pd: Theory

M. Viviani

INFN, Sezione di Pisa &
Department of Physics, University of Pisa
Pisa (Italy)

FemTUM 2022

August 31 – September 2, 2022
TUM, Garching FS, Munich, Germany

Istituto Nazionale di Fisica Nucleare

Outline

Collaborators

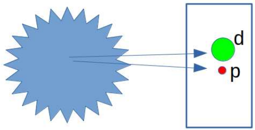
- A. Kievsky & L.E. Marcucci - *INFN-Pisa & Pisa University, Pisa (Italy)*

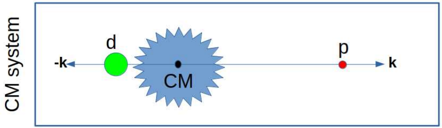
Introduction

Light nuclei formation in the ALICE experiment

Coalescence model: two-step process

- Formation of nucleon and anti-nucleons in the high-energy collision
- Formation of light-nuclei following the interaction of nucleons close in phase-space
- [Butler & Pearson, 1963], [Schwarzschild & Zupancic, 1963]





Thermodynamical models

- Direct creation of light-nuclei in the hot and dense fireball
- [Braun-Munzinger, Redlich & J. Stachel, 2004]

$p-d$ system

- Question: where is formed the deuteron?
- Optimal to study this issue (three-nucleon dynamics well under control)

Theoretical study of the $A = 3$ reactions

Very accurate numerical techniques for $A = 3$ bound and scattering states

- Faddeev methods [Glockle *et al.*, Phys. Rep. **274**, 107 (1996)], and many others
- Expansion on a basis: HH [Kievsky, Marcucci, MV, *et al.*, 2008]
- CDCC method [Austern, Yahiro, & Kawai, 1986]

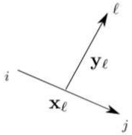
Nuclear interactions

- 1990's: Realistic NN interactions (phenomenological) (AV18, Nijmegen, CD-Bonn)
  - fit of the NN database with $\chi^2 \approx 1$
  - AV18 [Wiringa, Stoks, & Schiavilla, 1995]
  - +Urbana IX 3N force [Pudliner *et al.*, 1995]
- After 2000: chiral NN+3N interactions
- Expansion parameter Q/Λ_χ, $Q \sim m_\pi$, $\Lambda_\chi \approx 1$ GeV
- [Weinberg,1990-1992], [Ordoñez, Ray, & Van Kolck, 1996], [Epelbaum, Hammer, & Meissner, 2009] for a review
- Pionless interactions [Kaplan, Savage, & Wise, 1998]

pd calculation

Definitions

- Permutations: $ij\ell$=123, 231, 312
- Jacobi vectors
  - $\boldsymbol{x}_\ell = \boldsymbol{r}_j - \boldsymbol{r}_i$
  - $\boldsymbol{y}_\ell = \boldsymbol{r}_\ell - \frac{\boldsymbol{r}_j + \boldsymbol{r}_i}{2}$
- Hyperradius $\rho = \sqrt{\boldsymbol{x}_\ell^2 + \frac{4}{3}\boldsymbol{y}_\ell^2}$



$$E = \frac{k^2}{2\mu} - B_d = \frac{Q^2}{m},\ \mu = \frac{2}{3}m,\ \eta = \frac{\mu e^2}{k}$$

Asymptotic components

$$\Omega^F_{LS} = \sqrt{\frac{1}{3}} \sum_{ij\ell} \Big[Y_L(\hat{\mathbf{y}}_\ell) \otimes [\phi_d(ij) \otimes \chi_\ell]_S \Big]_{JJ_z} \frac{F_L(\eta, ky_\ell)}{ky_\ell}$$

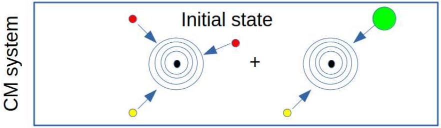
$$\Omega^G_{LS} = \sqrt{\frac{1}{3}} \sum_{ij\ell} \Big[Y_L(\hat{\mathbf{y}}_\ell) \otimes [\phi_d(ij) \otimes \chi_\ell]_S \Big]_{JJ_z} f_L(y_\ell) \frac{G_L(\eta, ky_\ell)}{ky_\ell}$$

