

# Electroweak corrections for LHC processes

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**MIAPbP Workshop, Gearing up for High-Precision LHC Physics  
August 25, 2022**

- 1 Introduction
- 2 Automated tools for NLO EW corrections
- 3 General aspects of EW corrections
- 4 Recent calculations for specific processes
- 5 Conclusion and outlook

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Generic size  $\mathcal{O}(\alpha) \sim \mathcal{O}(\alpha_s^2) \Rightarrow$  NLO EW  $\sim$  NNLO QCD

typical: few per cent for inclusive observables

systematic enhancements

- by (soft and/or collinear) photon emission:
  - kinematic effects, radiative tails
  - mass-singular logarithms  $\propto \alpha \ln(m_\mu/Q)$  for bare muons
  - $\Rightarrow$  huge effects ( $> 100\%$ ) possible (in radiative tails)
- at high energies:
  - EW Sudakov logarithms  $\propto (\alpha/s_w^2) \ln^2(M_W/Q)$  and subleading logs
  - $\Rightarrow$  EW corrections of several 10% in high-energy tails of distributions or cross sections dominated by high scales

$\Rightarrow$  NLO EW corrections can be sizeable

$\Rightarrow$  must be included in theoretical predictions

Good news:

automation of (fixed order) NLO EW corrections basically done

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## NLO EW matrix element providers $2 \rightarrow (5)6$ processes routinely available

tool	collaboration
GOSAM	Chiesa et al. 1407.0823
MADGRAPH5_AMC@NLO	Frixione et al. 1804.10017
NLOX	Honeywell et al. 1812.11925
OPENLOOPS	Pozzorini et al. 1907.13071
RECOLA	Actis et al. 1211.6316,1605.01090

### state-of-the-art applications:

- $2 \rightarrow 6$  processes

$pp \rightarrow e^+ \nu_e \mu^- \bar{\nu}_\mu b \bar{b} (t\bar{t})$  Denner, Pellen 1607.05571

$pp \rightarrow 4\ell jj$  (VBS) Denner et al. 1611.02951, 1708.00268, 1904.00882, 2009.00411, 2107.10688, 2202.10844

$pp \rightarrow \ell_1^- \bar{\nu}_{\ell_1} \ell_2^+ \nu_{\ell_2} \ell_3^+ \nu_{\ell_3}$  (WWW) Schönherr 1806.00307, Dittmaier et al. 1912.04117

$pp \rightarrow e^+ e^- \mu^+ \nu_\mu jj_b$  (tZj) Denner, Pelliccioli, Schwan 2207.11264
- $2 \rightarrow 7$  processes

$pp \rightarrow e^+ \nu_e \mu^- \bar{\nu}_\mu b \bar{b} H (t\bar{t}H)$  Denner, Lang, Pellen, Uccirati 1612.07138
- $2 \rightarrow 8$  processes

$pp \rightarrow e^+ \nu_e \tau^+ \nu_\tau \mu^- \bar{\nu}_\mu b \bar{b} (t\bar{t}W)$  Denner, Pelliccioli 2102.03246

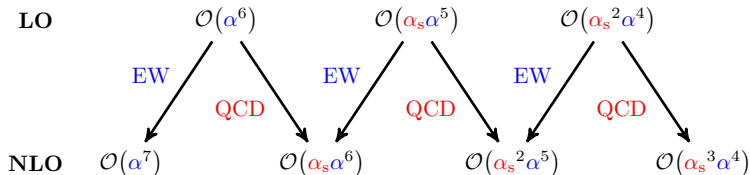
- 1 Introduction
- 2 Automated tools for NLO EW corrections
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Example:  $pp \rightarrow 4\ell jj$  (vector-boson scattering:  $pp \rightarrow VVjj$ )

LO: pure EW diagrams  $\mathcal{O}(e^6)$  and diagrams with gluons  $\mathcal{O}(e^4 g_s^2)$

