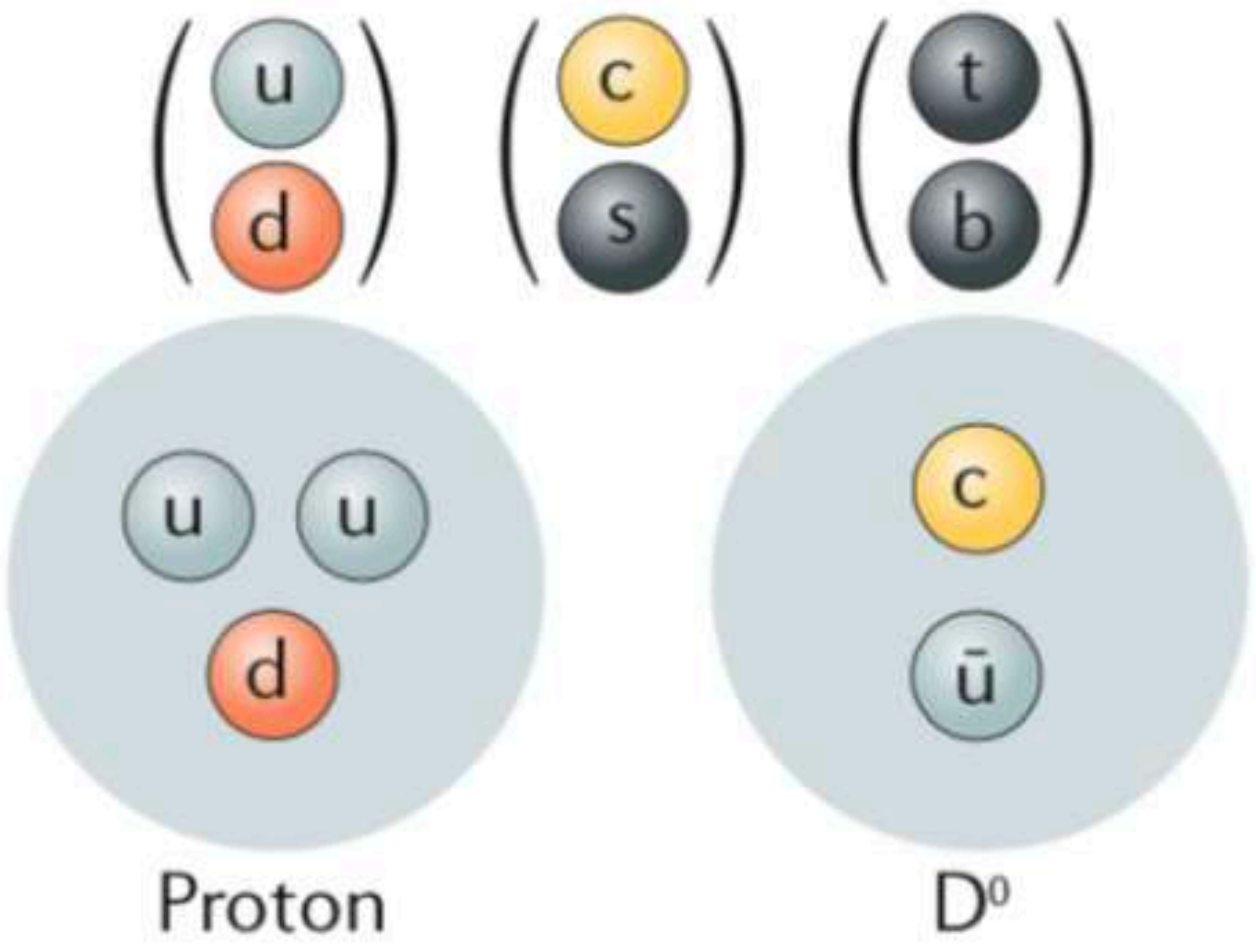
SM predictions for D Mixing

MIAPP

22.3.2022

Alexander Lenz

Charm Physics



	$D^0 = (\bar{u}c)$	$D^+ = (\bar{d}c)$	$D_s^+ = (\bar{s}c)$	$\Lambda_c = (udc)$
Mass (GeV)	1.86486	1.86962	1.96850	2.28646
Lifetime (ps)	0.4101	1.040	0.500	0.200

Charm Physics

Theoretical Peculiarities of Charm:

Thanks Tommaso and Patricia!

1. The strong coupling is strong

$$\alpha_s(m_c) = 0.33 \pm 0.01$$

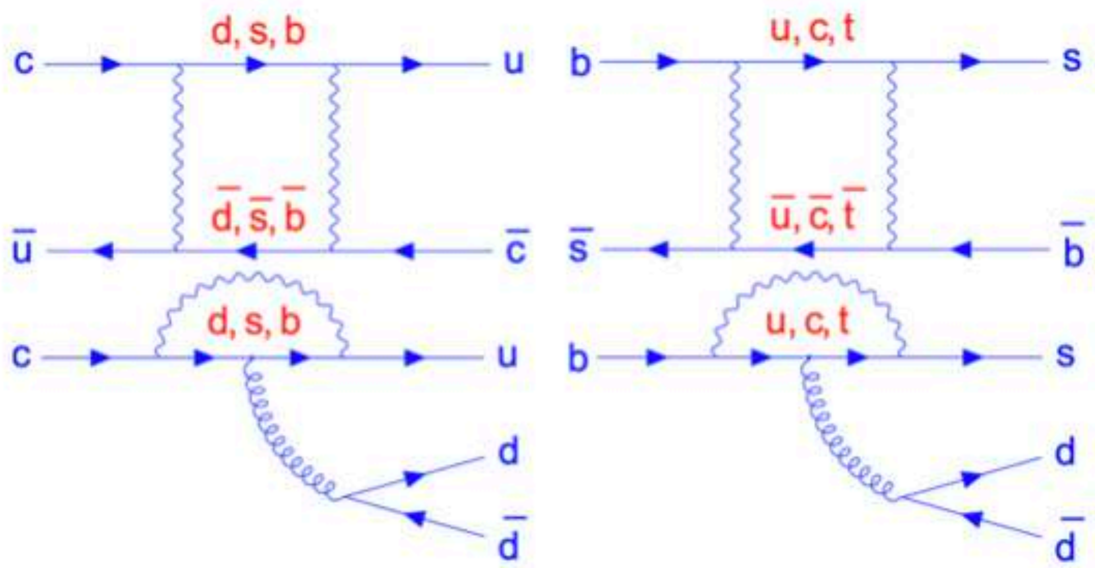
2. The charm quark is not really heavy

$$m_c^{\text{Pole}} = (1.67 \pm 0.07)\ \text{GeV}\ , \qquad \overline{m}_c(\overline{m}_c) = (1.27 \pm 0.02)\ \text{GeV}\ ,$$

3. There is almost no CPV in charm

$$V_{cd} = -0.2247 - 1.4 \cdot 10^{-4} I, \ V_{cs} = 0.97354 - 3.1 \cdot 10^{-5} I, \ V_{cb} = 0.0416$$

4. There are extremely pronounced GIM cancellations in the charm sector



$$\left(\frac{m_d}{M_W}\right)^2 \approx 0\,, \qquad \left(\frac{m_u}{M_W}\right)^2 \approx 0\,,$$
$$\left(\frac{m_s}{M_W}\right)^2 \approx 1.3 \cdot 10^{-6}\,, \qquad \left(\frac{m_c}{M_W}\right)^2 \approx 2.5 \cdot 10^{-4}\,,$$
$$\left(\frac{m_b}{M_W}\right)^2 \approx 2.8 \cdot 10^{-3}\,, \qquad \left(\frac{m_t}{M_W}\right)^2 \approx 4.5\,.$$

See e.g.
AL, G. Wilkinson
2011.04443

Cancellations

The charm system is theoretically more difficult than the b system since

$$\alpha_s(m_c) \approx 0.33 \quad \text{and} \quad \frac{\Lambda_{QCD}}{m_c} \approx 3 \frac{\Lambda_{QCD}}{m_b}$$