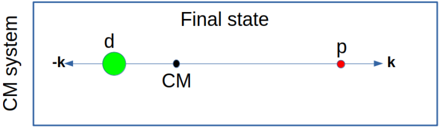
$f_L(y_\ell) = (1 - e^{-\beta y_\ell})^{2L+1}$ used to regularize G_L

pd wave function

$$\Psi^{(-)}_{LSJJ_z} = \sum_{[K]} \frac{u_{[K]}(\rho)}{\rho^{\frac{5}{2}}} \mathcal{Y}_{n,[K]} + \Omega^F_{LS} + \sum_{L'S'} \mathcal{T}^{(-),J}_{LS,L'S'} \left[\Omega^G_{L'S'} - i\Omega^F_{L'S'}\right]$$

- $\mathcal{Y}_{n,[K]}$ antisymmetrical hyperspherical harmonic (HH) functions – essentially, homegeneous polynomials of degree K
- They describe the configuration when the three particles are close + the ingoing three-particles amplitudes ($u(\rho) \sim e^{-iQ\rho}$)
- $\mathcal{T}^{(-),J}_{LS,L'S'}$ = T-matrix ($G - iF \sim e^{-iky_\ell}$)
- $u_{[K]}(\rho)$ and $\mathcal{T}^{(-),J}_{LS,L'S'}$ computed using the Kohn variational principle
- For more details, see [Kievsky *et al.* JPG **35**, 063101 (2008)], [Marcucci *et al.*, FIP **8**, 69 (2020)]





Asymptotic waves

Asymptotic waves

Possible L, S values for a given J^π (using the spectroscopic notation $^{2S+1}L_J$)
for a *pd* state $S = \frac{1}{2}, \frac{3}{2}$

state	$p - d$
$\frac{1}{2}^+$	$^2S_{\frac{1}{2}}, {}^4D_{\frac{1}{2}}$
$\frac{3}{2}^+$	$^2S_{\frac{3}{2}}, {}^2D_{\frac{3}{2}}, {}^4D_{\frac{3}{2}}$
$\frac{5}{2}^+$	$^2D_{\frac{5}{2}}, {}^2G_{\frac{5}{2}}, {}^4G_{\frac{5}{2}}$
$\frac{1}{2}^-$	$^2P_{\frac{1}{2}}, {}^4P_{\frac{1}{2}}$
$\frac{3}{2}^-$	$^2P_{\frac{3}{2}}, {}^4P_{\frac{3}{2}}, {}^4F_{\frac{3}{2}}$
$\frac{5}{2}^-$	$^4P_{\frac{5}{2}}, {}^2F_{\frac{5}{2}}, {}^4F_{\frac{5}{2}}$

S-waves: $\frac{1}{2}^+, \frac{3}{2}^+$
P-waves: $\frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^-$

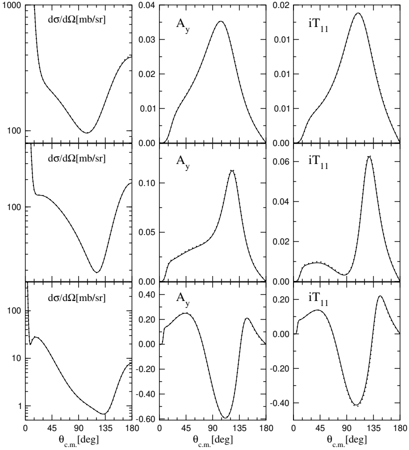
Benchmark test of 3N scattering calculations - *pd* scattering

Comparison with the Faddeed calculation by Deltuva [Deltuva *et al.*, 2005]

