Negative string tension of extended Schwinger model via digital quantum simulation and tensor network

Etsuko Itou (YITP, Kyoto U.)

M.Honda, E.I., Y.Kikuchi, L.Nagano, T.Okuda, Phys.Rev.D 105 (2022) 1, 014504
M.Honda, E.I., Y.Kikuchi, Y.Tanizaki, PTEP 2022 (2022) 3, 033B01
M.Honda, E.I., Y.Tanizaki, JHEP 11 (2022) 141

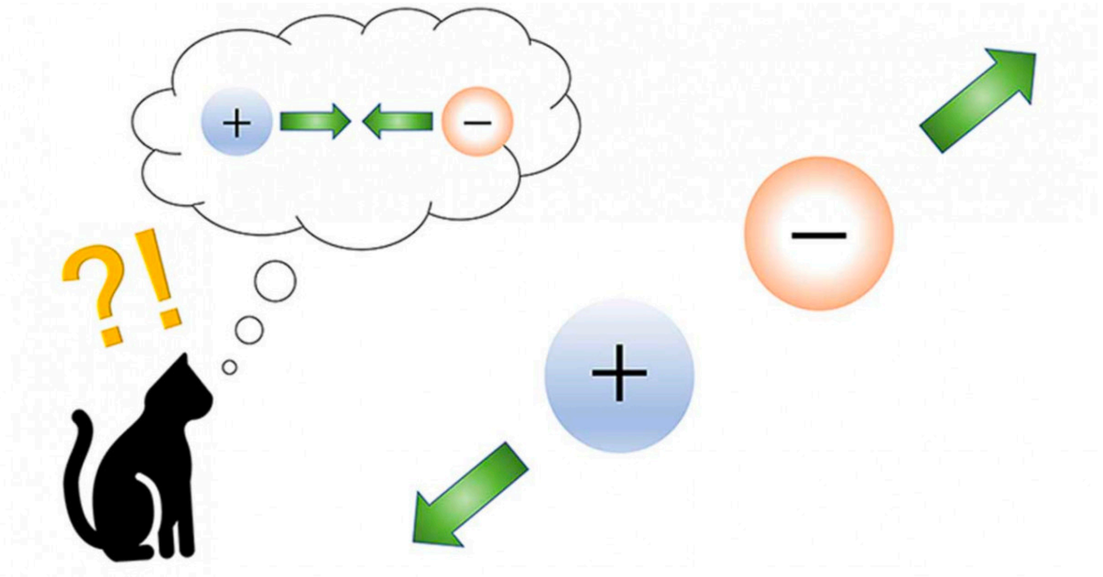
} Quantum computation

← Tensor network (DMRG)

QUANTUM COMPUTING METHODS FOR HIGH ENERGY PHYSICS, MIAPbP Munich, 27/04/2023

Outline

- Introduction
  Schwinger model
- Charge-q Schwinger model
- Summary and future directions



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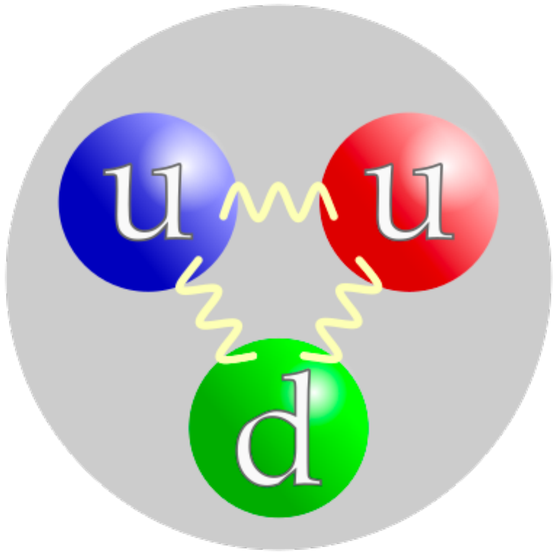
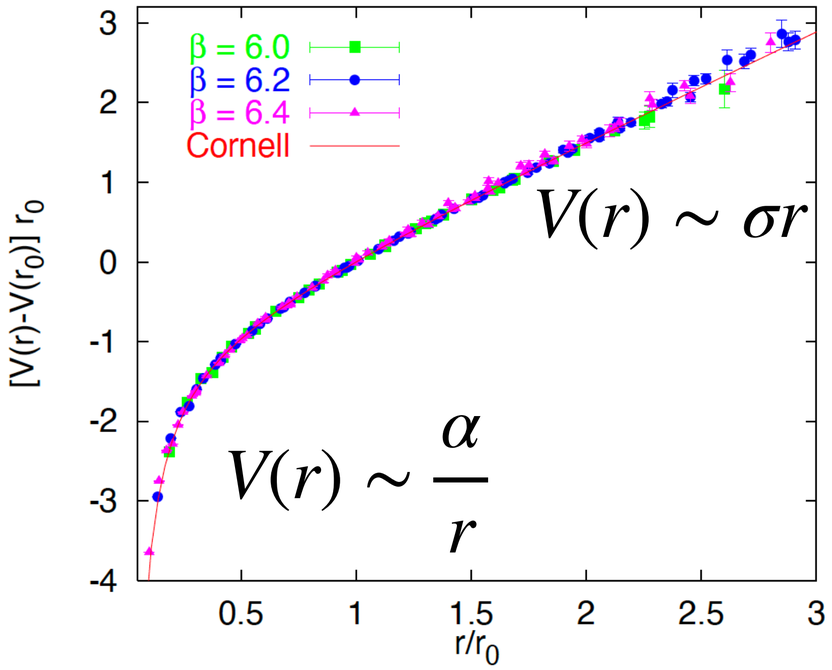
Lattice QCD started to study linear potential

- Confinement = linear potential



G.Bali, Phys.Rept.343:1 (2000)

pure YM (quenched QCD)



Monte Carlo, SU(2) gauge, 10^4 lattice
M.Creutz, PRD21 (1980) 2308

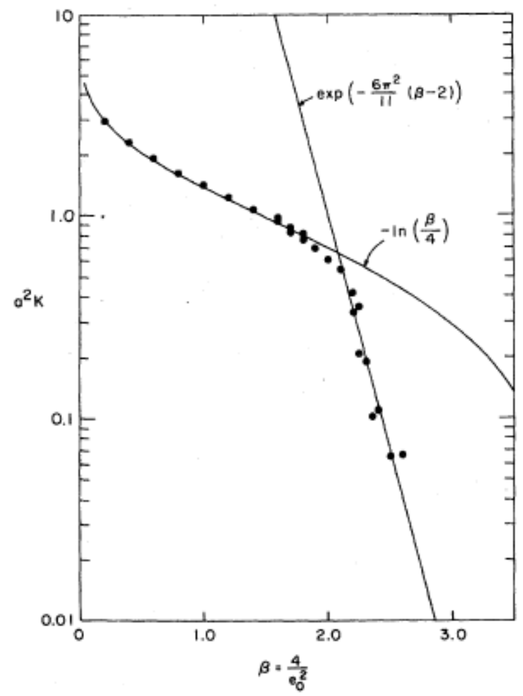


FIG. 6. The cutoff squared times the string tension as a function of β. The solid lines are the strong- and weak-coupling limits.

We study such a static potential for QFT (w/ the sign problem) in the Hamiltonian formalism

