Renormalization issues in BSM models

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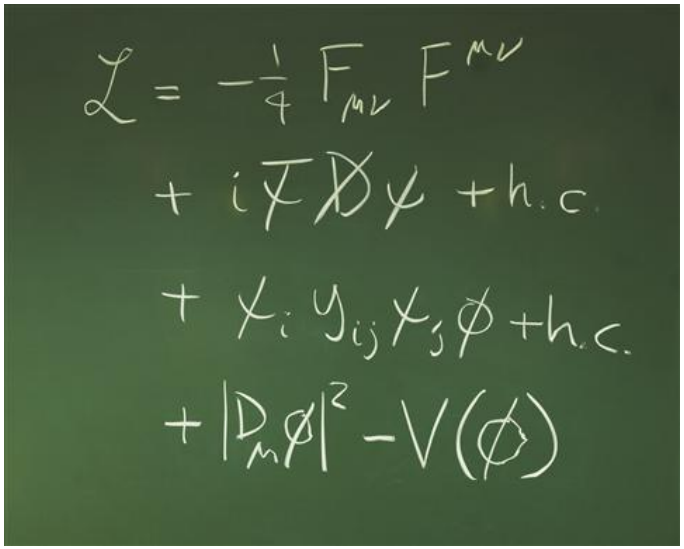
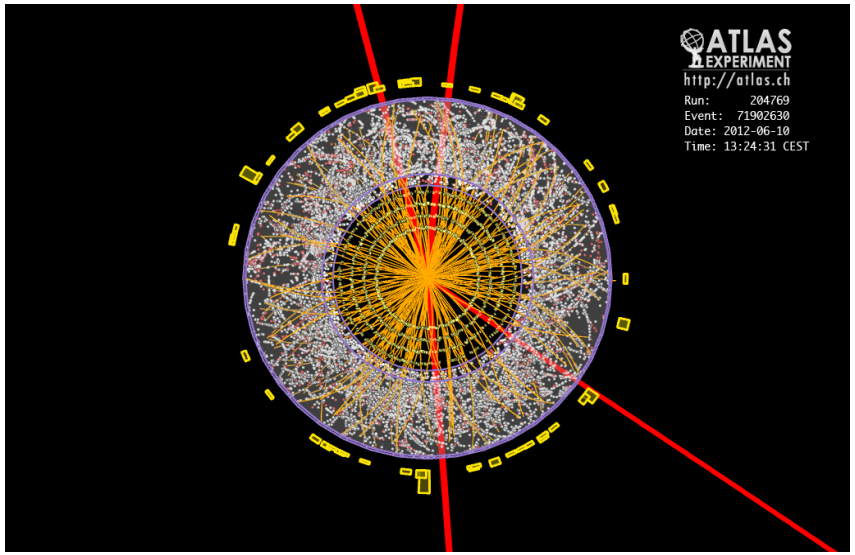
31 August 2022

