Quantum algorithms for the simulation of perturbative QCD processes

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Workshop: *Toward Quantum Advantage in High Energy Physics*
MIAPbP, Munich, 20th April 2023

Based on arXiv:2303.04818
In collaboration with Mathieu Pellen

Introduction

- Quantum simulation: a flagship application of quantum computers
- Recent years: proposals for quantum simulation of lattice QFTs (e.g. lattice QCD)
- Quantum simulation of perturbative QCD remains largely unexplored

Introduction

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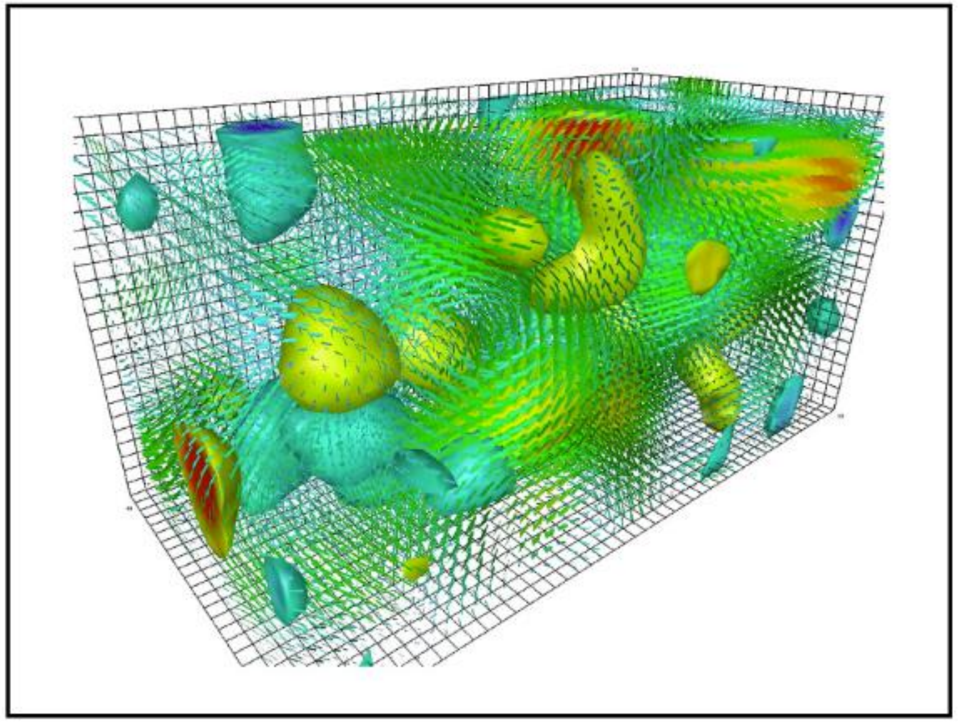
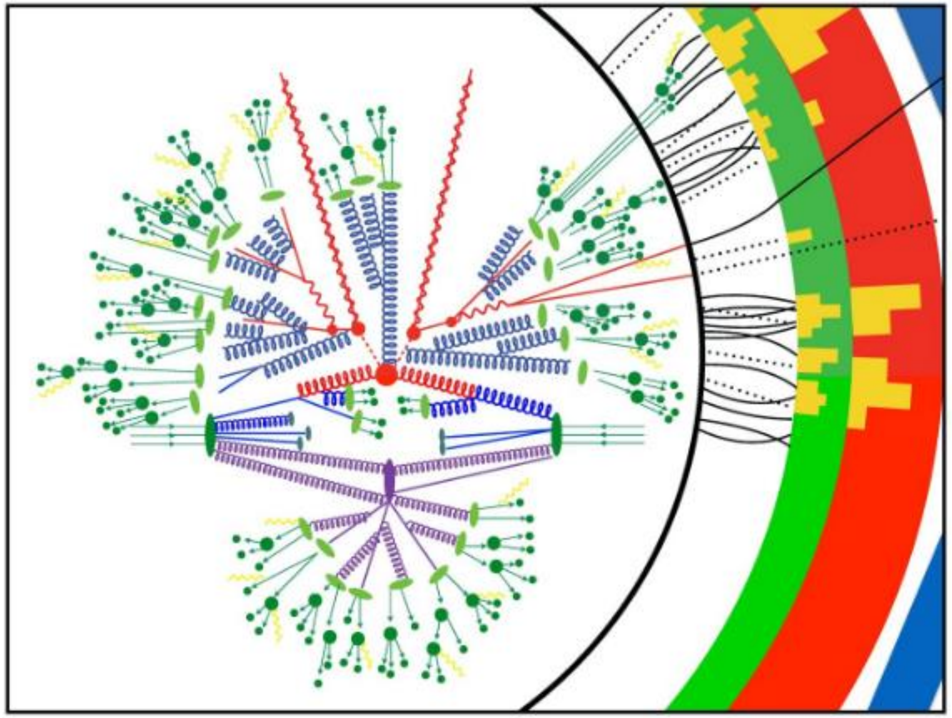
Slide from yesterday's opening talk