Nevertheless the **Heavy Quark Expansion** might still converge

But things will become very ugly, if in addition cancellations arising

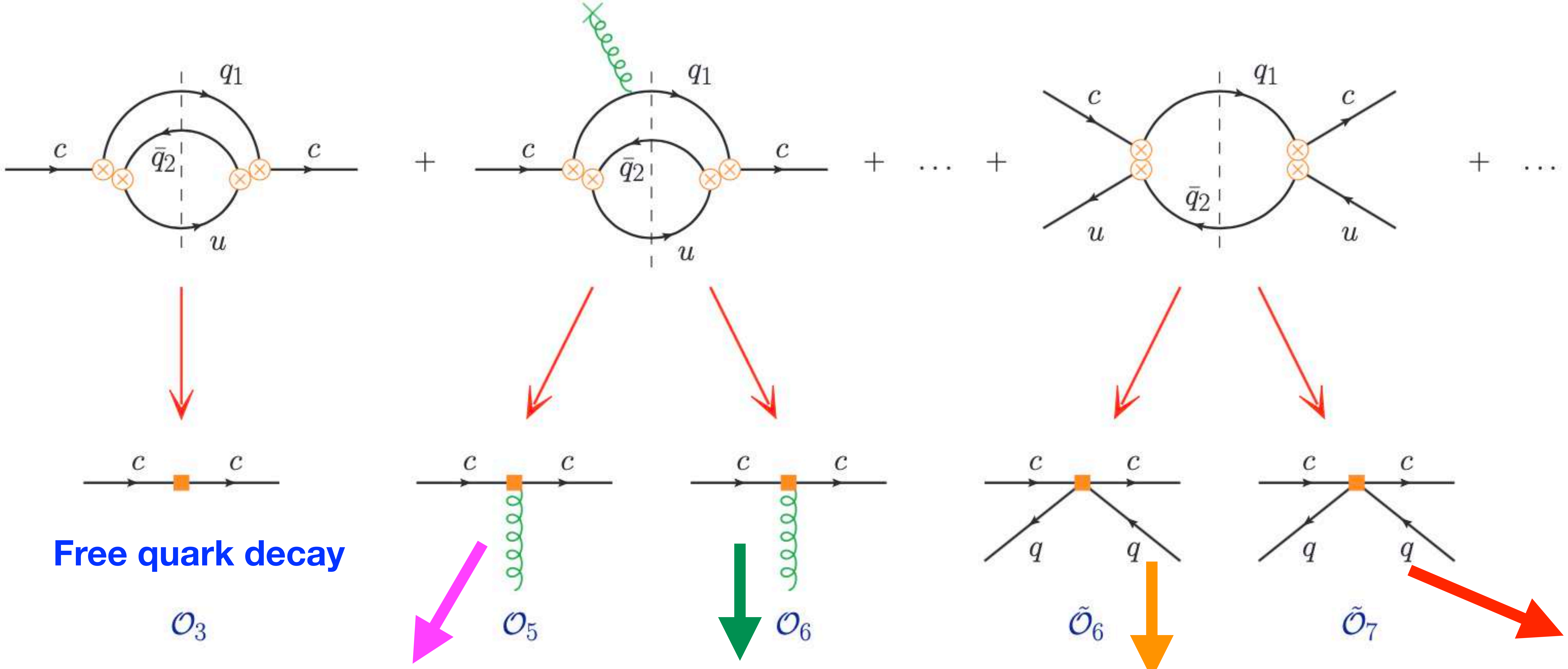
A. No cancellations, e.g. $\Gamma(D^0)$
B. Strong cancellations, e.g. $\Gamma(D^+)$
C. Crazy cancellations, e.g. D-mixing

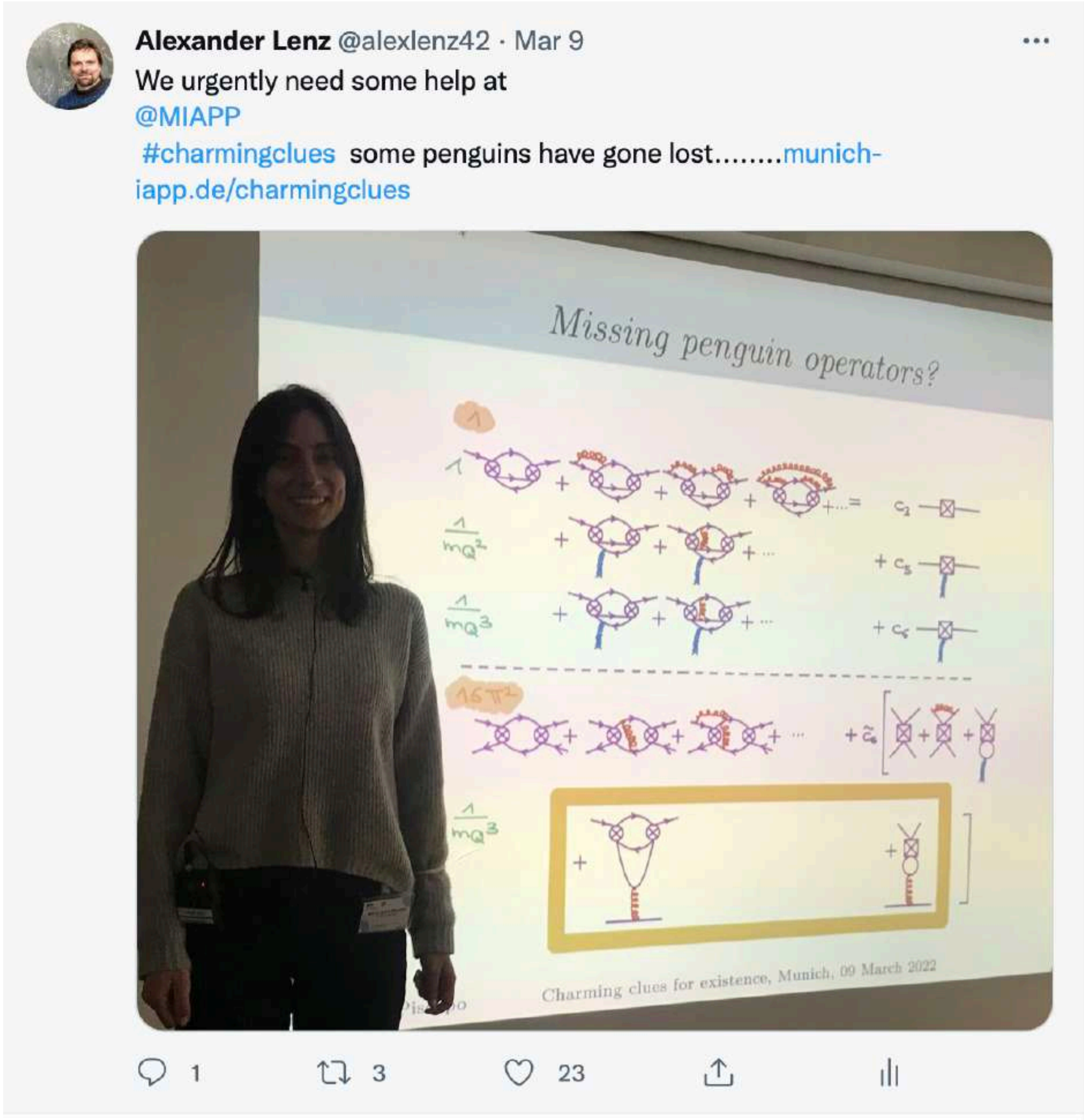
A. No Cancellations

$$\Gamma(D) = \Gamma_3 + \Gamma_5 \frac{\langle \mathcal{O}_5 \rangle}{m_c^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle}{m_c^3} + \ldots + 16\pi^2 \left(\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_c^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_c^4} + \ldots \right),$$

$$\Gamma_i = \Gamma_i^{(0)} + \frac{\alpha_s(m_c)}{4\pi} \Gamma_i^{(1)} + \left[\frac{\alpha_s(m_c)}{4\pi} \right]^2 \Gamma_i^{(2)} + \ldots .$$

For more details, see the talk of Maria Laura





Kinetic operator μ_π^2
Chromomagnetic operator μ_G^2

Darwin operator μ_π^2

4-quark operator B_i, ϵ_i
Eye contractions r

Dimension 7 operators
Vacuum insertion approximation

A. No Cancellations

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$$\Gamma(D^0) = 6.15\,\Gamma_0 \Big[1 + 0.48 - 0.13\,\frac{\mu_\pi^2(D)}{0.48\,\mathrm{GeV}^2} + 0.01\,\frac{\mu_G^2(D)}{0.34\,\mathrm{GeV}^2} + 0.31\,\frac{\rho_D^3(D)}{0.082\,\mathrm{GeV}^3}$$

$$- \underbrace{0.01}_{\mathrm{dim-6,VIA}} - 0.005\,\frac{\delta\tilde{B}_1^q}{0.02} + 0.005\,\frac{\delta\tilde{B}_2^q}{0.02} + 0.137\,\frac{\tilde{\epsilon}_1^q}{-0.04} - 0.125\,\frac{\tilde{\epsilon}_2^q}{-0.04} + \underbrace{0.00}_{\mathrm{dim-7,VIA}}$$

