SU(3)$_F$ in D decays to two pseudoscalars: branching ratios and CP asymmetries

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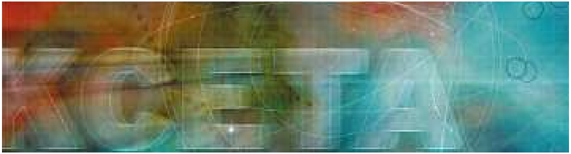


MIAPP Workshop *Charming Clues*, 23 Mar 2022, TUM, Garching, Germany

Contents

$SU(3)_F$ symmetry

Use the approximate $SU(3)_F$ symmetry of QCD: Owing to $m_{u,d,s} \ll \Lambda_{\rm QCD}$ hadronic amplitudes are approximately invariant under unitary rotations of

$$\begin{pmatrix} u \\ d \\ s \end{pmatrix}.$$

$\Rightarrow$ One can correlate various $D \to K\pi$ decays.

Example: In the limit of exact $SU(3)_F$ symmetry find

$$\mathcal{B}(D^0 \to \pi^+\pi^-) = \mathcal{B}(D^0 \to K^+K^-).$$

Data show $\mathcal{O}(30\%)$ $SU(3)_F$ breaking in the decay amplitudes.
It is possible to include $SU(3)_F$ breaking to first order (linear breaking) in the decomposition of the decay amplitudes in terms of $SU(3)_F$ representations.

I discuss hadronic two-body weak decays of D^+, D^0, D_s^+ mesons.

$$D^+ \sim c\overline{d}, \qquad D^0 \sim c\overline{u}, \qquad D_s^+ \sim c\overline{s},$$

Examples: $D^+ \to \overline{K}^0\pi^+$, $D^0 \to \pi^+\pi^-$, $D^+ \to K^0\pi^+$.

Decays are classified in terms of powers of the Wolfenstein parameter

$$\lambda \simeq |V_{us}| \simeq |V_{cd}| \simeq 0.22.$$

$$\text{Amplitude } A \propto \begin{cases} \lambda^0 & \text{Cabibbo-favoured} \\ \lambda^1 & \text{singly Cabibbo-suppressed} \\ \lambda^2 & \text{doubly Cabibbo-suppressed} \end{cases}$$

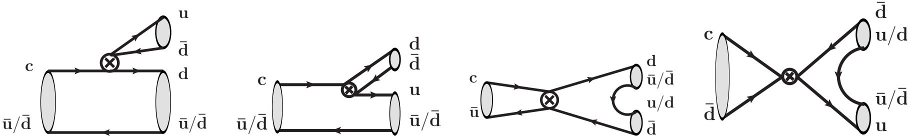
D, D^+, D_s^+ decays to two pseudoscalars

Goal: Get the most out of the measurements of the branching fractions of $D^0 \to K^+K^-$, $D^0 \to \pi^+\pi^-$, $D^0 \to K_SK_S$, $D^0 \to \pi^0\pi^0$, $D^+ \to \pi^0\pi^+$, $D^+ \to K_SK^+$, $D_s^+ \to K_S\pi^+$, $D_s^+ \to K^+\pi^0$, $D^0 \to K^-\pi^+$, $D^0 \to K_S\pi^0$, $D^0 \to K_L\pi^0$, $D^+ \to K_S\pi^+$, $D^+ \to K_L\pi^+$, $D_s^+ \to K_SK^+$, $D^0 \to K^+\pi^-$, $D^+ \to K^+\pi^0$,
and the $K^+\pi^-$ strong phase difference $\delta_{K\pi} = 6.45^\circ \pm 10.65^\circ$ to predict branching fractions and CP asymmetries in these decays.

S. Müller, UN, St. Schacht, PRD92 (2015) 1, 014004, arXiv:1503.06759
S. Müller, UN, St. Schacht, PRL 115 (2015) 25, 251802, arXiv:1506.04121
UN, St. Schacht, PRD 92 (2015) 5, 054036, arXiv:1508.00074
UN, St. Schacht, PRL 119 (2017) 25, 251801, arXiv:1708.03572

Combine topological amplitudes (Chau 1980,1982; Zeppenfeld 1981) with linear $SU(3)_F$ breaking (Gronau 1995).