NLO: EW and QCD corrections to both types of diagrams

at level of cross section:



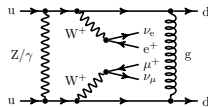
consequences:

- QCD and EW corrections cannot be separated in general
- QCD corrections to leading LO terms well defined
- consider well-defined orders  $\mathcal{O}(\alpha_s^n \alpha^m)$
- automation must deal with expansion in different couplings



Virtual diagrams mix QCD and EW corrections:

- EW correction to LO QCD amplitude
- QCD correction to LO EW amplitude
- QED and QCD IR singularities



⇒ separation into QCD and EW is not well-defined at NLO

real subtraction terms with both gluons and photons needed

For energies  $Q \lesssim 300$  GeV:

- corrections related to the running of the electromagnetic coupling  $\alpha(Q) \propto \alpha \log(m_f/Q)$   
 $\Rightarrow$  incorporated by suitable choice of renormalisation of  $\alpha$ 
  - $\alpha(0)$  for external isolated photons
  - $\alpha(M_Z)$  or  $\alpha_{G_\mu}$  otherwise

$$\alpha_{G_\mu} = \frac{\sqrt{2}}{\pi} G_\mu M_W^2 \left( 1 - \frac{M_W^2}{M_Z^2} \right)$$

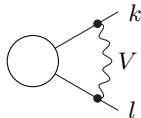
- corrections originating from soft photons or collinear massless fermion–antifermion or (anti)fermion–photon pairs  $\propto \alpha \log(m_f/Q)$ 
  - YFS resummation (Yennie–Frautschi–Suura)
  - electromagnetic parton showers
- top-mass corrections  $\propto \alpha m_t^2 / (M_W^2 s_w^2)$   
 $\Rightarrow$  (partially) incorporated by using  $\alpha_{G_\mu}$

For energies  $Q \gtrsim 300$  GeV in addition:

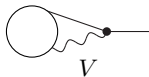
- logarithmic electroweak corrections involving  $\alpha \ln(Q/M_W)$  and  $\alpha \ln^2(Q/M_W)$

## Origin of leading-logarithmic virtual EW corrections

- double logarithms from soft-collinear singular diagrams:  $(\alpha/s_w^2) \ln^2(s_{kl}/M_W^2)$   
 $\Rightarrow$  angular-dependent logarithms of the form  
 $\ln \frac{s_{kl}}{s} \ln \frac{s}{M_W^2}, \ln \frac{t}{u} \ln \frac{s}{M_W^2}$



- single logarithms from collinear-singular diagrams and wave-function renormalisation (self-energies):  $(\alpha/s_w^2) \ln(Q/M_W)$



- single logarithms from coupling renormalisation at scale  $M_W \ll \sqrt{s}$   
 $\Rightarrow$  running of EW couplings ( $e, s_w, \lambda, g_{\text{Yukawa}}$ ) from  $M_W$  to  $\sqrt{s}$

Leading-logarithmic EW corrections depend only on gauge structure of model, external lines and their polarisations, (and on the running of the couplings).

$\Rightarrow$  Leading-logarithmic EW corrections are universal.

## Real emission of EW vector bosons

- separate IR finite contribution, experimentally identifiable
- can be included as extra LO process if needed

General results for EW logarithmic corrections to arbitrary non-mass suppressed processes exist Denner, Pozzorini '00

EW corrections in logarithmic approximation implemented in

- ALPGEN (specific processes) Chiesa et al. 1305.6837
- MCFM (specific processes) Campbell et al. 1608.03356
- SHERPA (general processes) Bothmann, Napoletano 2006.14635
- MADGRAPH5\_AMC@NLO (general processes) Pagani, Zaro 2110.03714

optionally including some universal non-logarithmic terms

features

- simple formulas, complexity of tree-level calculation
- non-logarithmic terms neglected  $\Rightarrow$  typical accuracy: few percent
- often not useful for inclusive quantities  
 [dominated by small scales, small EW corrections of  $\mathcal{O}(\alpha/(s_w^2 \pi)) \sim 1\%$ ]
- quality needs to be checked case by case