$E = 3$ MeV

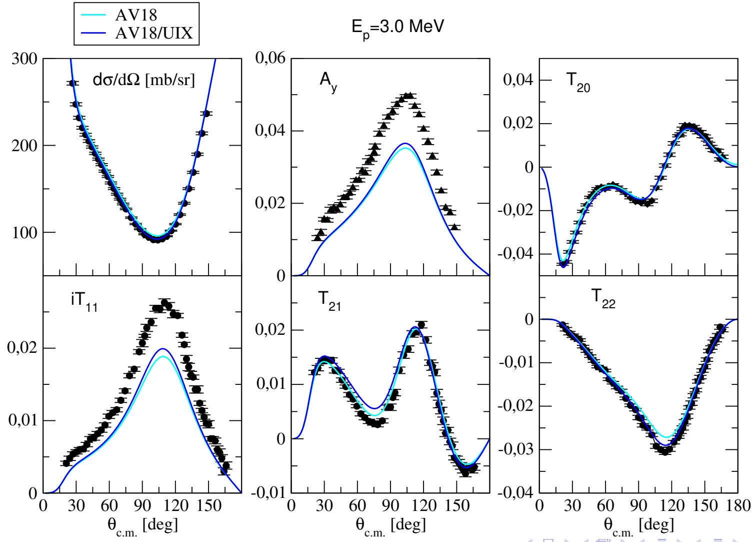
$E = 10$ MeV

$E = 65$ MeV



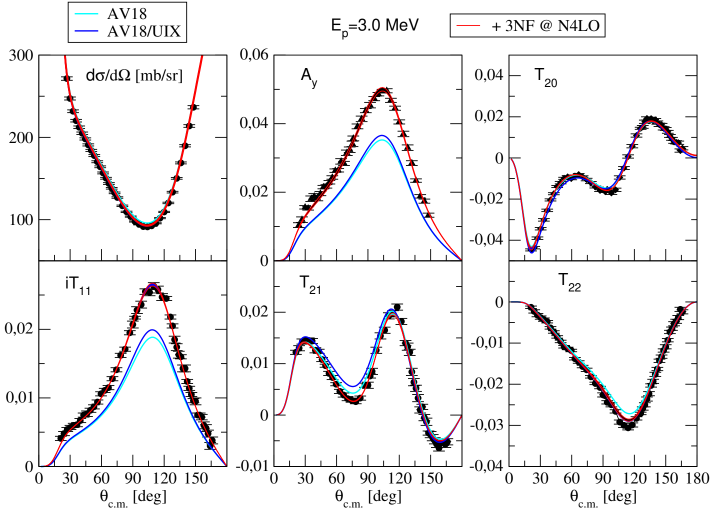
pd scattering at $E_p = 3.0$ MeV

A_y and iT_{11} puzzle



pd scattering at $E_p = 3.0$ MeV

Inclusion of 3NF at N4LO [Girlanda, Kievsky, Marcucci, & MV, 2018], [Witala, Golak, & Skibiński, 2022]

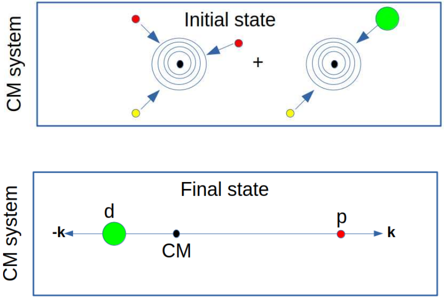


The *pd* wave function for the ALICE experiment

General idea: $\Psi^{(-)} =$ *pd* plane wave + incoming *pd* $+$ *ppn* spherical waves
(with the distortion induced by Coulomb properly taken into account)

In a time-dependet formalism, this corresponds to a state

- For $t \to -\infty$: incoming *pd* $+$ *ppn* spherical waves
- For $t \to +\infty$: *pd* clusters moving with relative impulse $\boldsymbol{k}$



The "free" *pd* wave function

$$\Psi^{(free)}_{m_2,m_1} = \frac{1}{\sqrt{3}} \sum_{ij\ell} \phi_{m_2}(ij)\chi_{m_1}(\ell)\Phi_c(\boldsymbol{k}, \boldsymbol{y}_\ell)$$

$\Phi_c(\boldsymbol{k}, \boldsymbol{y}_\ell)$ Coulomb-distorted plane wave solution of

$$\left[-\frac{1}{2\mu}\nabla^2 + \frac{e^2}{r}\right]\Phi_c(\boldsymbol{k}, \boldsymbol{y}_\ell) = E\Phi_c(\boldsymbol{k}, \boldsymbol{y}_\ell)$$

Using the spherical-wave expansion ($\eta = \frac{\mu e^2}{k}$)

$$\Phi_c(\boldsymbol{k}, \boldsymbol{y}_\ell) = \sum_{LM} 4\pi i^L Y^*_{LM}(\hat{k}) Y_{LM}(\hat{r}) e^{i\sigma_L} \frac{F_L(\eta, ky_\ell)}{ky_\ell}$$

and recoupling spin-angular $\Psi^{(free)}_{m_2,m_1}$ can be written as (here $\hat{k} = \hat{z}$)

$$\Psi^{(free)}_{m_2,m_1} = \sum_{LSJ} \sqrt{4\pi} i^L \sqrt{2L+1} e^{i\sigma_L} (1 m_2 \frac{1}{2} m_1 | S J_z)(L 0 S J_z | J J_z) \Omega^F_{LSJJ_z}$$

The "full" *pd* wave function

Just substitute $\Omega^F_{LSJJ_z}$ with $\Psi^{(-)}_{LSJJ_z}$!

But the effect of the long-range Coulomb interaction may be important also for large L . . .