SIMULATING HEP FROM FRIST-PRINCIPLES



* Z. Davoudi presents.

MODELING AND INTERPRETING HEP EXPERIMENTS



* M. Spannowsky presents.

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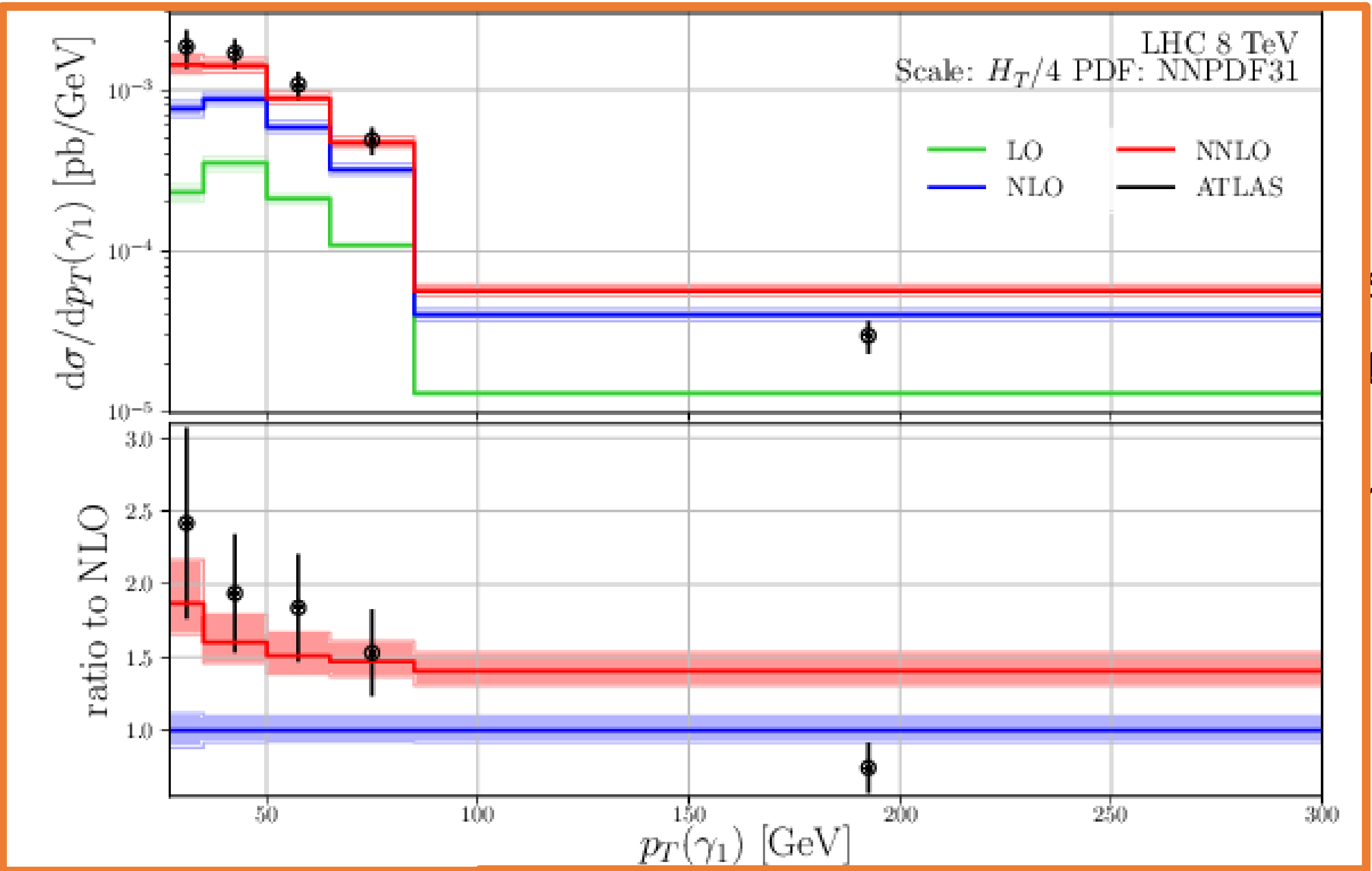