Implemented in **SHERPA**

Include only virtual corrections with IR singularities subtracted  
 e.g. via the  $I$  operator of Catani–Seymour subtraction  
 (and real corrections via a parton shower (PS)):

- definition and application to  $pp \rightarrow W + 1j, 2j$  [Kallweit et al. 1511.08692](#)
- application to  $pp \rightarrow t\bar{t}(+j)$  [Gütschow et al. 1803.00950](#)
- application to  $pp \rightarrow WW(+j)$  [Denner et al. 2005.12128](#)
- application to  $pp \rightarrow ZZ(+j)$  [Bothmann et al. 2111.13453](#)

features

- Includes all EW logarithms and renormalisation effects
- complexity of one-loop calculation
- neglects subtracted real corrections  
 $\Rightarrow$  neglects some non-logarithmic terms
- quality needs to be checked case by case
- bad approximation if real corrections are large
- simplifies PS matching and multi-jet merging

- Parton-distribution functions with QED/EW corrections

Photon PDFs: LUXqed

Manohar, Nason, Salam, Zanderighi 1607.04266, 1708.01256

For  $\mu_F > M_Z$ : all SM particles appear as partons

⇒ PDFs for massive vector bosons and Higgs boson

Bauer et al. 1703.0852, 1808.08831; Han et al. 2007.14300, 2103.09844

- EW effects in parton showers

Chen, Han, Tweedie, 1611.00788 (EW splitting functions);

Skands, Verheyen 2002.04939; Kleiss, Verheyen 2002.09248;

Gütschow, Schönherr 2007.15360;

Masouminia, Richardson 2108.10817; Darvishi, Masouminia 2112.15487

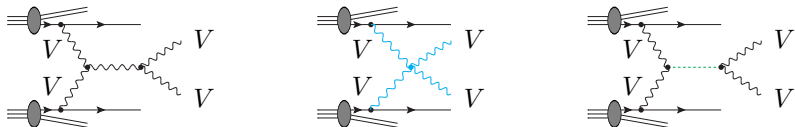
- Isolated photons within higher-order EW corrections

Pagani, Shao, Tsinikos, Zaro 2106.02059

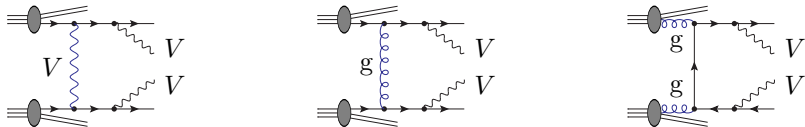
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Processes:  $pp \rightarrow VV + 2j \rightarrow 4\ell + 2j$

Vector-boson scattering (VBS) signal



Irreducible background to VBS



- **EW process:**  $\mathcal{O}(\alpha^4)$  for stable  $V$ s,  $\mathcal{O}(\alpha^6)$  with decays
- **QCD process**  $\mathcal{O}(\alpha_s^2\alpha^2)$  for stable  $V$ s,  $\mathcal{O}(\alpha_s^2\alpha^4)$  with decays
- non-vanishing **interferences** between EW and QCD contributions  
 $\mathcal{O}(\alpha_s\alpha^3)$  for stable  $V$ s,  $\mathcal{O}(\alpha_s\alpha^5)$  with decays
- **gluonic channels** for neutral final states