Introduction: Schwinger model

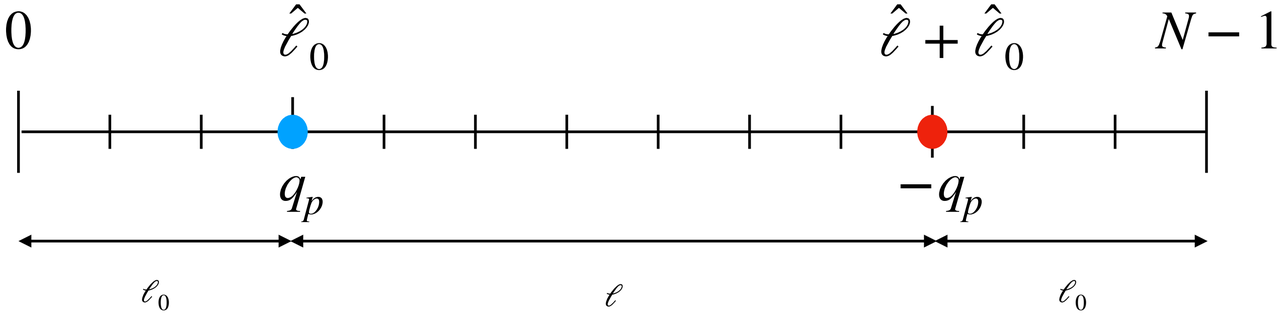
(1+1)dim. QED

M.Honda, E.I., Y.Kikuchi, L.Nagano, T.Okuda, Phys.Rev.D 105 (2022) 1, 01450

(charge-q) Schwinger model as a toy model of QCD

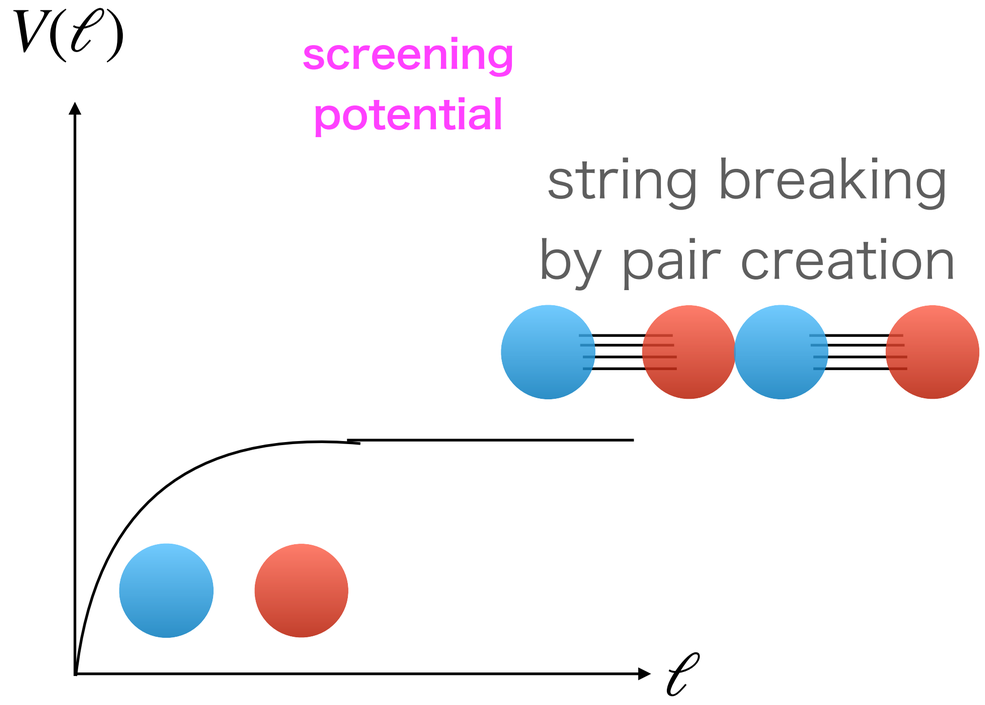
q : charge of dynamical fermion

q_p : probe charge

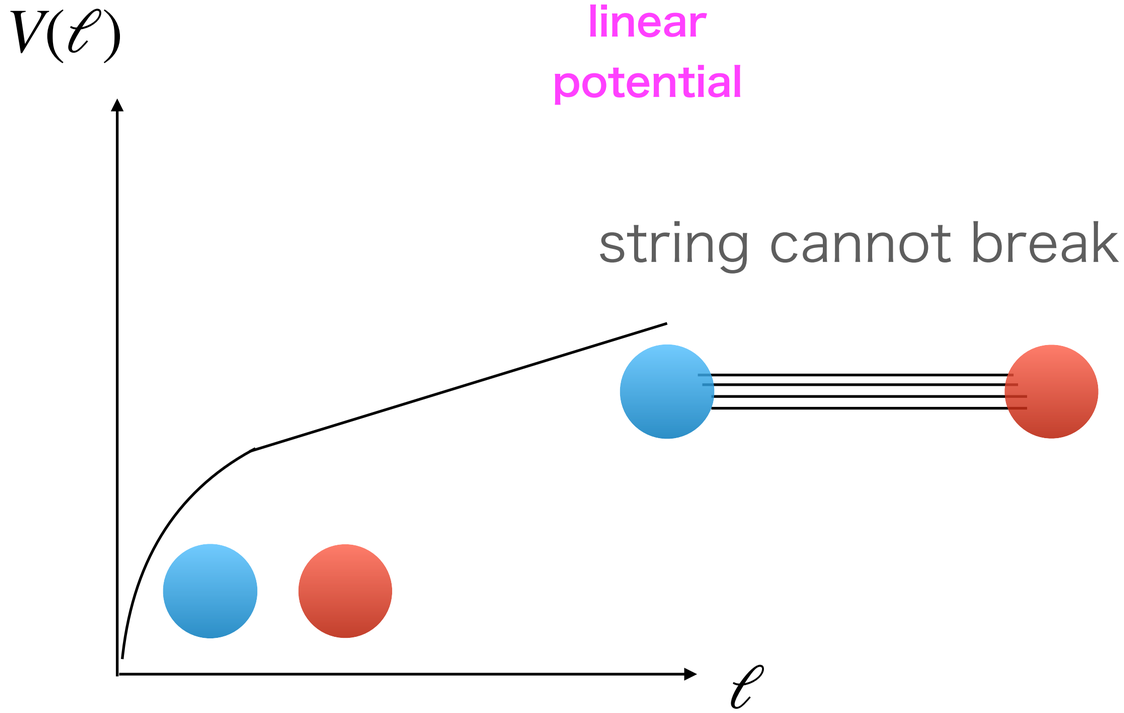


massless fermion

massive fermion $q_p/q \in \mathbb{Z}$



massive fermion with $q_p/q \notin \mathbb{Z}$



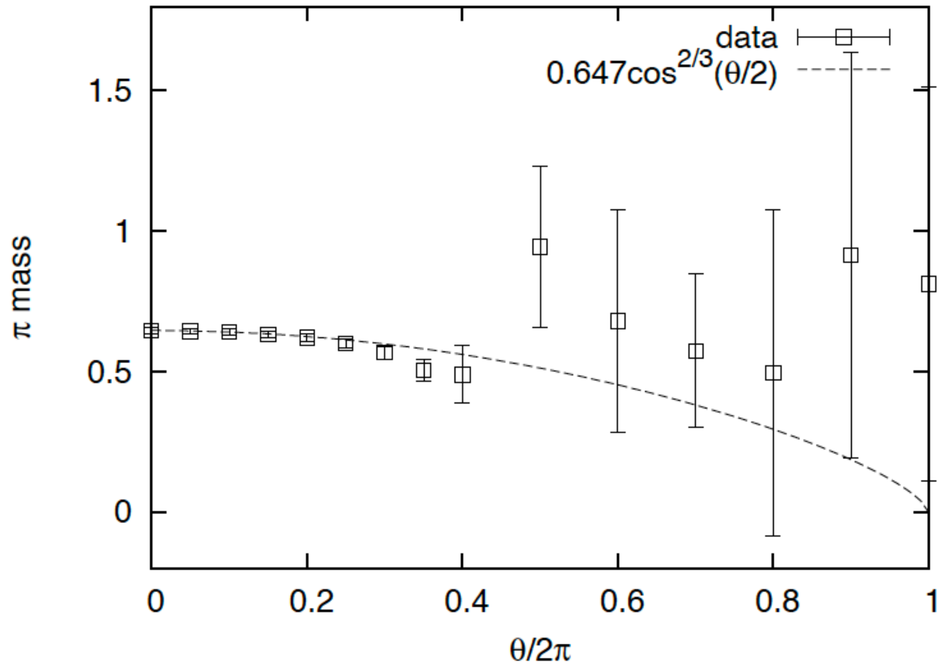
Schwinger model + θ term

- Lagrangian

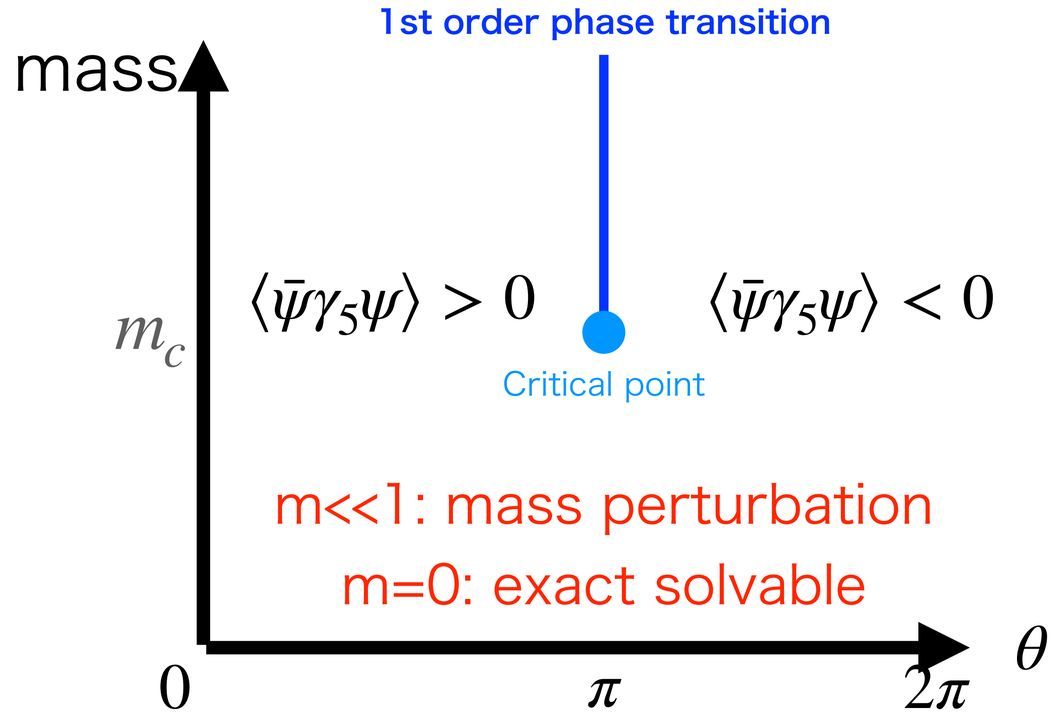
$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{g\theta_0}{4\pi}\epsilon_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}(\partial_{\mu} + igA_{\mu})\psi - m\bar{\psi}\psi$$

non-zero θ_0: Sign problem in conventional method

Fukaya and Onogi
Phys.Rev. D68 (2003) 074503



Phase diagram in $m - \theta$ plane



- Monte Carlo simulation for Nf=2 Schwinger model
- large θ regime, the sign problem becomes severe

Schwinger model + θ term

- Lagrangian

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{g\theta_0}{4\pi}\epsilon_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}(\partial_{\mu} + igA_{\mu})\psi - m\bar{\psi}\psi$$

non-zero θ_0: Sign problem in conventional method

(1) Gauge fixing

(2) Gauss law

$$(L_n - L_{n-1} = \chi_n^{\dagger}\chi_n - \frac{1-(-1)^n}{2})$$

(3) Open BC

$$(L_{-1} = 0)$$

(4) Jourdan-Winger trans.

