Quantum Mechanics to Quantum Darwinism

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Sep 2022

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Introduction

- usual axioms of quantum mechanics:
  - unitary evolution - time reversible
  - wavefunction collapse - non reversible
- avoid wavefunction collapse?[1]
- emergence of an objective classical reality[2]

[1] Ron Garret. *Quantum Mysteries Disentangled*. 2001 (latest revision 2016).

[2] Wojciech H. Zurek. "Quantum Darwinism, classical reality, and the randomness of quantum jumps". In: *Physics Today 67*, 10, 44 (2014).

Outline

Motivation

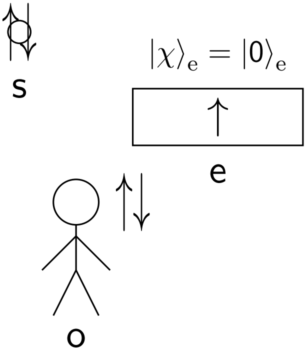
- electron spin measured
- three participants: system, observer and environment (no-deletion[3])
- system, observer and environment form a closed system
- system and observer entangled, environment absorbs observer state

[3] Samuel L. Braunstein Arun Kumar Pati. “Impossibility of deleting an unknown quantum state”. In: *Nature, 404: 164* (2000).

Motivation (A Schematic)

$$|\psi\rangle_{\mathrm{s}} = \psi_0 |0\rangle_{\mathrm{s}} + \psi_1 |1\rangle_{\mathrm{s}}$$

$$|\chi\rangle_{\mathrm{e}} = |0\rangle_{\mathrm{e}}$$



$$|\phi\rangle_{\mathrm{o}} = \phi_0 |0\rangle_{\mathrm{o}} + \phi_1 |1\rangle_{\mathrm{o}}$$

$$|\psi\rangle_{\mathrm{s}}, |\phi\rangle_{\mathrm{o}}, |\chi\rangle_{\mathrm{e}} \in \mathcal{H} = \mathrm{span}\{|0\rangle, |1\rangle\}$$

Motivation (The Measurement Procedure)

overall measurement procedure:

$$\begin{aligned} |\psi\rangle_{\mathrm{s}} |\phi\rangle_{\mathrm{o}} |\chi\rangle_{\mathrm{e}} &= (\psi_0 |0\rangle + \psi_1 |1\rangle)_{\mathrm{s}} (\phi_0 |0\rangle + \phi_1 |1\rangle)_{\mathrm{o}} |0\rangle_{\mathrm{e}} \\ &\to (\psi_0 |00\rangle + \psi_1 |11\rangle)_{\mathrm{so}} (\phi_0 |0\rangle + \phi_1 |1\rangle)_{\mathrm{e}} \\ &=: |\Psi\rangle_{\mathrm{so}} |\phi\rangle_{\mathrm{e}} \end{aligned}$$

$$\begin{aligned} |000\rangle_{\mathrm{soe}} &\to |000\rangle_{\mathrm{soe}} \\ |010\rangle_{\mathrm{soe}} &\to |001\rangle_{\mathrm{soe}} \\ |100\rangle_{\mathrm{soe}} &\to |110\rangle_{\mathrm{soe}} \\ |110\rangle_{\mathrm{soe}} &\to |111\rangle_{\mathrm{soe}} \end{aligned}$$

Motivation (Extension to Basis)

a natural unitary extension:

$$
\begin{aligned}
|001\rangle_{\mathrm{soe}} &\to |010\rangle_{\mathrm{soe}} \\
|011\rangle_{\mathrm{soe}} &\to |011\rangle_{\mathrm{soe}} \\
|101\rangle_{\mathrm{soe}} &\to |100\rangle_{\mathrm{soe}} \\
|111\rangle_{\mathrm{soe}} &\to |101\rangle_{\mathrm{soe}}
\end{aligned}
$$