So one should compute $\Psi^{(-)}_{LSJJ_z}$ for an infinite number of LSJ values . . .

For large values of L (i.e. J) the effect of nuclear interaction is negligible so the substitution $\Omega^F_{LSJJ_z} \to \Psi^{(-)}_{LSJJ_z}$ is adopted only up to $J \leq \overline{J}$

$$\Psi^{(-)}_{LSJJ_z} = \Omega^F_{LSJJ_z} + \overline{\Psi}^{(-)}_{LSJJ_z} \quad J \leq \overline{J} \qquad \Psi^{(-)}_{LSJJ_z} = \Omega^F_{LSJJ_z} \quad J > \overline{J}$$

The terms $\Omega^F_{LSJJ_z}$ can be resummed up to $L \to \infty$, obtaining

$$\Psi^{(full)}_{m_2,m_1} = \Psi^{(free)}_{m_2,m_1} + \widetilde{\Psi}^{(full)}_{m_2,m_1}$$

$$\widetilde{\Psi}^{(full)}_{m_2,m_1} = \sum_{J=\frac{1}{2}}^{\overline{J}} \sum_{LS} \sqrt{4\pi} i^L \sqrt{2L+1} e^{i\sigma_L} (1 m_2 \frac{1}{2} m_1 | S J_z)(L 0 S J_z | J J_z) \overline{\Psi}^{(-)}_{LSJJ_z}$$

Calculation of the *pd* correlation factor

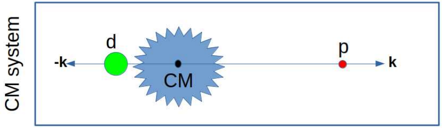
Standard formula

For two-body scattering [Koonin, 1977], [Pratt, 1986]

$$C(k) = \int d^3r\, S_R(r)|\psi(\mathbf{r})|^2 \qquad S_R(r) = \frac{1}{(2\pi R^2)^{\frac{3}{2}}} e^{-\left(\frac{r}{2R}\right)^2}$$

pd formula

- In the *pd* case, it "natural" to identify $r \to y_\ell$
- How to treat the antisymmetrization?



After several discussions with Alejandro., Laura F., Bwawani and many others

$$C_{pd}(k) = \frac{1}{6}\sum_{m_2,m_1} {\color{red}3} \sum_{m_2',m_1'} \int d^3y_3\, S_R(y_3)|\psi_{m_2',m_1',m_2,m_1}(k,\mathbf{y}_3)|^2 \qquad (1)$$

$$\psi_{m_2',m_1',m_2,m_1}(k,\mathbf{y}_3) = \int d^3x_3 \left[\phi_{m_2'}(1,2)\chi_{m_1'}(3)\right]^\dagger \Psi^{(full)}_{m_2,m_1}$$

$\phi(1,2) =$ deuteron wave function, $\chi(3) =$ spin state of the proton
namely, we "project" the full wave function onto the deuteron wave function
Suggestions well accepted!!!

Discussion (1)

Source $S_R(y_3)$, then

1. It is “natural” to apply the formula using an “effective” $p-d$ relative wave function
2. The detector “naturally” project onto a $p-d$ deuteron state
3. Mrówczyński’s formulation (for simplicitly here I neglect spin-isopin d.o.f)

$$C_{pd}(k)A_2 = \int d^3r_1 d^3r_2 d^3r_3\, S_{R_M}(r_1)S_{R_M}(r_2)S_{R_M}(r_3)|\Psi_{pd}|^2 \qquad (*)$$

$A_2 =$ “formation rate” of the deuteron, defined as by ($\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$)

$$A_2 = \int d^3r_1 d^3r_2\, S_{R_M}(r_1)S_{R_M}(r_2)|\phi_d(\mathbf{r})|^2 = \int d^3r\, S_{\sqrt{2}R_M}(r)|\phi_d(\mathbf{r})|^2$$

Mrówczyński assumes moreover $\Psi_{pd} \equiv \phi_d(\mathbf{r})\psi_{pd}(\mathbf{y}_3)$
Then, substituting in (*), integrating over the 3body CM coordinate, one obtains

$$C_{pd}(k)A_2 = \int d^3r\, d^3y_3\, S_{\sqrt{2}R_M}(r)S_{\sqrt{\frac{3}{2}}R_M}(y_3)|\phi_d(\mathbf{r})|^2|\psi_{pd}(\mathbf{y}_3)|^2$$

Integrating over d^3r, the A_2 term cancel out and we find again expression (1) with $R = \sqrt{\frac{3}{2}}R_M$

Discussion (1)

