - \tilde{\delta}_1^{qq'} + \frac{3}{4}\tilde{\delta}_3^{qq'} - \frac{3}{2}\tilde{\delta}_4^{qq'}\right)\right]$$

- Obtain $SU(3)_F$ breaking effects of $\sim 50\%$!
  - Overestimate of the size of the $SU(3)_F$ breaking effects?
  - Are there missing diagrams that might compensate this?

Missing penguin operators?



Thanks for the attention