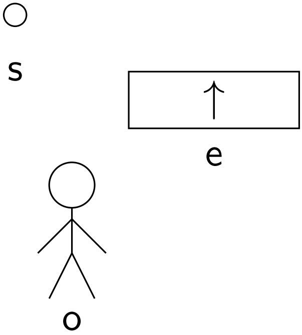
$$
|\psi\rangle_{\mathrm{s}} |\phi\rangle_{\mathrm{o}} |1\rangle_{\mathrm{e}} \to (\psi_0 |01\rangle + \psi_1 |10\rangle)_{\mathrm{so}} |\phi\rangle_{\mathrm{e}} =: |\bar{\Psi}\rangle_{\mathrm{so}} |\phi\rangle_{\mathrm{e}}
$$

Motivation (Generic State)

generic environment state:

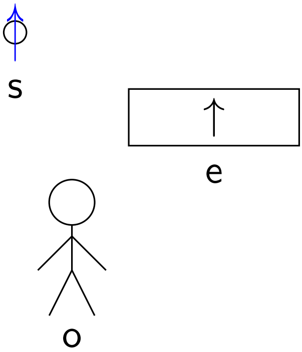
$$
\begin{aligned}
|\psi\rangle_{\mathrm{s}} |\phi\rangle_{\mathrm{o}} |\chi\rangle_{\mathrm{e}} &= (\psi_0 |0\rangle + \psi_1 |1\rangle)_{\mathrm{s}} (\phi_0 |0\rangle + \phi_1 |1\rangle)_{\mathrm{o}} (\chi_0 |0\rangle + \chi_1 |1\rangle)_{\mathrm{e}} \\
&\to (\chi_0 |\Psi\rangle_{\mathrm{so}} + \chi_1 |\bar{\Psi}\rangle_{\mathrm{so}}) |\phi\rangle_{\mathrm{e}}
\end{aligned}
$$

Motivation (Summarising)



North points up

Motivation (Summarising)



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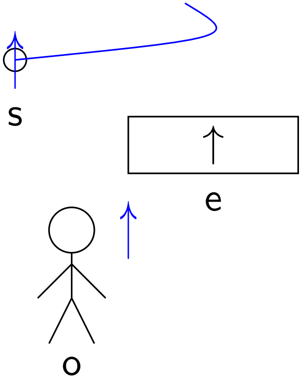

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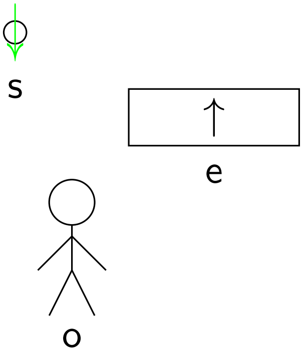

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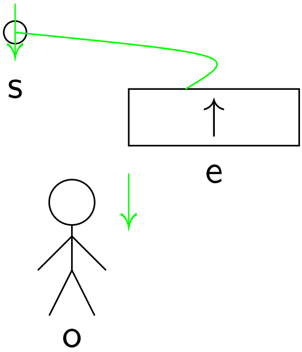

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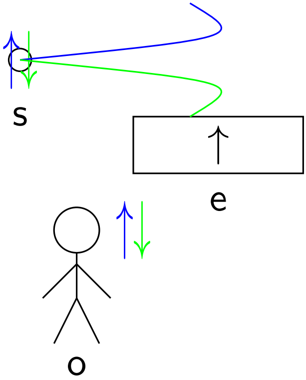

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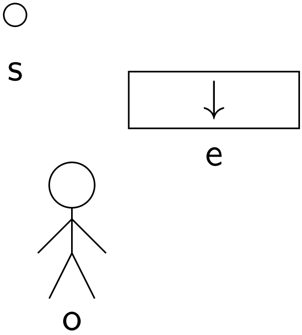

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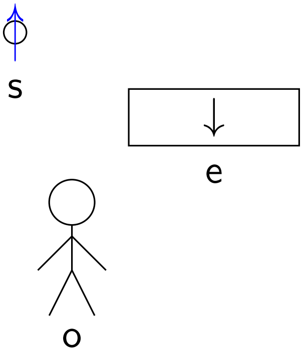
Motivation (Summarising)