Edited figure from Chawdhry, Czakon, Mitov, Poncelet (2019)

- 10-20 years per order in a_s
- high precision/accuracy

Introduction

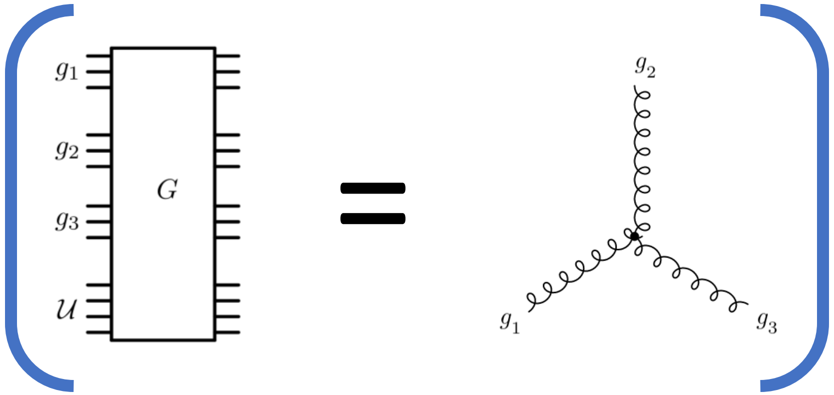
- Quantum simulation: a flagship application of quantum computers
- Recent years: proposals for quantum simulation of lattice QFTs (e.g. lattice QCD)
- Quantum simulation of perturbative QCD remains largely unexplored
  - Notable exception: several papers on parton showers
- This talk: first steps towards generic perturbative QCD processes
  - Quantum simulation of colour in perturbative QCD

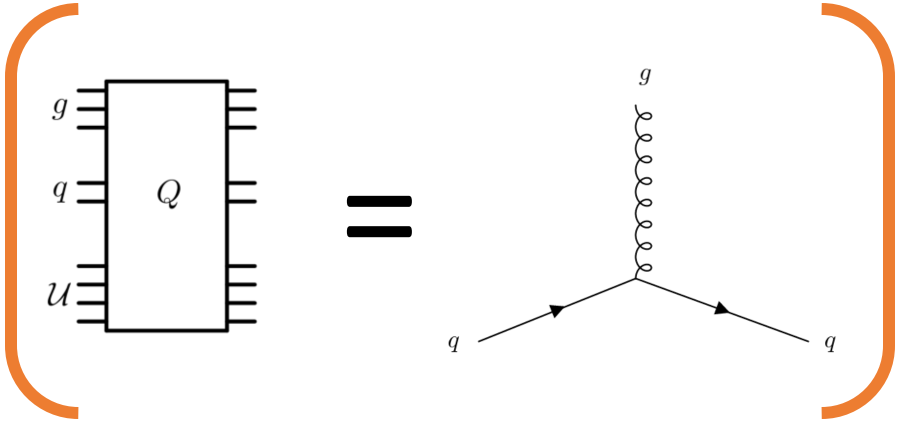
Motivation

1. Perturbative QCD requires quantum-coherent combination of contributions from many unobservable intermediate states
   - natural candidate to exploit superpositions of quantum states in quantum computers
2. Processes with high-multiplicity final states, with full interference effects
3. Improve speed/precision of perturbative QCD predictions by exploiting known quantum algorithms
   - e.g. quantum amplitude estimation; quantum Monte Carlo

Key results of this work

- Two quantum gates (*G* and *Q*) to simulate colour parts of the interactions of quarks and gluons