Renormalization

Original parameters $\longleftrightarrow$ Physical observables





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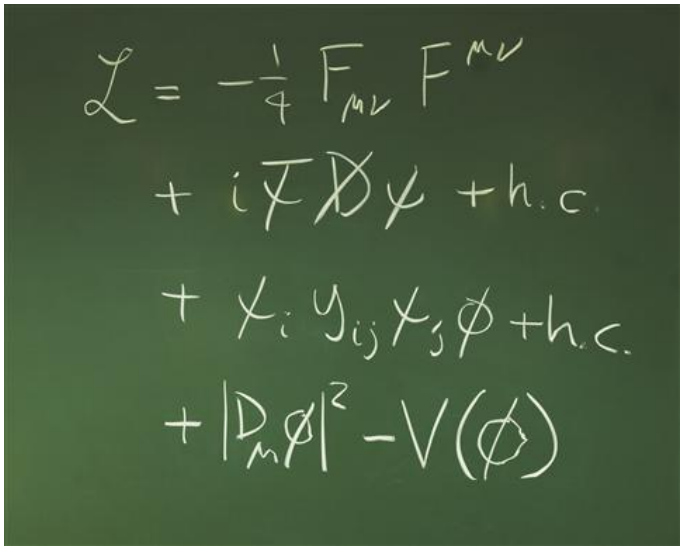
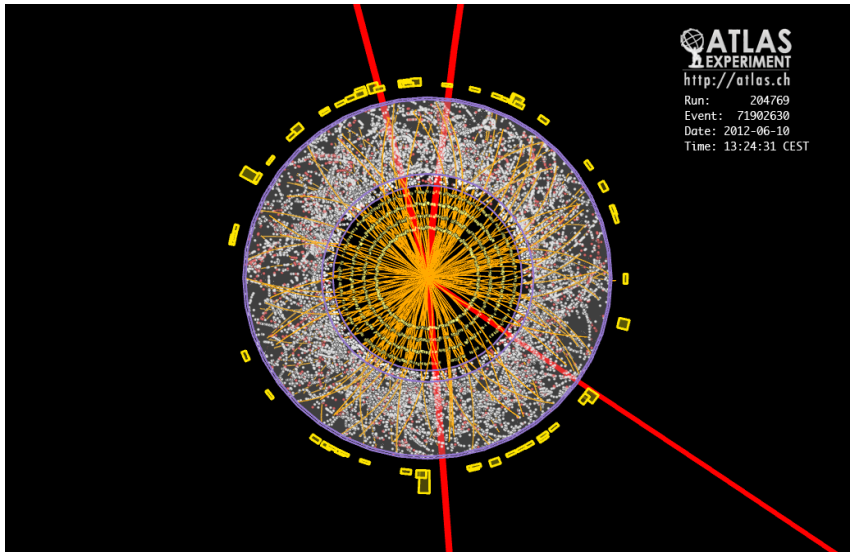
Original parameters $\xleftrightarrow{\text{quantum corrections}}$ Physical observables





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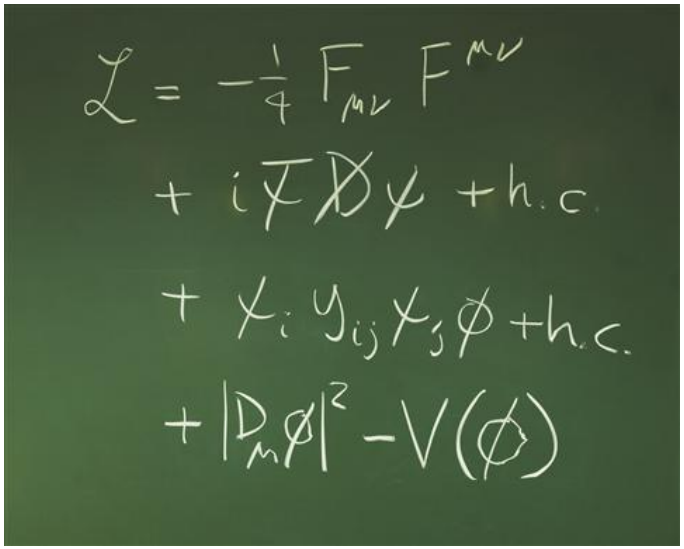