$$- 0.0045\,r_1^{qq} - 0.0004\,r_2^{qq} - 0.0035\,r_3^{qq} + 0.0000\,r_4^{qq}$$

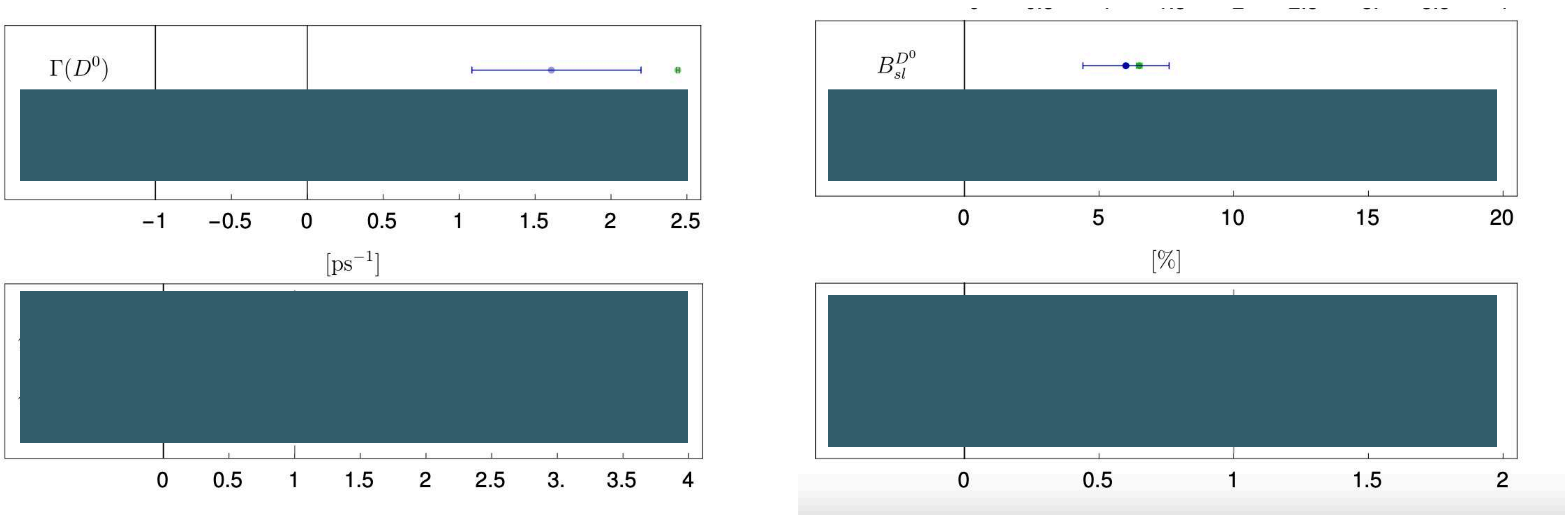
$$- 0.0109\,r_1^{sq} - 0.0079\,r_2^{sq} - 0.0000\,r_3^{sq} + 0.0001\,r_4^{sq} \Big].$$

Revisiting Inclusive Decay Widths of Charmed Mesons #5

Daniel King (Durham U., IPPP and Durham U.), Alexander Lenz (Siegen U.), Maria Laura Piscopo (Siegen U.), Thomas Rauh (U. Bern, AEC), Aleksey V. Rusov (Siegen U.) et al. (Sep 27, 2021)

e-Print: 2109.13219 [hep-ph]

A. No Cancellations



- Values of $\mu_\pi^2, \mu_G^2, \rho_D^3$ almost unknown
- NNLO-QCD corrections to free quark decay in progress

  Fael, Steinhauser,...

- NNLO-QCD corrections to spectator effects in progress

  Nierste, Steinhauser,...

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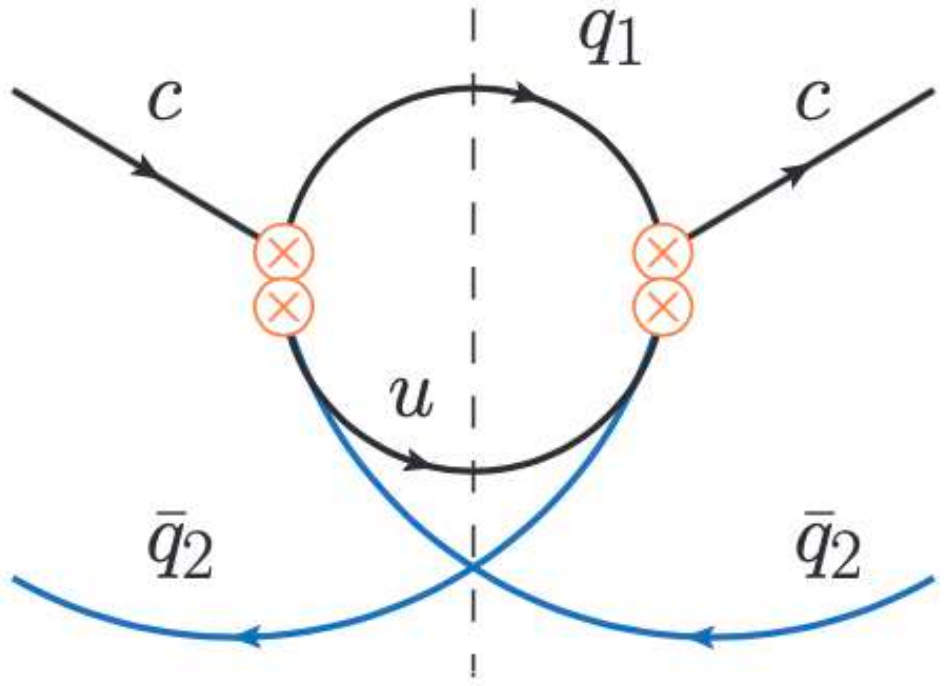
B. Strong Cancellations

$$\Gamma(D^+) = 6.15\,\Gamma_0 \Big[1 + 0.48 - 0.13 \frac{\mu_\pi^2(D)}{0.48\,\mathrm{GeV}^2} + 0.01 \frac{\mu_G^2(D)}{0.34\,\mathrm{GeV}^2} + 0.31 \frac{\rho_D^3(D)}{0.082\,\mathrm{GeV}^3}$$

$$- \underbrace{2.66}_{\mathrm{dim-6,VIA}} - 0.055 \frac{\delta \tilde{B}_1^q}{0.02} + 0.002 \frac{\delta \tilde{B}_2^q}{0.02} - 0.546 \frac{\tilde{\epsilon}_1^q}{-0.04} + 0.009 \frac{\tilde{\epsilon}_2^q}{-0.04} + \underbrace{1.10}_{\mathrm{dim-7,VIA}}$$

$$- 0.0000\, r_1^{qq} - 0.0000\, r_2^{qq} + 0.0011\, r_3^{qq} + 0.0008\, r_4^{qq}$$

$$- 0.0109\, r_1^{sq} - 0.0080\, r_2^{sq} - 0.0000\, r_3^{sq} + 0.0001\, r_4^{sq} \Big],$$



Huge effects due to Pauli interference

Revisiting Inclusive Decay Widths of Charmed Mesons #5
Daniel King (Durham U., IPPP and Durham U.), Alexander Lenz (Siegen U.), Maria Laura Piscopo (Siegen U.), Thomas Rauh (U. Bern, AEC), Aleksey V. Rusov (Siegen U.) et al. (Sep 27, 2021)
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B. Strong Cancellations

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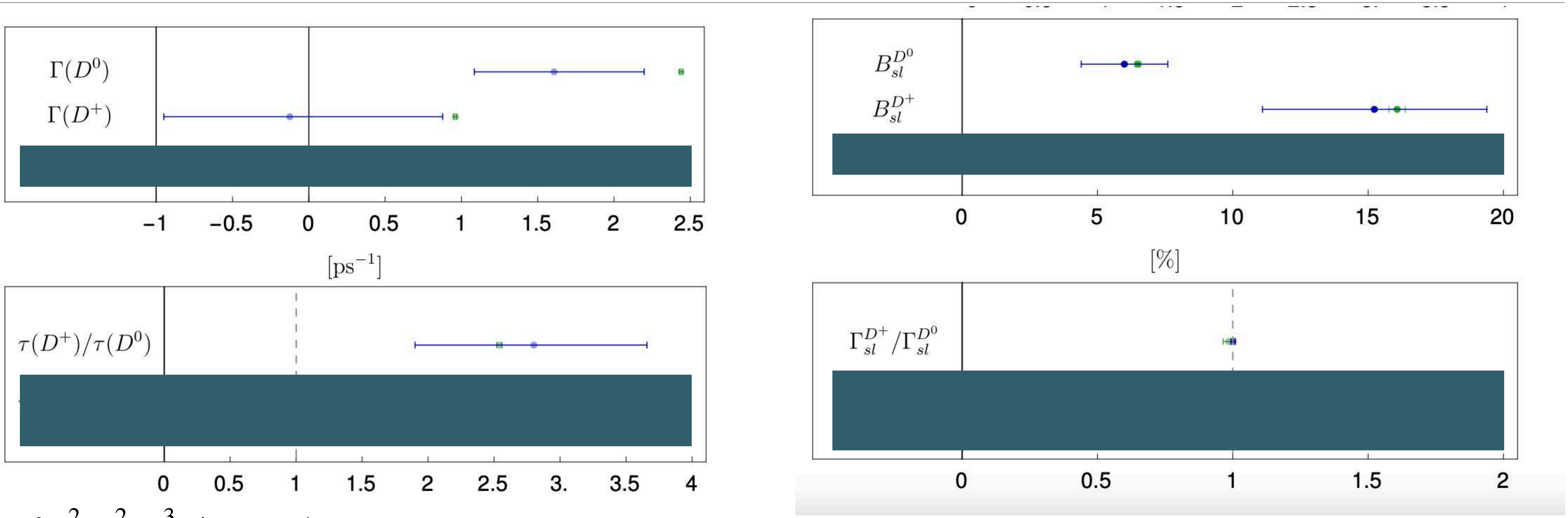