- Hamiltonian by spin variables

$$H = J\sum_{n=0}^{N-2}\left[\sum_{i=0}^{n}\frac{Z_i + (-1)^i}{2} + \frac{\theta_0}{2\pi}\right]^2 + \frac{w}{2}\sum_{n=0}^{N-2}\left[X_nX_{n+1} + Y_nY_{n+1}\right] + \frac{m}{2}\sum_{n=0}^{N-1}(-1)^nZ_n$$

kinetic term of electric field

kinetic/mass terms of electron

- θ_0 is constant shift of electric field

- all-to-all interaction of Z

  Gaped system (even in massless case)

Simulation setup

- IBM Qiskit library, Simulator (not real quantum device)
- lattice size: N=15 - 25, open b.c.(remove d.o.f. of gauge field)
- # of shots = 1,000,000
- ground state is generated by adiabatic state preparation

$$|\Omega\rangle = \lim_{T\to\infty} \mathcal{T} \exp\left(-i \int_0^T dt\ H_A(t) \right) |\Omega\rangle_0$$

initial state : Neel state (# of particle - # of anti-particle = 0)

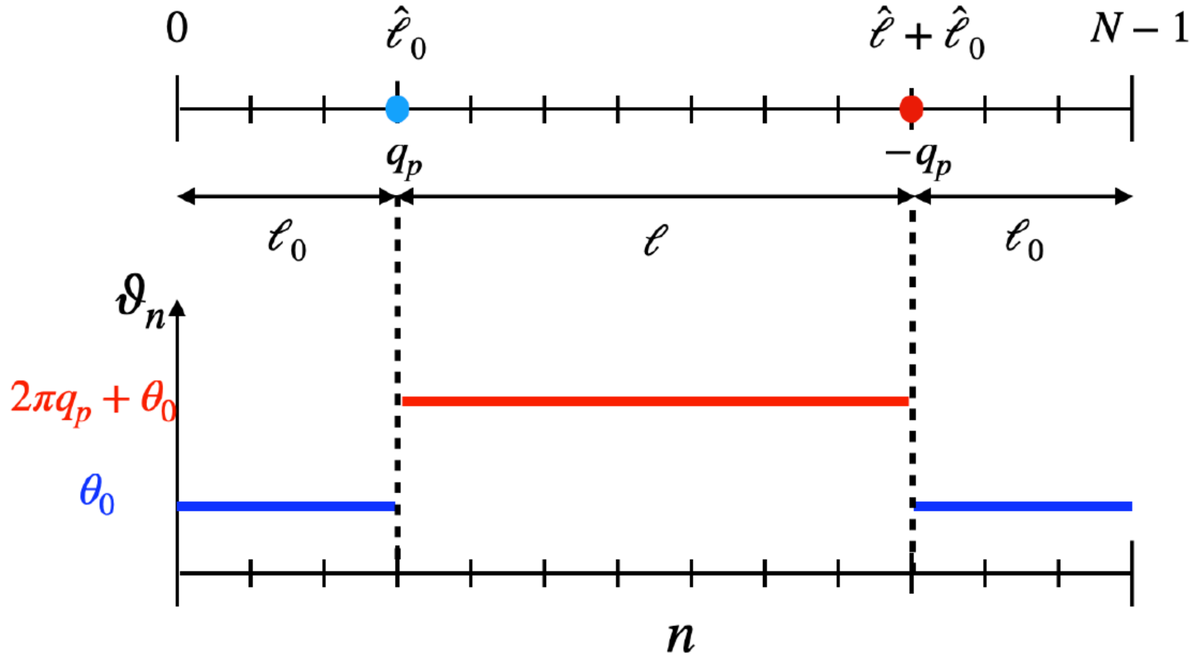
Potential between probe charges

- Charge insertion induces additional electric flux between particles

- Hamiltonian by spin variables

$$H = J\sum_{n=0}^{N-2}\left[\sum_{i=0}^{n}\frac{Z_i+(-1)^i}{2}+\frac{\vartheta_n}{2\pi}\right]^2+\frac{w}{2}\sum_{n=0}^{N-2}\left[X_nX_{n+1}+Y_nY_{n+1}\right]+\frac{m}{2}\sum_{n=0}^{N-1}(-1)^nZ_n$$

Insertion of Wilson loop can be realized by site-dependent ϑ_n



- Measure the energy,

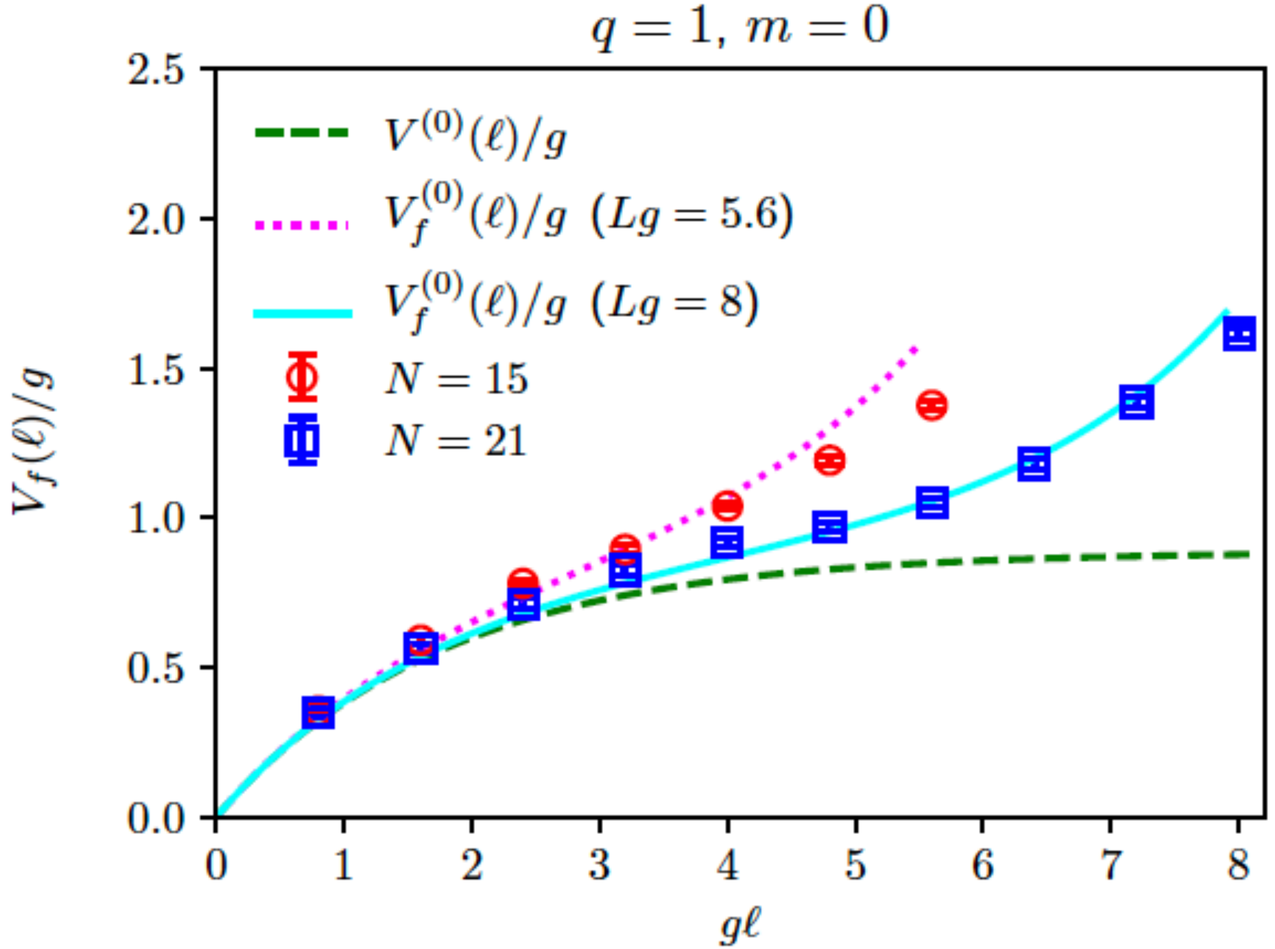
$E(\ell)=\langle\Omega|H(\ell)|\Omega\rangle$ with several ℓ

potential $V(\ell)=E(\ell)-E(0)$

Screening potential (q=1 case)

M.Honda, E.I., Y.Kikuchi, L.Nagano, T.Okuda,
Phys.Rev.D 105 (2022) 1, 01450

Numerical data is consistent with analytical result in finite volume



- upto O(m) in finite volume.

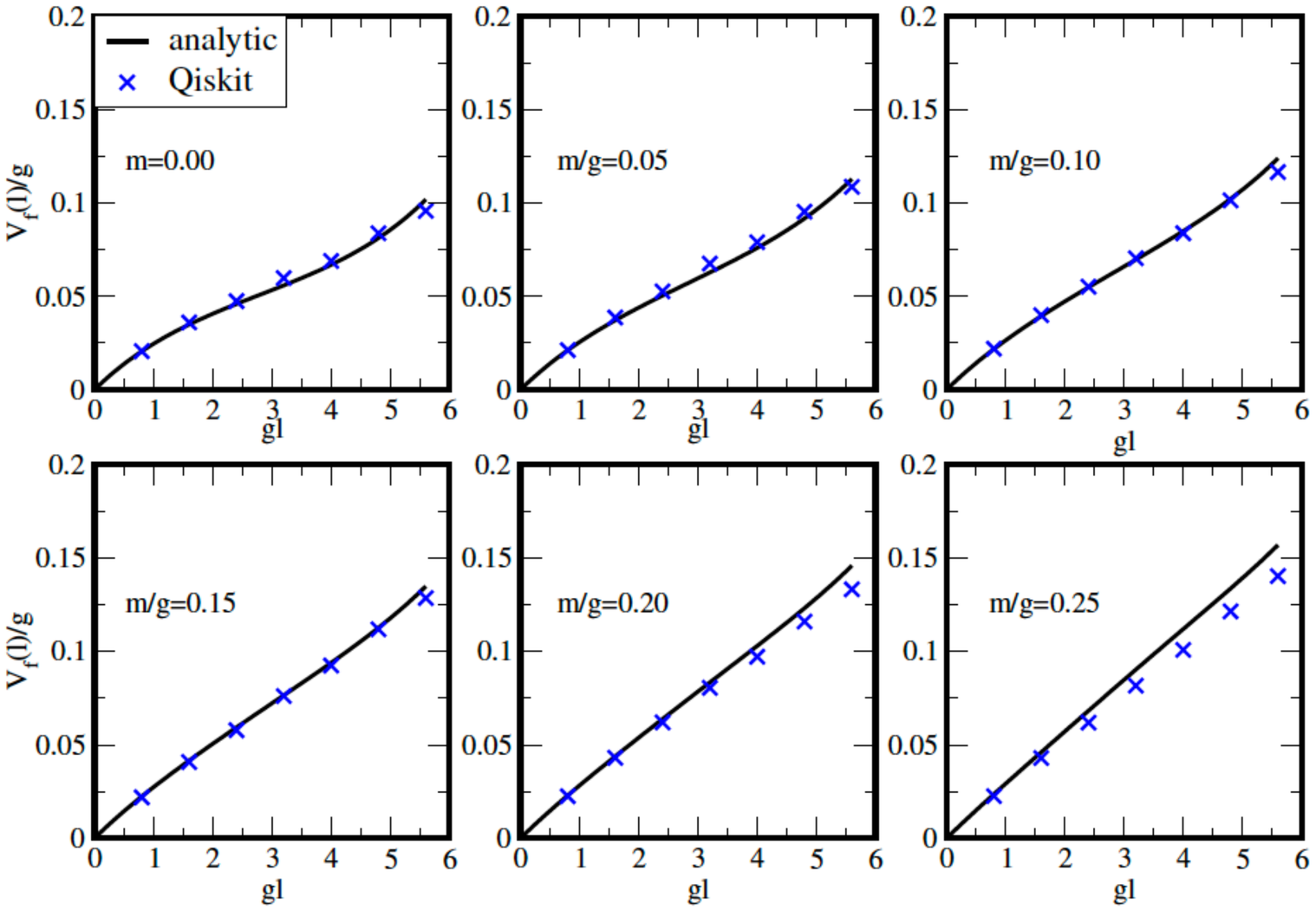
$$V_f^{(0)}(\ell) = \frac{\sqrt{\pi} q^2 g}{2} \frac{(1 - \mathrm{e}^{-\frac{g\ell}{\sqrt{\pi}}})(1 + \mathrm{e}^{-\frac{g(L-\ell)}{\sqrt{\pi}}})}{1 + \mathrm{e}^{-\frac{gL}{\sqrt{\pi}}}}.$$