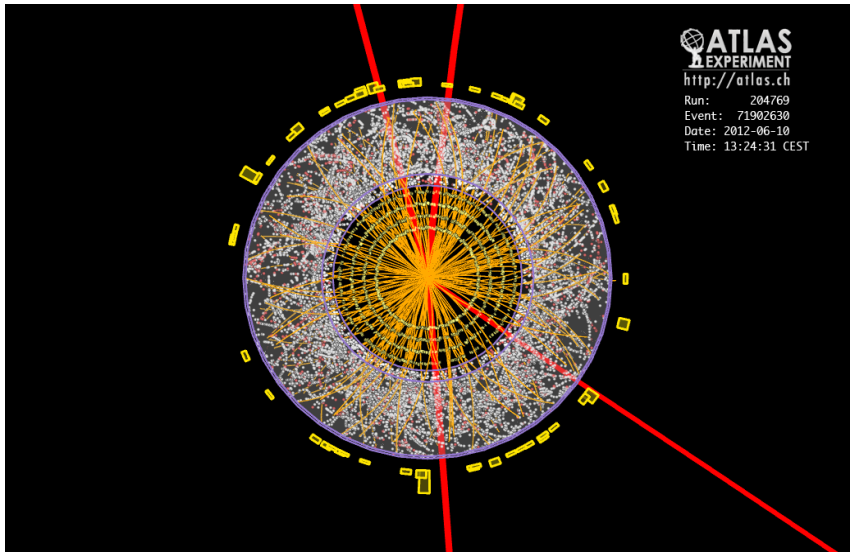

$\Rightarrow$ Establish meaning of parameters

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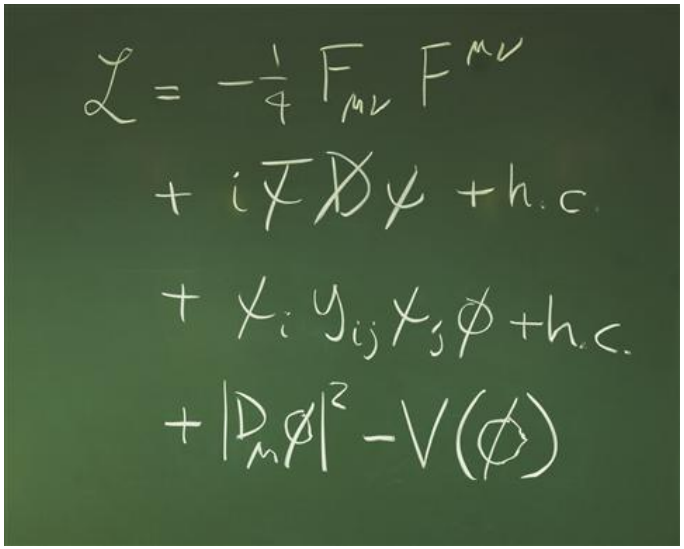

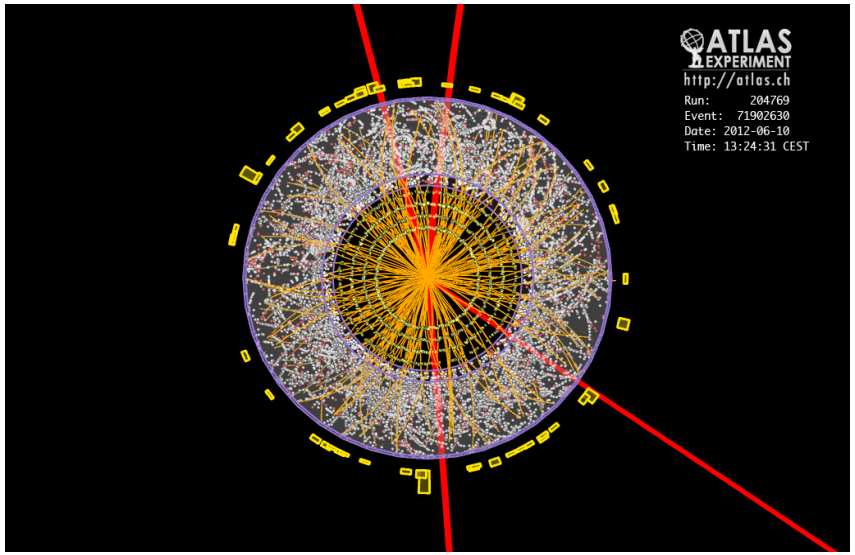

$\Rightarrow$ Establish meaning of parameters

$\Rightarrow$ Choice of expansion point

Assumptions for the following

- Chosen regularization scheme is good
  (see e.g. [Gnendinger et al, arXiv:1705.01827]),
  in particular:
  - ⋆ mathematically consistent
  - ⋆ compatible with QFT-properties of unitarity and causality
  - ⋆ symmetries are preserved
    otherwise: symmetry-restoring counterterms might be needed

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→ Multiplicative renormalization:

Parameters: $g_j^0 = \quad Z_{g_j} g_j \quad = \quad g_j + \delta g_j^{(1)} + \delta g_j^{(2)} + \dots$