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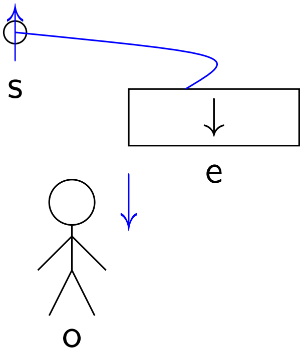

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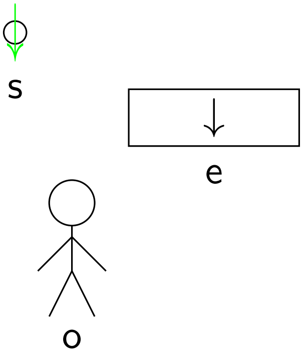

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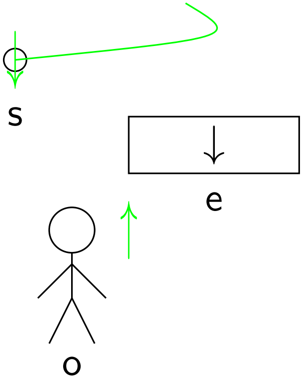

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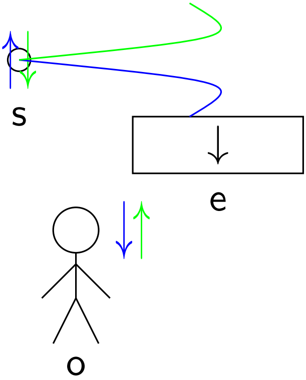

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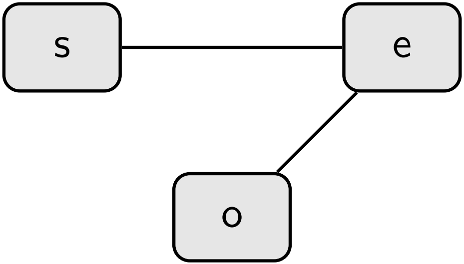
Local Operators



Local Operators (Imprint)

$\mathcal{I}_{\mathrm{a}\to\mathrm{b}}$:

$$\begin{aligned}
\mathcal{I}_{\mathrm{a}\to\mathrm{b}} |00\rangle_{\mathrm{ab}} &= |00\rangle_{\mathrm{ab}} \\
\mathcal{I}_{\mathrm{a}\to\mathrm{b}} |01\rangle_{\mathrm{ab}} &= |01\rangle_{\mathrm{ab}} \\
\mathcal{I}_{\mathrm{a}\to\mathrm{b}} |10\rangle_{\mathrm{ab}} &= |11\rangle_{\mathrm{ab}} \\
\mathcal{I}_{\mathrm{a}\to\mathrm{b}} |11\rangle_{\mathrm{ab}} &= |10\rangle_{\mathrm{ab}}
\end{aligned}$$

$\equiv$ CNOT gate
in condensed notation:

$$\mathcal{I}_{\mathrm{a}\to\mathrm{b}} |i\rangle_{\mathrm{a}} |j\rangle_{\mathrm{b}} = |i\rangle_{\mathrm{a}} |j+i\rangle_{\mathrm{b}}$$

Local Operators (Swap)

$\mathcal{S}_{\mathrm{a}\leftrightarrow\mathrm{b}}$:

$$\begin{aligned}
\mathcal{S}_{\mathrm{a}\leftrightarrow\mathrm{b}} |00\rangle_{\mathrm{ab}} &= |00\rangle_{\mathrm{ab}} \\
\mathcal{S}_{\mathrm{a}\leftrightarrow\mathrm{b}} |01\rangle_{\mathrm{ab}} &= |10\rangle_{\mathrm{ab}} \\
\mathcal{S}_{\mathrm{a}\leftrightarrow\mathrm{b}} |10\rangle_{\mathrm{ab}} &= |01\rangle_{\mathrm{ab}} \\
\mathcal{S}_{\mathrm{a}\leftrightarrow\mathrm{b}} |11\rangle_{\mathrm{ab}} &= |11\rangle_{\mathrm{ab}}
\end{aligned}$$