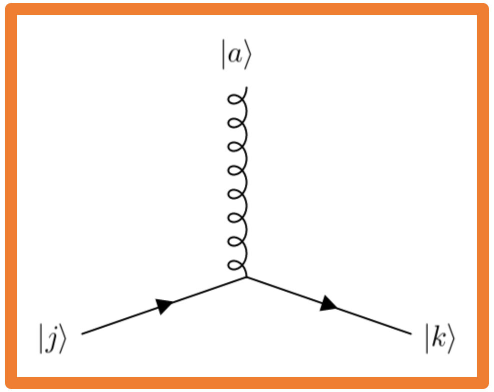


- Detailed construction of the gates: see our paper (arXiv:2303.04818)
- This short talk: will give illustrative example of application of these gates

Methods

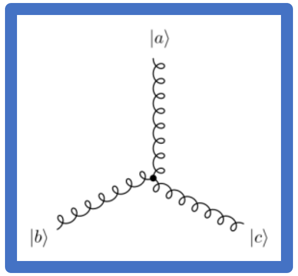
- Quark colours: represented by 2 qubits ($2^2 = 4$ basis states, of which 1 is unused)
- Gluon colours: represented by 3 qubits ($2^3 = 8$ basis states)
- Quark-gluon interaction gate is designed such that

$$Q\,|a\rangle_g\,|k\rangle_q\,|\Omega\rangle_{\mathcal{U}} = \sum_{j=1}^{3} T^a_{jk}\,|a\rangle_g\,|j\rangle_q\,|\Omega\rangle_{\mathcal{U}} + (\text{terms orthogonal to } |\Omega\rangle_{\mathcal{U}})$$



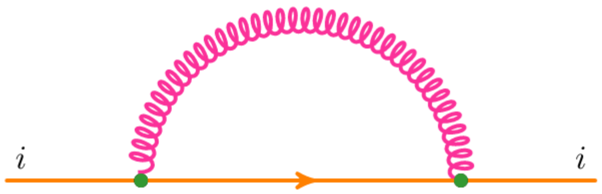
- Triple-gluon interaction gate is designed such that

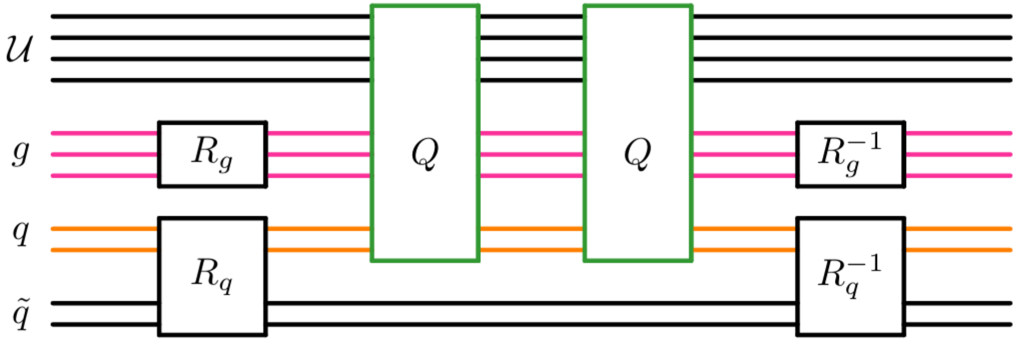
$$G\,|a\rangle_{g_1}\,|b\rangle_{g_2}\,|c\rangle_{g_3}\,|\Omega\rangle_{\mathcal{U}} = f^{abc}\,|a\rangle_{g_1}\,|b\rangle_{g_2}\,|c\rangle_{g_3}\,|\Omega\rangle_{\mathcal{U}} + (\text{terms orthogonal to } |\Omega\rangle_{\mathcal{U}})$$



- Note: $|\Omega\rangle_{\mathcal{U}}$ is a reference state of a "Unitarisation register", which we introduce because in SU(3), T^a_{jk} and f^{abc} are non-unitary.