Source $S_R(y_3)$, then

1. It is "natural" to apply the formula using an "effective" $p-d$ relative wave function
2. The detector "naturally" project onto a $p-d$ deuteron state
3. Mrówczyński's formulation (for simplicitly here I neglect spin-isopin d.o.f)

$$C_{pd}(k)A_2 = \int d^3r_1 d^3r_2 d^3r_3 \, S_{R_M}(r_1)S_{R_M}(r_2)S_{R_M}(r_3)|\Psi_{pd}|^2 \qquad (*)$$

A_2 = "formation rate" of the deuteron, defined as by ($\boldsymbol{r} = \boldsymbol{r}_2 - \boldsymbol{r}_1$)

$$A_2 = \int d^3r_1 d^3r_2 \, S_{R_M}(r_1)S_{R_M}(r_2)|\phi_d(\boldsymbol{r})|^2 = \int d^3r \, S_{\sqrt{2}R_M}(r)|\phi_d(\boldsymbol{r})|^2$$

Mrówczyński assumes moreover $\Psi_{pd} \equiv \phi_d(\boldsymbol{r})\psi_{pd}(\boldsymbol{y}_3)$
Then, substituting in (*), integrating over the 3body CM coordinate, one obtains

$$C_{pd}(k)A_2 = \int d^3r \, d^3y_3 \, S_{\sqrt{2}R_M}(r)S_{\sqrt{\frac{3}{2}}R_M}(y_3)|\phi_d(\boldsymbol{r})|^2|\psi_{pd}(\boldsymbol{y}_3)|^2$$

Integrating over d^3r, the A_2 term cancel out and we find again expression (1) with $R = \sqrt{\frac{3}{2}}R_M$

Discussion (1)

Source $S_R(y_3)$, then

1. It is "natural" to apply the formula using an "effective" $p-d$ relative wave function
2. The detector "naturally" project onto a $p-d$ deuteron state
3. Mrówczyński's formulation (for simplicitly here I neglect spin-isopin d.o.f)

$$C_{pd}(k)A_2 = \int d^3r_1 d^3r_2 d^3r_3 \, S_{R_M}(r_1)S_{R_M}(r_2)S_{R_M}(r_3)|\Psi_{pd}|^2 \qquad (*)$$

A_2 = "formation rate" of the deuteron, defined as by ($\boldsymbol{r} = \boldsymbol{r}_2 - \boldsymbol{r}_1$)

$$A_2 = \int d^3r_1 d^3r_2 \, S_{R_M}(r_1)S_{R_M}(r_2)|\phi_d(\boldsymbol{r})|^2 = \int d^3r \, S_{\sqrt{2}R_M}(r)|\phi_d(\boldsymbol{r})|^2$$

Mrówczyński assumes moreover $\Psi_{pd} \equiv \phi_d(\boldsymbol{r})\psi_{pd}(\boldsymbol{y}_3)$
Then, substituting in (*), integrating over the 3body CM coordinate, one obtains

$$C_{pd}(k)A_2 = \int d^3r \, d^3y_3 \, S_{\sqrt{2}R_M}(r)S_{\sqrt{\frac{3}{2}}R_M}(y_3)|\phi_d(\boldsymbol{r})|^2|\psi_{pd}(\boldsymbol{y}_3)|^2$$

Integrating over d^3r, the A_2 term cancel out and we find again expression (1) with $R = \sqrt{\frac{3}{2}}R_M$

Discussion (1)

Source $S_R(y_3)$, then

1. It is "natural" to apply the formula using an "effective" $p-d$ relative wave function
2. The detector "naturally" project onto a $p-d$ deuteron state
3. Mrówczyński's formulation (for simplicitly here I neglect spin-isopin d.o.f)

$$C_{pd}(k)A_2 = \int d^3r_1 d^3r_2 d^3r_3 \, S_{R_M}(r_1)S_{R_M}(r_2)S_{R_M}(r_3)|\Psi_{pd}|^2 \qquad (*)$$

A_2 = "formation rate" of the deuteron, defined as by ($\boldsymbol{r} = \boldsymbol{r}_2 - \boldsymbol{r}_1$)

$$A_2 = \int d^3r_1 d^3r_2 \, S_{R_M}(r_1)S_{R_M}(r_2)|\phi_d(\boldsymbol{r})|^2 = \int d^3r \, S_{\sqrt{2}R_M}(r)|\phi_d(\boldsymbol{r})|^2$$

Mrówczyński assumes moreover $\Psi_{pd} \equiv \phi_d(\boldsymbol{r})\psi_{pd}(\boldsymbol{y}_3)$
Then, substituting in (*), integrating over the 3body CM coordinate, one obtains