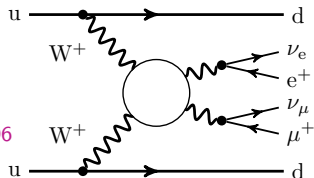
## Large NLO EW corrections to VBS processes

process	$\sigma_{\text{LO}}^{\mathcal{O}(\alpha^6)}$ [fb]	$\Delta\sigma_{\text{NLO,EW}}^{\mathcal{O}(\alpha^7)}$ [fb]	$\delta_{\text{EW}}$ [%]
Biedermann et al. 1708.00268 $pp \rightarrow \mu^+ \nu_\mu e^+ \nu_e jj$ ( $W^+ W^+$ )	1.4178(2)	-0.2169(3)	-15.3
Denner et al. 1904.0088 $pp \rightarrow \mu^+ \mu^- e^+ \nu_e jj$ ( $ZW^+$ )	0.25511(1)	-0.04091(2)	-16.0
Denner et al. 2009.00411 $pp \rightarrow \mu^+ \mu^- e^+ e^- jj$ ( $ZZ$ )	0.097681(2)	-0.015573(5)	-15.9
Denner et al. 2202.10844 $pp \rightarrow \mu^+ \mu^- e^+ e^- jj$ ( $W^+ W^-$ )	2.6988(3)	-0.307(1)	-11.4

- EW corrections similar for all processes and rather independent of cuts  
 $\Rightarrow$  **intrinsic feature of VBS process**
- smaller corrections to  $W^+ W^-$  due to Higgs resonance in fiducial phase space  
 (Higgs contribution about 25%, corresponding EW corrections  $-6.5\%$ )
- $\sigma^{\text{LO}}$  receives sizeable contributions involving large invariants  $s_{ij} \gg M_W$

Double-pole approximation (DPA) for outgoing W bosons  
effective vector-boson approximation (EVBA) for incoming W bosons

- DPA and EVBA reduce discussion to  $V_1 V_2 \rightarrow V_3 V_4$
- DPA accurate for cross section within 1%
- EVBA crude approximation **Kuss, Spiesberger '96** sufficient to understand dominant effects



high-energy, logarithmic approximation for  $V_1 V_2 \rightarrow V_3 V_4$  **Denner, Pozzorini '00**

$$d\sigma_{LL} = d\sigma_{LO} \left[ 1 - \frac{\alpha}{4\pi} 4C_W^{EW} \log^2 \left( \frac{Q^2}{M_W^2} \right) + \frac{\alpha}{4\pi} 2b_W^{EW} \log \left( \frac{Q^2}{M_W^2} \right) \right]$$

$$C_W^{EW} = \frac{2}{s_w^2}, \quad b_W^{EW} = \frac{19}{6s_w^2} \quad \text{for transverse W bosons,} \quad Q \rightarrow M_{4\ell}$$

(double EW logs, collinear single EW logs, and single logs from parameter renormalisation included) (angular-dependent logarithms omitted,  $\log \frac{t}{u} \log \frac{Q}{M_W}$ )

large NLO EW corrections intrinsic feature of VBS

Simple formula for total cross section

$$d\sigma_{LL} = d\sigma_{LO} \left[ 1 - \frac{\alpha}{4\pi} 4C_W^{EW} \log^2 \left( \frac{Q^2}{M_W^2} \right) + \frac{\alpha}{4\pi} 2b_W^{EW} \log \left( \frac{Q^2}{M_W^2} \right) \right]$$

process	$\delta_{EW}$ [%]	$\delta_{EW}^{\log, \text{int}}$ [%]	$\delta_{EW}^{\log, \text{diff}}$ [%]	$\langle M_{4\ell} \rangle$ [GeV]
$pp \rightarrow \mu^+ \nu_\mu e^+ \nu_e jj$	<b>-16.0</b>	-16.1	-15.0	<b>390</b>
$pp \rightarrow \mu^+ \mu^- e^+ \nu_e jj$	<b>-16.0</b>	-17.5	-16.4	<b>413</b>
$pp \rightarrow \mu^+ \mu^- e^+ e^- jj$	<b>-15.9</b>	-15.8	-14.8	<b>385</b>