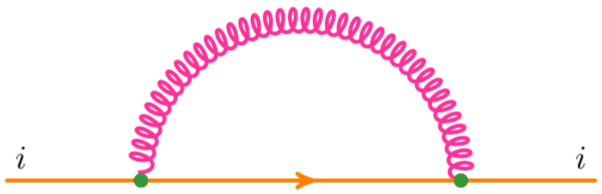
Calculating colour factors: illustrative example

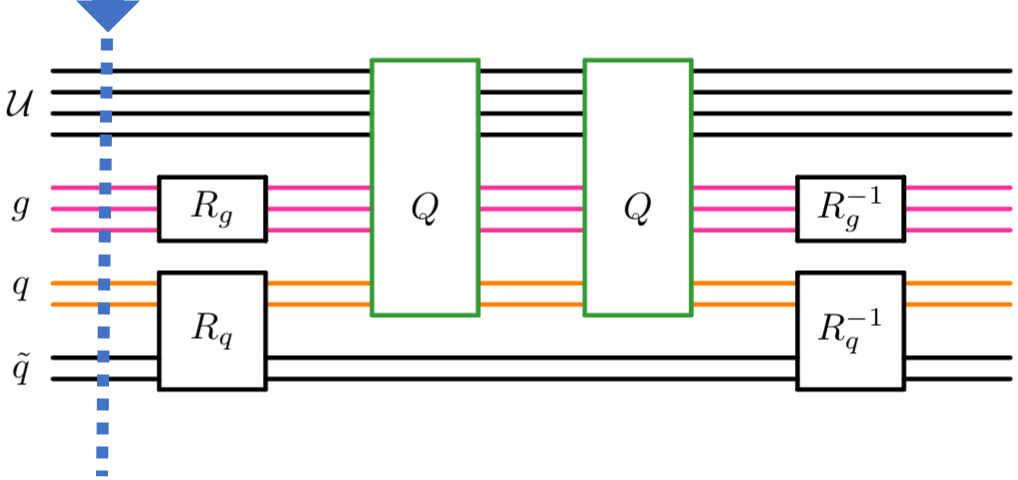




Calculating colour factors: illustrative example

$$|\Omega\rangle_g \, |\Omega\rangle_q \, |\Omega\rangle_{\tilde{q}} \, |\Omega\rangle_{\mathcal{U}}$$





Calculating colour factors: illustrative example

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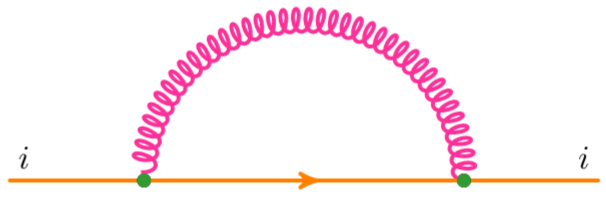
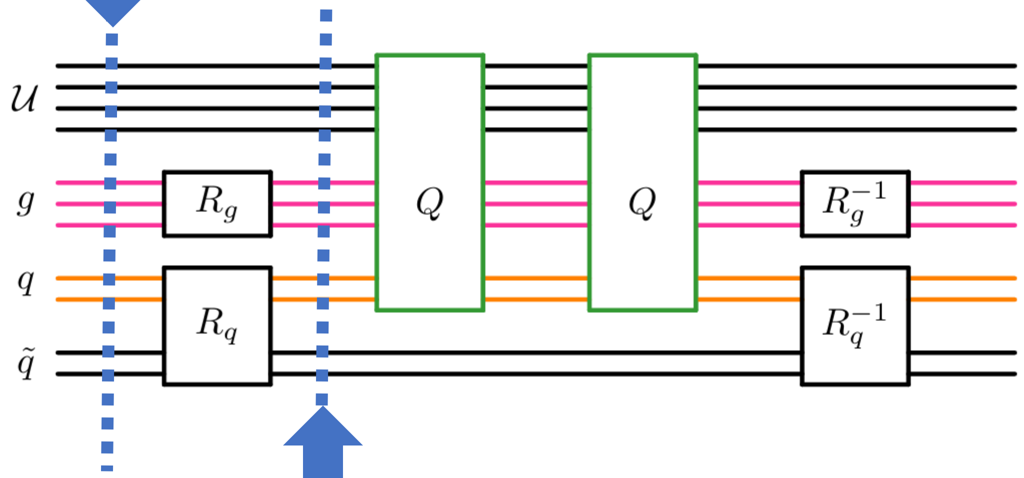