- Values of $\mu_\pi^2, \mu_G^2, \rho_D^3$ almost unknown
- NNLO-QCD corrections to free quark decay in progress

Fael, Steinhauser,…

- NNLO-QCD corrections to spectator effects in progress

Nierste, Steinhauser,…

- Check of HQET sum rule results with lattice

Black, Witzel,…RBC-UK

- First non-perturbative determination of dimension 7

Revisiting Inclusive Decay Widths of Charmed Mesons #5

Daniel King (Durham U., IPPP and Durham U.), Alexander Lenz (Siegen U.), Maria Laura Piscopo (Siegen U.), Thomas Rauh (U. Bern, AEC), Aleksey V. Rusov (Siegen U.) et al. (Sep 27, 2021)

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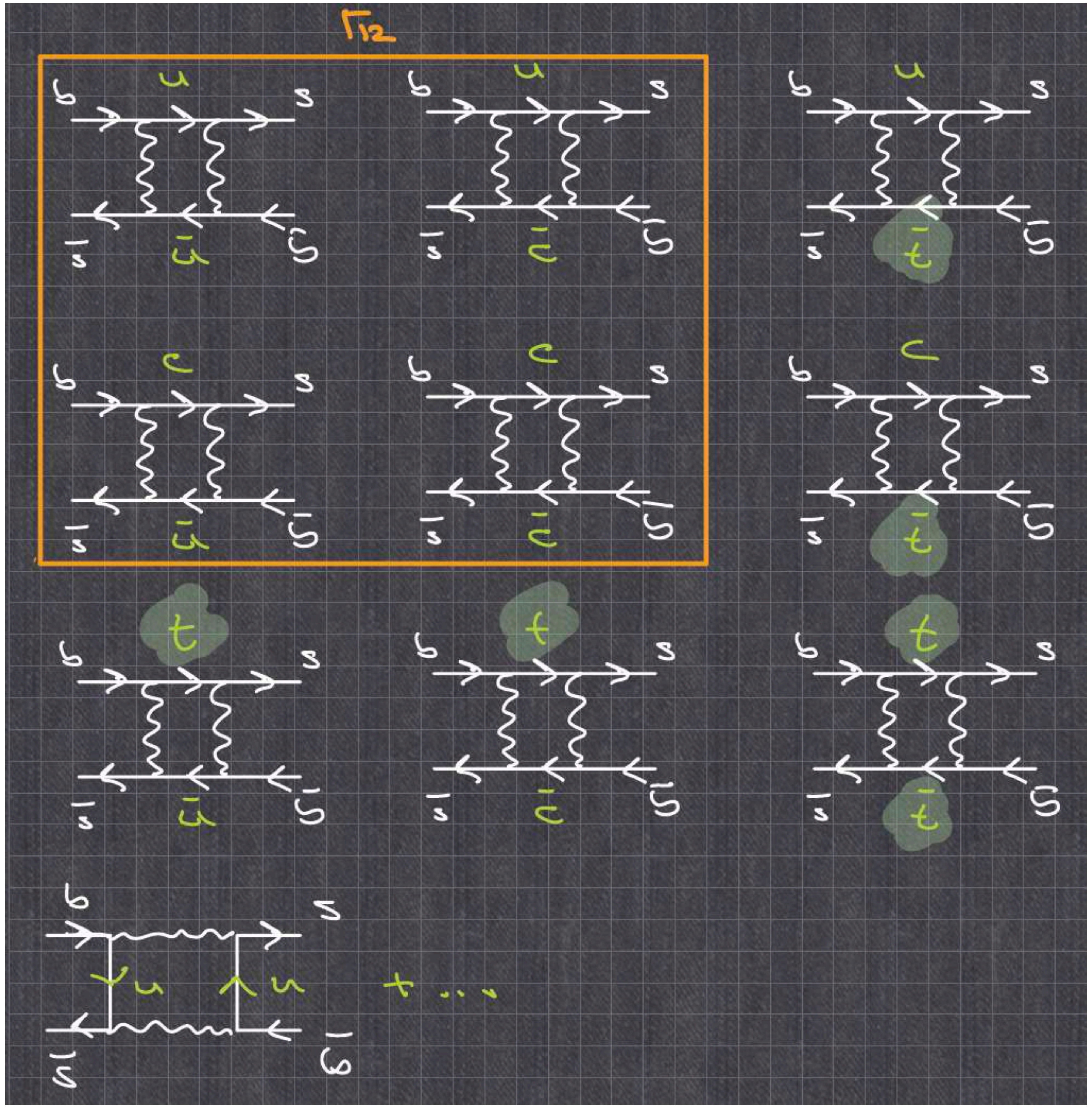
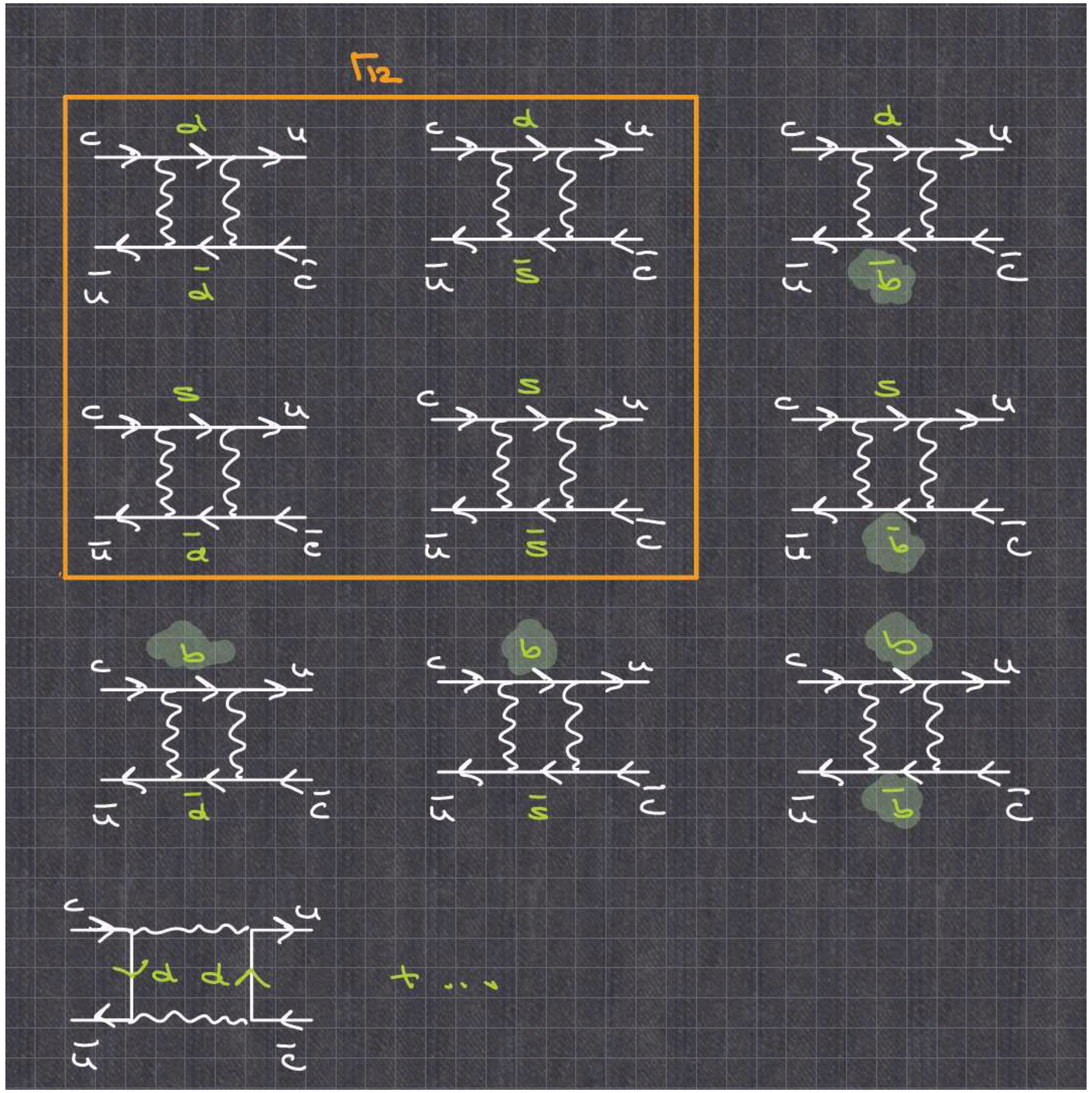
C: Crazy Cancellations