$\equiv$ SWAP gate
in condensed notation:

$$\mathcal{I}_{\mathrm{a}\rightarrow\mathrm{b}} |i\rangle_{\mathrm{a}} |j\rangle_{\mathrm{b}} = |j\rangle_{\mathrm{a}} |i\rangle_{\mathrm{b}}$$

Local Operators

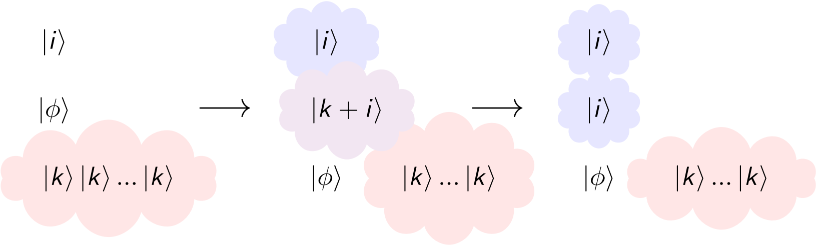
$$
\begin{array}{ccccc}
 & \mathcal{I}_{s\to e} & & \mathcal{S}_{o\leftrightarrow e} & \\
|000\rangle_{\text{soe}} & \longrightarrow & |000\rangle_{\text{soe}} & \longrightarrow & |000\rangle_{\text{soe}} \\
|010\rangle_{\text{soe}} & \longrightarrow & |010\rangle_{\text{soe}} & \longrightarrow & |001\rangle_{\text{soe}} \\
|100\rangle_{\text{soe}} & \longrightarrow & |101\rangle_{\text{soe}} & \longrightarrow & |110\rangle_{\text{soe}} \\
|110\rangle_{\text{soe}} & \longrightarrow & |111\rangle_{\text{soe}} & \longrightarrow & |111\rangle_{\text{soe}} \\
|001\rangle_{\text{soe}} & \longrightarrow & |001\rangle_{\text{soe}} & \longrightarrow & |010\rangle_{\text{soe}} \\
|011\rangle_{\text{soe}} & \longrightarrow & |011\rangle_{\text{soe}} & \longrightarrow & |011\rangle_{\text{soe}} \\
|101\rangle_{\text{soe}} & \longrightarrow & |100\rangle_{\text{soe}} & \longrightarrow & |100\rangle_{\text{soe}} \\
|111\rangle_{\text{soe}} & \longrightarrow & |110\rangle_{\text{soe}} & \longrightarrow & |101\rangle_{\text{soe}}
\end{array}
$$

Learning the Environment

starting with "classical" environment:

$$|\chi\rangle_{\mathrm{e}} = \sum_k \chi_k |k\rangle_{\mathrm{e1}} |k\rangle_{\mathrm{e2}} |k\rangle_{\mathrm{e3}} \dots |k\rangle_{\mathrm{eN}}$$

perfect entanglement obtained



clouds indicate entanglement!

Learning the Environment (The Procedure)

$$\mathcal{S}_{o \leftrightarrow e_1} \circ \mathcal{I}_{s \to e_1}$$

$$\begin{aligned} \mathrm{s} &: |i\rangle_{\mathrm{s}} \\ \mathrm{o} &: |\phi\rangle_{\mathrm{o}} \\ \mathrm{e} &: |k\rangle_{\mathrm{e_1}} \, |k\rangle_{\mathrm{e_2}} \ldots |k\rangle_{\mathrm{e_N}} \end{aligned}$$

$$\hookrightarrow$$

$$\begin{aligned} \mathrm{s} &: |i\rangle_{\mathrm{s}} \\ \mathrm{o} &: |k+i\rangle_{\mathrm{o}} \\ \mathrm{e} &: |\phi\rangle_{\mathrm{e_1}} \, |k\rangle_{\mathrm{e_2}} \ldots |k\rangle_{\mathrm{e_N}} \end{aligned}$$