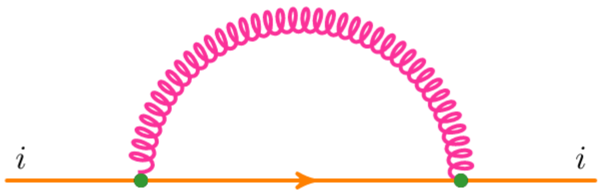


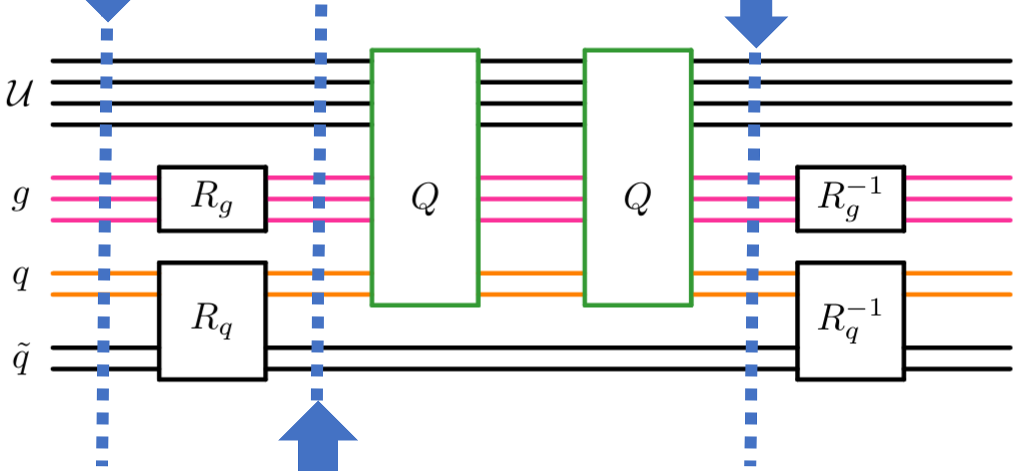
$$\frac{1}{\sqrt{24}} \sum_{a=1}^{8} \sum_{k=1}^{3} |a\rangle_g \, |k\rangle_q \, |k\rangle_{\tilde{q}} \, |\Omega\rangle_{\mathcal{U}}$$

Calculating colour factors: illustrative example

$$|\Omega\rangle_g\,|\Omega\rangle_q\,|\Omega\rangle_{\tilde{q}}\,|\Omega\rangle_{\mathcal{U}}$$

$$\frac{1}{\sqrt{24}}\sum_{\substack{a\in\{1,\ldots,8\}\\ i,j,k\in\{1,2,3\}}} T^a_{ij}T^a_{jk}\,|a\rangle_g\,|i\rangle_q\,|k\rangle_{\tilde{q}}\,|\Omega\rangle_{\mathcal{U}} + (\text{terms orthogonal to } |\Omega\rangle_{\mathcal{U}})$$





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Calculating colour factors: illustrative example

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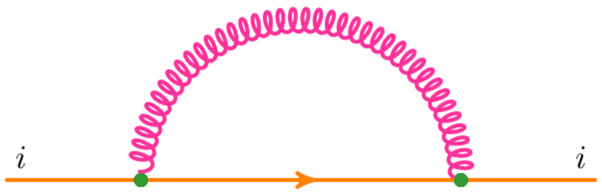
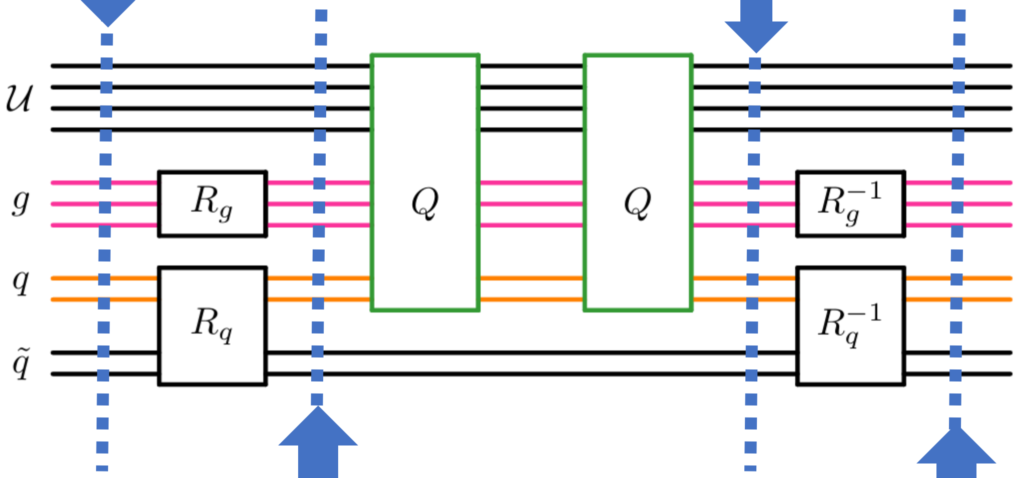