B-mixing



D-mixing



C: Crazy Cancellations

B-mixing

$$M_{12} = \lambda_u^2 F(u,u) + \lambda_u\lambda_c F(u,c) + \lambda_u\lambda_t F(u,t)$$
$$+ \lambda_c\lambda_u F(c,u) + \lambda_c^2 F(c,c) + \lambda_c\lambda_t F(c,t)$$
$$+ \lambda_t\lambda_u F(t,u) + \lambda_t\lambda_c F(t,c) + \lambda_t^2 F(t,t)$$

$\lambda_u + \lambda_c + \lambda_t = 0$

$$= \lambda_u^2 \left[F(c,c) - 2F(u,c) + F(u,u)\right]$$
$$+ 2\lambda_u\lambda_t \left[F(c,c) - F(u,c) + F(u,t) - F(c,t)\right]$$
$$+ \lambda_t^2 \left[F(c,c) - 2F(c,t) + F(t,t)\right]$$

	B_d	B_s
λ_u	$\lambda^{3.8}$	$\lambda^{4.8}$
λ_c	λ^3	λ^2
λ_t	λ^3	λ^2

$m_u^2/m_W^2 \simeq 0$

$m_c^2/m_W^2 \simeq 2.5 \cdot 10^{-4}$

$m_t^2/m_W^2 \simeq 4.5$

CKM dominant ≡ GIM dominant

CKM suppressed ≡ GIM suppressed

D-mixing

$$M_{12} = \lambda_d^2 F(d,d) + \lambda_d\lambda_s F(d,s) + \lambda_d\lambda_b F(d,b)$$
$$+ \lambda_s\lambda_d F(s,d) + \lambda_s^2 F(s,s) + \lambda_s\lambda_b F(s,b)$$
$$+ \lambda_b\lambda_d F(b,d) + \lambda_b\lambda_s F(b,s) + \lambda_b^2 F(b,b)$$

$\lambda_d + \lambda_s + \lambda_b = 0$

$$= \lambda_d^2 \left[F(d,d) - 2F(d,s) + F(s,s)\right]$$
$$+ 2\lambda_s\lambda_b \left[F(s,s) - F(d,s) + F(d,b) - F(s,b)\right]$$
$$+ \lambda_b^2 \left[F(s,s) - 2F(s,b) + F(b,b)\right]$$

	D
λ_d	λ^1
λ_s	λ^1
λ_b	$\lambda^{5.8}$

$m_d^2/m_W^2 \simeq 0$

$m_s^2/m_W^2 \simeq 1.3 \cdot 10^{-6}$

$m_b^2/m_W^2 \simeq 2.8 \cdot 10^{-3}$

CKM suppressed ≡ GIM dominant

CKM dominant ≡ GIM suppressed

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C: Crazy Cancellations

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$$x \equiv \frac{\Delta M_D}{\Gamma_D} = 4.09^{+0.48}_{-0.49} \cdot 10^{-3}$$

HFLAV

$$\frac{\Delta \Gamma_D}{2\Gamma_D} = 6.15^{+0.56}_{-0.55} \cdot 10^{-3}$$

$$\frac{\Delta \Gamma_D^{\text{naive HQE}}}{\Delta \Gamma_D^{\text{EXp.}}} \approx 10^{-5} \ldots 10^{-4}$$

AL, Piscopo, Vlahos 2007.03022

1955 — K^0: *Behavior of neutral particles*, e.g. Phys.Rev. 97 (1955) 1387

1987 — B^0: ARGUS — *Observation of B^0 oscillations*, Phys.Lett.B 192 (1987) 245

2006 — B^0_s: CDF — *Observation of B_s^0 oscillations*, Phys.Rev.Lett. 97 (2006) 242003

2007 — D^0: Belle & BaBar — *Evidence of D^0 oscillations*, Phys.Rev.Lett. 98 (2007) 211802, Phys.Rev.Lett. 98 (2007) 211803

2012: $\Delta\Gamma_s$ by LHCb

2013 — D^0: LHCb — *Observation of D^0 oscillations*, Phys.Rev.Lett. 110 (2013) 101802

2021 — D^0: LHCb — *Observation of D^0 mass difference*, LHCb-PAPER-2021-009

8 June 2021: First observation of the mass difference between neutral charm mesons.

$x = (3.98^{+0.56}_{-0.54}) \times 10^{-3}$

$m_1 - m_2 = 6.4 \times 10^{-6}$ eV = 0.00000000000000000000000000000000000001 grams (1×10^{-38}g)

$(m_1 - m_2)/(D^0 \text{mass}) = 3 \times 10^{-15}$

Today, the LHCb Collaboration submitted a paper for publication that reports the first observation of the mass difference between neutral charm mesons (or rather their mass eigenstates for experts). The result is also presented today at the CERN seminar and was reported last week at the 10th International Workshop on CHARM Physics. This mass difference determines the

Exclusive approaches

Estimate phase space effects for y: Falk et al. 0110317
- **assume pert. SU(3)F breaking**
- **neglect 3rd family**
- **neglect SU(3)F breaking in matrix elements - no QCD calculation**

$y \approx 1\%$

Mass difference from a dispersion relation Falk et al. 0402204 $x \approx y$

Exp. data Cheng, Chiang 1005.1106 $x \propto \mathcal{O}(0.1\%)$ $y \propto \mathcal{O}(few\ 0.1\%)$

U-Spin sum rule Gronau, Rosner 2012

Factorisation-assisted topological amplitude approach Jiang et al. 1705.07335 $y \approx 0.2\%$

C: Crazy Cancellations

The HQE is successful in the B system and for D meson lifetimes

=> apply it for D-mixing

$$y_D^{\text{HQE}} \approx \lambda_s^2 \left(\Gamma_{12}^{ss} - 2\Gamma_{12}^{sd} + \Gamma_{12}^{dd} \right) \approx 10^{-5} y_D^{\text{Exp.}}$$

How can this be?

Look only at a single diagram:

$$y_D^{\text{HQE}} \neq \lambda_s^2 \Gamma_{12}^{ss} \tau_D = 3.7 \cdot 10^{-2} \approx 5.6 y_D^{\text{Exp.}}$$

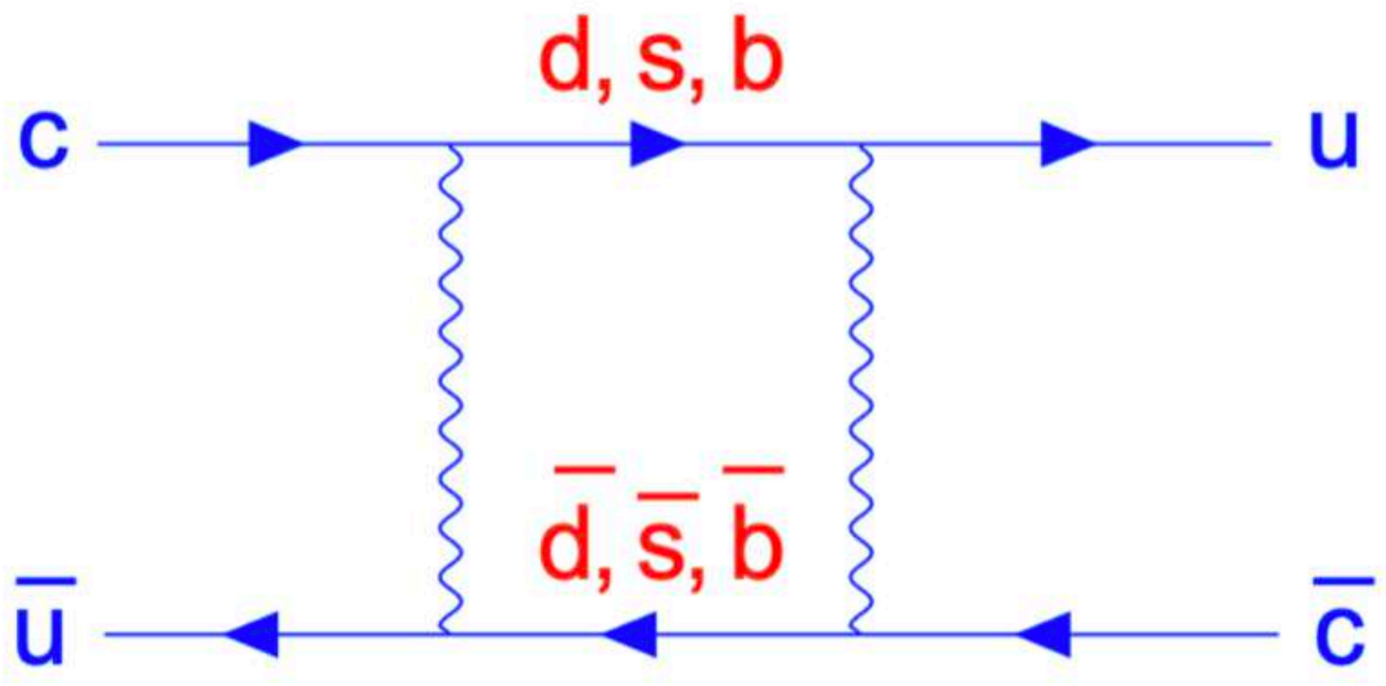
pert. calculation: Bobrowski et al 1002.4794
lattice input: ETM 1403.7302; 1505.06639; FNAL/MILC 1706.04622
HQET sum rules: Kirk, AL, Rauh 1711.02100