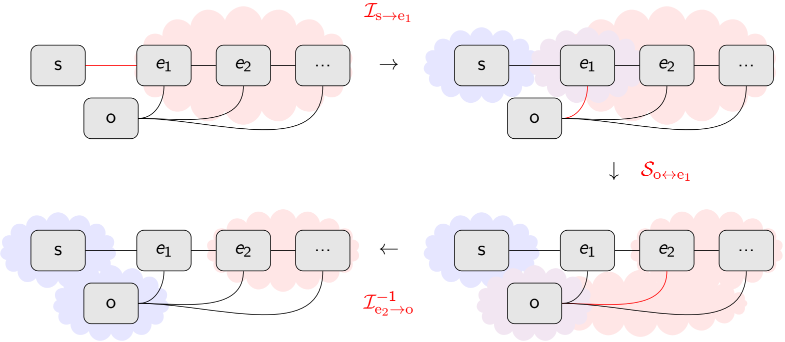
Learning the Environment (The Procedure)

$$\mathcal{I}^{-1}_{e_2 \to o} \quad \begin{aligned} &\mathrm{s} : |i\rangle_{\mathrm{s}} \\ &\mathrm{o} : |k+i\rangle_{\mathrm{o}} \\ &\mathrm{e} : |\phi\rangle_{\mathrm{e_1}} \, |k\rangle_{\mathrm{e_2}} \ldots |k\rangle_{\mathrm{e_N}} \end{aligned}$$

$$\hookrightarrow \quad \begin{aligned} &\mathrm{s} : |i\rangle_{\mathrm{s}} \\ &\mathrm{o} : |k+i-k\rangle_{\mathrm{o}} \\ &\mathrm{e} : |\phi\rangle_{\mathrm{e_1}} \, |k\rangle_{\mathrm{e_2}} \ldots |k\rangle_{\mathrm{e_N}} \end{aligned}$$

$$= \quad \begin{aligned} &\mathrm{s} : |i\rangle_{\mathrm{s}} \\ &\mathrm{o} : |i\rangle_{\mathrm{o}} \\ &\mathrm{e} : |\phi\rangle_{\mathrm{e_1}} \, |k\rangle_{\mathrm{e_2}} \ldots |k\rangle_{\mathrm{e_N}} \end{aligned}$$

Summary of Measurement Procedure



process of entangling system and observer to achieve measurement

Summary of Measurement Procedure

- perfect entanglement between system and observer
- classical environment required for entanglement
- wavefunction also contains counterfactual branches but proliferation is local

Multiple Observers

$$\sum_i \psi_i \left|i\right\rangle_{\mathrm{s}} \left|\phi_1\right\rangle_{\mathrm{o}_1} \left|\phi_2\right\rangle_{\mathrm{o}_2} \left|\phi_3\right\rangle_{\mathrm{o}_3} \ldots^{4}$$

$$\xrightarrow{\text{o}_1\text{ measures}} \sum_i \psi_i \left|i\right\rangle_{\mathrm{s}} \left|i\right\rangle_{\mathrm{o}_1} \left|\phi_2\right\rangle_{\mathrm{o}_2} \left|\phi_3\right\rangle_{\mathrm{o}_3} \ldots$$

$$\xrightarrow{\text{o}_2\text{ measures}} \sum_i \psi_i \left|i\right\rangle_{\mathrm{s}} \left|i\right\rangle_{\mathrm{o}_1} \left|i\right\rangle_{\mathrm{o}_2} \left|\phi_3\right\rangle_{\mathrm{o}_3} \ldots$$

$$\xrightarrow{\text{o}_3\text{ measures}} \left|\Psi\right\rangle_{\mathrm{so}_1\mathrm{o}_2\ldots} = \sum_i \psi_i \left|i\right\rangle_{\mathrm{s}} \left|i\right\rangle_{\mathrm{o}_1} \left|i\right\rangle_{\mathrm{o}_2} \left|i\right\rangle_{\mathrm{o}_3} \ldots$$