$$E_f^{(1)}(\theta_0, q, \ell) = -m\Sigma \int_0^L dx\ \lambda(x) \times \cos\left[2\sqrt{\pi}\phi_0 - \sum_{n=1}^{\infty} \frac{\mu^2 \Theta_n^{(q)}}{\mu^2 + k_n^2} \sin(k_n x)\right]$$

It is hard to see exact plateau because of finite vol. effect

Numerical result for charge-1 Schwinger model

M.Honda, E.I., Y.Kikuchi, L.Nagano, T.Okuda,
Phys.Rev.D 105 (2022) 1, 01450

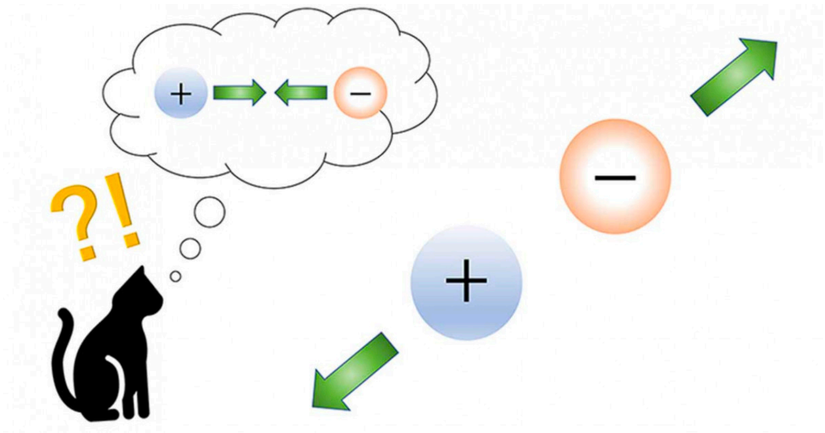


- N = 15, q_p= 0.25

  Qiskit simulator

- Easy to see a linear potential, massive fermion with fractional probe charge in small box

Charge-q Schwinger model

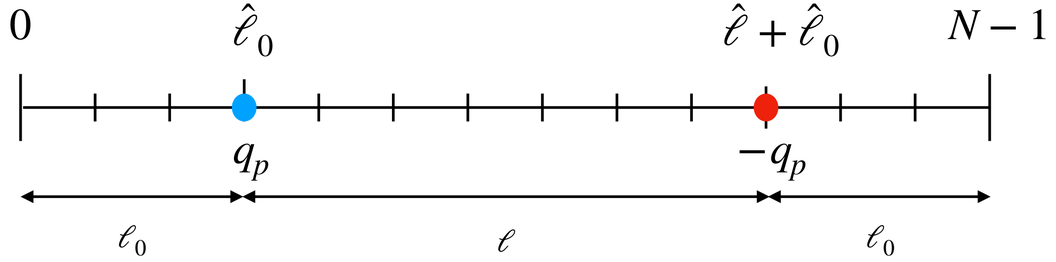


Charge-q Schwinger model

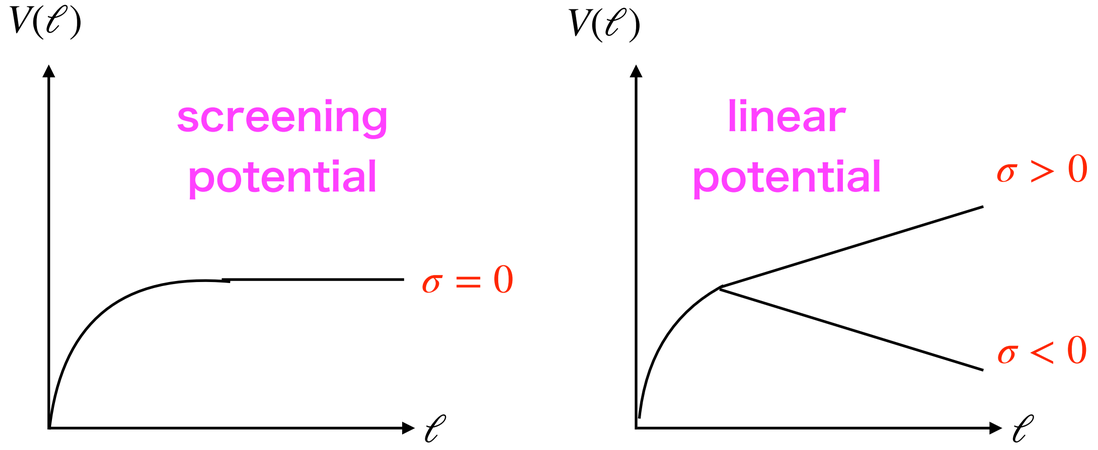
T.Misumi, Y.Tanizaki, M.Unsal, JHEP 07 (2019)018

- Schwinger model = 1+1 dim. U(1) gauge theory

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{g\theta_0}{4\pi}\epsilon_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}(\partial_{\mu} + iqgA_{\mu})\psi - m\bar{\psi}\psi$$



q>1 massive Schwinger model
theoretical predictions in [0,L]



q, q_p, θ_0によって$\sigma = 0,\ \sigma > 0,\ \sigma < 0$がある

Charge-q Schwinger model

T.Misumi, Y.Tanizaki, M.Unsal, JHEP 07 (2019) 018
M.Honda, E.I., Y.Kikuchi, Y.Tanizaki, PTEP 2022 (2022) 033B01

- $$H = J\sum_{n=0}^{N-2}\left[q\sum_{i=0}^{n}\frac{Z_i+(-1)^i}{2}+\frac{\theta_0}{2\pi}\right]^2+\frac{w}{2}\sum_{n=0}^{N-2}\left[X_nX_{n+1}+Y_nY_{n+1}\right]+\frac{m}{2}\sum_{n=0}^{N-1}(-1)^nZ_n$$

- dynamical charge q>1 case

  U(1) 1form symmetry (pure Maxwell)

  -> $\mathbb{Z}_q$ 1form symmetry due to charge-q fermions

  Hilbert space decomposes into q sectors $\mathscr{H}=\bigoplus_{k=0}^{q-1}\mathscr{H}_k,\quad k=0,\cdots,q-1$

- Ground energy is sector specific: in sector k, $E_k(\theta_0)=-m\frac{e^\gamma qg}{2\pi^{3/2}}\cos\left(\frac{\theta_0-2\pi k}{q}\right)$