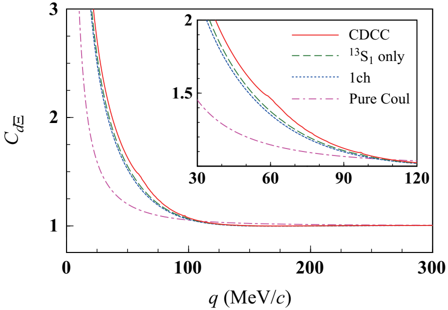
$$C_{pd}(k)A_2 = \int d^3r \, d^3y_3 \, S_{\sqrt{2}R_M}(r)S_{\sqrt{\frac{3}{2}}R_M}(y_3)|\phi_d(\boldsymbol{r})|^2|\psi_{pd}(\boldsymbol{y}_3)|^2$$

Integrating over d^3r, the A_2 term cancel out and we find again expression (1) with $R = \sqrt{\frac{3}{2}}R_M$

Discussion (2)

Calculation for $d\Xi$ by [Ogata *et al.*, 2021]

1. CDCC calculation of $\Psi_{d\Xi}(\boldsymbol{x}_3, \boldsymbol{y}_3)$ (here particle 3 is the K-meson)
2. $\Psi_{d\Xi}(\boldsymbol{x}_3, \boldsymbol{y}_3)$ projected over all *np* states (bound+scattering)



Effect of the scattering states rather small

Discussion (3)

Projection on the *pn* scattering states

Schematically (spin-isospin d.o.f. neglected)
$\phi_\alpha =$ all the $p - n$ states ($\alpha = 0$ deuteron, etc)

$$C_{pd}(k) \sim \sum_\alpha \int d^3y_3 \; S_R(y_3) |\langle \phi_\alpha(1,2) | \Psi_{pd} \rangle|^2$$

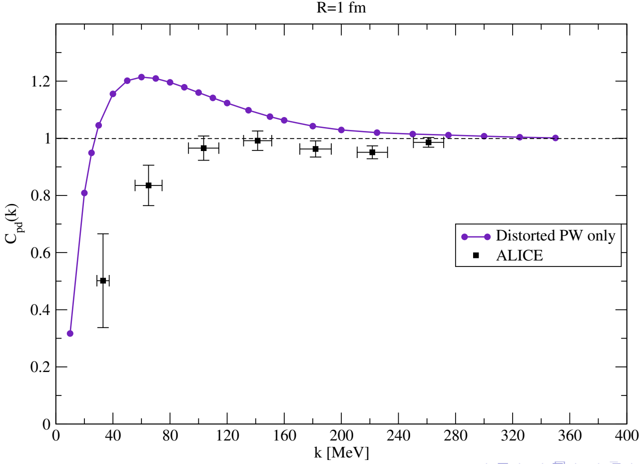
Using closure

$$C_{pd}(k) \sim \int d^3y_3 \; S_R(y_3) |\Psi_{pd}|^2$$

$\Psi_{pd} \sim \phi_d(1,2) e^{i\mathbf{k}\cdot\mathbf{y}_3} + \phi_d(2,3) e^{i\mathbf{k}\cdot\mathbf{y}_1} + \phi_d(3,1) e^{i\mathbf{k}\cdot\mathbf{y}_2}$
it can be shown that for $k \to \infty$, $C_{pd}(k) \neq 1$

Convergence

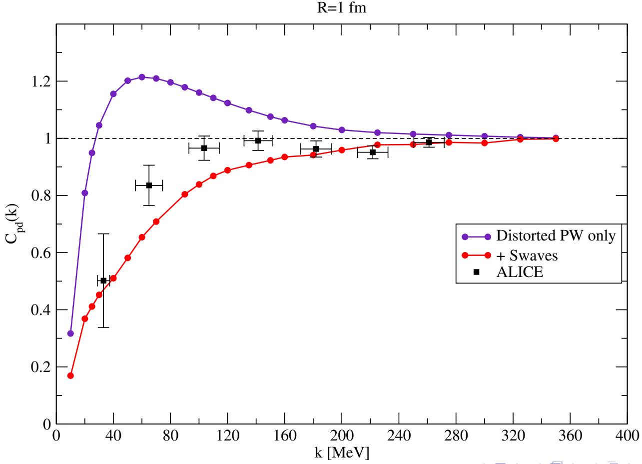
Inclusion of $\Psi^{(free)}_{m_2,m_1}$ and the various components of $\widetilde{\Psi}^{(full)}_{m_2,m_1}$