$\implies$ objective classical reality!
only the states in this basis “survive”
superposition states “perish”

[4] the set of observers now forms the environment

Mutual Information

starting from:

$$|\Psi\rangle_{\mathrm{so_1o_2\ldots}} = \sum_i \psi_i \, |i\rangle_{\mathrm{s}} \, |i\rangle_{\mathrm{o_1}} \, |i\rangle_{\mathrm{o_2}} \, |i\rangle_{\mathrm{o_3}} \ldots$$

s, $\mathrm{o_1}$, $\mathrm{o_2}$... all contain the same information!

$$\rho_{\mathrm{I}} = \sum_i |\psi_i|^2 \, |i\rangle \langle i|_{\mathrm{I}}, \mathrm{I} \in \{\mathrm{s}, \mathrm{o_1}, \mathrm{o_2}, \ldots\}$$

indeed this is true even for other subsets of systems:

$$\rho_{\mathrm{I}} = \sum_i |\psi_i|^2 \, |ii(i)\rangle \langle ii(i)|_{\mathrm{I}}, \mathrm{I} \in \{\mathrm{so_1}, \mathrm{so_2}, \mathrm{so_1(o_2)}, \ldots\}$$

$$\Longrightarrow \mathsf{H}_{\mathrm{I}} = -\,\mathrm{tr}(\rho_{\mathrm{I}} \ln(\rho_{\mathrm{I}})), \mathrm{I} \in \{\mathrm{s}, \mathrm{o_1}, \mathrm{o_2}, \mathrm{so_1}, \mathrm{so_2}, \mathrm{so_1o_2}, \ldots\}$$

however, the whole system is pure

$$\rho_{\mathrm{so_1o_2\ldots}} = |\Psi\rangle \langle \Psi|_{\mathrm{so_1o_2\ldots}}$$

$$\Longrightarrow \mathsf{H}_{\mathrm{so_1o_2\ldots}} = 0$$

Mutual Information

$$M_{I:J} := H_I + H_J - H_{IJ}$$

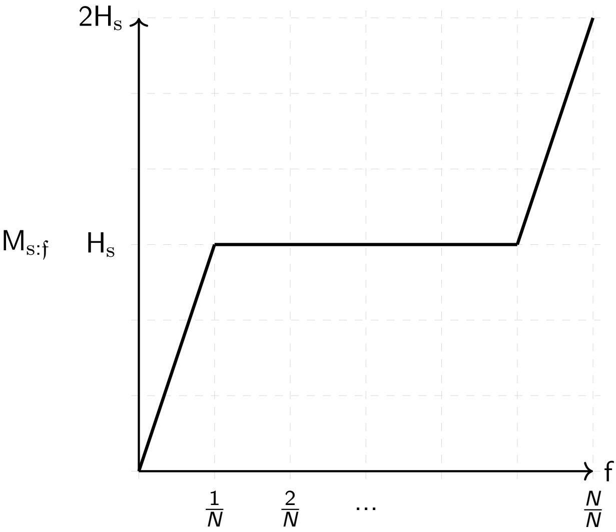
for example:

$$\begin{aligned} M_{s:o_1o_2} &= H_s + H_{o_1o_2} - H_{so_1o_2} \\ &= H_s + H_s - H_s = H_s \end{aligned}$$

