Spectral reconstruction: brief intro

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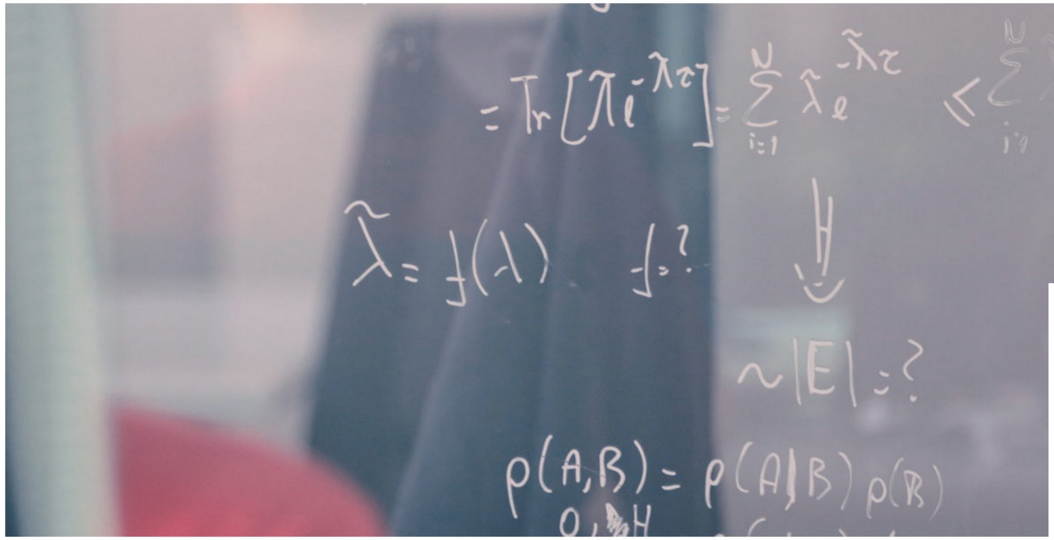
Munich, Feb 2023

https://www.ectstar.eu/activities/workshops/

MACHINE LEARNING FOR LATTICE FIELD THEORY AND BEYOND



26 June 2023 — 30 June 2023

ECT* - Villa Tambosi

Strada delle Tabarelle, 286

Trento - Italy

Registration

Registration available from 26/04/2023 until 31/05/2023.

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Spectral functions in QFT

- plethora of propagators (or in general n-point functions):

  Feynman, retarded, Wightman, Euclidean, …

- choice depends on physical problem:

  scattering, linear response, non-equilibrium, computable, …

- typically related by dispersion relation or analytical continuation

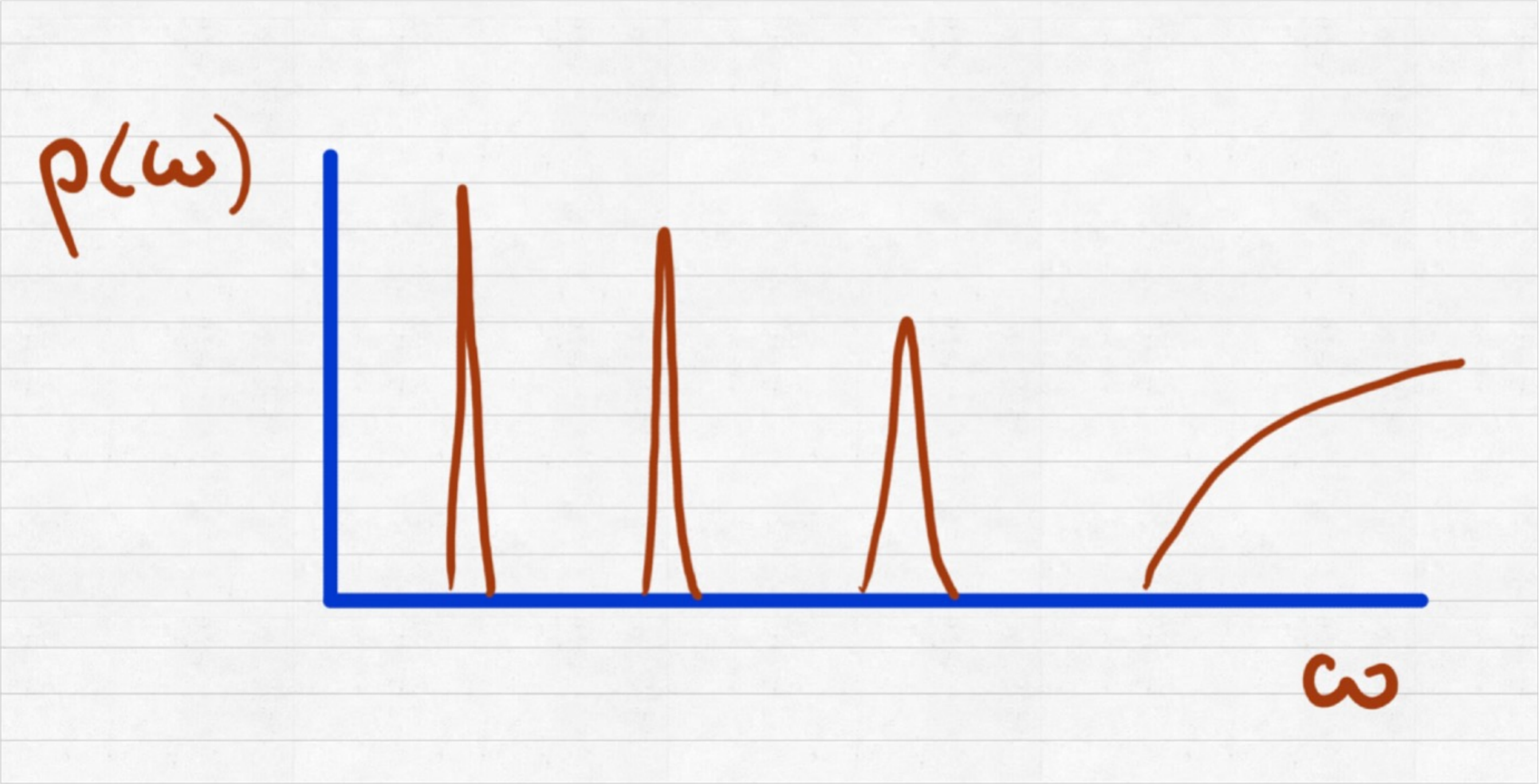
$$G^R(\omega) = \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{\rho(\omega')}{\omega' - \omega - i\epsilon} \qquad G^E(\omega_n) = \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{\rho(\omega')}{\omega' - i\omega_n}$$