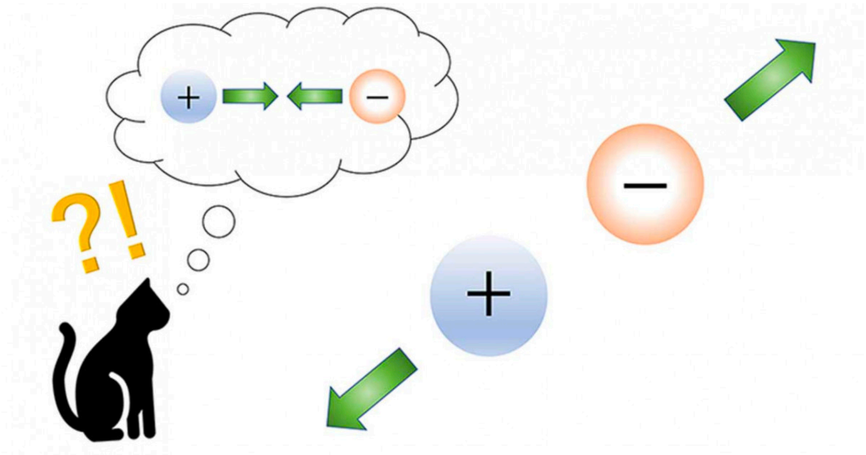
Charge-q Schwinger model : massive case

- If $q_p/q \notin \mathbb{Z}_q$, linear potential

  Slope depends on θ_0 term

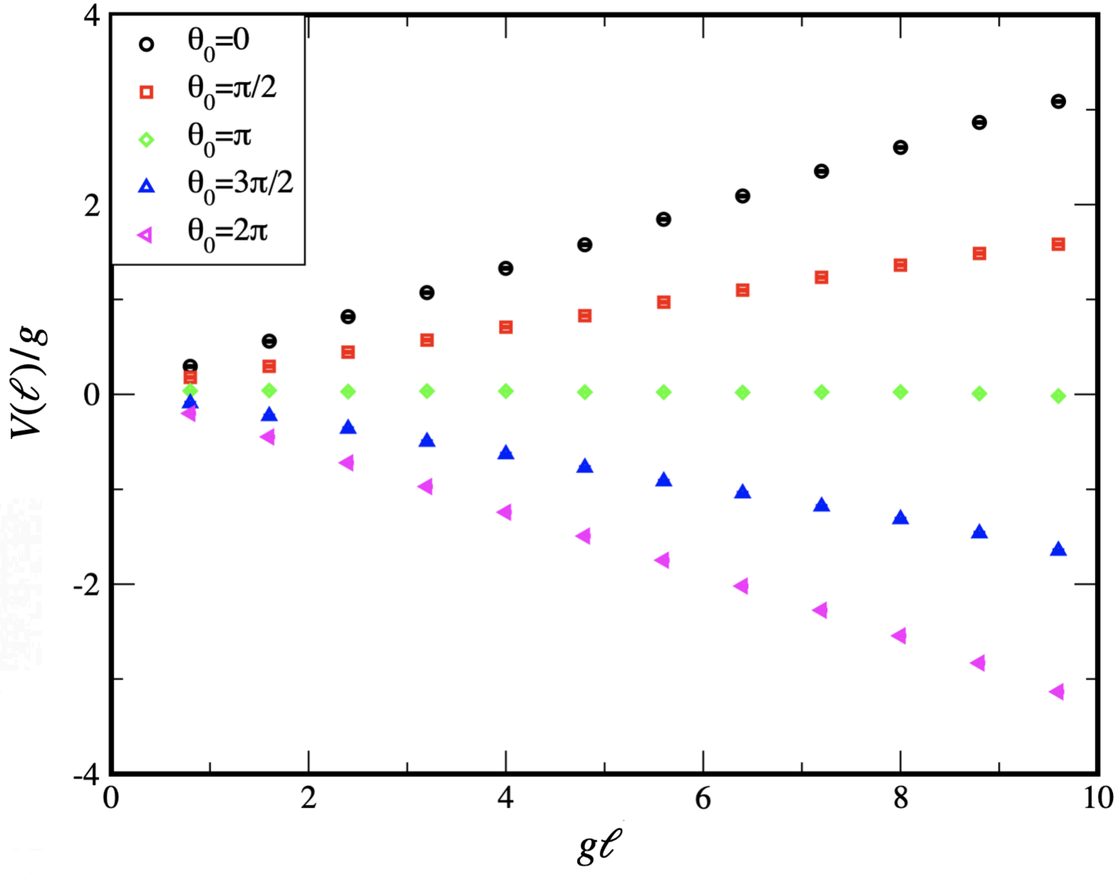
- In large θ_0, linear potential with negative string tension appears?!

  $V(\ell) \sim \sigma\ell,\ \sigma < 0$

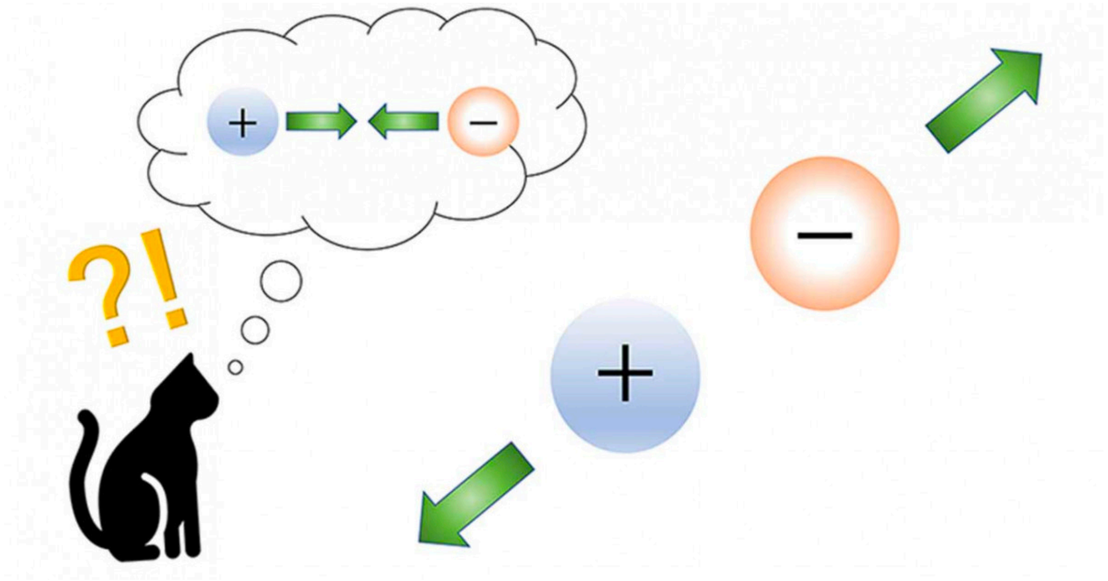


Qiskit simulator results

Lattice size N=25

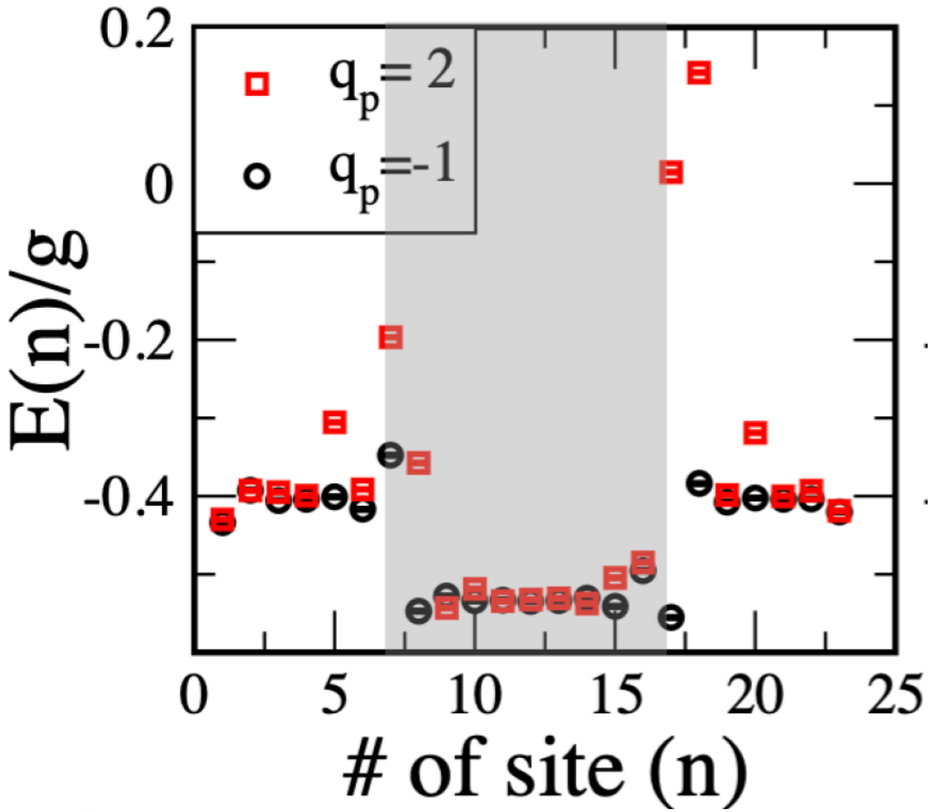


Local energy at each site: massive case



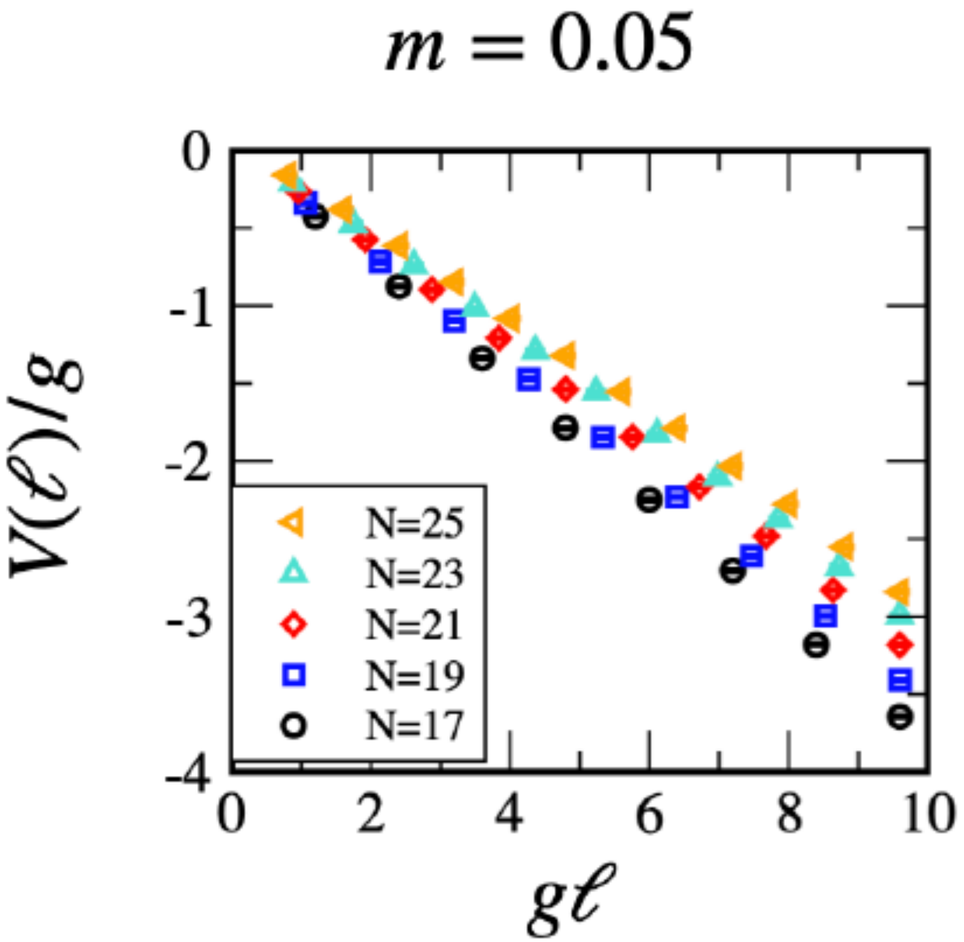
$$E_k(\theta_0) = -m\frac{e^{\gamma}qg}{2\pi^{3/2}}\cos\left(\frac{\theta_0 - 2\pi k}{q}\right)$$