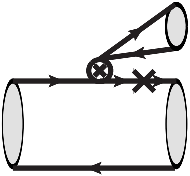
$SU(3)_F$ limit:



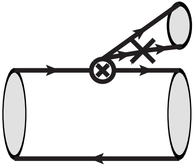
tree (T) color-suppressed tree (C) exchange (E) annihilation (A)

$SU(3)_F$ breaking

Feynman rule from $H_{\cancel{\mathrm{SU(3)}}_F} = (m_s - m_d)\bar{s}s$: dot on s-quark line.
Find 14 new topological amplitudes such as



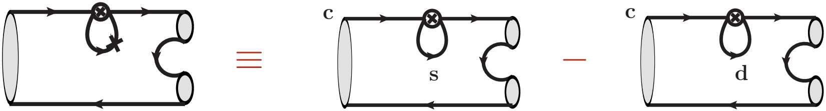
T_1



T_2

…

Important:



penguin (P_{break})

Steps:

i) Invoke colour counting to justify factorisation of tree and annihilation amplitudes à la

$$T = T^{\rm fac}\left[1 + \mathcal{O}\left(\frac{1}{N_c^2}\right)\right]$$

with $T^{\rm fac}$ expressed in terms of decay constants and form factors.

ii) Null hypothesis of a Frequentist analysis: Global fit to data permitting up to 50% $SU(3)_F$-breaking and $\frac{1}{N_c^2}$-corrections of up to 15%. $\rightarrow$ find multi-dimensional valley with perfect $\chi^2 = 0$.

iii) Perform likelihood tests for various hypothesis.
Race horse: *my*Fitter, M. Wiebusch 2012

Branching ratios

The individual branching ratios predicted from the global fit are not more precise than the current measurements.

Exception: $\mathcal{B}(D_s^+ \to K_L K^+)$, which was unmeasured in 2015 was predicted as

$$\mathcal{B}(D_s^+ \to K_L K^+) = 0.012^{+0.007}_{-0.002} \qquad \text{at } 3\sigma$$

2022 PDG value:

$$\mathcal{B}(D_s^+ \to K_L K^+) = 0.0149 \pm 0.0006$$

However, the 2015 fit predicted non-trivial correlations among branching ratios.

Example:
$D^0 \to K_S\pi^0$ and $D^0 \to K_L\pi^0$ are superpositions of the Cabbibbo-favoured amplitude $A(D^0 \to \overline{K}^0\pi^0)$ and the doubly Cabbibbo-suppressed amplitude $A(D^0 \to K^0\pi^0)$.

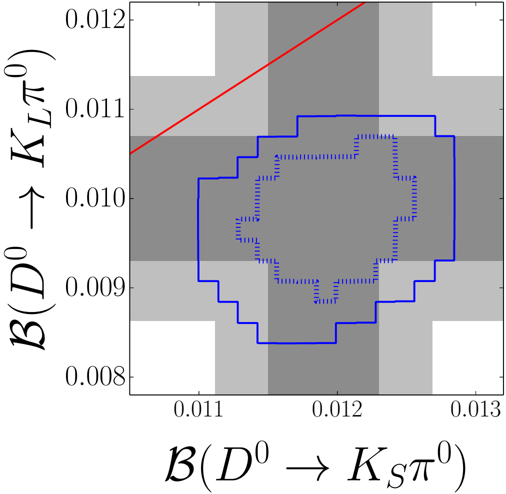
$\Rightarrow$ Their correlation probes $A(D^0 \to K^0\pi^0)$.