The problem seems to originate in the extreme GIM cancellations

C: Crazy Cancellations

CPV

GIM cancellation vs CKM hierarchy: $|\lambda_b| \ll |\lambda_s|$, but complex!!!



survives in SU(3)F limit!

dominant for B mixing

$$\Gamma_{12}^D = \quad -\lambda_s^2 \left(\Gamma_{ss}^D - 2\Gamma_{sd}^D + \Gamma_{dd}^D\right) + 2\lambda_s\lambda_b \left(\Gamma_{sd}^D - \Gamma_{dd}^D\right) - \lambda_b^2 \Gamma_{dd}^D \,,$$

$$M_{12}^D = \lambda_s^2 \left[M_{ss}^D - 2M_{sd}^D + M_{dd}^D\right] + 2\lambda_s\lambda_b \left[M_{bs}^D - M_{bd}^D - M_{sd}^D + M_{dd}^D\right] + \lambda_b^2 \left[M_{bb}^D - 2M_{bd}^D + M_{dd}^D\right] \,.$$

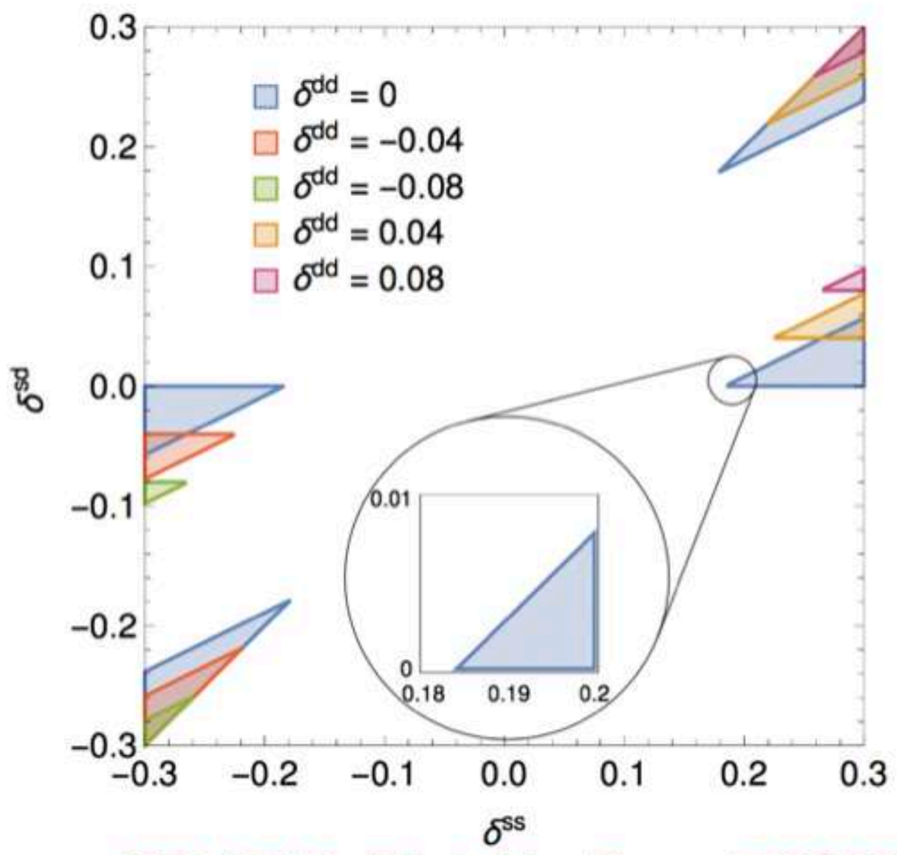
C: Crazy Cancellations

1. **Duality violations - break down of HQE**

$$\Gamma_{12}^{ss} \to \Gamma_{12}^{ss}(1+\delta^{ss}) \,,$$
$$\Gamma_{12}^{sd} \to \Gamma_{12}^{sd}(1+\delta^{sd}) \,,$$
$$\Gamma_{12}^{dd} \to \Gamma_{12}^{dd}(1+\delta^{dd}) \,,$$

20% of duality violation is sufficient to explain experiment

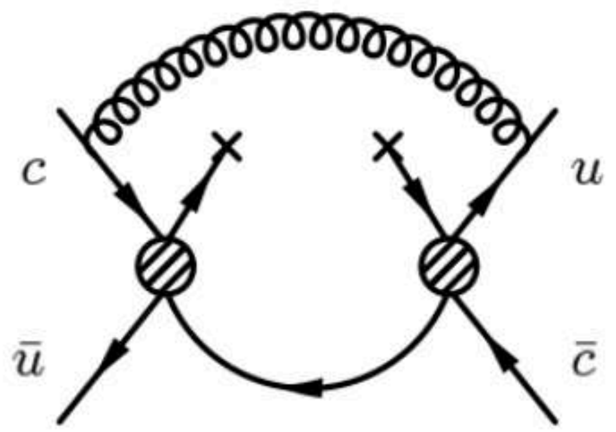
Jubb, Kirk, AL, Tetlalmatzi-Xolocotzi 2016

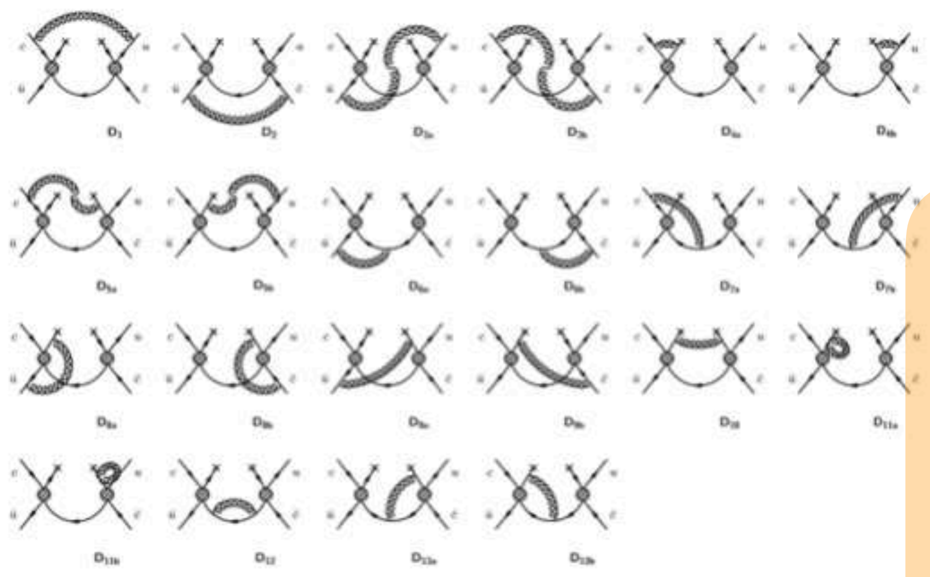


2. **Higher dimensions** Georgi 9209291; Ohl, Ricciardi,Simmons 9301212; Bigi, Uraltsev 0005089

Idea: GIM cancellation is lifted by higher orders in the HQE - overcompensating the 1/mc suppression.

Partial calculation of D=9 yields an enhancement - but not to the experimental value Bobrowski, AL, Rauh 2012





3. **Renormalisation scale setting:** AL, Piscopo, Vlahos 2020

$$\mu_x^{ss} = \mu_x^{sd} = \mu_x^{dd}$$

Implicitly assumes a precision of 10^-5!