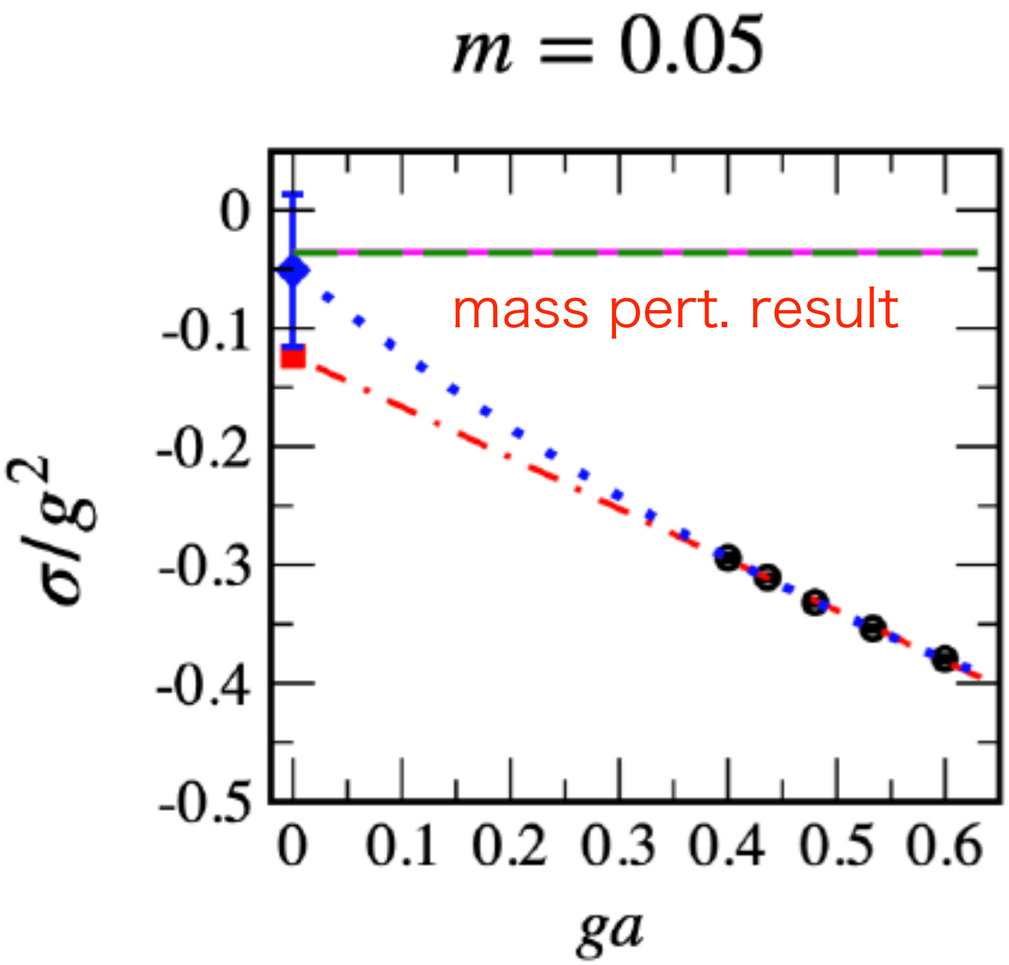
Ground state energy at each site
in negative string tension case
Qiskit simulation, N=25



Insertion of q_p corresponds to $\theta_0 \to \theta_0 + 2\pi q_p$

Continuum limit and comparison with analytical result





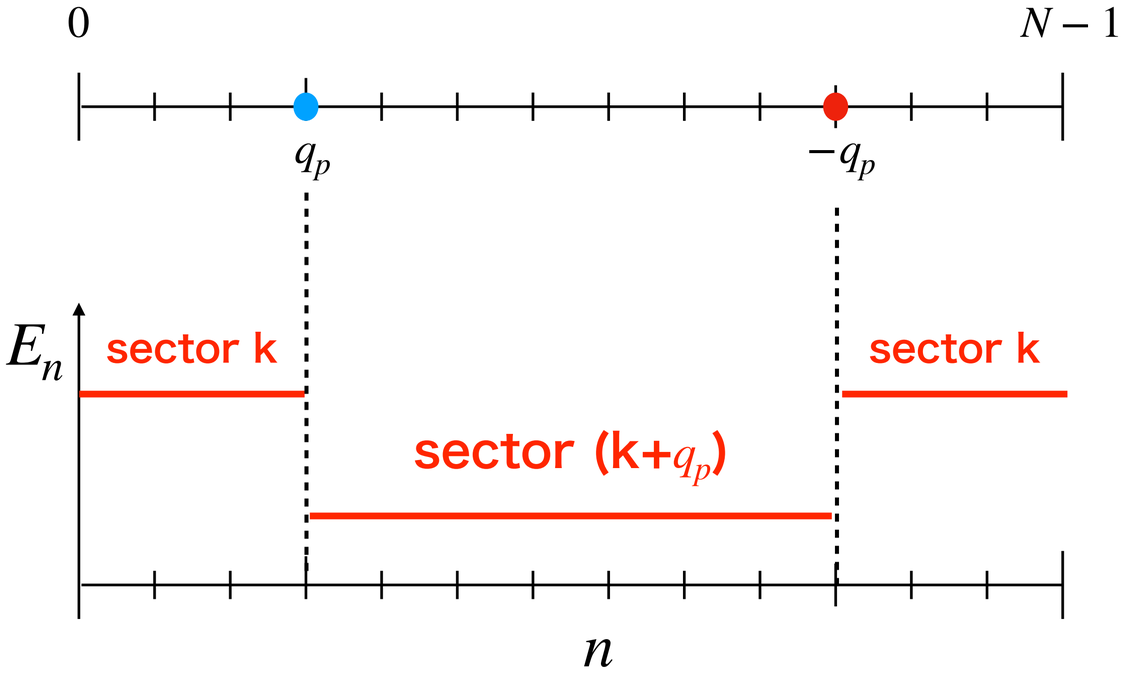
The string tension after continuum extrapolation is consistent with analytical result.

But the error is large… Can we do the more precise consistency check?

DMRG analysis for large N

M.Honda, E.I., Y.Tanizaki, JHEP 11 (2022) 141

- DMRG calculation for charge-3 Schwinger model w/ massless fermion
- Using ITensor (open code), perform N=101 - 801 simulation and take a cont. limit
- Investigate the order parameter of higher-form symmtry
  (discrete chiral sym. + 1-form background field = θ term)



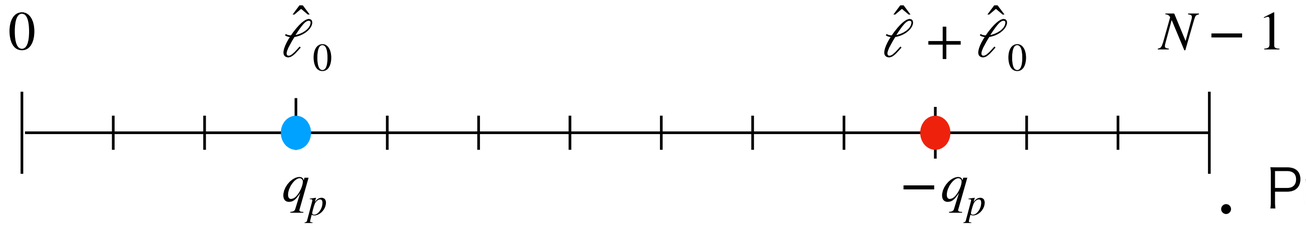
Order parameter:

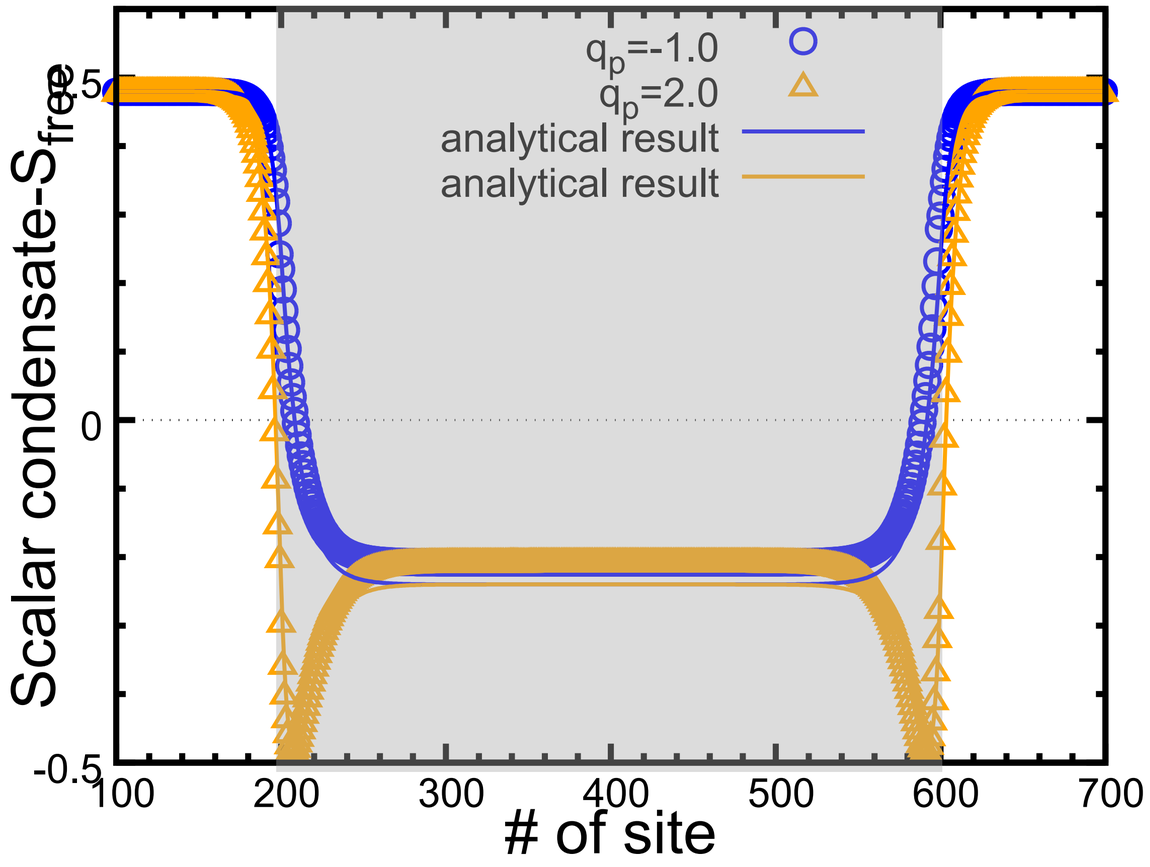
$$S(x) + iP(x) = \bar{\psi}\psi(x) \pm \bar{\psi}\bar{\gamma}_{\mathrm{E}}\psi(x)$$