Fields: $f_j^0 = (\sqrt{Z_f})_{jk} f_k = (\delta_{jk} + \frac{1}{2}(\delta^{(1)} Z_f)_{jk} + \dots) f_k$

Issues to be discussed

- Bookkeeping and number of parameters
- Mixing angles
- Treatment of tadpole contributions
- Issues in mixed schemes at higher orders

Bookkeeping

Renormalization procedure for a (B)SM:

Step 1:

Count the number of the original independent parameters in your model:

- Be careful to count parameters

  that are vanishing by choice, not by symmetry

Example: Georgi-Machacek (GM) Model

SM extension with:

Two Higgs triplets with hypercharge $Y = 0$ and $Y = 2$, respectively

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Different contributions to W- and Z-mass: see e.g. [Logan 1406.1786]

$$M_W^2 = \frac{g^2}{4}\left(v_{\text{doubl.}}^2 + 4{v_{Y=0}}^2 + 4{v_{Y=2}}^2\right) \quad M_Z^2 = \frac{g^2}{4c_w^2}\left(v_{\text{doubl.}}^2 + 8{v_{Y=2}}^2\right)$$

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Way out: $SU(2) \times SU(2)$ symmetric potential at tree-level

$$\Rightarrow v_{Y=0} = v_{Y=2} \Rightarrow \rho = 1$$

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- Some parameters might not be physical, e.g. some phases

  $\Rightarrow$ can be eliminated

Choice of input parameters

Step 2:

Choose your input parameters:

Original parameters $\rightarrow$ e.g. parameters related to mass

Example:

Higgs and gauge sector in the SM:

$$\underbrace{g, \quad g',}_{\text{gauge couplings}} \quad \underbrace{\lambda,}_{\text{quartic Higgs coupling}} \quad \underbrace{\mu^2}_{\text{Higgs mass parameter}} \quad \rightarrow \quad \underbrace{M_W, \quad M_Z,}_{\text{gauge boson masses}} \quad \underbrace{M_H,}_{\text{Higgs mass}} \quad \underbrace{e}_{\text{electric charge}}$$

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Step 3:

Find renormalization conditions

Step 2 and 3 are related.

Criteria for reasonable conditions

- Numerical stability of the perturbative expansion should be ensured.
- No "dead corners" in parameter space
  $\rightarrow$ avoid singularities in physical observables
- S-matrix elements
  = gauge-independent functions of renormalized parameters
- Simple: Avoid dependence on a specific physical process

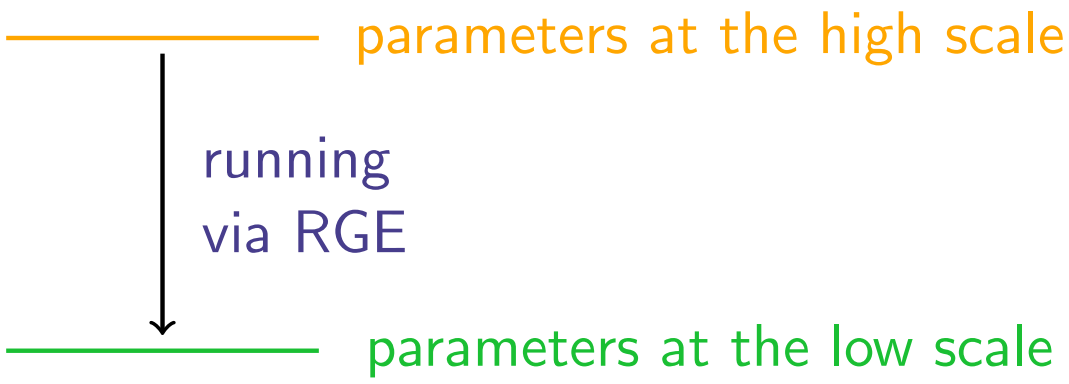
formulated for mixing angles in [Denner, Dittmaier, Lang 1808:03466; Freitas, Stöckinger hep-ph/9409375]

Renormalization conditions in BSM models: Options

In BSM models: Additional parameters e.g. couplings

Possible renormalization conditions:

- High-scale scenario $\rightarrow$ effects at low scale:



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- Further parameters?
  - $\star$ $\overline{\text{MS}}/\overline{\text{DR}}$ scheme?
  - $\star$ Via vertices?
  - $\star$ Via ratios?
  - $\star$ (Via full processes?)