4. **New Physics is present and we cannot prove it yet:-)**

1) Vary $\mu^{ss,dd}$ and μ^{ds} independently between 1 GeV and 2 m_c
   ⇒ uncertainty increases and exp. value is covered
2) Choose scales somehow phase space inspired as

$$\mu^{ss} = m_c - 2\epsilon$$
$$\mu^{sd} = m_c - \epsilon$$
$$\mu^{dd} = m_c$$

⇒ exp. value is covered

Exclusive and inclusive approaches can over the experimental regions

No precision determination possible

C: Crazy Cancellations

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Exclusive approach

$$\Gamma_{12}^D = \sum_n \rho_n \langle \bar{D}^0 | \mathcal{H}_{eff.}^{\Delta C=1} | n \rangle \langle n | \mathcal{H}_{eff.}^{\Delta C=1} | D^0 \rangle \,,$$

$$M_{12}^D = \sum_n \langle \bar{D}^0 | \mathcal{H}_{eff.}^{\Delta C=2} | D^0 \rangle + P \sum_n \frac{\langle \bar{D}^0 | \mathcal{H}_{eff.}^{\Delta C=1} | n \rangle \langle n | \mathcal{H}_{eff.}^{\Delta C=1} | D^0 \rangle}{m_D^2 - E_n^2} \,,$$

Cannot be calculated yet

Estimate phase space effects for y: Falk et al. 0110317
- assume pert. SU(3)F breaking
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Factorisation-assisted topological amplitude approach
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C: Crazy Cancellations

Direct lattice determination

Still a very long way!
But not completely crazy anymore!

Multiple-channel generalization of Lellouch-Luscher formula

Maxwell T. Hansen (Washington U., Seattle), Stephen R. Sharpe (Washington U., Seattle) (Apr, 2012)

Published in: *Phys.Rev.D* 86 (2012) 016007 • e-Print: 1204.0826 [hep-lat]

pdf DOI cite 259 citation:

Status of multi-hadron matrix elements in LQCD…

physical system	Method to get it from LQCD
$\pi\pi \to \pi\pi$, $\sqrt{s} < 4M_\pi$ ($\mathbf{P} \neq 0$ in finite-volume frame)*	Lüscher (1986, 1991) Rummukainen and Gottlieb (1995)*
$K \to \pi\pi$ (relies on $M_K < 4M_\pi$) ($\mathbf{P} \neq 0$ in finite-volume frame)*	Lellouch and Lüscher (2001) Kim, Sachrajda and Sharpe (2005)*, Christ, Kim and Yamazaki (2005)*
$\pi\pi \to K\overline{K}$, $\sqrt{s} < 4M_\pi$ (not possible for physical masses)	Bernard et al. (2011), Fu (2012), Briceño and Davoudi (2012)
$D \to \pi\pi$, $K\overline{K}$ (ignores four-particle states)	MTH and Sharpe (2012)
$NN \to NN$, $N\pi \to N\pi$ (energies below three-particle production)	Detmold and Savage (2004) Göckeler et al. (2012) Briceño (2014)
$\gamma^* \to \pi\pi$, $\pi\gamma^* \to \pi\pi$, $N\gamma^* \to N\pi$ $B \to K^*(\to K\pi)\ell\ell$ (energies below three-particle production)	Meyer (2011), Bernard et al. (2012), A. Agadjanov et al. (2014), Briceño, MTH and Walker-Loud (2014) Briceño and MTH (2015)

slide by Max Hansen

#Charmingclues

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You Retweeted

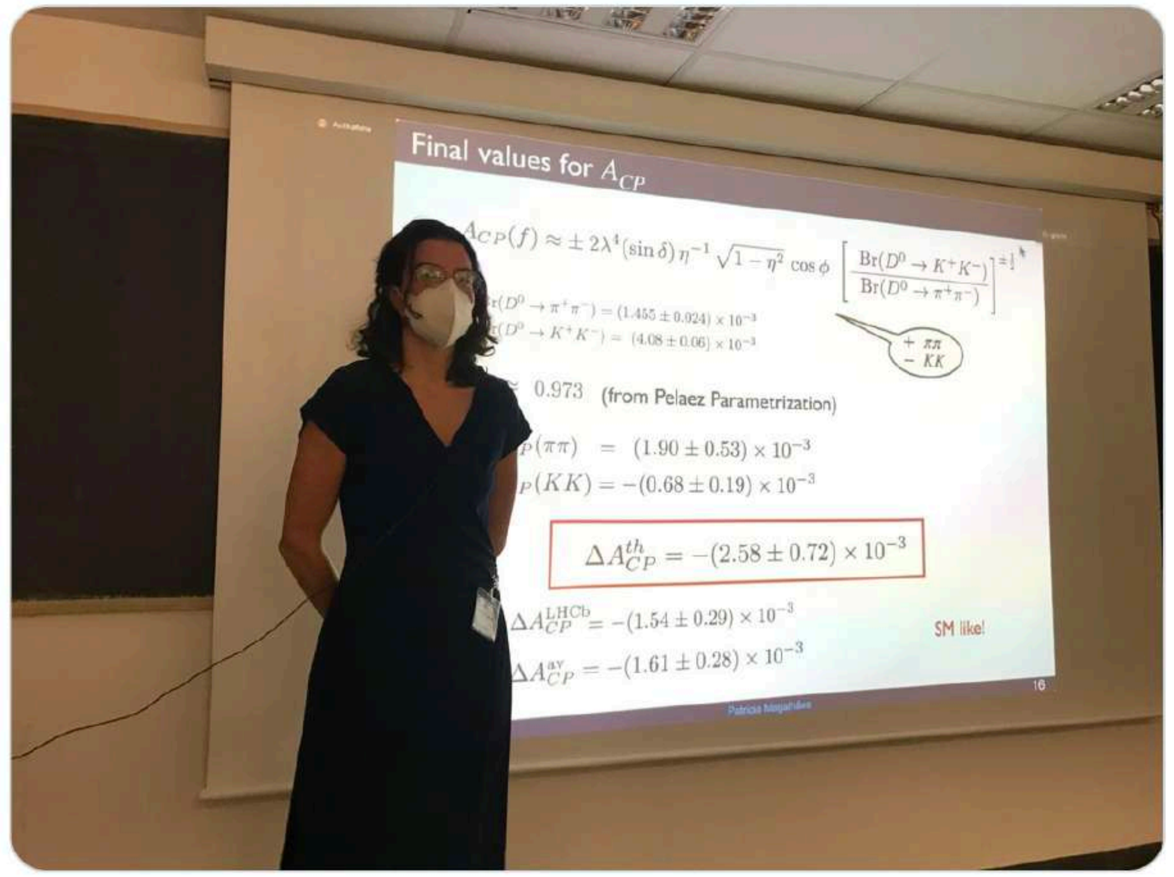
Marco Gersabeck (he/him) @MarcoGersabeck · Mar 17

Glad to witness the first seminar of my PhD student Holly McGrath @UoMparticle, which she gave at the #CharmingClues programme at MIAPP. Great job! 👏 Pity I couldn't join in person.

3 · 3 · 30

Alexander Lenz @alexlenz42 · Mar 16

Is CP violation in decays of D mesons a sign of physics beyond the Standard Model? Patricia says no at #charmingclues