across the Wilson loop, it obtains a complex phase $e^{i\frac{2\pi q_p}{q}}$

Scalar condensate

DMRG simulation, N=801



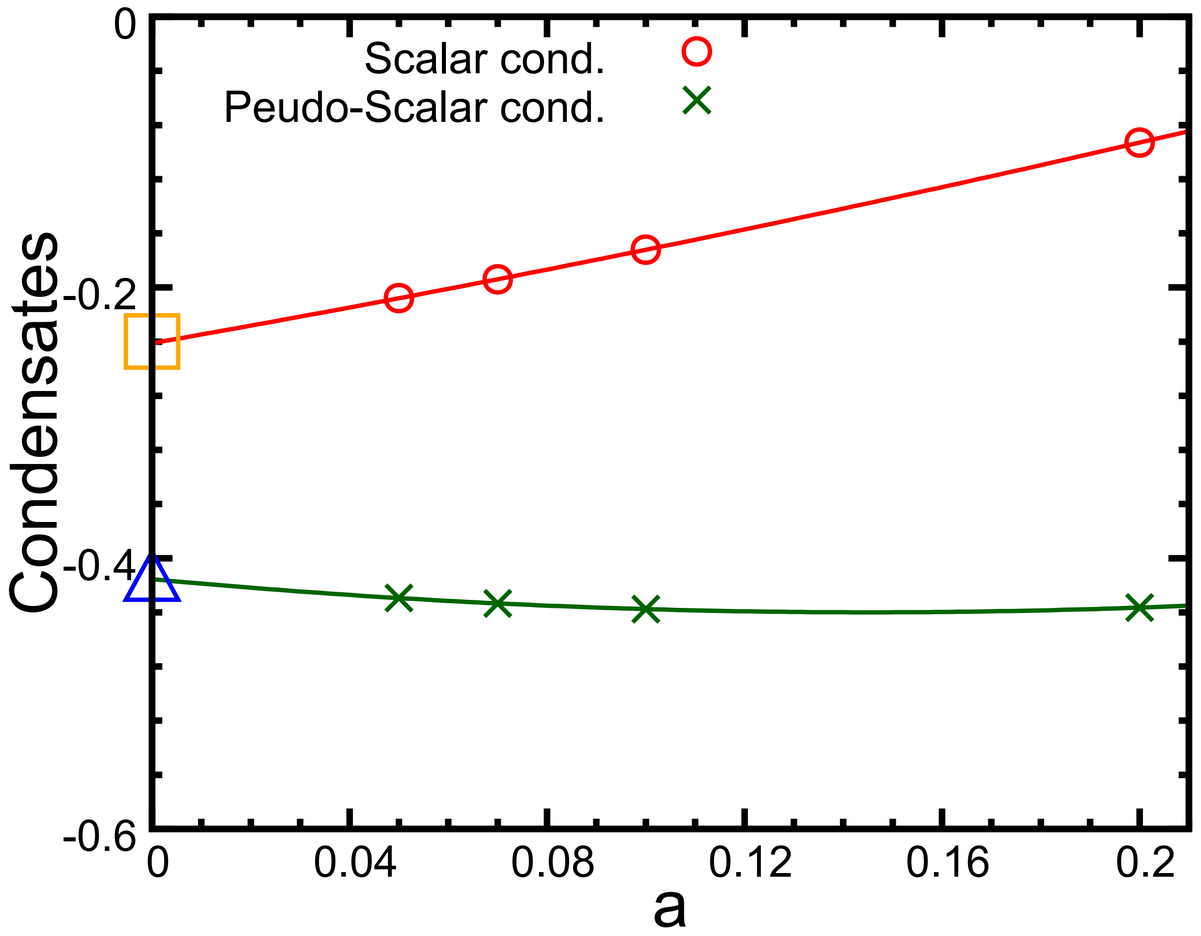


- Put probe charges $+q_p, -q_p$ at n=200,600
- q=3 Schwinger model (Z_3 sym.)
  $q_p = -1$ is the same sector with $q_p = 2$ (mod q)
- Solid curve：analytical result by bosonization
  (it is massless simulation)
- Even though massless theory, gapped theory
  $\mu^2 = \frac{q^2 g^2}{\pi}$: entangled regime is finite

Scalar and Pseudo-Scalar condensates

□: theoretical prediction for scalar cond.

△: theoretical prediction for pseudo-scalar cond.



In the cont. lim., our data is consistent with the theoretical prediction (within 3-digit accuracy)

O(a) correction for staggered fermion in Hamiltonian formalism

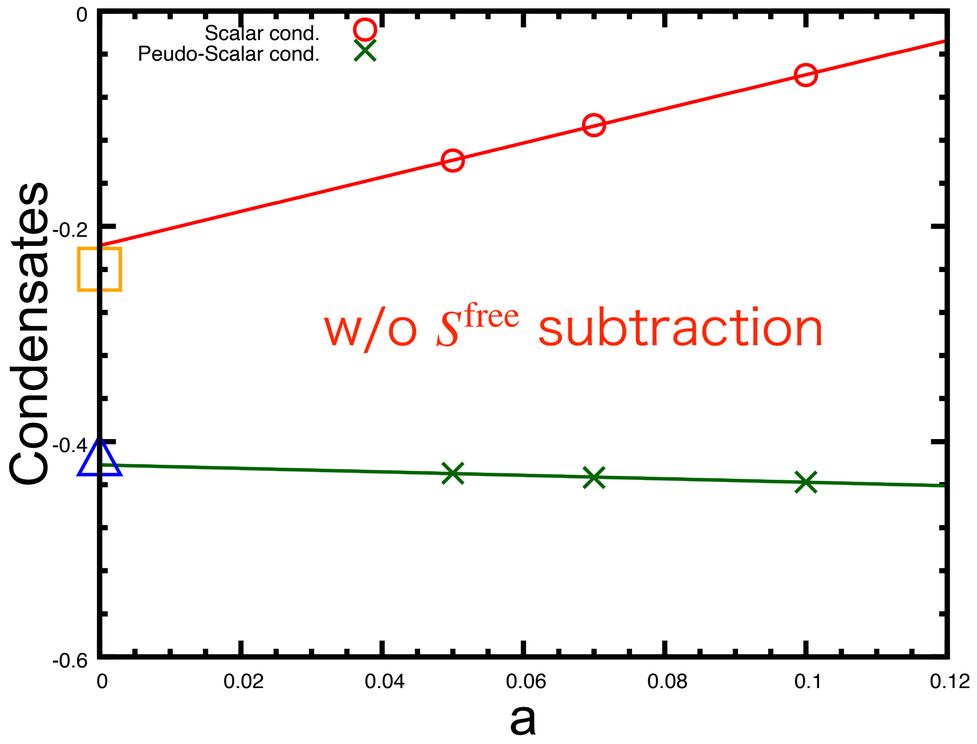
No log-divergence in massless case?

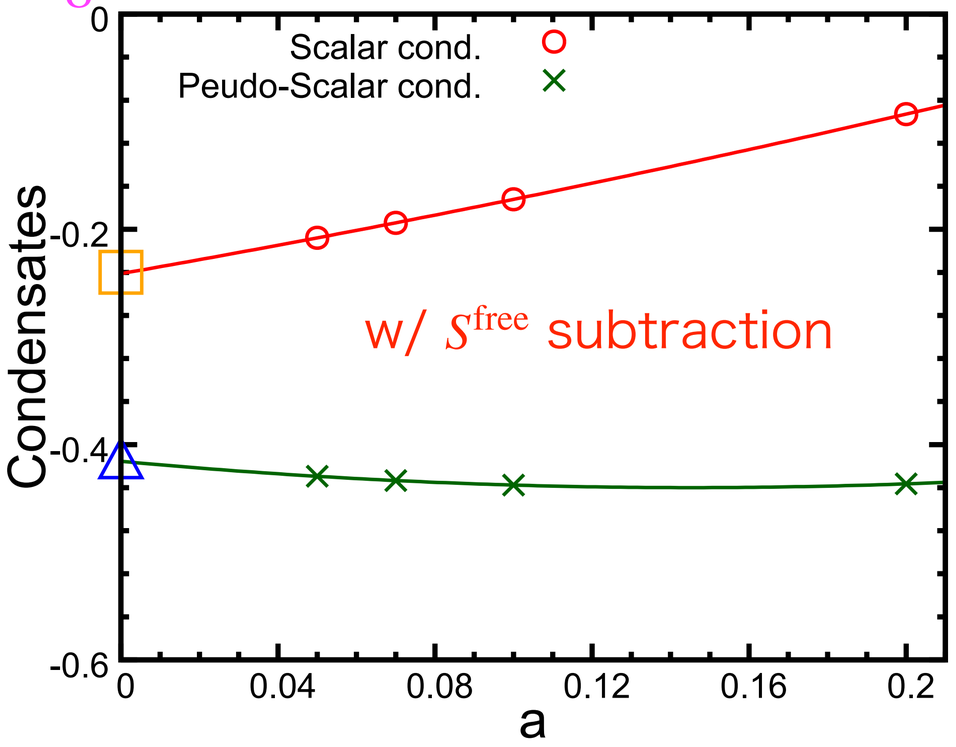
$$S^{\text{free}} = -\frac{m}{\pi\sqrt{1+(\mu a)}} K\left(\frac{1}{\sqrt{1+(\mu a)^2}}\right)$$