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- What about mixing angles?

Mixing angles

A priori: at NLO not well-defined:

can be absorbed by field-renormalization constants

$\rightarrow$ no need to renormalize mixing angles

Mixing angles

Transformation of bare interaction fields into bare mass eigenstates fields:

$c_x = \cos x, s_x = \sin x$

$$\begin{pmatrix} \varphi_{1,0} \\ \varphi_{2,0} \end{pmatrix} = \mathbf{R}_\varphi(\theta_0) \begin{pmatrix} \phi_{1,0} \\ \phi_{2,0} \end{pmatrix} = \begin{pmatrix} c_{\theta,0} & -s_{\theta,0} \\ s_{\theta,0} & c_{\theta,0} \end{pmatrix} \begin{pmatrix} \phi_{1,0} \\ \phi_{2,0} \end{pmatrix},$$

Renormalization of interaction fields and mixing angle θ:

$$\begin{pmatrix} \phi_{1,0} \\ \phi_{2,0} \end{pmatrix} = \begin{pmatrix} 1 + \frac{1}{2}\delta Z_{\phi_1\phi_1} & \frac{1}{2}\delta Z_{\phi_1\phi_2} \\ \frac{1}{2}\delta Z_{\phi_2\phi_1} & 1 + \frac{1}{2}\delta Z_{\phi_2\phi_2} \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \qquad \theta_0 = \theta + \delta\theta.$$

Field renormalization of bare mass eigenstate fields:

$$\begin{pmatrix} \varphi_{1,0} \\ \varphi_{2,0} \end{pmatrix} = \left[\begin{pmatrix} c_\theta & -s_\theta \\ s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} 1 + \frac{1}{2}\delta Z_{\phi_1\phi_1} & \frac{1}{2}\delta Z_{\phi_1\phi_2} \\ \frac{1}{2}\delta Z_{\phi_2\phi_1} & 1 + \frac{1}{2}\delta Z_{\phi_2\phi_2} \end{pmatrix} + \begin{pmatrix} -s_\theta & -c_\theta \\ c_\theta & -s_\theta \end{pmatrix} \delta\theta \right] \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

$$= \begin{pmatrix} c_\theta & -s_\theta \\ s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} 1 + \frac{1}{2}\delta Z_{\phi_1\phi_1} & \frac{1}{2}(\delta Z_{\phi_1\phi_2} - 2\delta\theta) \\ \frac{1}{2}(\delta Z_{\phi_2\phi_1} + 2\delta\theta) & 1 + \frac{1}{2}\delta Z_{\phi_2\phi_2} \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}.$$

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careful bookkeeping needed

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Can help to avoid singularities

Vacuum expectation values and tadpole contributions

Higgs potential in the SM:

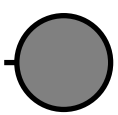
$$V = -\frac{\mu_2^2}{2}\mathrm{tr}\left[\mathbf{\Phi}^\dagger\mathbf{\Phi}\right] + \frac{\lambda_2}{16}\left(\mathrm{tr}\left[\mathbf{\Phi}^\dagger\mathbf{\Phi}\right]\right)^2$$

with Higgs doublet (matrix notation)

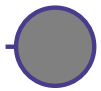
$$\mathbf{\Phi} = \frac{1}{\sqrt{2}}\left[(v+\eta)\mathbb{1} + 2\mathrm{i}\phi\right], \qquad \phi \equiv \frac{\phi_j\sigma_j}{2} = \frac{\vec{\phi}\cdot\vec{\sigma}}{2}$$

Beyond leading order:

Tadpole contributions in V occur:

$\Gamma^\eta = T_\eta =$  $\neq 0$

Convenient:

$\Gamma^\eta_R = T_\eta + \delta t_\eta =$  $+$ $\overset{!}{=} 0$

Issues with tadpole contributions

- On-shell conditions
  (= parameters defined via physical observables):

  Tadpole contributions cancel
  $\Rightarrow$ independent on tadpole treatment

- $\overline{\text{MS}}/\overline{\text{DR}}$ conditions:

  - $\star$ $\overline{\text{MS}}/\overline{\text{DR}}$ masses (also in SM!)
  - $\star$ $\overline{\text{MS}}/\overline{\text{DR}}$ mixing angles

  depend on tadpole treatment.