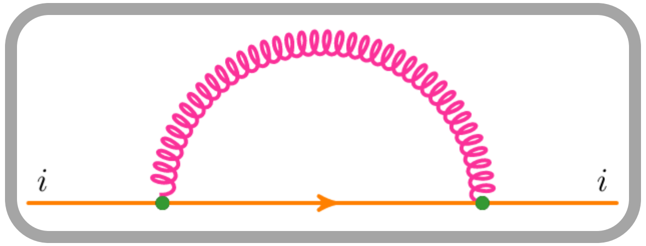
$$\frac{1}{\sqrt{24}} \sum_{a=1}^{8} \sum_{k=1}^{3} |a\rangle_g |k\rangle_q |k\rangle_{\tilde{q}} |\Omega\rangle_{\mathcal{U}}$$

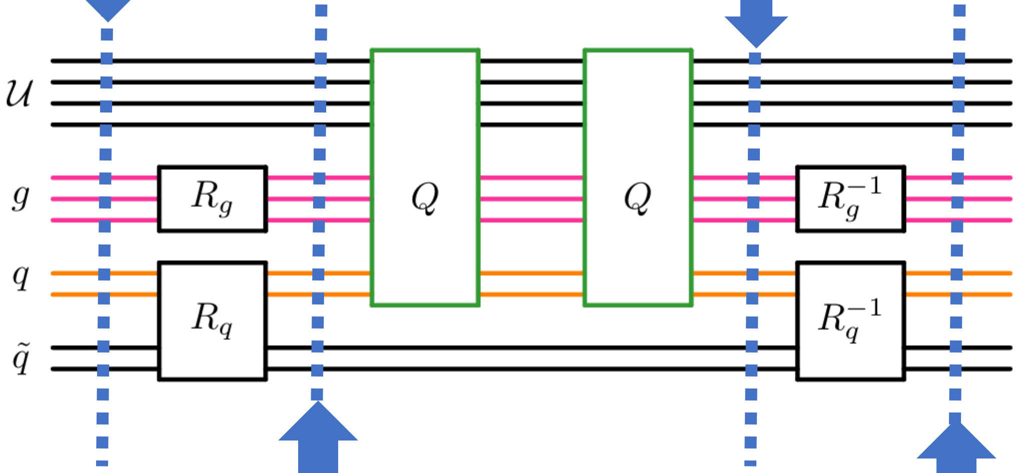
$$\frac{1}{24} \left(\sum_{\substack{a\in\{1,\dots,8\}\\ i,j\in\{1,2,3\}}} T^a_{ij} T^a_{ji} \right) |\Omega\rangle_g |\Omega\rangle_q |\Omega\rangle_{\tilde{q}} |\Omega\rangle_{\mathcal{U}} + \left(\text{terms orthogonal to } |\Omega\rangle_g |\Omega\rangle_q |\Omega\rangle_{\tilde{q}} |\Omega\rangle_{\mathcal{U}}\right)$$