Spectral functions play a special role

- clear exposition of physics, especially in thermal context

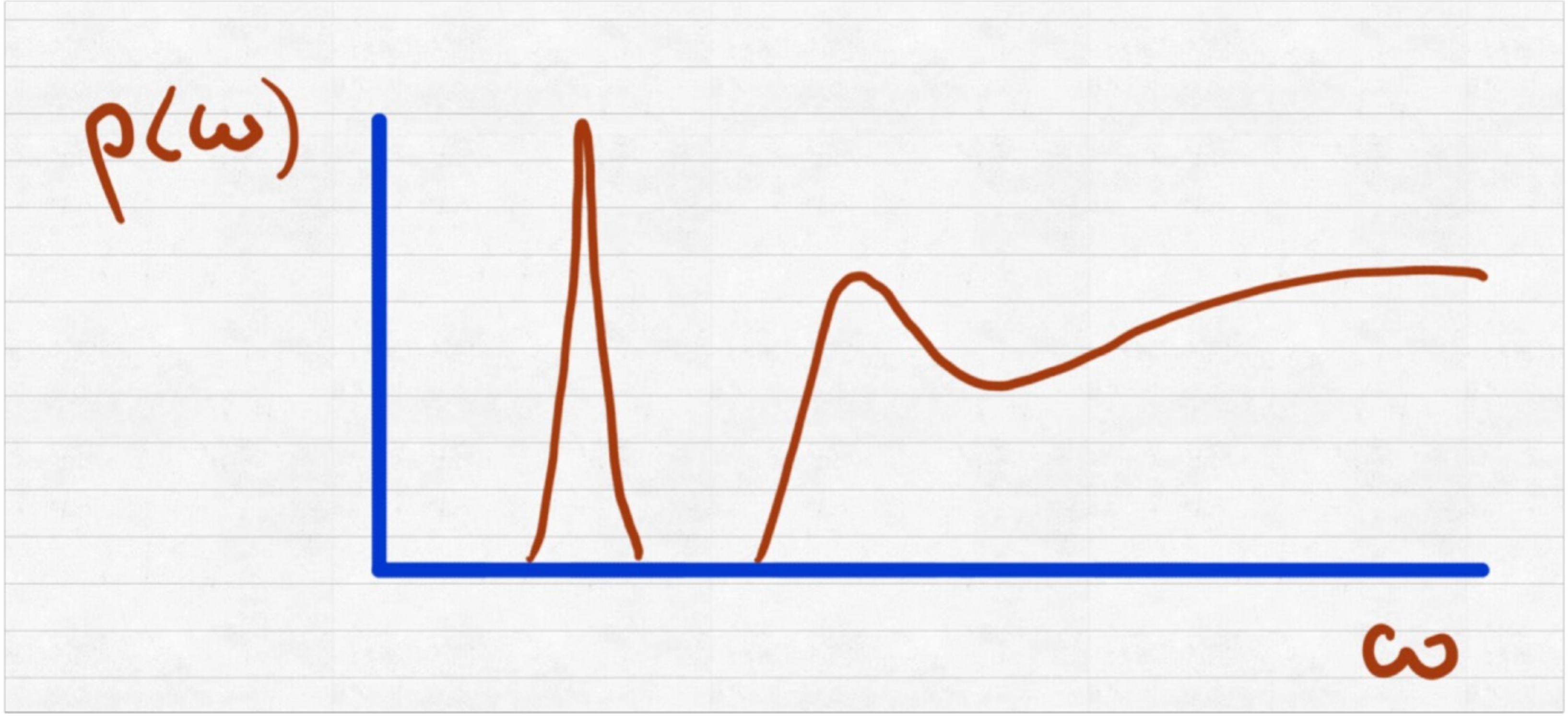
Spectral functions display physical context