Common treatments of tadpoles

Parameter Renormalized Tadpole Scheme (PRTS): [Böhm, Hollik, Spiesberger 86; Denner 0709.1075]

- Expansion about the corrected minimum of the effective potential:

$$t_\eta + \underbrace{\delta t_\eta^{\text{PRTS}} + T_\eta}_{=0} = 0 \Rightarrow t_\eta = 0$$

$$t_{\eta,0} = t_\eta + \delta t_\eta^{\text{PRTS}} = \delta t_\eta^{\text{PRTS}}$$

$\Rightarrow$ gauge dependent $\delta t_\eta^{\text{PRTS}}$ enters the relation between parameters and physical observables

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Fleischer-Jegerlehner Tadpole Scheme (FJTS): [Fleischer, Jegerlehner 80; Actis, Ferroglia, Passera, Passarino hep-ph/0612122]

- Expansion about the minimum of the potential at leading order:

$t_{\eta,0} \stackrel{!}{=} 0 \Rightarrow$ no gauge dependencies

Ensuring $\delta t_\eta^{\text{FJTS}} + T_\eta = 0$ by $\eta \to \eta + \Delta^{\text{FTJS}} v \Rightarrow -M_h^2 \Delta^{\text{FTJS}} v + T_\eta = 0$

$\Rightarrow$ relatively large corrections

Gauge-Invariant Vacuume expectation value Scheme

Idea: [Dittmaier, HR 2203.07236; 2206.01479]

- As in PRTS:
  Expansion about the corrected minimum of the effective potential

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- Use field basis in which $v + h(x)$ is invariant

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Gauge-Invariant Vacuume expectation value Scheme (GIVS):

$\rightarrow$ Switch to non-linear (nl) representation:

$$\mathbf{\Phi} = \frac{1}{\sqrt{2}}(v + h)U(\zeta) \quad \text{with } U(\zeta) \equiv \exp\left(\frac{2i\zeta}{v}\right)$$

h = physical Higgs field, $\zeta \equiv \frac{\zeta_j \sigma_j}{2}$ = Goldstone matrix

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$\rightarrow$ Determine $\delta t^{\text{GIVS}} = \delta t_1^{\text{GIVS}} + \delta t_2^{\text{GIVS}}$ via:

PRTS part: $\delta t_1^{\text{GIVS}} = -T_h^{\text{nl}}$

FJTS part: $\delta t_2^{\text{GIVS}} = -M_h^2 \Delta v^{\text{GIVS}} = T_h^{\text{nl}} - T_\eta$

Comparison of Schemes

Conversion from on-shell mass M^{OS} to $M^{\overline{\mathrm{MS}}}$ via bare mass M_0:

$$M_0 = M^{\mathrm{OS}} + \delta M^{\mathrm{OS}} = M^{\overline{\mathrm{MS}}} + \delta M^{\overline{\mathrm{MS}}}$$

Difference between on-shell and $\overline{\mathrm{MS}}$ mass:

$$\Delta M^{\overline{\mathrm{MS}}-\mathrm{OS}} = \delta M^{\mathrm{OS}} - \delta M^{\overline{\mathrm{MS}}} = \delta M^{\mathrm{OS}}|_{\mathrm{finite}}$$

	$M^{\mathrm{OS}}[\mathrm{GeV}]$	$\Delta M_{\mathrm{EW}}^{\overline{\mathrm{MS}}-\mathrm{OS}}[\mathrm{GeV}]$		
		FJTS	PRTS	GIVS
W boson	80.379	-2.22	0.82	0.74
Z boson	91.1876	-0.77	1.25	1.14
Higgs boson	125.1	6.34	3.16	2.80
top quark	172.4	10.75	0.99	0.54
bottom quark	4.93	-1.79	0.10	0.13
τ lepton	1.77686	-0.93	-0.028	-0.015

GIVS leads to much smaller corrections than FJTS!