$SU(3)_F$ limit ($\lambda = 0.22$ is the Wolfenstein parameter):

$$\mathcal{B}(D^0 \to K_S \pi^0) \propto |E - C|^2 + 2\lambda^2 |E - C|^2$$

$$\mathcal{B}(D^0 \to K_L \pi^0) \propto |E - C|^2 - 2\lambda^2 |E - C|^2$$

Can $SU(3)_F$ breaking change the $SU(3)_F$-limit prediction
$\mathcal{B}(D^0 \to K_L \pi^0) < \mathcal{B}(D^0 \to K_S \pi^0)$?



Gray:
68% CL and 95% CL measurements

Blue:
68% CL and 95% CL fit regions

Red line:
$\mathcal{B}(D^0 \to K_L\pi^0) = \mathcal{B}(D^0 \to K_S\pi^0)$

While $SU(3)_F$ breaking can be sizable, $\mathcal{B}(D^0 \to K_L\pi^0) < \mathcal{B}(D^0 \to K_S\pi^0)$ was predicted with a significance of more than 4σ.

2022 PDG data:

$$\mathcal{B}(D^0 \to K_S \pi^0) = (1.239 \pm 0.022) \cdot 10^{-2}$$
$$\mathcal{B}(D^0 \to K_L \pi^0) = (1.00 \pm 0.07) \cdot 10^{-2}$$

$> 3\sigma$ evidence for $\mathcal{B}(D^0 \to K_S \pi^0) > \mathcal{B}(D^0 \to K_L \pi^0)$.

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Predict CP asymmetries in D decays

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Every measurement hinting at some non-zero CP asymmetry was successfully postdicted offering interpretations both

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Every measurement hinting at some non-zero CP asymmetry was successfully postdicted offering interpretations both

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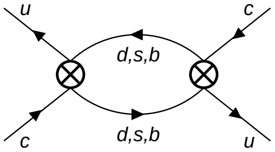
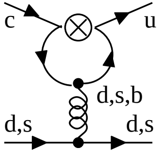
And we are not stubborn at all: After new measurements we eagerly change our opinions!

Detectable CP asymmetries stem from the interference of a tree diagram with a

box,

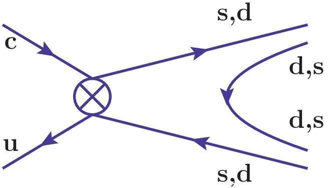


penguin,



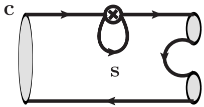
or tree-level diagram:

"(colour favoured) tree" (T),
"colour suppressed tree" (C),
or "exchange" (E).

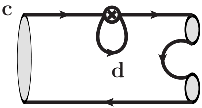


Generic problem: CP asymmetries involve new hadronic quantities which are not constrained by branching fractions. E.g. new SU(3) representations or, in our analysis, new topological-amplitudes.

With



and



Penguins P_s and P_d

(and analogously defined P_b), the amplitude A_b entering CP asymmetries like $a_{CP}^{\mathrm{dir}}(D^0 \to K^+K^-, \pi^+\pi^-)$ involves

$$P \equiv P_d + P_s - 2P_b.$$

Correlate CP asymmetries

Strategy: Build combinations out of several CP asymmetries containing only those topological amplitudes which can be extracted from the global fit to the branching ratios.

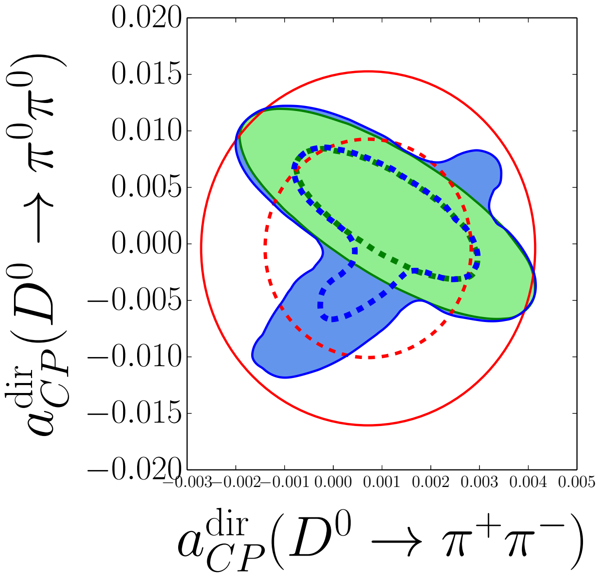
$\rightarrow$ sum rules among CP asymmetries.