- **surprisingly good agreement with complete calculation** ( $\delta_{EW} = -16.0\%$ )
- large EW corrections are due to large gauge couplings of vector bosons ( $C^{EW}$ ) and large scale  $Q \sim \langle M_{4\ell} \rangle \sim 400$  GeV
- **angular-dependent logarithms** different for different processes  $\sim 1-2\%$  owing to cancellations

large NLO EW corrections intrinsic feature of VBS

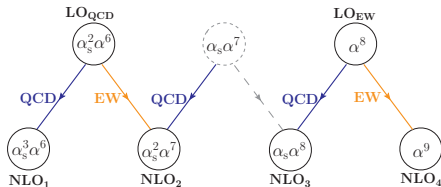
Process:

$$pp \rightarrow e^+ \nu_e \tau^+ \nu_\tau \mu^- \bar{\nu}_\mu b \bar{b}$$

$$(pp \rightarrow t \bar{t} W^+) (2 \rightarrow 8)$$

Denner, Pelliccioli 2102.03246

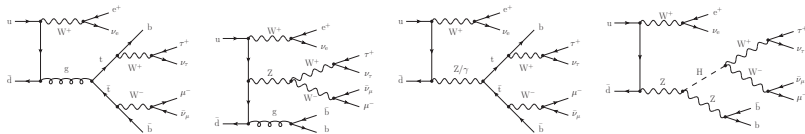
contributions to cross section



LO:

- QCD and EW contributions
- interference of order  $\mathcal{O}(\alpha_s \alpha^7)$  vanish for unit quark-mixing matrix

Sample diagrams for LO<sub>QCD</sub> (left) and to LO<sub>EW</sub> (right)



NLO<sub>1</sub>: QCD corrections to LO QCD

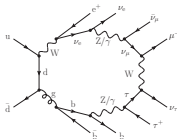
Bevilaqua et al. 2005.9427; Denner, Pelliccioli 2007.12089

## NLO<sub>2</sub>: EW corrections to LO<sub>QCD</sub> and QCD corrections to LO interference

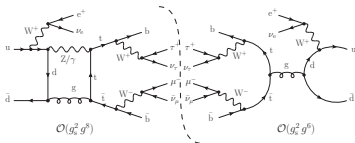
Denner, Pelliccioli 2102.03246

### Virtual corrections

- up 10-point functions

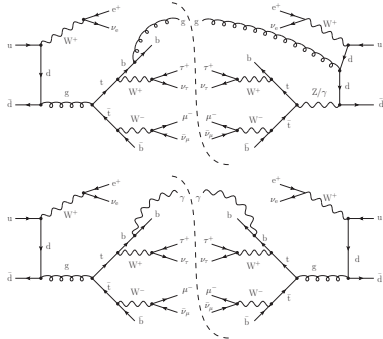


- Classification as QCD or EW corrections ambiguous



### Real corrections

- 2 → 9 process
- large number of IR singular regions
- real QCD correction to LO interference non-zero



Virtual IR-singularities in  $\mathcal{O}(g_s^2 g^8) \times \mathcal{O}(g_s^2 g^6)$  cancelled by both classes of real corr.

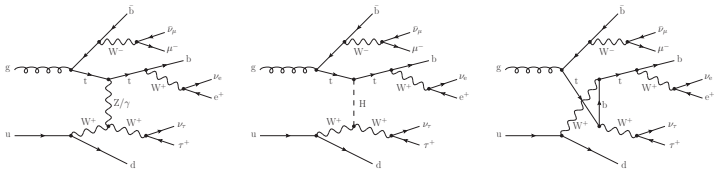
### NLO<sub>3</sub>: Pure QCD corrections to LO<sub>EW</sub>