GIVS for BSM

GIVS can be formulated for BSM:

Already available for: [Dittmaier, HR 2206.01479]

- Singlet extension of the SM
- Two-Higgs-Doublet Model (2HDM)
  $\rightarrow$ non-linear representation worked out

$\Rightarrow$ Improvement for $\overline{\text{MS}}$ defined mixing angles

Depending on process and parameter region:

On-shell definitions via ratios of form factors still better

e.g. for $H \rightarrow 4\ f$ H = heavy CP-even Higgs boson in 2HDM

[Denner, Dittmaier, Lang 1808:03466]

Mixed schemes at higher orders

Mixed scheme = on-shell + $\overline{\text{MS}}/\overline{\text{DR}}$ conditions

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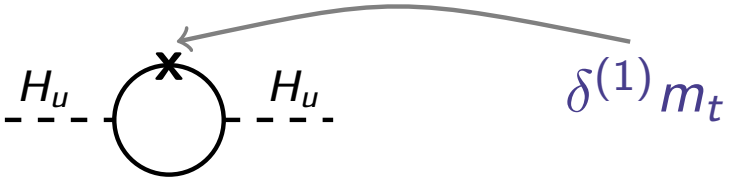
Example: Possible counterterm for $\tan\beta$ in the MSSM at $\mathcal{O}(\alpha_t\alpha_s)$:

$$\delta^{(2)}\tan\beta = \frac{1}{2}\tan\beta\,\delta^{(2)}Z_{H_u} = -\frac{1}{2}\tan\beta\frac{\partial\Sigma_{H_uH_u}(p^2)}{\partial p^2}|_{\text{div.}}$$

$\rightarrow$ depends on subloop renormalization:



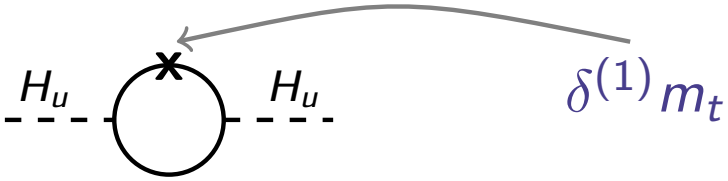
Mixed schemes at higher orders

Mixed scheme = on-shell + $\overline{\text{MS}}/\overline{\text{DR}}$ conditions

Example: Possible counterterm for $\tan\beta$ in the MSSM at $\mathcal{O}(\alpha_t\alpha_s)$:

$$\delta^{(2)}\tan\beta = \frac{1}{2}\tan\beta\,\delta^{(2)}Z_{H_u} = -\frac{1}{2}\tan\beta\frac{\partial\Sigma_{H_uH_u}(p^2)}{\partial p^2}|_{\text{div.}}$$

$\rightarrow$ depends on subloop renormalization:



Relation between $\overline{\text{DR}}$ and on-shell mass via bare mass:

$$m_t^{\overline{\text{DR}}} + \delta^{(1)}m_t^{\overline{\text{DR}}}|_{\frac{1}{\epsilon}} = m_t^{\text{OS}} + \delta^{(1)}m_t^{\overline{\text{DR}}}|_{\frac{1}{\epsilon}} + \delta^{(1)}m_t^{\text{OS}}|_{\epsilon^0} + \delta^{(1)}m_t^{\text{OS}}|_{\epsilon^1}$$

Conversion of Z factor

Using

$$m_t^{\overline{\mathrm{DR}}} + \delta^{(1)} m_t^{\overline{\mathrm{DR}}}|_{\frac{1}{\epsilon}} = m_t^{\mathrm{OS}} + \delta^{(1)} m_t^{\overline{\mathrm{DR}}}|_{\frac{1}{\epsilon}} + \delta^{(1)} m_t^{\mathrm{OS}}|_{\epsilon^0} + \delta^{(1)} m_t^{\mathrm{OS}}|_{\epsilon^1}$$