Our finding: Two sum rules each correlating three direct CP asymmetries in

I $D^0 \rightarrow K^+K^-$, $D^0 \rightarrow \pi^+\pi^-$, and $D^0 \rightarrow \pi^0\pi^0$,
and

II $D^+ \rightarrow \overline{K}^0K^+$, $D_s^+ \rightarrow K^0\pi^+$, and $D_s^+ \rightarrow K^+\pi^0$.

Unfortunately: only works to zeroth order in $SU(3)_F$ breaking.
Still: theoretical accuracy of new-physics tests only limited by the assumed size of $SU(3)_F$ breaking; great progress compared to the $\mathcal{O}(1000\%)$ spread of past predictions.



Red solid:
95% CL measurement

Red dashed:
68% CL measurement

Present data:

Light blue:
95% CL from global fit

Dark blue dashed:
68% CL from global fit

Future scenario:
assume $\sqrt{50}$ better branching ratios, but $a_{CP}^{\text{dir}}(D^0 \to K^+K^-)$ as today.

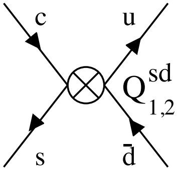
Light green:
95% CL from global fit

Dark green dashed:
68% CL from global fit

Operators from W exchange, e.g.

$$Q_1^{sd} = \overline{s}_L^j \gamma_\mu c_L^k \, \overline{u}_L^k \gamma^\mu d_L^j$$
$$Q_2^{sd} = \overline{s}_L^j \gamma_\mu c_L^j \, \overline{u}_L^k \gamma^\mu d_L^k$$

with colour indices j, k.



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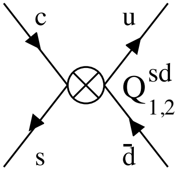

Strip CKM factors off the amplitudes:

$$\mathcal{A}^{\mathrm{CF}} \equiv V_{cs}^* V_{ud} A$$
$$\mathcal{A}^{\mathrm{DCS}} \equiv V_{cd}^* V_{us} A.$$

In the SCS amplitudes three CKM structures appear:
$\lambda_d = V_{cd}^* V_{ud}$, $\lambda_s = V_{cs}^* V_{us}$, $\lambda_b = V_{cb}^* V_{ub}$ and CKM unitarity $\lambda_d + \lambda_s + \lambda_b = 0$ is invoked to eliminate one of these.

Commonly used

$$\mathcal{A}^{\rm SCS} \equiv \lambda_{sd} A_{sd} - \frac{\lambda_b}{2} A_b$$

with

$$\lambda_{sd} = \frac{\lambda_s - \lambda_d}{2} \qquad \text{and} \qquad -\frac{\lambda_b}{2} = \frac{\lambda_s + \lambda_d}{2}$$

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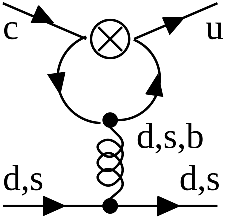
with

$$\lambda_{sd} = \frac{\lambda_s - \lambda_d}{2} \qquad \text{and} \qquad -\frac{\lambda_b}{2} = \frac{\lambda_s + \lambda_d}{2}$$

In view of $|\lambda_b|/|\lambda_{sd}| \sim 10^{-3}$ only A_{sd} is relevant for branching ratios.

Penguin loop contributions to A_{sd} are GIM-suppressed (naively: $\propto (m_s^2 - m_d^2)/m_c^2$).

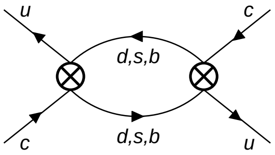


CP asymmetries of hadronic charm decays ...