EW corrections do not change colour structure of LO interference

Naively expected to be subleading but **larger than NLO<sub>2</sub>**

Frederix et al. 1711.02116, 1804.10017

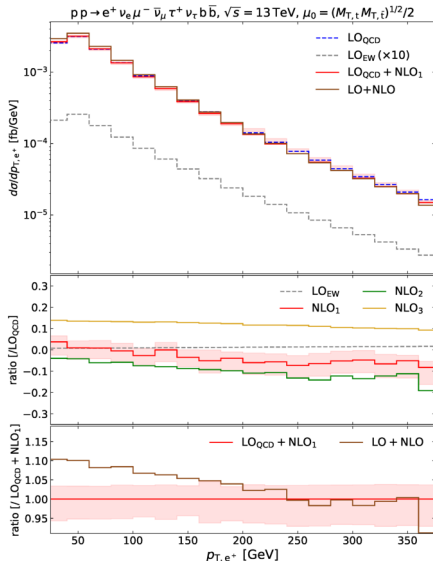
Dominated by **gq** channel contribution involving **tW → tW** scattering



### NLO<sub>4</sub>: EW corrections to LO<sub>EW</sub>

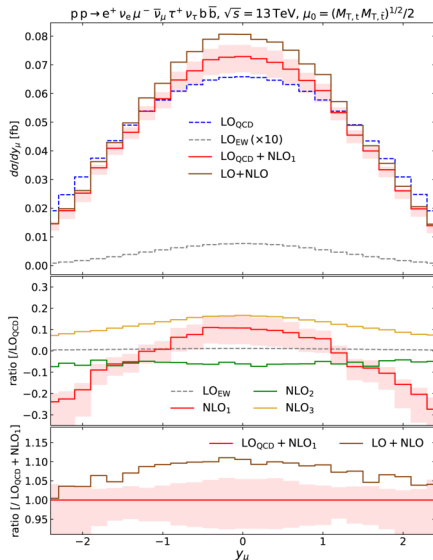
0.04% of LO<sub>QCD</sub> ⇒ negligible

Frederix et al. 1711.02116



Denner, Pelliccioli 2102.03246

- $p_{T,e}$  proxy for  $p_{T,t}$
- $NLO_1$  = NLO QCD corrections vary by 10%
- $NLO_2$  = NLO EW corrections vary between  $-3\%$  and  $-15\%$
- $NLO_3 = \mathcal{O}(\alpha^2/\alpha_s)$  corrections vary between  $14\%$  and  $10\%$
- corrections beyond  $NLO_1$  exceed QCD scale uncertainty



Denner, Pelliccioli 2102.03246

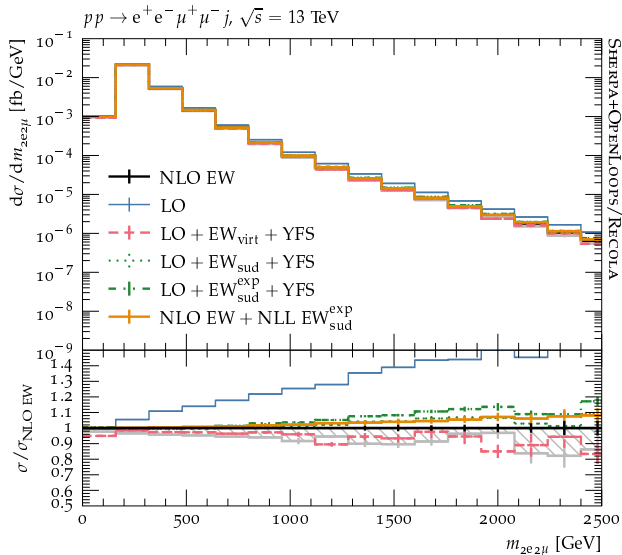
- $y_\mu$  proxy for  $y_{\bar{t}}$
- **NLO<sub>1</sub>** = NLO QCD corrections vary by 35%
- **NLO<sub>2</sub>** = NLO EW corrections vary between  $-4\%$  and  $-8\%$
- **NLO<sub>3</sub>** =  $\mathcal{O}(\alpha^2/\alpha_s)$  corrections vary between  $8\%$  and  $16\%$
- **corrections beyond NLO<sub>1</sub>** up 11% exceed QCD scale uncertainty