Results (1)

Convergence

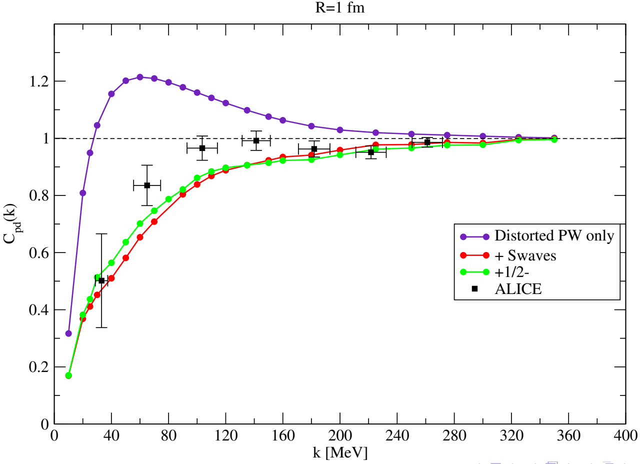
Inclusion of $\Psi^{(free)}_{m_2,m_1}$ and the various components of $\widetilde{\Psi}^{(full)}_{m_2,m_1}$



Results (1)

Convergence

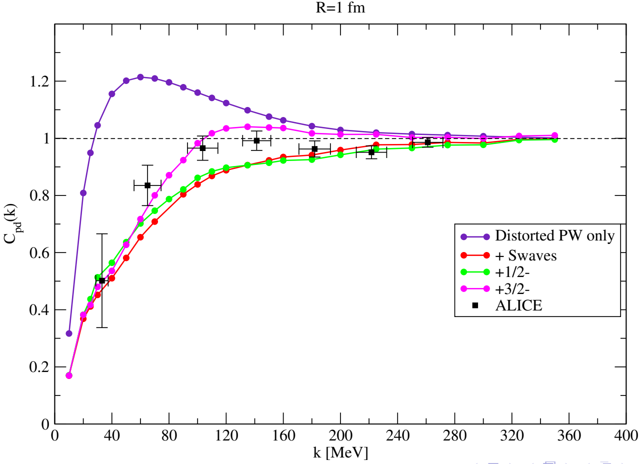
Inclusion of $\Psi^{(free)}_{m_2,m_1}$ and the various components of $\widetilde{\Psi}^{(full)}_{m_2,m_1}$



Results (1)

Convergence

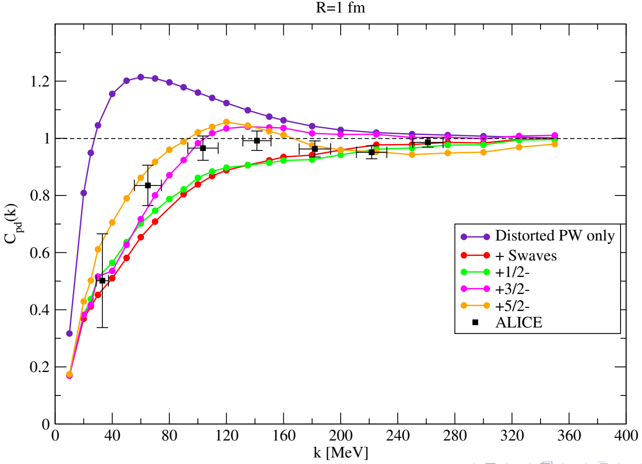
Inclusion of $\Psi^{(free)}_{m_2,m_1}$ and the various components of $\widetilde{\Psi}^{(full)}_{m_2,m_1}$



Results (1)

Convergence

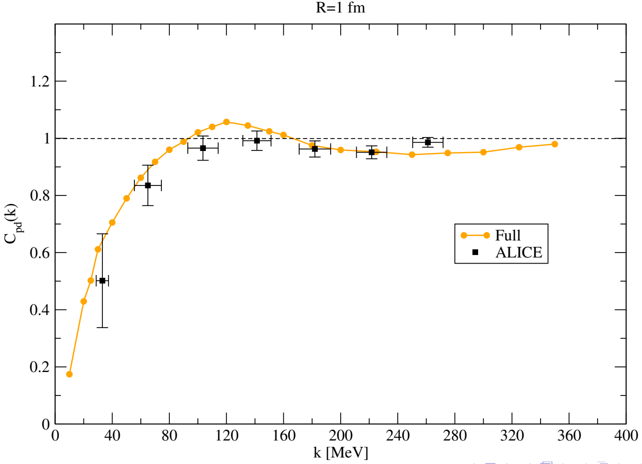
Inclusion of $\Psi^{(free)}_{m_2,m_1}$ and the various components of $\widetilde{\Psi}^{(full)}_{m_2,m_1}$