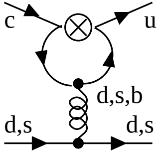
- ... are proportional to $\mathrm{Im}\,\frac{\lambda_b}{\lambda_{sd}} = -6 \cdot 10^{-4}$ in the Standard Model
  - ... and probe new physics in flavour transitions of up-type quarks,
- ... are very difficult to predict in the Standard Model.

Observables (examples):

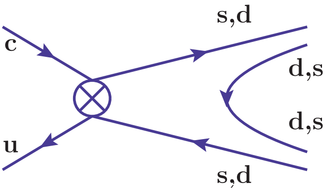
$$a_{CP}^{\text{mix}}(D^0(t) \to K^+\pi^-)$$



$$a_{CP}^{\text{dir}}(D^0 \to \pi^+\pi^-)$$
$$a_{CP}^{\text{dir}}(D^0 \to K^+K^-)$$



$$a_{CP}^{\text{dir}}(D^0 \to K_S K_S)$$
$$a_{CP}^{\text{dir}}(D^0 \to K_S K^{*0})$$



In all cases find:

$$a_{CP} \propto \frac{\lambda_b}{\lambda_{sd}} = -6 \cdot 10^{-4}$$

The prefactor multiplying $\frac{\lambda_b}{\lambda_{sd}}$ cannot be precisely calculated in all three cases.

Direct CP asymmetries in singly Cabibbo-suppressed decays:
With $\mathcal{A}^{\mathrm{SCS}} = \mathcal{A}$ write

$$\mathcal{A} = \lambda_{sd} A_{sd} - \frac{\lambda_b}{2} A_b,$$

CP-conjugate decay: $$\overline{\mathcal{A}} = -\lambda_{sd}^* A_{sd} + \frac{\lambda_b^*}{2} A_b.$$

Find

$$a_{CP}^{\mathrm{dir}} \equiv \frac{|\mathcal{A}|^2 - |\overline{\mathcal{A}}|^2}{|\mathcal{A}|^2 + |\overline{\mathcal{A}}|^2} = \mathrm{Im}\frac{\lambda_b}{\lambda_{sd}}\,\mathrm{Im}\frac{A_b}{A_{sd}}.$$

Branching ratios only fix $|A_{sd}| = |\mathcal{A}|/|\lambda_{sd}|$.

$\Rightarrow$ still need $\mathrm{Im}\frac{A_b}{A_{sd}}$ to predict a_{CP}^{dir}.

CPV discovery channels in the SM

$$a_{CP}^{\text{dir}} = \text{Im}\frac{\lambda_b}{\lambda_{sd}}\ \text{Im}\frac{A_b}{A_{sd}}$$
$$= -6 \cdot 10^{-4}\ \underbrace{\text{Im}\frac{A_b}{A_{sd}}}$$

can be $\mathcal{O}(10)$ in the SM,
if A_{sd} is suppressed.

Typical SM values of a_{CP}^{dir} are below 10^{-3}, thus identifying decays with large $\left|\frac{A_b}{A_{sd}}\right|$ is important. (The phase $\arg\frac{A_b}{A_{sd}}$ is unpredictable, so one must be lucky.)

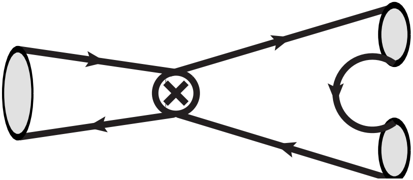
CP violation in tree-tree interference

Whenever $c \to u\bar{d}d$ and $c \to u\bar{s}s$ interfere, the decay can have a non-vanishing direct CP asymmetry proportional to