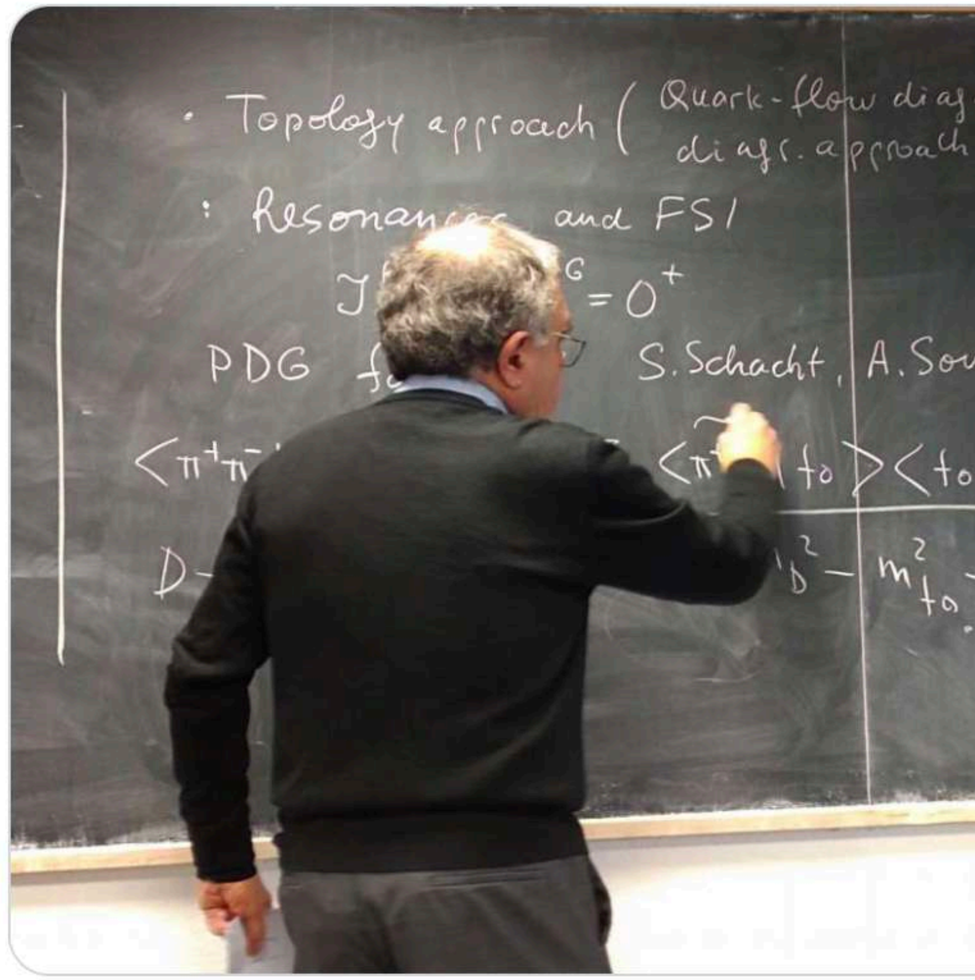
4 · 11 · 123

Alexander Lenz @alexlenz42 · Mar 19

OMG! Friday morning at #charmingclues in Munich - what is going on? White Sausages?

Alexander Lenz @alexlenz42 · Mar 10

Is this the charming clue for existence?
@MIAPP #charmingclues
munich-iapp.de/charmingclues



2 · 10

Alexander Lenz @alexlenz42 · Mar 15

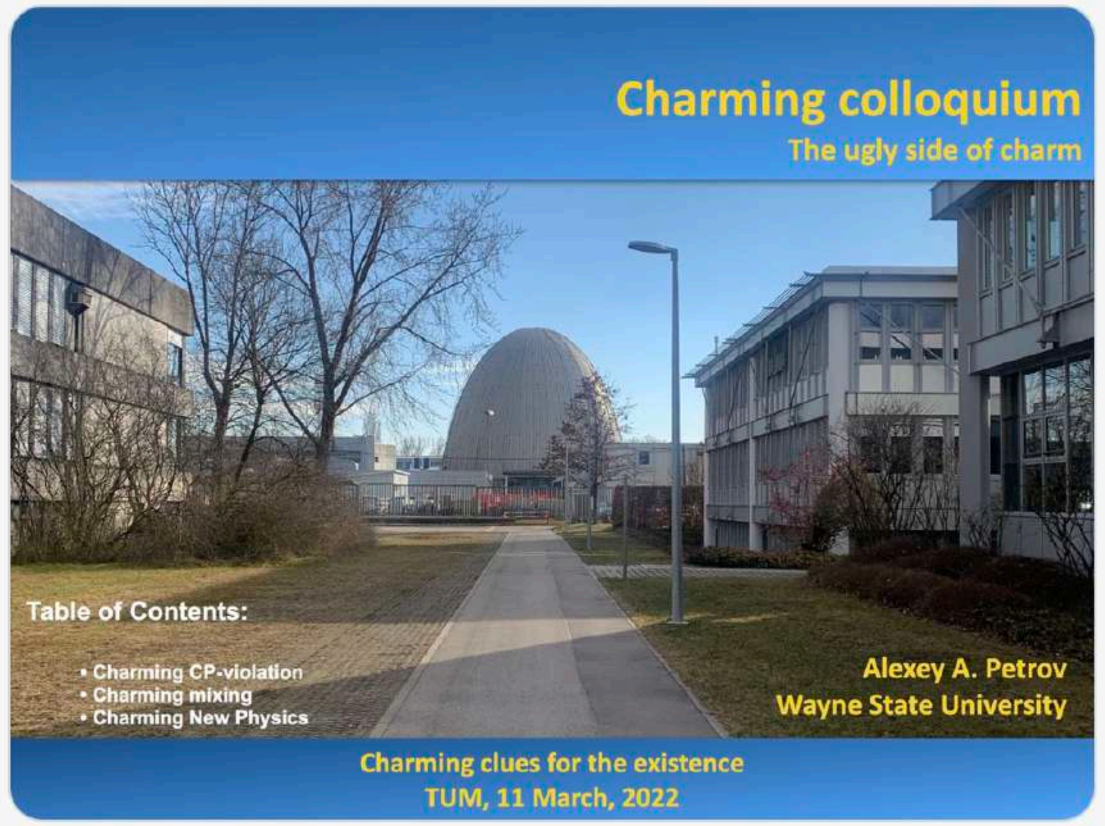
Serious physics discussions about the fundamental laws of physics all night long @MIAPP #charmingclues

3 · 2 · 43

You Retweeted

Alexey Petrov @AlexeyPetrov · Mar 12

I gave a colloquium at TUM as part of the "Charming clues for existence" program. I think I'll use this color scheme from now on, what do you think? #charmingclues



2 · 10

Alexander Lenz @alexlenz42 · Mar 11

Back to real life meetings with a charming dog, 27 charming colleagues and myself @MIAPP
#charmingclues
munich-iapp.de/charmingclues

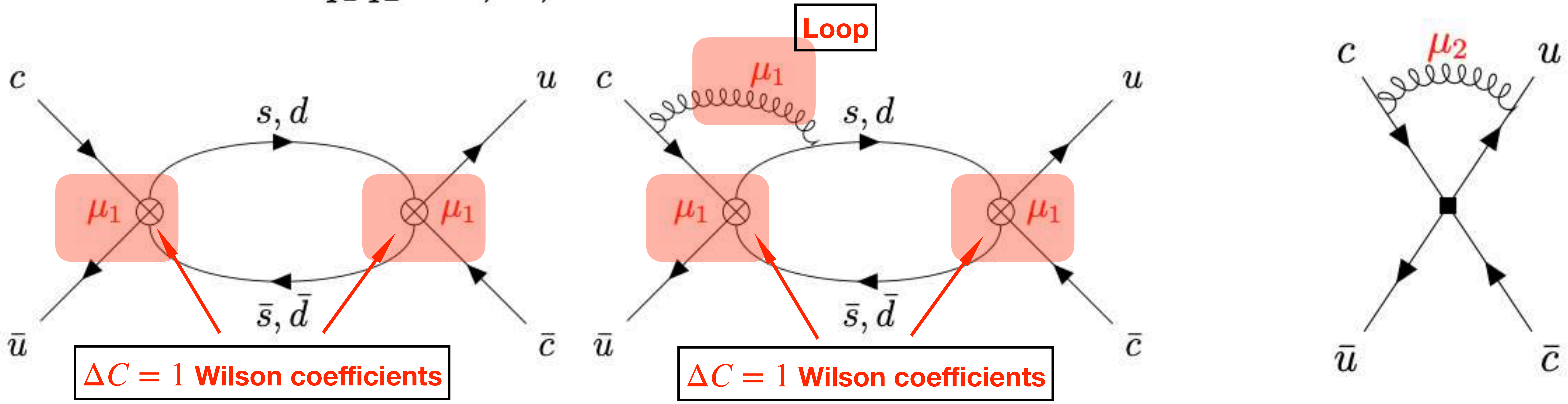
1 · 5 · 13

#Charmingclues

Charm mixing - Theory

Renormalisation scale setting?

$$\Gamma_{12} = \sum_{q_1 q_2 = ss, sd, dd} \Gamma_3^{q_1 q_2}(\mu_1^{q_1 q_2}, \mu_2^{q_1 q_2}) \langle Q \rangle (\mu_2^{q_1 q_2}) \frac{1}{m_c^3} + \ldots$$



μ_1 and μ_2 cancel within the ss, sd and dd contributions independently

Is there any requirement to set exactly $\mu_1^{ss} = \mu_1^{sd} = \mu_1^{dd}$ (also during scale variation)?

ss and dd might be related via re-scattering, but sd is physically different from ss!

Charm mixing - Theory

Renormalisation scale setting?

$$\Delta\Gamma_D > 0.028\text{ps}^{-1} \Rightarrow \Omega \equiv \frac{2|\Gamma_{12}|^{\text{SM}}}{0.028\text{ps}^{-1}} \Rightarrow \Omega \approx 1$$ **means HQE can describe Experiment**

Two scenarios:

1. Vary $\mu^{ss}, \mu^{sd}, \mu^{dd}$ independently around m_c between 1GeV and 2 m_c:

$$\Omega \in [4.6 \cdot 10^{-5}, 1.3]$$

2. Phase space inspired scale choice

$$\mu^{ss} = m_c - 2\epsilon$$
$$\mu^{sd} = m_c - \epsilon$$
$$\mu^{dd} = m_c$$