- spectral quantities at low temperature



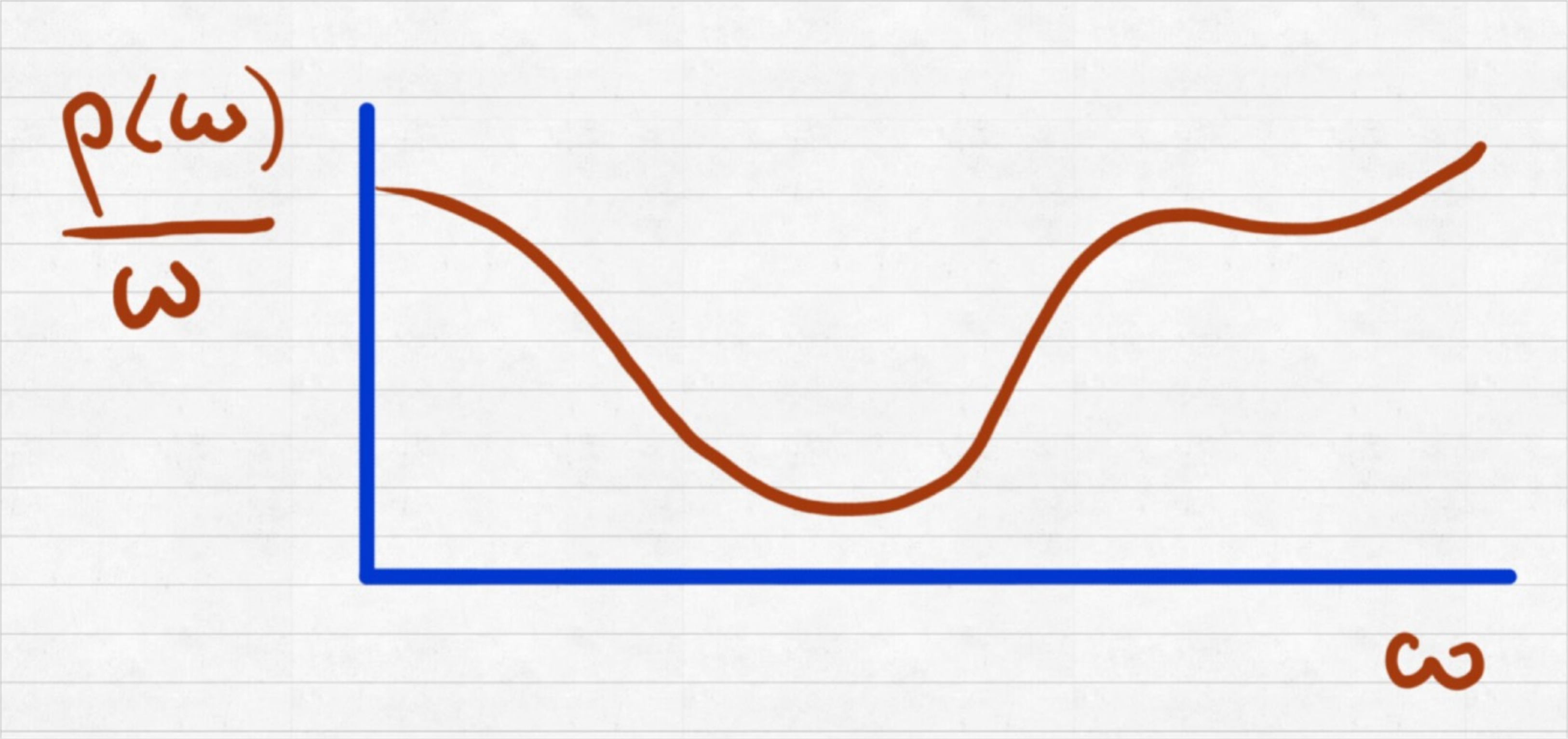
Spectral functions display physical context

- spectral quantities at higher temperature



Spectral functions display physical context

- transport properties at small frequencies



Spectral function reconstruction

- lattice QCD/QFT computes Euclidean correlator $G(\tau)$ $\quad (0 \leq \tau < 1/T)$

- dispersion relation expressed as integral equation $$G(\tau) = \int d\omega\, K(\tau, \omega) \rho(\omega)$$

- with kernel $$K(\tau, \omega) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)} = e^{-\omega\tau} + \text{p.b.c}$$ (bosons)

- associated pair: $G(\tau) \longleftrightarrow \rho(\omega)$

Spectral function reconstruction (numerically)

- given $\rho(\omega)$: computation of $G(\tau)$ is easy

$$G(\tau) = \int d\omega\, K(\tau, \omega)\rho(\omega)$$

- given $G(\tau)$: computation of $\rho(\omega)$ is hard, ill-posed inversion problem

standard argument:

- $G(\tau)$ known numerically at $\mathrm{O}(16 - 64)$ points
- $\rho(\omega)$ in principle continuous function, with sharp and broad structures
- integral over kernel washes out information

Spectral function reconstruction (numerically)

$$G(\tau) = \int d\omega\, K(\tau, \omega)\rho(\omega)$$

matrix problem: discretise

- $0 \leq \tau < N_\tau\,,\ 0 \leq \omega < N_\omega$: $\qquad G_\tau = \sum_\omega K_{\tau\omega}\, \rho_\omega$
- $K_{\tau\omega}$ *very* rectangular matrix, ill-conditioned
- how to invert, regularise, …?

Longstanding problem in many fields

- Bayesian inference and the analytic continuation of imaginary-time quantum Monte Carlo data, M. Jarrell, J.E. Gubernatis, *Phys. Rept.* 269 (1996) 133

- Maximum entropy analysis of the spectral functions in lattice QCD, M. Asakawa, T. Hatsuda, Y. Nakahara, *Prog. Part. Nucl. Phys.* 46 (2001) 459

- Maximum Entropy, Bayesian Reconstruction, Ansatz, Backus-Gilbert, Tikhonov regularisation, …

- Machine Learning?