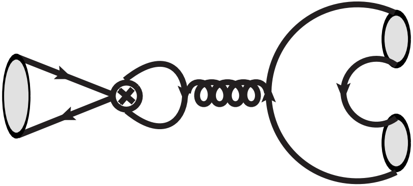
$$\mathrm{Im}\,\frac{V_{ud}V_{cd}^*}{V_{us}V_{cs}^*} = \mathrm{Im}\,\frac{-V_{us}V_{cs}^* - V_{ub}V_{cb}^*}{V_{us}V_{cs}^*} = -\mathrm{Im}\,\frac{V_{ub}V_{cb}^*}{V_{us}V_{cs}^*} \simeq -\mathrm{Im}\,\frac{\lambda_b}{\lambda_{sd}} \simeq 6 \cdot 10^{-4}$$

Tree-tree interference occurs for final states with $\eta^{(\prime)}, \omega \ldots$ or with a pair of neutral Kaons like $K_S K_S, K_S K^{*0}, \ldots$ or for multibody final states containing all four $s, \bar{s}, d, \bar{d}$ quarks like $K^+K^-\pi^+\pi^-$.

A_{sd} is suppressed in $D^0 \to K_S K_S$.
$a_{CP}^{\rm dir}(D^0 \to K_S K_S)$ receives contributions at tree level, from the (sizable!) exchange diagram:



exchange diagram



penguin annihilation diagram

Other CP asymmetry with tree-tree interference:
$a_{CP}^{\rm dir}(D^0 \to \overline{K}^{*0} K^0) \approx -a_{CP}^{\rm dir}(D^0 \to K^{*0} \overline{K}^0)$

Extracting *E* amplitudes from branching ratio data we find

$$|a_{CP}^{\rm dir}(D^0 \to K_S K_S)| \leq 1.1\% \qquad @95\% \text{ C.L.}$$

UN, St. Schacht, Phys.Rev.D92(2015) 054036

and

$$|a_{CP}^{\rm dir}(D^0 \to \overline{K}^{*0} K_S)| \leq 0.003$$

. UN, St. Schacht, Phys.Rev.Lett. 119 (2017) 251801

Precise predictions are not possible because we have no information on the strong phase $\arg(A_b/A_{sd})$.

How to best discover CP violation in tree-tree interference

UN, St. Schacht, Phys.Rev.Lett. 119 (2017) 251801

Two $D^0 \to KK^*$ decays:

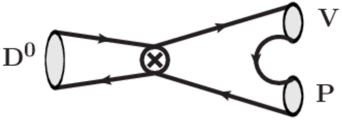
$$D^0 \to \overline{K}^{*0}[\to K^-\pi^+]K^0$$
$$D^0 \to K^{*0}[\to K^+\pi^-]\overline{K}^0$$

with the K^0, $\overline{K}^0$ hadronising into K_S.

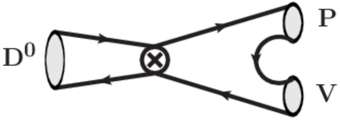
Write shortly:

$$\mathcal{A}(\overline{K}^{*0}) \equiv \mathcal{A}(D^0 \to \overline{K}^{*0}K^0)$$
$$\mathcal{A}(K^{*0}) \equiv \mathcal{A}(D^0 \to K^{*0}\overline{K}^0).$$

Each diagram comes in two variants, e.g.



E_P



E_V

Topological amplitudes:

$$\mathcal{A}_{sd}(K^{*0}) = \quad E_P - E_V + E_{P3} - E_{V1} - E_{V2} - PA_{PV}^{\text{break}}$$

$$\mathcal{A}_b(K^{*0}) = -E_P - E_V - E_{P3} - E_{V1} - E_{V2} - PA_{PV}$$

$$= \mathcal{A}_{sd}(K^{*0}) - 2E_P - 2E_{P3} - PA_{PV} + PA_{PV}^{\text{break}},$$

$$\mathcal{A}_{sd}(\overline{K}^{*0}) = -E_P + E_V - E_{P1} - E_{P2} + E_{V3} - PA_{PV}^{\text{break}},$$

$$\mathcal{A}_b(\overline{K}^{*0}) = -E_P - E_V - E_{P1} - E_{P2} - E_{V3} - PA_{PV}$$