Pointing out that scaling is better when O(a) correction is added.

$$m_{\text{lat}} = m - \frac{q^2 g^2 a}{8}$$

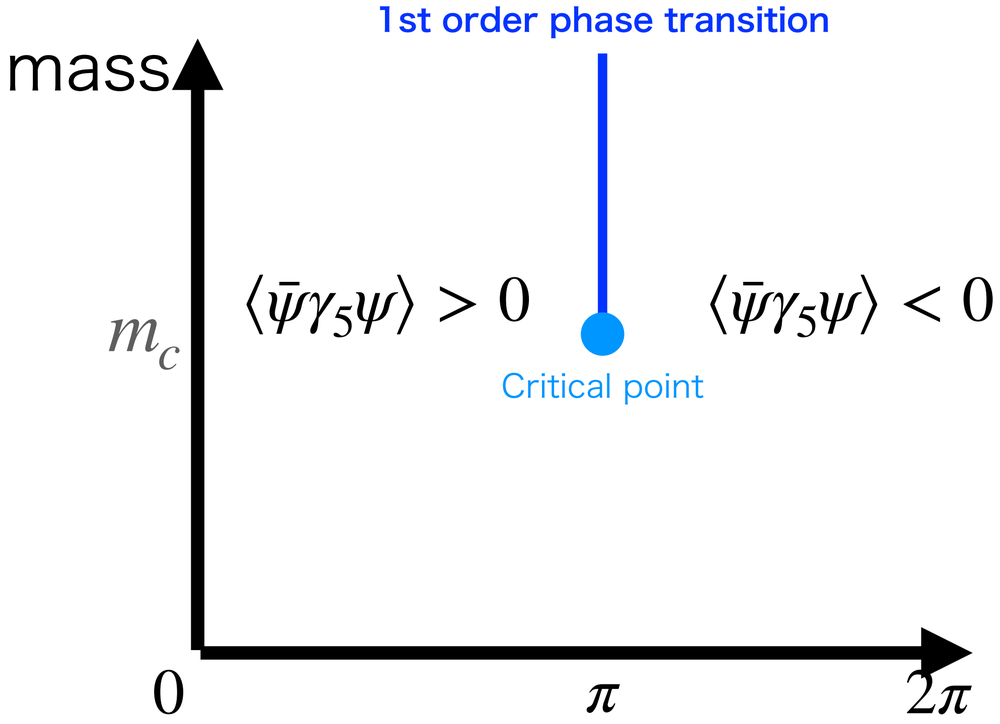
Dempsey, Klebanov, Pufu, Zan: arXiv:2206.05308



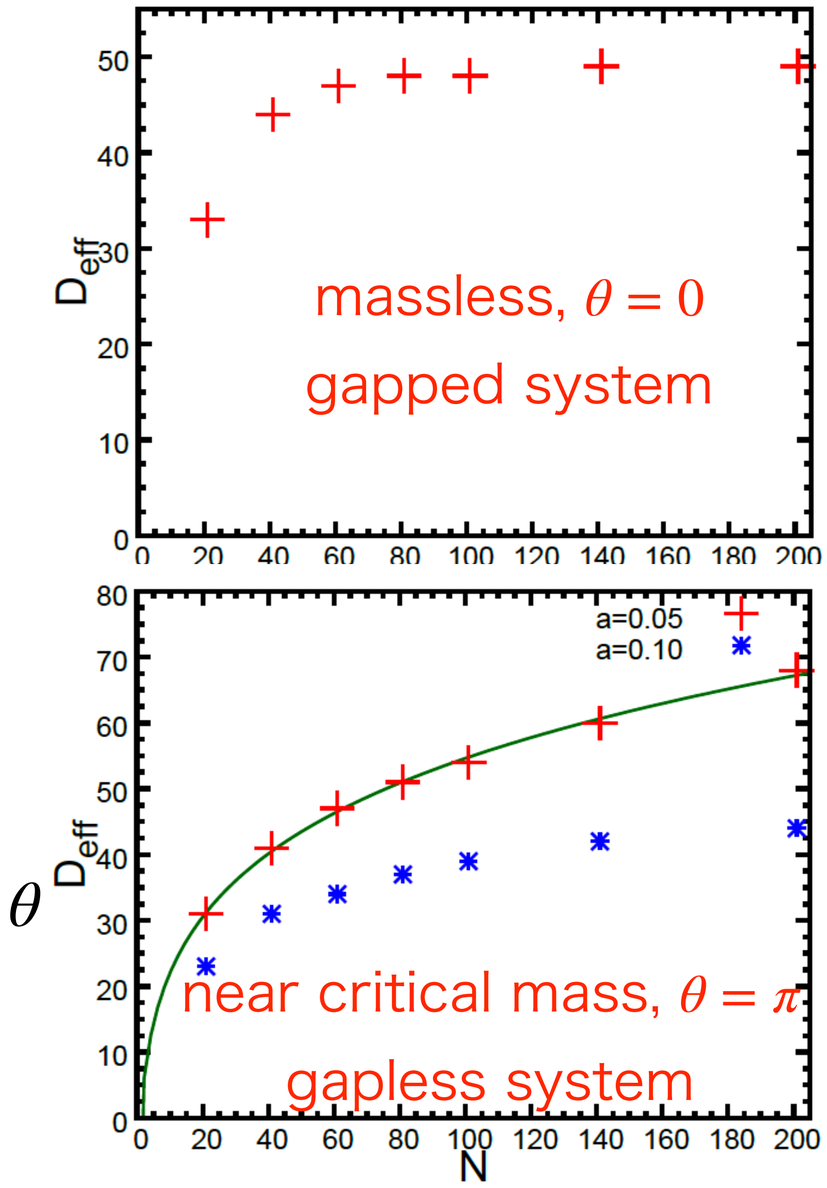


Entanglement and scaling of N

Phase diagram in $m-\theta$ plane



bond dim scaling of N with fixed accuracy ($\epsilon = 10^{-10}$)



entanglement entropy gives lower bound for bond dim.

ln (D) > S_{EE}

1+1 d CFT

$$S_{EE} \sim \frac{c}{3} \ln N$$

Our data can be fitted

$D_{eff} = c_1 N^{1/6} + c_2$

=> c=1/2 (critical Ising)

Summary

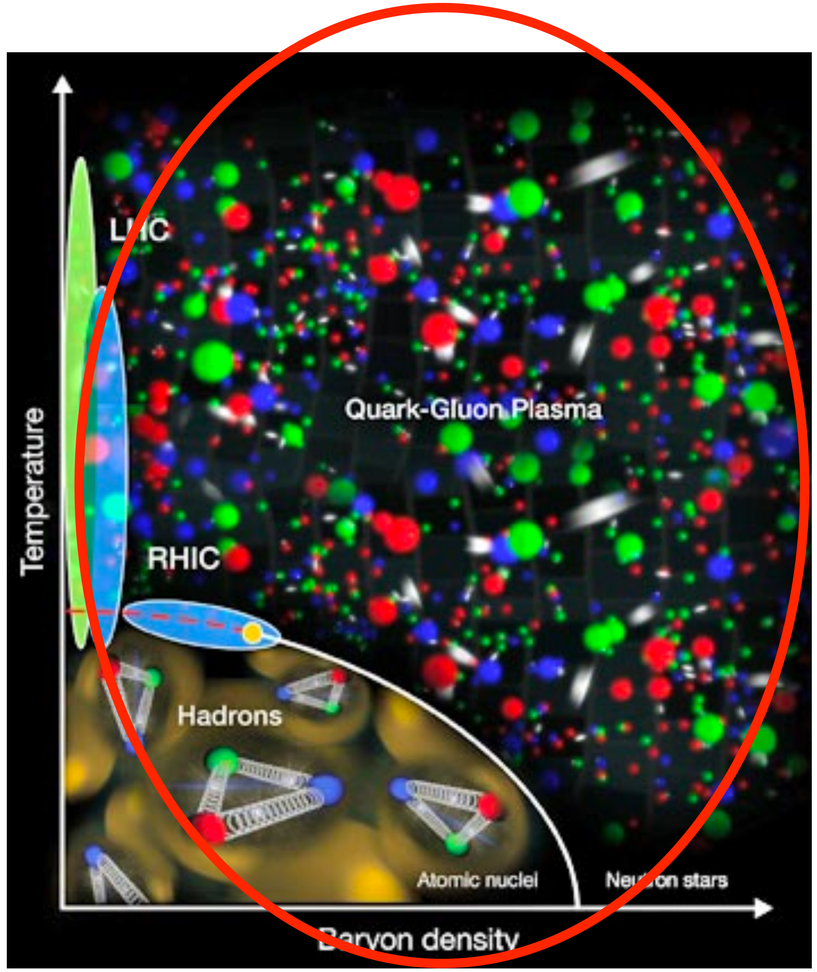
Summary

- Study potential between probe charges using site-dependent θ-term
- (extended) Schwinger model is getting attention as a testing ground of new calculation methods
- We demonstrate the simulations for the model with large θ using quantum algorithm (adiabatic state preparation) and DMRG
- A precise consistency of the order parameter of the higher form sym. (’t Hooft anomaly) with the theoretical prediction
  Hamiltonian formalism looks promising for simulating quantum field theory with sign problem in conventional method

Future directions

- We want to know physics of QCD-like theory with the sign problem in Lattice QCD
  - Confinement/chiral sym. breaking
  - Hadron mass
  - Hadron interaction
  - Thermodynamics

- Theoretically, we need good description for
  - non-abelian gauge theory
  - higher dimension



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