because s and o_1o_2 are just copies of each other!
for the entire system

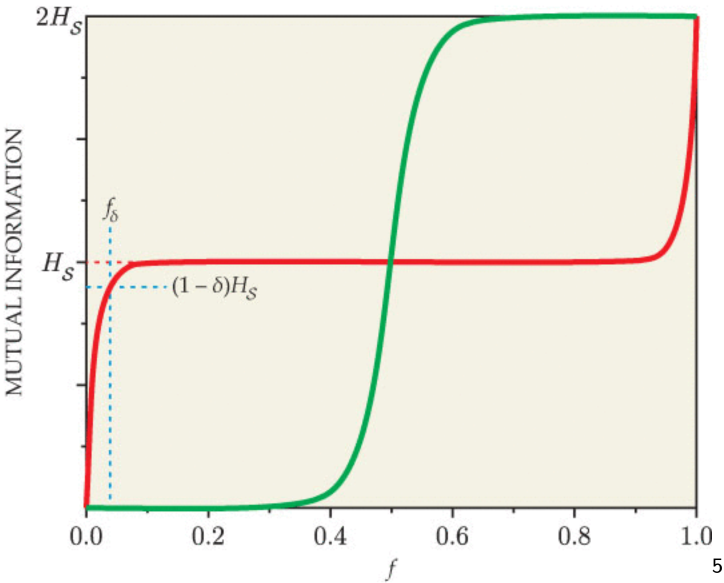
$$M_{s:o_1o_2\ldots} = H_s + H_{o_1o_2\ldots} - H_{so_1o_2\ldots} = 2H_s$$

Mutual Information



Mutual information between system and a fraction of the observers, $M_{s:\mathfrak{f}}$*,*
as a function of the fraction of observers, f.

Mutual Information



[5]

[5]Image from Zurek, "Quantum Darwinism, classical reality, and the randomness of quantum jumps".

Remarks

- states in the "good" basis survive and those in superposition states perish
- multiple observers agree on what is reality making it objective
- how does this basis get selected?

Thanks

Thanks for listening!

Measurement in a different Basis

consider $\left\{|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle, |-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right\}$
o_1 measures s in $\{0, 1\}$ basis:

$$\psi_0 |0\rangle_\mathrm{s} + \psi_1 |1\rangle_\mathrm{s} \to \psi_0 |0\rangle_\mathrm{s} |0\rangle_{\mathrm{o}_1} + \psi_1 |1\rangle_\mathrm{s} |1\rangle_{\mathrm{o}_1}$$

o_2 measures s in $\{+, -\}$ basis:

$$\begin{aligned} &\to |+\rangle_\mathrm{s} \frac{1}{\sqrt{2}} \left(\psi_0 |0\rangle_{\mathrm{o}_1} + \psi_1 |1\rangle_{\mathrm{o}_1}\right) |+\rangle_{\mathrm{o}_2} \\ &+ |-\rangle_\mathrm{s} \frac{1}{\sqrt{2}} \left(\psi_0 |0\rangle_{\mathrm{o}_1} - \psi_1 |1\rangle_{\mathrm{o}_1}\right) |-\rangle_{\mathrm{o}_2} \end{aligned}$$

density matrix for o_2 is:

$$\rho_{\mathrm{o}_2} = \frac{1}{2} |+\rangle \langle +|_{\mathrm{o}_2} + \frac{1}{2} |-\rangle \langle -|_{\mathrm{o}_2}$$

Measurement in a different Basis

o_3' measures o_1 in $\{+,-\}$ basis:

$$\stackrel{\text{quantum } \mathrm{o}_1}{\rightarrow} \frac{1}{2}\left[(\psi_0+\psi_1)\left|+\right\rangle_{\mathrm{s}}\left|+\right\rangle_{\mathrm{o}_2}+(\psi_0-\psi_1)\left|-\right\rangle_{\mathrm{s}}\left|-\right\rangle_{\mathrm{o}_2}\right]\left|+\right\rangle_{\mathrm{o}_1}\left|+\right\rangle_{\mathrm{o}_3'}$$
$$\frac{1}{2}\left[(\psi_0-\psi_1)\left|+\right\rangle_{\mathrm{s}}\left|+\right\rangle_{\mathrm{o}_2}+(\psi_0+\psi_1)\left|-\right\rangle_{\mathrm{s}}\left|-\right\rangle_{\mathrm{o}_2}\right]\left|-\right\rangle_{\mathrm{o}_1}\left|-\right\rangle_{\mathrm{o}_3'}$$

$\Longrightarrow$ correlations destroyed! no objective reality!