$$= \mathcal{A}_{sd}(\overline{K}^{*0}) - 2E_V - 2E_{V3} - PA_{PV} + PA_{PV}^{\text{break}}.$$

$$\Rightarrow \quad a_{CP}^{\text{dir}}(D^0 \to \overline{K}^{*0} K^0) = \operatorname{Im}\frac{\lambda_b}{\lambda_{sd}} \operatorname{Im}\frac{A_b(\overline{K}^{*0})}{A_{sd}(\overline{K}^{*0})}$$

$$\approx -\operatorname{Im}\frac{\lambda_b}{\lambda_{sd}} \operatorname{Im}\frac{A_b(K^{*0})}{A_{sd}(K^{*0})} = -a_{CP}^{\text{dir}}(D^0 \to K^{*0}\overline{K}^0) = a_{CP}^{\text{dir}}(\overline{D}^0 \to \overline{K}^{*0} K^0)$$

$a_{CP}^{\rm dir}(D^0 \to \overline{K}^{*0} K^0) \approx a_{CP}^{\rm dir}(\bar{D}^0 \to \overline{K}^{*0} K^0)$ means that no flavour tagging is needed:

$$a_{CP}^{\rm dir}(\overset{(-)}{D} \to K_S K^{0*}) \approx a_{CP}^{\rm dir}(D^0 \to K_S K^{0*})$$

Using

$$\mathcal{B}^{\rm exp}(D^0 \to K^{*0} K_S) = (1.1 \pm 0.2) \cdot 10^{-4}\,,$$
$$\mathcal{B}^{\rm exp}(D^0 \to \overline{K}^{*0} K_S) = (0.9 \pm 0.2) \cdot 10^{-4}\,.$$

from experiment to determine $|E_P - E_V| = (1.6 \pm 0.2) \cdot 10^{-6}$ we find

$$|a_{CP}^{\text{dir, untag}}| \lesssim 0.003\,.$$

The maximum corresponds to $\arg(E_V/E_P) = 0.14\,\pi$.

Another goodie: One can scan the $K^+\pi^- K_S$ Dalitz plot near the K^{*0} resonance for a favourable $\arg(E_V/E_P)$.

Summary

- Global fit of $D \to PP'$ branching ratios to topological amplitudes including linear $\mathrm{SU}(3)_F$ breaking gives multiple degenerate best-fit solutions.
- In 2015 we predicted:

  $\mathcal{B}(D_s^+ \to K_L K^+) = 0.012^{+0.007}_{-0.002}$ at 3σ

  $\mathcal{B}(D^0 \to K_L \pi^0) < \mathcal{B}(D^0 \to K_S \pi^0)$ at 4σ
- CP asymmetries involve topological amplitudes not constrained by the fit. These can be eliminated by forming judicious combinations of several CP asymmetries $\rightarrow$ sum rules .
- The sum rules test the quality of $\mathrm{SU}(3)_F$ in penguin amplitudes and/or new physics.
- The small CKM factor $\mathrm{Im}\,\frac{\lambda_b}{\lambda_{sd}} \simeq -6 \cdot 10^{-4}$ renders CP asymmetries in the charm sector sensitive to new physics.

Summary

- Next to discover: CP violation in tree-box interference (i.e. related to D mixing)?
- Next-to-next discover: CP violation in tree-tree interference: $a_{CP}^{\text{dir}}(D^0 \to K_S K_S)$ and $a_{CP}^{\text{dir}}(D^0 \to \overline{K}^{*0} K_S)$:
  - Within the Standard Model the direct CP asymmetry in the charm decay in $D^0 \to K_S K_S$ can be as large as 1.1%.
    $a_{CP}^{\text{dir}}(D^0 \to K_S K_S)$ is dominated by the exchange diagram, which involves no penguin loop.
  - The same is true for $D^0 \to K^{*0} K_S$, which moreover requires no tagging to measure a_{CP}^{dir}. $a_{CP}^{\text{dir, untag}}(D^0 \to K^{*0} K_S)$ can be as large as 0.3%.