Starting with $\overline{\mathrm{DR}}$ subloop renormalization ($\hat{=}$ RGE definition):

$$\delta Z_{H_u}^{\overline{\mathrm{DR}}} = \underbrace{-\frac{3(m_t^2)^{\overline{\mathrm{DR}}}}{8\pi^2 v^2 \sin^2\beta}\frac{1}{\epsilon}}_{\text{one-loop}} + \underbrace{-\frac{3(m_t^2)^{\overline{\mathrm{DR}}}\alpha_s}{8\pi^2 v^2 \sin^2\beta}\left(\frac{1}{\epsilon^2} - \frac{1}{\epsilon}\right)}_{\text{two-loop}}$$

converting to on-shell subloop renormalization:

$$\delta Z_{H_u}^{\mathrm{OS}} = \underbrace{-\frac{3(m_t^2)^{\mathrm{OS}}}{8\pi^2 v^2 \sin^2\beta}\frac{1}{\epsilon}}_{\text{one-loop}} + \underbrace{-\frac{3(m_t^2)^{\mathrm{OS}}\alpha_s}{8\pi^2 v^2 \sin^2\beta}\left(\frac{1}{\epsilon^2} - \frac{1}{\epsilon}\right) - \frac{3(m_t)^{\mathrm{OS}}\delta^{(1)} m_t^{\mathrm{OS}}|_{\epsilon^0}}{4\pi^2 v^2 \sin^2\beta}\frac{1}{\epsilon}}_{\text{two-loop}}$$

$$- \frac{3 m_t^{\mathrm{OS}}\delta^{(1)} m_t^{\mathrm{OS}}|_{\epsilon^1}}{4\pi^2 v^2 \sin^2\beta} \quad \leftarrow \quad \text{finite two-loop contribution}$$

Different conditions for $\tan\beta$

That means:

$$(\tan\beta)^{\text{pure }\overline{\text{DR}}} \neq (\tan\beta)^{\text{OS subloop}}$$

with

$$\delta^{(2)\overline{\text{DR}}\text{ subloop}}\tan\beta = -\frac{1}{2}\tan\beta\frac{\partial\Sigma^{\overline{\text{DR}}\text{ subloop}}_{H_uH_u}(p^2)}{\partial p^2}\Big|_{\text{div.}}$$

$$\delta^{(2)\text{OS subloop}}\tan\beta = -\frac{1}{2}\tan\beta\frac{\partial\Sigma^{\text{OS subloop}}_{H_uH_u}(p^2)}{\partial p^2}\Big|_{\text{div.}}$$

but

$$(\tan\beta)^{\text{pure }\overline{\text{DR}}} = (\tan\beta)^{\text{OS subloop}} + \frac{3\tan\beta\, m_t^{\text{OS}}\delta^{(1)}m_t^{\text{OS}}|_{\epsilon^1}}{8\pi^2 v^2\sin^2\beta}$$

Mixed schemes at higher orders

$\rightarrow$ can induce dependence on ϵ^1 terms of counterterms in subloop renormalization

see results and discussions in

[Borowka, Hahn, Heinemeyer, Heinrich, Hollik 1404.7074; 1505.03133;

Degrassi, Di Vita, Slavich 1410.3432

Mühlleitner, Nhung, HR, Walz 1412.0918] $\leftarrow$ dropped ϵ^1 terms in counterterms

Summary

- Bookkeeping is important!
- Challenge:
  Find simple gauge-independent conditions
  $\rightarrow$ numerically stable results in the whole parameter space
- Mixing angles:
  Different treatments possible
  $\rightarrow$ best options might depend on model
- Treatment of tadpoles:
  $\rightarrow$ in pure OS scheme not relevant
  $\rightarrow$ in $\overline{\text{MS}}/\overline{\text{DR}}$ schemes:
  Hybrid scheme GIVS combines advantages of PRTS and FJTS:
  Numerical stability and gauge independence
- In mixed schemes: ϵ^1 terms of counterterms can lead to scheme changes via subloop renormalization.