Calculating colour factors: illustrative example

$$|\Omega\rangle_g |\Omega\rangle_q |\Omega\rangle_{\tilde{q}} |\Omega\rangle_{\mathcal{U}}$$

$$\frac{1}{\sqrt{24}} \sum_{\substack{a\in\{1,\dots,8\}\\ i,j,k\in\{1,2,3\}}} T^a_{ij} T^a_{jk} |a\rangle_g |i\rangle_q |k\rangle_{\tilde{q}} |\Omega\rangle_{\mathcal{U}} + (\text{terms orthogonal to } |\Omega\rangle_{\mathcal{U}})$$





$$\frac{1}{\sqrt{24}} \sum_{a=1}^{8} \sum_{k=1}^{3} |a\rangle_g |k\rangle_q |k\rangle_{\tilde{q}} |\Omega\rangle_{\mathcal{U}}$$

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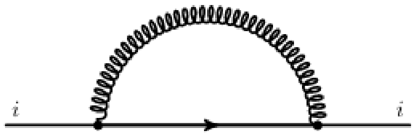
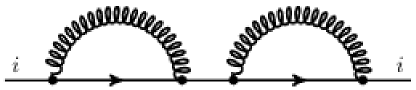
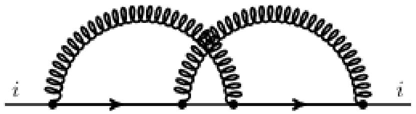
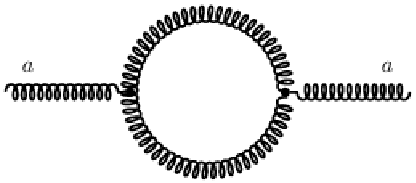
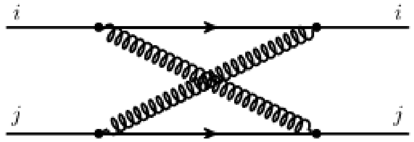
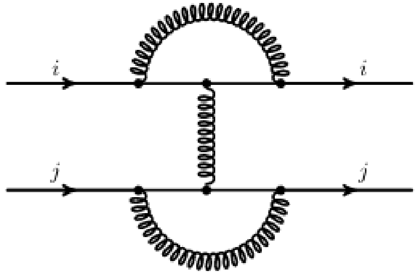
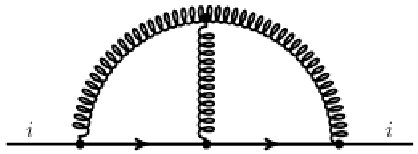
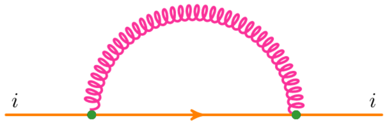
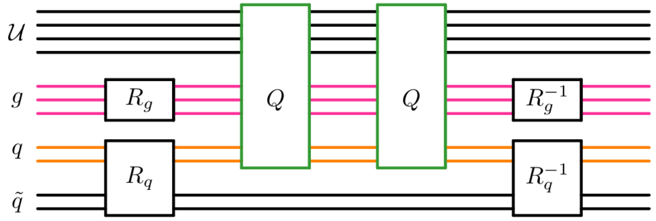
Validation

- Implemented using Qiskit (IBM)
- Simulated various diagrams
  - Simulated noiseless quantum computer
    - These examples use up to 30 qubits
  - Ran each diagram 10^8 times
  - Measured output to infer colour factor
  - Full agreement with analytic expectation
- Circuits are relatively small:
  - #qubits is linear in #quarks + #gluons
  - #gates is linear in #vertices

Diagram	Analytical	Numerical
	$C_F N = 4$	3.9988 ± 0.0012
	${C_F}^2 N = \frac{16}{3}$	5.331 ± 0.010
	$\frac{C_F}{2} = \frac{2}{3}$	0.673 ± 0.010
	$N(N^2-1) = 24$	23.95 ± 0.03
	$\frac{(N^2-1)}{4} = 2$	2.00 ± 0.03
	0	$0.0^{+0.5}_{-0.0}$
	$\frac{C_F N^2}{2} = 6$	5.92 ± 0.08

Summary and outlook





- Designed quantum circuits to simulate colour part of perturbative QCD
  - Example application: colour factors for arbitrary Feynman diagrams
  - First step towards a full quantum simulation of generic perturbative QCD processes
- Natural avenues for follow-up work:
  - Kinematic parts of Feynman diagrams
  - Interference of multiple Feynman diagrams
  - Use in a quantum Monte Carlo calculation of cross-sections
    - Quadratic speed-up over classical Monte Carlo