Process  $pp \rightarrow e^+e^-\mu^+\mu^-$ ,  $e^+e^-\mu^+\mu^-j$

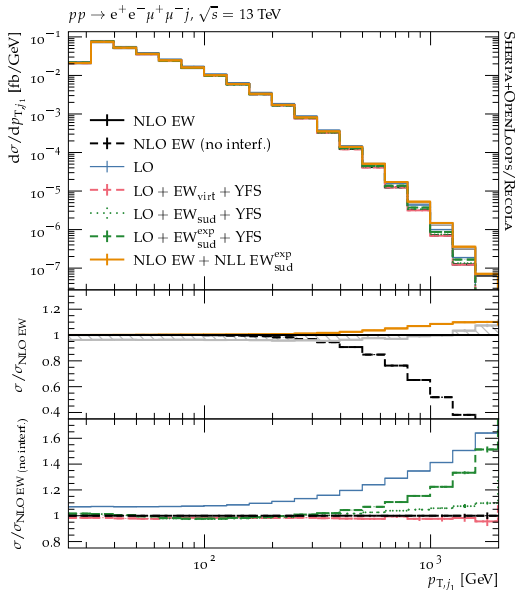
Bothmann, Napoletano, Schönherr, Schumann, Villani 2111.13453 (SHERPA)

- Comparison of exact NLO EW results with EW virtual and NLL Sudakov approximation:
  - NLO EW: full virtual EW corrections  $V_{EW}$  and full real corrections
  - $EW_{virt}$ :  $V_{EW} + I_{EW}$ , full virtual EW + integrated real corrections in soft-collinear limit (Catani–Seymour  $I$ -operator)
  - $EW_{sud}$ : NLL part of  $V_{EW}$ :  $\delta_{sud}^{EW}$ , also exponentiated  $\exp(\delta_{sud}^{EW})$
  - NLO EW + NLL  $EW_{sud}^{exp}$ :  
 NLL part beyond NLO included via exponentiation  
 relative correction:  $\delta_{virt}^{EW} + \exp(\delta_{sud}^{EW}) - \delta_{sud}^{EW}$   
 $\Rightarrow$  matching of NLO EW to resummed Sudakov logarithms
  - + YFS: soft photon resummation in the YFS scheme
- $EW_{virt}$  works well in phase-space regions not dominated by real photon radiation
- resummation of NLL  $\Rightarrow$  reduction of Sudakov suppression of 5–10%
- inclusion of leading EW corrections in MEPS@NLO in SHERPA



Bothmann et al. 2111.13453

- Sudakov suppression up to 50%
- approximations work within 5–10%
- grey band: difference between  $G_\mu$  and  $\alpha(M_Z)$  schemes



Bothmann et al. 2111.13453

NLO EW (no interf.):

real corrections from interference of orders  $\mathcal{O}(g_s^2 g^4)$  and  $\mathcal{O}(g^6)$  for  $pp \rightarrow e^+ e^- \mu^+ \mu^- jj$  neglected

(not included in approximations)

$\Rightarrow$  strong discrepancy between approximations and full NLO EW for  $p_{T,j_1} > 300 \text{ GeV}$