Results (1)

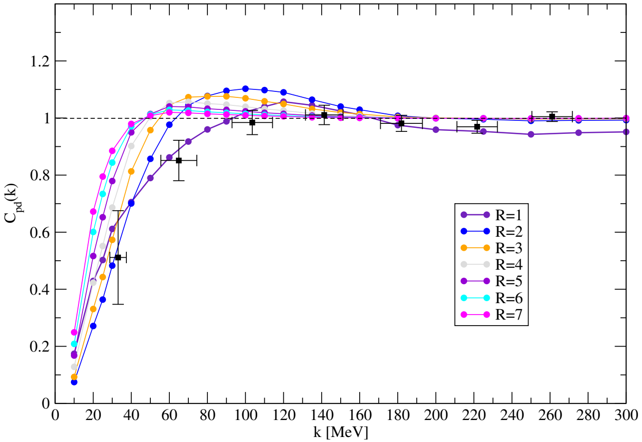
Convergence

Inclusion of $\Psi^{(free)}_{m_2,m_1}$ and the various components of $\widetilde{\Psi}^{(full)}_{m_2,m_1}$



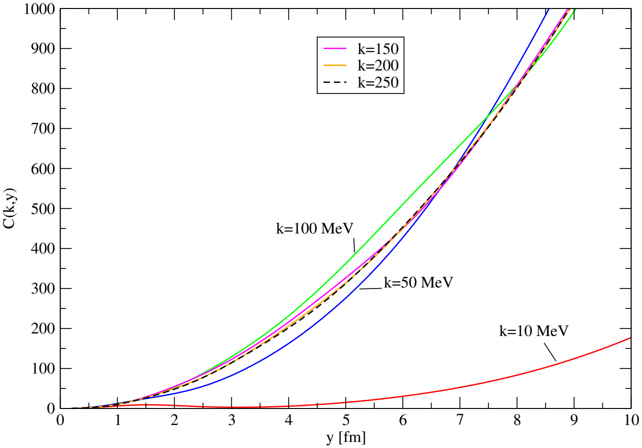
Results (2)

Dependence on R



Results (3)

Functions $C_{pd}(k,y)$: $C_{pd}(k) = \int_0^\infty dy\ S_R(y) C_{pd}(k,y)$

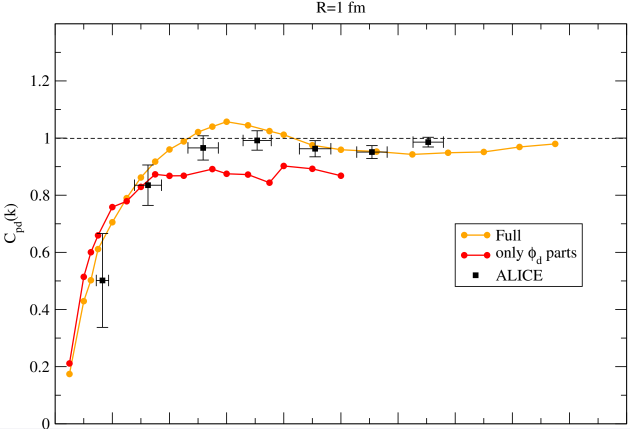


Results (4)

Importance of the deuteron component

Reduce the wave function to:

$$\Psi^{(-)}_{LSJJ_z} \to \Omega^F_{LS} + \sum_{L'S'} \mathcal{T}^{(-),J}_{LS,L'S'} \left[\Omega^G_{L'S'} - i\Omega^F_{L'S'}\right]$$



Conclusions and perspectives

Analysis of *pd* correlation factor

- "projection" formula
- Contribution of P-waves very significant
- Good agreement between the theoretical and the experimental data for $R \approx 1$ fm

Perspectives

- Calculation with other interactions
- Comparison with the calculations by S. Koenig
- More tests:
  - study the effect of the "free" part
  - inclusion of more J states (increase $\bar{J}$)
- Applications for $p - {}^3\mathrm{He}$ and $d - d$:
  - $p - {}^3\mathrm{He}$: [Bazak& Mrówczyński, 2020]
  - $d - d$: [Mrówczyński & Slon, 2021]
  - HH calculations [MV *et al*, 2020 & 2022]
- Applications for *pppt* and $pp\Lambda$: See Alejandro's talk

Thank you for your attention!