approximations reproduce NLO EW (no interf.) well

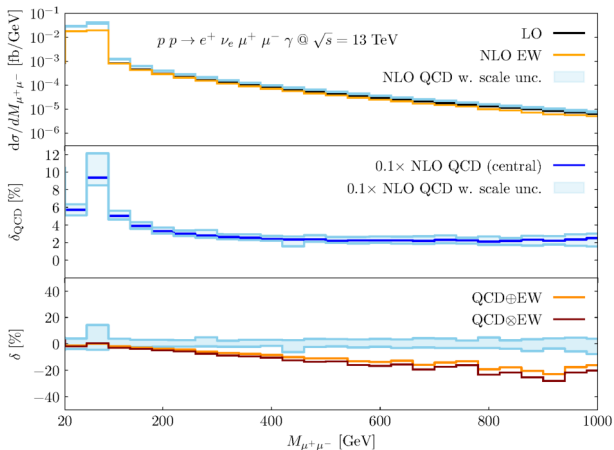
$$pp \rightarrow e^+\nu_e\mu^+\mu^-\gamma \text{ (W}^+\text{Z}\gamma\text{)}$$

Cheng, Wackerth 2112.12052

- tools
  - **RECOLA** for virtual corrections *Actis et al.*
  - **MadDipole** for real corrections *Frederix, Gehrmann, Greiner '08 –'10*
- isolated photons
  - **mixed renormalisation scheme** with  $\alpha(0)$  for on-shell photons
  - **Frixione isolation** *Frixione hep-ph/9801442*
- results for **EW corrections**
  - small ( $\sim 1\%$ ) for total cross section
  - up to 20%–40% for differential distributions
  - **larger than QCD scale uncertainty** for some distributions
  - **cancellations** between EW corrections from quark-induced and photon-induced processes

Cheng, Wackerth 2112.12052

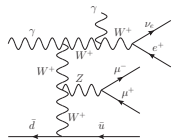
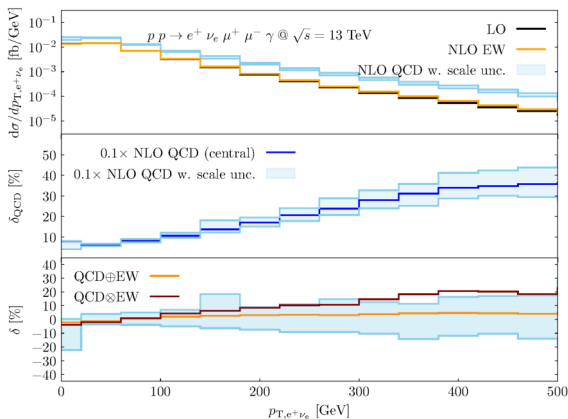
## Distribution in invariant mass of $\mu^+ \mu^-$ pair (Z boson)



- EW corrections reach  $\approx -25\%$  at 1 TeV (Sudakov logarithms)
- photon-induced corrections at most 5%
- EW corrections larger than QCD scale uncertainty

Cheng, Wackerath 2112.12052

## Distribution in transverse momentum of $e^+ \nu_e$ pair (W boson)



- (photon-induced) EW corrections  $\approx +20\%$  at 500 GeV, opening of new  $\gamma q$  channels with hard jets, similar to  $pp \rightarrow e^+ \nu_e \gamma$  [Denner et al. 1412.7421](#)
- differences between **multiplicative** and **additive** combinations of QCD and EW corrections

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## Status of fixed-order EW corrections

- EW corrections automated in several codes
  - EW corrections to  $2 \rightarrow 5(6)$  processes easily available
  - present frontier  $2 \rightarrow 7(8)$  processes
- EW corrections typically  $\lesssim 5\text{--}10\%$  for inclusive observables
- large EW corrections possible
  - in radiative tails ( $> 100\%$ )
  - in high-energy tails of distributions ( $\mathcal{O}(40\%)$ )
  - in fiducial cross sections for specific processes ( $\mathcal{O}(20\%)$  for VBS)
- naively suppressed coupling orders may be important due to opening of new kinematic channels

## Ongoing work

- matching of EW corrections with parton showers
- PDFs and parton showers including EW effects
- automation for processes with polarized vector bosons and/or top quarks  
 Pelliccioli et al., Baglio et al., Buarque Franzosi et al. (Madgraph)